ELEVATION AND AZIMUTH ESTIMATION IN ARBITRARY PLANAR MONO-STATIC MIMO RADAR VIA TENSOR DECOMPOSITION

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ABSTRACT

Elevation and azimuth estimation in arbitrary planar mono-static multiple-input-multiple-output (MIMO) radar via tensor decomposition is proposed. Transmit beamspace design is used to map a planar array into a Khatri-Rao product of two perpendicular desired uniform linear arrays within a range bin of interest, and to suppress sidelobes outside of the range bin of interest. Each desired array has rotational invariance property with respect to elevation and azimuth separately. Then the received MIMO radar data are folded into a fourth-order tensor along each spatial and temporal dimension. A computationally efficient tensor-based target localization method is proposed. Our simulation results demonstrate the effectiveness of the proposed method and its superiority over the matrix-based counterpart.

Index Terms— Mono-static MIMO radar, tensor decomposition, parameter estimation, transmit beamspace.

1. INTRODUCTION

In the last decade, target localization problem with multiple-input-multiple-output (MIMO) radar has received considerable attention [1]-[11]. Unlike conventional phased-array radar, each antenna in MIMO radar emits spatial orthogonal waveforms. It enables the receive array to separate multiple transmit beams from each other after matched filtering [1],[12],[13]. This property offers MIMO radar many merits such as better resolution and flexible transmit beamspace design, to name a few. For a planar transmit array MIMO radar, elevation and azimuth estimation problem has been considered in [8]-[11]. Matrix-based target localization techniques have been developed for MIMO radar. However, these two-dimensional (2D) parameter estimation techniques do not capture the multi-linear structure of the received radar data which holds with respect to different parameters.

To take advantage of the multidimensional property inherent in the MIMO radar data, the concept of multi-linear algebra has been used in [2] for 1D arrays. A tensor modeling of received MIMO radar data is built by collecting several pulses and then arranging them along angles, respectively. Furthermore, the corresponding tensor decomposition techniques, e.g., parallel factor analysis (PARAFAC) and higher-order singular value decomposition (HOSVD) have been shown to have many benefits over matrix-based decomposition methods. Among such benefits, it is worth mentioning relaxed uniqueness condition and providing more accurate signal subspace estimation, which eventually improves the parameter estimation performance [14]-[15].

Tensor structure, however, is not so obvious for an arbitrary planar mono-static MIMO radar due to the fact that the elevation and azimuth information is mixed in the steering vector. Recently, a novel 2D transmit beamspace (TB) design has been proposed to improve the transmit gain while mapping the array structure into a virtual array that preserves rotational invariance property with respect to each parameter i.e., elevation or azimuth, separately [4],[11]. This 2D TB design suppresses the sidelobes while minimizing difference between the virtual array and real array within the range bin of interest. In this paper, 2D TB design is used to map the transmit array to a Cartesian product, i.e., Khatri-Rao product of two perpendicular virtual linear arrays (ULAs) bearing with azimuth and elevation information, respectively. Then we establish a fourth-order tensor modeling of an arbitrary planar MIMO radar collected over several pulses. Finally, we derive a tensor-based HOSVD estimation of signal parameters via rotational invariance techniques (ESPRIT) [16] to estimate the elevation and azimuth. We also show that tensor-based ESPRIT significantly outperforms its matrix-counterpart by simulation results.

2. NOTATION AND MIMO RADAR MODEL

2.1. Notation

Scalars are denoted by italic letters, column vectors by lower-case bold letters, matrices by upper-case bold letters, tensors by calligraphic bold-face letters, respectively. The
(i_1, i_2) \text{-element of a matrix } A \text{ is denoted as } a_{i_1, i_2} \text{ and the } (i_1, i_2, \ldots, i_n) \text{-element of an } N \text{-order tensor } A \text{ as } a_{i_1, i_2, \ldots, i_n}. \text{ We use the notations } (\cdot)^T, (\cdot)^H, (\cdot)^{-1}, \otimes, \circ, \triangledown, \text{ and vec}(\cdot), \text{ to represent complex conjugate, transpose, Hermitian transpose, matrix inverse, outer product, Khatri-Rao product, pseudo-inverse, and vectorization, respectively, and } I_M \text{ symbolizes an } M \times M \text{ identity matrix, while diag}(\alpha) \text{ denotes a diagonal matrix that holds the entries of } \alpha \text{ on its diagonal.}

\text{Definition 1 (The scalar product): The scalar product of two tensors } A \text{ and } D \in \mathbb{C}^{I_1 \times I_2 \times \ldots \times I_N} \text{ is given by}
\begin{equation}
c_{ij} = \langle A, D \rangle = \sum_{i_1=1}^{I_1} \sum_{i_2=1}^{I_2} \cdots \sum_{i_n=1}^{I_n} b_{i_1, i_2, \ldots, i_n}^* \cdot a_{i_1, i_2, \ldots, i_n}. \tag{1}
\end{equation}

\text{Definition 2 (The } n \text{-mode product): The } n \text{-mode product of a tensor } A \in \mathbb{C}^{I_1 \times I_2 \times \ldots \times I_N} \text{ and a matrix } D \in \mathbb{C}^{J_n \times I_n} \text{ is given by}
\begin{equation}
C \triangleq A \times_n D
\end{equation}
\begin{equation}
c_{i_1, i_2, \ldots, i_{n-1}, j_n, i_{n+1}, \ldots, i_N} = \sum_{i_n=1}^{I_n} a_{i_1, i_2, \ldots, i_n} \cdot d_{j_n, i_n}
\end{equation}
where } C \in \mathbb{C}^{M_1 \times M_2 \times \ldots \times M_N - 1 \times J_n \times M_{n+1} \times \ldots \times M_N}.

\text{A matrix unfolding of a tensor } A \text{ along the } R \text{-th mode is denoted as } \mathcal{Y}(R). \text{ Setting } A_{i_n=k} = A_k \text{, we obtain a subtensor of } A.

\text{Finally, the higher-order norm of a tensor is given by}
\begin{equation}
||A||_H \triangleq \sqrt{\langle A, A \rangle}.
\end{equation}

\textbf{2.2. MIMO radar model}

\text{Consider a mono-static MIMO radar system with } M \text{ co-located transmit antennas and } N \text{ co-located receive antennas. The transmit and receive antennas are assumed to be arbitrary planar, and they are used for localizing } K \text{ targets in the range bin of interest. The transmit antennas are assumed to be located at } p_m \triangleq [x_m, y_m]^T, m = 1, 2, \ldots, M. \text{ Thus, the } M \times 1 \text{ steering vector of the transmit array can be expressed as}
\begin{equation}
a(\theta, \phi) = [e^{-j2\pi x_1 \sin \theta}, e^{-j2\pi x_2 \sin \theta}, \ldots, e^{-j2\pi x_M \sin \theta}]^T \tag{4}
\end{equation}
where } u(\theta, \phi) \triangleq [\sin \theta \cos \phi, \sin \theta \sin \phi]^T \text{ represents the propagation vector, } \theta \text{ and } \phi \text{ are the elevation and azimuth angles, respectively. Similarly, the steering vector of the receive array is expressed as}
\begin{equation}
b(\theta, \phi) = [e^{-j2\pi x_1 \sin \theta}, e^{-j2\pi x_2 \sin \theta}, \ldots, e^{-j2\pi x_N \sin \theta}]^T \tag{5}
\end{equation}
\text{Let } E \triangleq [e_1, \ldots, e_M] \text{ be a mapping matrix, such that}
\begin{equation}
E^H a(\theta, \phi) = \tilde{a}(\theta, \phi) \tag{6}
\end{equation}
i.e., } a(\theta, \phi) \text{ is mapped to } \tilde{a}(\theta, \phi) \text{ that has a desired structure. The mapping matrix } E \text{ can be designed through 2D TB techniques [4]. Here, we map the transmit array steering vector to a coordinate product, i.e., Khatri-Rao product, of two perpendicular ULAs with } M_1 \text{ and } M_2 \text{ elements, placed along vertical and horizontal directions, respectively. The vertical and horizontal ULAs carry the elevation and azimuth, respectively. The steering vector } \tilde{a}(\theta, \phi) \text{ is designed according to (6) and can be expressed as}
\begin{equation}
\tilde{a}(\theta, \phi) = c(\theta) \odot d(\phi) \tag{7}
\end{equation}
\text{where}
\begin{equation}
c(\theta) = [1, e^{-j2\pi x_1 \sin \theta}, \ldots, e^{-j2\pi x_{M-1} \sin \theta}]^T \tag{8}
\end{equation}
\begin{equation}
d(\phi) = [1, e^{-j2\pi y_1 \sin \phi}, \ldots, e^{-j2\pi y_{M-1} \sin \phi}]^T \tag{9}
\end{equation}
i.e., } c(\theta) \text{ and } d(\phi) \text{ represent the steering vectors of the mapped horizontal and vertical virtual arrays, respectively. The desired inter-element spacing is set as half-wavelength to avoid the ambiguity in angle estimation.}

\text{Let } s(t) \triangleq [s_1(t), s_2(t), \ldots, s_{M_1 M_2}(t)]^T \text{ be the } M_1 M_2 \times 1 \text{ vector of transmit waveforms, each waveform has unit energy, and they are orthogonal to each other during one pulse, i.e., } \int_P s(t) s^H(t) dt = I_M \text{ where } I_M \text{ is the identity matrix of size } M_1 M_2\times M_1 M_2. \text{ The signal radiated towards a spatial region of interest is given by}
\begin{equation}
\zeta(t, \theta, \phi) = a^T(\theta, \phi) s(t). \tag{10}
\end{equation}
\text{We assume that radar cross section (RCS) coefficients obey Swerling II model [17]. It means that the RCS coefficients remain unchanged during one pulse while vary for different pulses. Therefore, the received MIMO observation vector can be expressed as}
\begin{equation}
x(t, \tau) = \sum_{k=1}^{K} \beta_k(\tau) a^T(\theta_k, \phi_k) \cdot \tilde{a}(\theta_k, \phi_k) + n(t, \tau) \tag{11}
\end{equation}
where } t \text{ and } \tau \text{ represent the fast and slow time indices, respectively, } \beta_k(\tau) \text{ is the RCS coefficient of } k \text{-th target with variance } \sigma_{\beta_k}^2, \text{ and } n(t, \tau) \text{ is the noise vector modeled as complex spatial and temporal white Gaussian process. After the matched-filter, the } \tilde{m}_n \text{-th } M_1 M_2 \times 1 \text{ received vector is expressed as}
\begin{equation}
\tilde{y}(t) = \sum_{k=1}^{K} \beta_k(\tau) \cdot \tilde{a}(\theta_k, \phi_k) \cdot \tilde{a}^T(\theta_k, \phi_k) + z(t) = A_{\tilde{m}} \beta(\tau) + z(t) \tag{12}
\end{equation}
where } A_{\tilde{m}} \triangleq [\tilde{a}_{\tilde{m}}(\theta_1, \phi_1), \ldots, \tilde{a}_{\tilde{m}}(\theta_K, \phi_K)] \text{ is the desired steering matrix, } \beta(\tau) \triangleq [\beta_1(\tau), \beta_2(\tau), \ldots, \beta_K(\tau)]^T, \text{ and } z(\tau) \text{ is the noise vector whose covariance matrix is given by } \sigma_z^2 I_{M_1 M_2 N}. \text{ Hence, the whole received vector, i.e., the vector}
that is obtained by stacking $\tilde{y}_m(\tau), \tilde{m} = 1, \ldots, M_1M_2$, one under another, can be obtained as
\[
\tilde{y}(\tau) = \tilde{A}\tilde{\beta}(\tau) + z(\tau)
\] (13)
where $\tilde{y}(\tau) \in \mathbb{C}^{M_1M_2N \times 1}$
\[
\tilde{A} \triangleq [\tilde{a}(\theta_1, \phi_1), \ldots, \tilde{a}(\theta_K, \phi_K)] \in \mathbb{C}^{M_1M_2N \times K}
\] (14)
\[
\tilde{a}(\theta_k, \phi_k) = c(\theta_k) \odot d(\phi_k) \odot b(\theta_k, \phi_k)
\] (15)
denote the array steering matrix and each steering vector, respectively. It is worth noting that (15) does not fully capture the multidimensional structured information hidden in the TB based MIMO radar model. In the following section, we show how to fold the 2D TB-based MIMO radar data into a tensor form and derive a tensor-based ESPRIT method for the elevation and azimuth direction finding.

3. TENSOR FORMULATION AND SIGNAL SUBSPACE ESTIMATION

3.1. Steering vector and steering tensor

The following relationship between a rank-one three-way tensor and its vector expression is of interest
\[
\mathbf{a} \odot \mathbf{b} \odot \mathbf{c} = \text{vec}((\mathbf{a} \odot \mathbf{b} \odot \mathbf{c})).
\] (16)
Defining the steering tensor of $k$-th signal as
\[
\mathbf{A}_k(\theta_k, \phi_k) = c(\theta_k) \odot d(\phi_k) \odot b(\theta_k, \phi_k)
\] (17)
where $\mathbf{A}_k \in \mathbb{C}^{M_1 \times M_2 \times N}$, we obtain by using (16), the following relationship between the steering vector (15) and the steering tensor
\[
\tilde{a}(\theta_k, \phi_k) = \text{vec}(\mathbf{A}_k(\theta_k, \phi_k)).
\] (18)
It is clear that the array steering tensor obeys the rotational invariance property with respect to $c(\theta_k)$ or $d(\phi_k)$ if $\tilde{a}(\theta_k, \phi_k)$ does, which makes tensor-based ESPRIT methods applicable for this scheme.

3.2. MIMO radar tensor modeling

Here, we operate directly on the received data, which is known as direct data approach. Consider that there are $Q$ pulses, then the 2D TB based MIMO radar data matrix is given as
\[
\mathbf{Y} = \tilde{A} \cdot \mathbf{B} + \mathbf{Z}
\] (19)
where $\mathbf{Y} \in \mathbb{C}^{M_1M_2N \times Q}$, $\mathbf{B} \triangleq [\beta^T(1), \beta^T(2), \ldots, \beta^T(Q)]^T$ contains $K$ targets RCS coefficients of $Q$ pulses, and $\mathbf{Z}$ represents the noise. The corresponding receive MIMO radar tensor model is therefore given as
\[
\mathbf{Y} = \mathbf{A} \times_1 \mathbf{B} + \mathbf{Z}
\] (20)
where $\mathbf{Y} \in \mathbb{C}^{M_1 \times M_2 \times N \times Q}$, steering tensor $\mathbf{A} \in \mathbb{C}^{M_1 \times M_2 \times N \times K}$ is composed by stacking $K$ targets’ steering tensor $\mathbf{A}_k$ together, and $\mathbf{Z}$ stands of the noise samples the same as that in (19). In doing so, we obtain the tensor form representation of the MIMO radar data. This form allows to use some computationally efficient methods, e.g., ESPRIT, for elevation and azimuth direction finding. Comparing with search-based methods which require exhaustive searching over the whole spatial sector, the proposed method reduces the computation burden, especially when the model has a large scale.

The relation between a tensor and its matrix unfolding is expressed as
\[
\mathbf{Y} = \mathbf{Y}^{[R+1]}(R+1)
\] (21)

3.3. Target localization

As long as we write the 2D TB-based MIMO radar data model as a tensor form, it allows for a more accurate elevation and azimuth estimation through tensor decompositions, e.g., HOSVD [18]. The HOSVD of the tensor (20) is given by
\[
\mathbf{Y} = \mathbf{S} \times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \times_3 \mathbf{U}_3 \times_4 \mathbf{U}_4.
\] (22)
The tensor signal subspace is obtained as
\[
\mathbf{U}^{(\mathbf{s})} = \mathbf{S}^{(\mathbf{s})} \times_1 \mathbf{U}^{(\mathbf{s})}_1 \times_2 \mathbf{U}^{(\mathbf{s})}_2 \times_3 \mathbf{U}^{(\mathbf{s})}_3 \times_4 \mathbf{U}^{(\mathbf{s})}_4.
\] (23)
We have the following relationship between the steering tensor and the signal subspace
\[
\mathbf{A} = \mathbf{U}^{(\mathbf{s})} \times_4 \mathbf{T}
\] (24)
where $\mathbf{T}$ denotes a full-rank transform matrix.

The shift invariance along each virtual array of $\mathbf{Y}$ can be expressed as
\[
\mathbf{A} \times_1 \mathbf{J}_1^{(1)} \times_4 \mathbf{\Theta} = \mathbf{A} \times_1 \mathbf{J}_2^{(1)}
\] (25)
\[
\mathbf{A} \times_2 \mathbf{J}_2^{(2)} \times_4 \mathbf{\Phi} = \mathbf{A} \times_2 \mathbf{J}_2^{(2)}
\] (26)
where
\[
\mathbf{\Theta} \triangleq \text{diag}([e^{-j\pi \sin(\theta_1)}, \ldots, e^{-j\pi \sin(\theta_K)}])
\] (27)
\[
\mathbf{\Phi} \triangleq \text{diag}([e^{-j\pi \sin(\phi_1)}, \ldots, e^{-j\pi \sin(\phi_K)}])
\] (28)
and $\mathbf{J}_1^{(i)}$ and $\mathbf{J}_2^{(i)}$ stand for the selection matrices of each virtual array, which they are expressed as
\[
\mathbf{J}_1^{(i)} \triangleq [\mathbf{I}_{M_1}, \mathbf{0}_{M_1 \times 1}]
\] (29)
\[
\mathbf{J}_2^{(i)} \triangleq [\mathbf{0}_{M_1 \times 1}, \mathbf{I}_{M_1}]
\] (30)
where $i = 1, 2$. The selection matrices are used here to show clearly the rotation invariance property along $i$-th virtual array. In fact, in (25) and (26) the rotational invariance property is satisfied in between the left and right-hand sides of the equation.
Fig. 1. Mono-static MIMO radar transmit beampattern

expressions. The problems of finding $\Theta$ and $\Phi$ can then be expressed as tensor least squares problems, i.e.,

$$\Theta = \arg \min_\Theta \| \mathbf{U}^s \times_1 \mathbf{J}^{(1)} \times_{R+1} \mathbf{U}^s \times_1 \mathbf{J}^{(1)} \|_H$$

$$\Phi = \arg \min_\Phi \| \mathbf{U}^s \times_2 \mathbf{J}^{(2)} \times_{R+1} \mathbf{U}^s \times_2 \mathbf{J}^{(2)} \|_H.$$  

By solving these two tensor-based least squares problems, we find the elevation and azimuth estimations. Finally, the auto-pairing has to be performed, which has been addressed in [19] as well as in other papers, and it is not discussed here.

4. NUMERICAL EXAMPLES

We assume a mono-static MIMO radar with $M = 64$ transmit antennas. Their X- and Y-components are randomly drawn form an interval $[0, 4]$ in wavelength. The elements of the receive array are randomly chosen from $M$ elements, and we set $N = 8$. Two virtual ULAs with $M_1 = 4$ and $M_2 = 4$ elements are placed on Y-axis and X-axis, respectively. The desired azimuth and elevation sectors are set as $\Theta_a = [20^\circ, 35^\circ]$ and $\Theta_s = [55^\circ, 70^\circ]$, respectively. We apply 2D TB design to obtain the virtual arrays [4]. From Fig. 1, we observe that the main beam of the beampattern indeed focuses on the desired spatial area in both elevation and azimuth domain, while suppressing the transmit power outside the designed area. Also, this non-adaptive transmit beamforming is designed off-line.

We assume that there are two far-field targets located at $[21^\circ, 52^\circ]$ and $[33^\circ, 66^\circ]$, respectively. We apply both the matrix-based ESPRIT and tensor-based ESPRIT to estimate the elevation and azimuth of the targets in small pulses scenario. The number of pulses is set to $Q = 2$ and $Q = 4$ in Figs. 2 and 3, respectively. It can be observed from both figures that the proposed tensor-based ESPRIT estimation method offers better root mean square error (RMSE) performance than its matrix-counterpart in medium and high signal-to-noise ratio (SNR) regions. In the high SNR region, the RMSE versus SNR curves go flat because the mapping errors dominate the target elevation and azimuth estimation errors.

Fig. 2. RMSE versus SNR, monostatic MIMO radar with two pulses, $M_1 = M_2 = 4$, $N = 8$, $K = 2$.

Fig. 3. RMSE versus SNR, monostatic MIMO radar with four pulses, $M_1 = M_2 = 4$, $N = 8$, $K = 2$.

5. CONCLUSION

Elevation and azimuth estimation problem with arbitrary planar mono-static MIMO radar has been addressed in this paper. A fourth-order tensor modeling of an arbitrary planar mono-static 2D TB-based MIMO radar data has been proposed. Specifically, by utilizing the 2D TB design, the transmit array has been mapped into a Khatri-Rao product of two perpendicular virtual ULAs along X-axis and Y-axis, respectively. Moreover, the corresponding radar data tensor obeys the rotational invariance property with respect to elevation or azimuth. Then a tensor-based ESPRIT elevation and azimuth estimation method has been derived. The HOSVD has been used to obtain a better estimation of the signal subspace. Our simulation results verify the effectiveness of the proposed method as compared with the matrix-based counterpart.
6. REFERENCES


