5 стр.

Adaptive Recognition of the States of Dynamic Object with Periodic Output Signal[1]

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Abstract—A new approach to solving the problem of recognizing the states of a dynamic object with periodic output signal is suggested. A synthesized algorithm for recognition of such states of objects constitutes a two-level procedure. At the first level, the analyzed signal described by a model of the Fourier series type is put into correspondence with a finite set of autoregressive models with complex-conjugate roots; a specific state is revealed with the use of adaptive weighting of outputs of all the models in the bank with the constraints imposed on unbiasedness of the weighted output relative to the actual signal. At the second level, the probabilities of specific faults corresponding to probabilities of hypotheses related to specific models of the first level are determined.

INTRODUCTION

For a wide class of control objects (such as electric motors and generators), the harmonics of a constant frequency may be regarded as the output signal under normal operating conditions: when a malfunction develops, higher multiple-frequency harmonics appear and are superimposed on the main signal. The problem of recognizing the states of such objects is reduced to detection of the emergence of higher harmonics with rather small amplitudes and is complicated by the fact that the checked signal can be distorted by fairly intense noise.

In accordance with the above, a dynamic object can be in various states corresponding to different diagnoses. For the majority of technical systems, it is reasonable to assume that the number of states is finite because either the number of malfunctions, which can develop in the object, is limited or the number of malfunctions foreseen beforehand is specified.

Consider a set of possible states of the object

$$S = \{s_i:\ i = \overline{0, n-1}\},$$

and assume that the state s_0 corresponds to normal operating conditions.

The states of the system are described by a set of features. Thus, for an object with polyharmonic output signal, it is reasonable to take the parameters of corresponding harmonics as the above features.

Let the output signal of the object be described by the model equation

$$y(\tau) = \sum_{k=0}^{n} a_k \sin(k\omega_0\tau + F_k) + e(\tau), \qquad (1)$$

which can be easily transformed to

$$y(\tau) = \sum_{k=0}^{n} (a_k \cos F_k \sin k\omega_0\tau + a_k \sin F_k \cos k\omega_0\tau) + e(\tau)$$

$$= \sum_{k=0}^{n} (b_k \sin k\omega_0\tau + c_k \cos k\omega_0\tau) + e(\tau),$$

where $b_k = a_k \cos F_k$ and $c_k = a_k \sin F_k$ are the coefficients of the harmonics for the frequencies $\omega_k = k\omega_0$; ω_0 is the fundamental frequency; F_k is the phase of the kth harmonic; $e(\tau)$ is an interference in measurements of the output signal and is such that $E\{e(\tau)\} = 0$ and $E\{e^2(\tau)\} = \sigma_e^2 < \infty$, with $E\{\cdot\}$ being the symbol of expectation value; and τ is continuous time.

Introducing the vector of parameters with the dimension of $(2n + 1) \times 1$

$$C(\tau) = (b_0(\tau), b_1(\tau), c_1(\tau), \ldots, b_n(\tau), c_n(\tau))^T,$$

and the vector of the same dimension

$$\psi(\tau) = (1, \sin\omega_0\tau, \cos\omega_0\tau, \ldots, \sin\omega_0 n\tau, \cos\omega_0 n\tau)^T,$$

we can rewrite model equation (1) in the vector form as

$$y(\tau) = c^T(\tau)\psi(\tau) + e(\tau). \qquad (2)$$

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1. REPRESENTATION OF THE FOURIER SERIES MODEL IN THE FORM OF AUTOREGRESSIVE MODEL

We can put the following autoregressive model equation with complex-conjugate roots [1] into correspondence with (2):

$$\tilde{y}(t) = d_0 + \sum_{k=1}^{2n} d_i y(t-i) + e(t). \qquad (3)$$

Here, t is the current discrete time ($t = 0, 1, 2, \ldots$). Introducing the $[(2n+1)\times 1]$-dimensional vector of coefficients of autoregressive model (3)

$$\theta(t) = (d_0(t), d_1(t), \ldots, d_{2n}(t))^{\mathrm{T}},$$

and the history vector

$$\varphi(t) = (1, y(t-1), \ldots, y(t-2n))^{\mathrm{T}},$$

we can write model equation (3) in the vector form as

$$\tilde{y}(t) = \theta^{\mathrm{T}}(t)\varphi(t) + e(t). \qquad (4)$$

Relations between the parameters of models (1) and (4) [for example, for a model corresponding to the normal operating conditions ($n = 1$)] can easily be derived in the following way.

The following description corresponds to the normal operating conditions where only the fundamental harmonic is present in the output signal:

$$\tilde{y}(t) = d_1 y(t-1) + d_2 y(t-2) + e(t). \qquad (5)$$

The characteristic polynomial of the differential equation of the form

$$y(\tau)(D^2 - Dd_1 - d_2) = e(\tau),$$

where $D = dy/d\tau$. which is used to derive the difference equation (5), takes the following form after sampling the output variable with the frequency of $T_0 = 1$:

$$\lambda^2 - d_1\lambda - d_2 = 0.$$

We use the condition for the complex conjugacy of the roots

$$\frac{d_1^2}{4} - d_2 < 0.$$

to determine the roots of the characteristic polynomial as

$$\lambda_{1,2} = \frac{d_1}{2} \pm i\sqrt{-\frac{d_1^2}{4} - d_2} = |\lambda| e^{\pm i\omega_0} = e^{\ln|\lambda| \pm i\omega_0},$$

where $\ln|\lambda| = \frac{1}{2}\ln(-d_2) = F$.

The corresponding equation of the type of (1) for case of normal operating conditions can be written the following form after sampling:

$$y(t) = a\sin(\omega_0 t + F) + e(t)$$
$$= b\sin\omega_0 t + c\cos\omega_0 t + e(t).$$

In view of the fact that

$$a = \sqrt{-d_2},$$

we finally have

$$b = a\cos F = \sqrt{-d_2}\cos\left(\frac{1}{2}\ln(-d_2)\right),$$

$$c = a\sin F = \sqrt{-d_2}\sin\left(\frac{1}{2}\ln(-d_2)\right),$$

$$\lambda_1 + \lambda_2 = d_1 = |\lambda|(e^{i\omega_0} + e^{-i\omega_0}) = 2|\lambda|\cos\omega_0.$$

Consequently,

$$\omega_0 = \arccos\frac{d_1}{2|\lambda|} = \frac{d_1}{2\sqrt{-d_2}};$$

in turn, d_1 and d_2 are defined as

$$d_1 = 2\sqrt{-d_2}\cos\omega_0 = 2e^F\cos\omega_0,$$

$$d_2 = -e^{2F}.$$

Each possible state of the object corresponds to specific diagnostic model given by

$$y_k(\tau) = C_k^{\mathrm{T}}(\tau)\psi_k(\tau). \qquad (6)$$

where $C_k(\tau) = (b_0(\tau), b_1(\tau), c_1(\tau), \ldots, b_k(\tau), c_k(\tau))^{\mathrm{T}}$, and $\psi(\tau) = (1, \sin\omega_0\tau, \cos\omega_0\tau, \ldots, \sin\omega_0 k\tau, \cos\omega_0 k\tau)^{\mathrm{T}}$ are the vectors with a dimension of $(2k+1)\times 1$. Thus, state's change, manifesting itself in the emergence of new multiple-frequency harmonics in the output signal of the object, corresponds to an increase in the order of the autoregressive model. The latter, in the case of superposition of k harmonics of multiple frequencies on the output signal, takes the form of the autoregressive model with complex-conjugate roots; i.e., we have

$$\tilde{y}_k(t) = \hat{\theta}_k^{\mathrm{T}}(t-1)\varphi_k(t). \qquad (7)$$

where $\hat{\theta}_k(t-1) = (\hat{d}_0(t-1), \hat{d}_1(t-1), \ldots, \hat{d}_{2k}(t-1))$ is the parameter-estimate vector with a dimension of $(2k+1)\times 1$ and

$$\varphi_k(t) = (1, y(t-1), \ldots, y(t-2k))^{\mathrm{T}}, \quad k = 1, 2, \ldots, n.$$

2. RECURSIVE ESTIMATION OF PARAMETERS OF AN AUTOREGRESSIVE MODEL WITH ALLOWANCE MADE FOR THE POSSIBLE INCREASE IN THE ORDER

In order to estimate the parameters of models given by (7), with allowance made for a possible increase in the model's order and for the fact that the diagnostics is performed in real time, we suggest the following recursive algorithms.

(I) The Kaczmarz algorithm takes the following form with allowance made for the possible increase in the autoregression order:

$$\hat{\theta}_{k+1}(t) = \hat{\theta}_{k+1}(t-1) + \frac{y(t) - \hat{\theta}^{T}_{k+1}(t-1)\varphi_{k+1}(t)}{\|\varphi_k(t)\|^2 + y^2(t-2k-1)}\varphi_{k+1}(t). \quad (8)$$

Here,

$$\|\varphi_{k+1}(t)\|^2 = \|\varphi_k(t)\|^2 + y^2(t-2k-1)$$

$$= \gamma_k^{-1}(t) + y^2(t-2k-1) = \gamma_{k+1}^{-1}(t),$$

$$y(t) - \hat{\theta}^{T}_{k+1}(t-1)\varphi_{k+1}(t) = \varepsilon_{k+1}(t)$$

$$= y(t) - \hat{\theta}^{T}_{k}(t-1)\varphi_k(t) - \hat{a}_{2k+1}(t-1)y(t-2k-1)$$

$$= \varepsilon_k(t) - \hat{a}_{2k+1}(t-1)y(t-2k-1).$$

(II) The recursive least-squares method has the following form if allowance is made for an increase in the autoregressive model's order:

$$\hat{\theta}_{k+1}(t) = \hat{\theta}_{k+1}(t-1)R^{-1}_{k+1}(t) \times (y(t) - \hat{\theta}_{k+1}(t-1)\varphi_{k+1}(t))\varphi_{k+1}(t), \quad (9)$$

$$R^{-1}_{k+1}(t) = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}. \quad (10)$$

$$R_k^{-1}(t) = R_k^{-1}(t-1) - \frac{R_k^{-1}(t-1)\varphi_k(t)\varphi_k^{T}(t)R_k^{-1}(t-1)}{1 + \varphi_k^{T}(t)R_k^{-1}(t-1)\varphi_k(t)}. \quad (11)$$

Here,

$$\beta_{k+1}(t) = \alpha_{k+1}(t) + v_{k+1}(t)R_k^{-1}(t)u_{k+1}(t);$$

$$A_{11} = R_k^{-1}(t) + \frac{R_k^{-1}(t)u_{k+1}(t)v_{k+1}(t)R_k^{-1}(t)}{\beta_{k+1}(t)};$$

$$A_{12} = -\frac{R_k^{-1}(t)u_{k+1}(t)}{\beta_{k+1}(t)};$$

$$A_{21} = -\frac{v_{k+1}(t)R_k^{-1}(t)}{\beta_{k+1}(t)};$$

$$A_{22} = 1/\beta_{k+1}(t);$$

and $u_{k+1}(t)$, $v_{k+1}(t)$, and $\alpha_{k+1}(t)$ are the column, row, and scalar, respectively, of the following bordered matrix:

$$R_{k+1}(t) = \begin{bmatrix} R_k(t) & u_{k+1}(t) \\ v_{k+1}(t) & \alpha_{k+1}(t) \end{bmatrix}.$$

In deriving the algorithm defined by (9)–(11), we used the formulas for calculating the matrix reciprocal to the covariance matrix $R_{k+1}(t)$ with allowance made for an increase in dimension [2].

As mentioned above, a fault in the object manifests itself by the emergence of higher frequency harmonics in the output signal, which corresponds to an increase in the autoregressive-model order. Thus, if at step t the quantity $\omega(t) = y(t) - \tilde{y}_k(t)$ represents the white noise, then it signifies that the model $\tilde{y}_k(t)$ correctly describes the output signal. However, if at the $(t + h)$th step the quantity $\omega(t+h) = y(t+h) - \tilde{y}_{k+s}(t+h)$ represents the white noise, whereas the quantity $\omega(t+h) = y(t+h) - \tilde{y}_k(t+h)$ does not, then this signifies that the higher frequency harmonics appear in the output signal and the diagnosed object is in the state corresponding to the model $\tilde{y}_{k+s}(t)$.

3. CALCULATION OF THE PROBABILITY OF FAULTS

Each of the models corresponds to a certain fault in the object and is in direct correspondence with the number of harmonics in the output signal. Furthermore, each model is based on a specific hypothesis H_k $(k = 1, 2, \dots, n)$ with respect to the nature of the faults. Consequently, the truth of the hypothesis H_k indicates that the object has a fault corresponding to the kth diagnostic model, which, in turn, is indicative of the presence of the kth frequency harmonic in the output signal.

In order to identify the true hypothesis about the nature of faults in the object, we introduce a multimodel filtered sequence [3] as

$$\check{y}(t) = \bar{p}^{T}(t)\bar{Y}(t), \quad (12)$$

where $\bar{P}(t) = (\bar{p}_1(t), \bar{p}_2(t), \dots, \bar{p}_n(t))^T$ is an $(n \times 1)$-dimensional vector of unknown adjustable weighting coefficients, which define the closeness of the output signal of the model $\tilde{y}_k(t)$ to the actual signal $y(t)$ and satisfy the conditions for unbiasedness

$$E^{T}\bar{P}(t) = 1, \quad (13)$$

where E is an $(n \times 1)$-dimensional vector consisting of unities and $\bar{Y}(t) = (\tilde{y}_1(t), \tilde{y}_2(t), \dots, \tilde{y}_n(t))^T$ is an $(n \times 1)$-dimensional vector composed of the magnitudes of output signals corresponding to the diagnostic models $\tilde{y}_k(t)$.

Taking into account conditions (13) and the conditions for nonnegativity of the coefficients $\tilde{p}_k(t)$ in model (12), we formulate the Lagrangian with respect to the residual function

$$W(t) = y(t) - \tilde{y}(t) = \tilde{p}^{\mathrm{T}}(t)(Ey(t) - \tilde{Y}(t)) = \tilde{P}^{\mathrm{T}}(t)V(t), \quad (14)$$

$$V(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_n(t) \end{bmatrix} = \begin{bmatrix} y(t) - \tilde{y}_1(t) \\ y(t) - \tilde{y}_2(t) \\ \vdots \\ y(t) - \tilde{y}_n(t) \end{bmatrix};$$

this Lagrangian has the form

$$L(\tilde{P}, \lambda, \mu) = \tilde{P}^{\mathrm{T}}(t)H(t)\tilde{P}(t) + \lambda(\tilde{P}^{\mathrm{T}}E - 1) - \mu^{\mathrm{T}}\tilde{P}(t), \quad (15)$$

where $H(t) = \sum_{i=1}^{t} V(i)V^{\mathrm{T}}(i)$, λ is the indeterminate Lagrange multiplier, and μ is the vector of nonnegative indeterminate Lagrange multipliers with the dimension of $(n \times 1)$.

Using the Arrow–Hurwitz–Uzawa procedure [4], we find the value of the Langrangian (15) saddle point as

$$\tilde{P}(t) = P(t) + 0.5\left(I - \frac{H^{-1}(t)EE^{\mathrm{T}}}{E^{\mathrm{T}}H^{-1}(t)E}\right)H^{-1}(t)\mu(t-1), \quad (16)$$

$$P(t) = \frac{H^{-1}(t)E}{E^{\mathrm{T}}H^{-1}(t)E}, \quad (17)$$

$$\mu(t) = \mathrm{Pr}_{+}(\mu(t-1) - \gamma_{\mu}(t)\tilde{P}(t)), \quad (18)$$

where I is an identity matrix,

$$\gamma_{\mu}(t) = \mathrm{diag}(\gamma_1(t), \gamma_2(t), \ldots, \gamma_n(t)).$$

$\mathrm{Pr}_{+}(\cdot)$ is the projector on positive ortant, and the matrix reciprocal to the covariance matrix of errors of the models $H^{-1}(t)$ is calculated using the Sherman–Morrison recursive formula [5] as

$$H^{-1}(t) = H^{-1}(t-1) - \frac{H^{-1}(t-1)V(t)V^{\mathrm{T}}(t)H^{-1}(t-1)}{1 + V^{\mathrm{T}}(t)H^{-1}(t-1)V(t)}. \quad (19)$$

Since $\sum_{k=1}^{n} \tilde{p}_k(t) = 1$ and $\tilde{p}_k(t) \geq 0$, the quantity $\tilde{p}_k(t)$ has a significance for the probability of the hypothesis H_k with respect to the nature of the faults because the corresponding model $\tilde{y}_k(t)$ actually describes the output harmonic signal of the object $y(t)$.

It is noteworthy that the value of the saddle point in Lagrangian (15) ensures minimization of the test

$$J = \tilde{P}^{\mathrm{T}}(t)H(t)\tilde{P}(t) \quad (20)$$

with allowance made for the conditions for unbiasedness (13) and nonnegativity of the coefficients of model (12).

We now supplement the procedure defined by (16)–(19) by an obvious rule for the choice of the hypothesis H_k concerning the nature of the fault. This rule is as follows: the hypothesis H_k is regarded as true if it corresponds to the highest probability $\tilde{p}_k(t)$. To put it formally, if $\tilde{p}_k(t) > \tilde{p}_i(t)$ ($\forall i = \overline{1, n}$, and $i \neq k$), the hypothesis H_k is true.

4. EXAMPLE

As an example, we report the results of a numerical experiment concerning the recognition of the states of a dynamic object with a periodic output signal.

Normal operating conditions correspond to model M1 defined as

$$y_1(\tau) = 1\sin 60\tau + e(\tau),$$

where noise $e(\tau)$ has the mean value $E\{e(\tau)\} = 0$ and the standard deviation $\sigma_e = 0.05$.

The object can be in two other states characterized by emergence of additional higher frequency harmonics in the output signal; these two states correspond to models M2 and M3 given by

$$y_2(\tau) = 1\sin 60\tau + a_2\sin 2 \times 60\tau + e(\tau),$$

$$y_3(\tau) = 1\sin 60\tau + a_2\sin 2 \times 60\tau + a_3\sin 3 \times 60\tau + e(\tau),$$

where the amplitude coefficients of additional harmonics a_2 and a_3 can take various values.

The autoregressive model having complex-conjugate roots and corresponding to the model of normal-operating conditions M1 is written as

$$\tilde{y}_1(t) = 1y(t-1) - 1y(t-2) + e(t).$$

The structure of autoregressive models corresponding to models M2 and M3 coincides with the structure of (7) for $k = 2$ and $k = 3$, where

$$\hat{\theta}_2(t) = (1, -1, d_3(t), d_4(t))^{\mathrm{T}},$$

$$\varphi_2(t) = (y(t-1), y(t-2), y(t-3), y(t-4))^{\mathrm{T}},$$

$$\hat{\theta}_3(t) = (1, -1, d_3(t), \ldots, d_6(t))^{\mathrm{T}},$$

$$\varphi_3(t) = (y(t-1), y(t-2), \ldots, y(t-6))^{\mathrm{T}}.$$

In order to adjust the vectors of parameters $\hat{\theta}_2(t)$ and $\hat{\theta}_3(t)$ in the models $\tilde{y}_2(t)$ and $\tilde{y}_3(t)$, we used a modification of the Kaczmarz algorithm with allowance made for an increase in the order of autoregression (8).

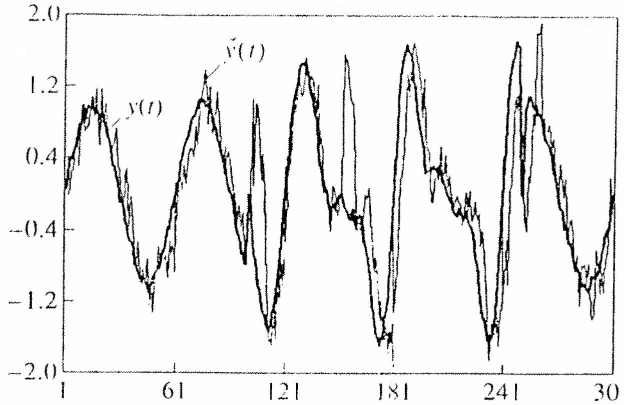

Fig. 1.

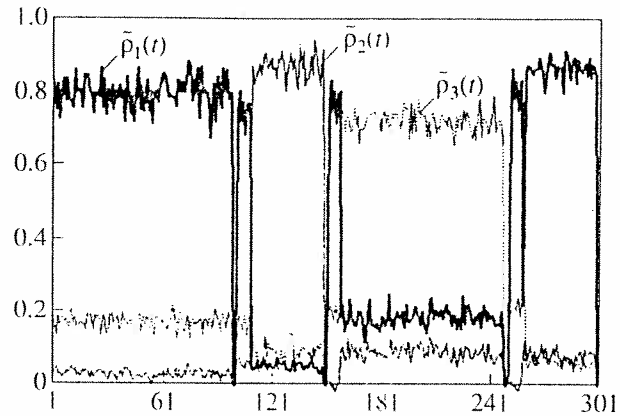

Fig. 2.

Furthermore, the variables of the model $\tilde{y}_1(t)$ were taken as initial parameters.

A switchover between models M1–M3 occurs at random points in time; the problem consists in the accurate recognition of the current object state described by one of the above models and in the precise simulation of the output signal.

In order to calculate the probability of the object being in a specific state, we used the procedure given by (16)–(19).

The results of simulation are shown in graphic form in Figs. 1 and 2. Figure 1 shows the plots of the diagnosed-object output signal $y(t)$ and the simulating signal $\tilde{y}(t)$ calculated by formula (12) with $n = 3$. In Fig. 2, the plots for probabilities of the object being in specific states $\tilde{p}_1(t)$, $\tilde{p}_2(t)$, and $\tilde{p}_3(t)$ are shown. The largest error in simulating the output signal occurs at the instants of emergence of additional higher frequency harmonics. Within the time intervals (when the object is in a specific state), the error does not exceed 10% of the variation range for the output signal. The longest delay in recognition of the object state amounts to 9 time steps.

Thus, for the given level of disturbances, reasonable precision of simulating the output signal and the speed of recognizing the current state of the object are ensured.

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