

Tensor and AI for intelligent 6G wireless systems

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Abstract

Driven by the unprecedented demands of emerging applications, next-generation (6G) wireless networks must evolve beyond existing networks into an intelligent, environment-aware, and interactive ecosystem to support a fully digital society. Although the architectural and algorithmic foundations for this vision are taking shape, critical bottlenecks restrict their practical deployment. To this end, this thesis focuses specifically on the challenges of waveform design for vehicular integrated sensing and communications (ISAC) systems and power control in cell-free massive MIMO (CFmMIMO) networks. This thesis develops tailored methodologies to resolve the performance bottlenecks imposed by stringent 6G requirements, including high-accuracy and wide-area sensing, performance trade-off between sensing and communications (S&C), and ultra-low latency.

First, a slow-time transmit beamspace design for MIMO radar systems is developed to optimize energy distribution while preserving waveform diversity. A corresponding tensor-based signal model is then formulated, and parallel factor (PARAFAC) decomposition is employed to enable accurate sensing parameter estimation and velocity disambiguation.

Second, a time-division ISAC framework for vehicular systems is introduced to balance S&C requirements. Within this framework, a multibeam sensing strategy is proposed to achieve wide-area coverage and higher sensing accuracy compared to conventional beam-scanning methods. The effect of beamforming-induced null directions on velocity estimation is analyzed, and a corresponding estimation technique is developed. Moreover, a new identifiability condition for tensors with Vandermonde-structured factor matrices is established to support PARAFAC-based parameter estimation.

Third, this thesis investigates power control in CFmMIMO systems under pilot contamination. To solve the max-min fairness power control problem, a self-supervised graph neural network enhanced with an attention mechanism is developed. By exploiting the graph-structured representations of the CFmMIMO network, the proposed model effectively captures interactions between access points and user equipments (UEs), offering scalability to large networks and adaptability to varying numbers of UEs. Experimental results show that the proposed approach significantly reduces computational complexity compared to conventional optimization methods while achieving superior performance, validating its capability in ultra-low-latency 6G real-time applications.

Overall, this thesis presents strategic insights that bridge the gap between theoretical developments and practical considerations for 6G deployment. The developed methodologies provide useful guidance for system design and optimization, potentially contributing to the advancement of next-generation intelligent wireless networks.

Keywords: transmit beamspace (TB) design, MIMO radar, ISAC, 6G, PARAFAC, graph neural network (GNN), power control in CFmMIMO, learning-based method

Preface

This thesis marks the end of the most significant chapter in my life so far. It is a milestone shaped and sustained by the guidance, support, and encouragement of many individuals to whom I am deeply and sincerely grateful.

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This thesis is dedicated to courage, whether born of strength or sustained by belief.

Preface

Espoo, October 6, 2026,

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List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

- I** T. Zhang, F. Xu, and S. A. Vorobyov. Transmit Energy Focusing for Parameter Estimation in Transmit BeamSpace Slow-Time MIMO Radar. In *IEEE International Conference on Acoustics, Speech, and Signal Processing*, Rhodes Island, Greece, pp. 1–5, June 2023.
- II** T. Zhang, S. A. Vorobyov, and F. Xu. Transmit Energy Focusing for Parameter Estimation in Slow-Time Transmit BeamSpace L-Shaped MIMO Radar. *IEEE Transactions on Signal Processing*, vol. 72, pp. 5228–5243, Nov. 2024.
- III** T. Zhang, T. Kim, S. A. Vorobyov, and D. J. Love. A Novel Multibeam Time-Division ISAC Approach with Accurate Sensing. In *IEEE International Conference on Acoustics, Speech, and Signal Processing*, Barcelona, Spain, pp. 22617–22621, May 2026.
- IV** T. Zhang, T. Kim, S. A. Vorobyov, and D. J. Love. A Novel Multibeam Time-Division ISAC Approach for Accurate Sensing Parameter Estimation. *IEEE Transactions on Wireless Communications*, vol. 25, pp. 16304–16319, May 2026.
- V** T. Zhang, S. A. Vorobyov, D. J. Love, T. Kim, and K. Dong. Pilot Contamination-Aware Graph Attention Network for Power Control in CFmMIMO. *IEEE Wireless Communications Letters*, vol. 15, pp. 1464–1468, Feb. 2026.

Author's Contribution

Publication I: “Transmit Energy Focusing for Parameter Estimation in Transmit Beamspace Slow-Time MIMO Radar”

The author developed the core idea, performed the analysis and simulation validation, and wrote and revised the article with the advice from other coauthors.

Publication II: “Transmit Energy Focusing for Parameter Estimation in Slow-Time Transmit Beamspace L-Shaped MIMO Radar”

The author developed the core idea, performed the analysis and simulation validation, and wrote and revised the article with the advice from other coauthors.

Publication III: “A Novel Multibeam Time-Division ISAC Approach with Accurate Sensing”

The author developed the idea, performed the analysis and simulation validation, and wrote and revised the article with the advice from other coauthors.

Publication IV: “A Novel Multibeam Time-Division ISAC Approach for Accurate Sensing Parameter Estimation”

The author developed the idea, performed the analysis and simulation validation, and wrote and revised the article with the advice from other coauthors.

Publication V: “Pilot Contamination-Aware Graph Attention Network for Power Control in CFmMIMO”

The author developed the idea initially proposed by the supervisor, conducted the analysis, designed the model architecture, carried out the theoretical development and simulation validation, and wrote and revised the article with the advice from other coauthors.

Abbreviations

1-D	1-Dimensional
2-D	2-Dimensional
3-D	3-Dimensional
5G	Fifth-Generation
6G	Sixth-Generation
AI	Artificial Intelligence
ALS	Alternating Least Squares
AP	Access Point
APG	Accelerated Projected Gradient
C-centric	Communication-centric
CDF	Cumulative Distribution Function
CFmMIMO	Cell-Free Massive Multiple-Input Multiple-Output
CPD	Canonical Polyadic Decomposition
CPI	Coherent Processing Interval
CPU	Central Processing Unit
CSI	Channel State Information
CRB	Cramér–Rao Bound
DDMA	Doppler Division Multiple Access
DFT	Discrete Fourier Transform
DOA	Direction of Arrival

Abbreviations

DOD	Direction of Departure
DoF	Degree of Freedom
ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
EVD	Eigenvalue Decomposition
FIM	Fisher Information Matrix
FoV	Field-of-View
GNN	Graph Neural Network
IID	Independent and Identically Distributed
ISAC	Integrated Sensing and Communications
LoS	Line-of-Sight
MIMO	Multiple-Input Multiple-Output
MUSIC	Multiple Signal Classification
PARAFAC	Parallel Factor
PD	Polyadic Decomposition
PRF	Pulse Repetition Frequency
QoS	Quality of Service
RCS	Radar Cross Section
S&C	Sensing and Communications
S-centric	Sensing-centric
SE	Spectral Efficiency
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SVD	Singular Value Decomposition
TB	Transmit Beamspace
TD	Time-Division
TDMA	Time Division Multiple Access
UE	User Equipment
ULA	Uniform Linear Array

Symbols

$\mathcal{CN}(\mu, \sigma^2)$	Complex Gaussian distribution with mean μ and variance σ^2
$\overline{\text{CRB}}$	Sensing performance metric for multiple targets in ISAC
f_k	Doppler frequency shift of the k th target
FIM	Fisher information matrix
M_r	Number of receive array antenna elements
M_t	Number of transmit array antenna elements
M_v	Number of virtual transmit array antenna elements
T_r	Pulse repetition interval
v_k	Velocity of the k th target
x	Scalar (lowercase non-bold letter)
x	Vector (lowercase bold letter)
X	Matrix (uppercase bold letter)
$\mathcal{X}$	Tensor (uppercase calligraphic letter)
$\mathbf{X}^{(n)}$	Matrix formed by selecting the top n rows of X
<u>X</u>	Matrix obtained by deleting the bottom row of X
$\overline{\mathbf{X}}$	Matrix obtained by deleting the top row of X
λ	Wavelength
φ_k	Direction of arrival for the k th target

Symbols

θ_k	Direction of departure for the k th target
$\mathbb{C}^{I_1 \times \cdots \times I_N}$	Set of complex-valued tensors of size $I_1 \times \cdots \times I_N$
$\mathbb{E}(\cdot)$	Expectation operator
$\mathbb{R}^{I_1 \times \cdots \times I_N}$	Set of real-valued tensors of size $I_1 \times \cdots \times I_N$
$\mathbb{R}_{++}^{I_1 \times \cdots \times I_N}$	Set of tensors of size $I_1 \times \cdots \times I_N$ with positive real-valued entries
$\mathbb{R}_+^{I_1 \times \cdots \times I_N}$	Set of tensors of size $I_1 \times \cdots \times I_N$ with nonnegative real-valued entries

1. Introduction

1.1 Background and Motivation

1.1.1 Evolution Toward 6G

We are entering an unprecedented era in which a larger number of smart products and applications with continuously increasing levels of interactivity and intelligence, such as fully autonomous driving, unmanned aerial vehicles, large-scale industrial automation, and immersive virtual and augmented reality, are rapidly emerging and evolving, thereby imposing enormous demands on wireless networks [1, 2, 3]. It is foreseeable that current fifth-generation (5G) wireless systems will struggle to accommodate the explosive growth in mobile traffic and increasingly stringent intelligence requirements expected in future wireless networks [1]. Therefore, sixth-generation (6G) wireless networks are envisioned to support emerging applications and services with much higher performance demands than previous generations. These demands include substantially higher data rates in terms of both peak throughput and cell-edge performance, denser connectivity with broader coverage, ultra-low latency, improved energy and spectral efficiency (SE), enhanced adaptability to mobility, as well as strengthened security and privacy [2, 3]. To meet these requirements, a range of novel paradigms and enabling technologies are being actively investigated for the next-generation wireless networks. These key innovations include integrated sensing and communications (ISAC), cell-free massive multiple-input multiple-output (MIMO) (CFmMIMO) architecture, artificial intelligence (AI), terahertz communications, reconfigurable intelligent surface, sparse array design, and quantum networking, among other emerging technologies. Beyond surpassing the performance benchmarks of current 5G network systems, these advances collectively aim to drive a transition toward environment-aware, interactive, and intelligent wireless

ecosystems capable of supporting the next generation of digital society.

Within the landscape of emerging 6G technologies, this thesis specifically focuses on ISAC, CFmMIMO architecture, and AI-driven optimization techniques within the framework of 6G wireless networks.

1.1.2 Integrated Sensing and Communications (ISAC)

ISAC is widely recognized as a key enabler of 6G wireless networks [4, 5, 6, 7]. By integrating sensing and communications (S&C) functionalities within a unified hardware platform, or even sharing the same waveform, this paradigm significantly improves both hardware utilization and system SE. Moreover, such integration enables synergistic interaction between the two functionalities, effectively transforming conventional wireless infrastructures into adaptive and intelligent systems anticipated for 6G [5, 7]. By blending real-time environmental perception with high-speed communications, ISAC catalyzes a diverse range of transformative applications, including autonomous navigation in smart transportation, vehicle-to-everything connectivity in smart cities, and Wi-Fi sensing in smart homes.

Depending on the dominant system functionality, ISAC system designs are generally classified into three categories:

- **Sensing-centric (S-centric) design:** Prioritizes sensing performance, typically suffering from limited communication throughput.
- **Communication-centric (C-centric) design:** Focuses on maximizing data rates using information-oriented waveforms and high-gain directional beamforming, sacrificing spatial coverage and sensing performance.
- **Joint design:** Establishes a unified framework in which both S&C functionalities are jointly optimized to achieve an optimal performance tradeoff.

An appropriate ISAC design choice depends on the scenario-specific deployment demands. Central to meeting these demands is the dual-purpose waveform design, which serves as the foundational mechanism to balance inherently competing objectives.

1.1.3 Waveform Design in ISAC

Due to their distinct operating principles and performance requirements, S&C functionalities impose different demands on waveform design. Specifically, communication waveforms are typically optimized to support high data rates, whereas sensing waveforms are designed to exhibit constant-envelope characteristics and superior correlation properties [8]. Moreover,

co-existing S&C systems inevitably generate mutual interference when operating simultaneously. Waveform design in ISAC must therefore account for interference mitigation while satisfying individual performance requirements, making it a challenging and critical area of research. Consistent with the ISAC design classification introduced in Subsection 1.1.2, existing waveform design approaches can be categorized according to their functional priorities into S-centric, C-centric, and joint ISAC waveform designs. S-centric ISAC waveform design [9, 10, 11, 12, 13] adopts legacy sensing waveforms as the primary transmission framework and embeds communication information into the sensing signals across different domains. In contrast, C-centric waveform design [14, 15, 16, 17, 18] primarily exploits or marginally modifies established communication waveforms to simultaneously enable secondary radar sensing functionality.

Compared with S-centric or C-centric ISAC waveform design, which mainly focuses on enabling the secondary functionality based on the signal formats of the primary functionality, joint waveform design is generally more challenging because it explicitly addresses the performance tradeoff between S&C functionalities. Nevertheless, without being restricted to existing sensing or communication waveforms, joint design exhibits more degrees of freedom (DoFs) and enhanced flexibility in balancing the requirements of both functionalities, potentially providing a better performance tradeoff between S&C. Joint waveform design can be conducted in one or more independent signal dimensions to multiplex S&C functionalities, and common multiplexing strategies include spatial-division multiplexing, time-division (TD) multiplexing, and frequency-division multiplexing [5, 19, 20]. To quantitatively characterize the performance tradeoff, various performance metrics for both S&C are jointly incorporated into the optimization objectives or constraints, such as Cramér-Rao Bound (CRB), mutual information, signal-to-interference-plus-noise ratio (SINR), and waveform similarity [5, 21, 22, 23].

Numerous emerging applications fall within the scope of joint waveform design, with vehicular ISAC systems being among the most prominent examples, where the core decision-making architecture simultaneously requires high-accuracy environmental sensing and low-latency communications to ensure reliable autonomous operation [7, 19, 24]. However, as a cornerstone of next-generation architecture, joint ISAC waveform design within 6G is still in its early but rapidly evolving stage, with many fundamental design principles and performance metrics still under active investigation. A more comprehensive study of the scenario-specific joint S&C requirements, together with the coordinated exploitation of multiple signal dimensions, may enable more flexible and efficient ISAC waveform design.

Within the framework of ISAC, this thesis first presents a transmit beamspace (TB) design approach for improved sensing performance in

Chapter 2, based on the work developed in Publication I and Publication II. Building upon this foundation, Chapter 3 introduces a waveform design for ISAC, together with a novel multibeam method for accurate sensing, as developed in Publication III and Publication IV.

1.1.4 AI-Enabled 6G

The remarkable success of AI across various domains has catalyzed and accelerated its integration into wireless network infrastructures. Future 6G networks are expected to transcend conventional mobile internet paradigms, fostering a ubiquitous ecosystem where AI-driven applications are seamlessly embedded from the network core all the way to the end-user devices. AI is poised to play a pivotal role in the design, management, and optimization of 6G architectures, communication protocols, and physical-layer operations in the future 6G systems [25].

Learning-based techniques, as a core branch of AI, have demonstrated their significant potential to improve both the effectiveness and efficiency of key wireless communication tasks, including network management, channel estimation, beamforming, and resource allocation [25, 26, 27, 28, 29]. Owing to unprecedented scale, ultra-high device density, and severe heterogeneity, future 6G ecosystems are expected to exhibit highly dynamic and complex behaviors. Traditionally, optimization problems in wireless networks have been addressed by rigid rule-based methods grounded in analytical models, domain expertise, and empirical observations. However, in large-scale and complex environments, the precise mapping between an optimization decision and its physical system response is often analytically intractable or computationally prohibitive to model. Consequently, conventional model-based optimization approaches become impractical for 6G networks. In contrast, learning-based techniques circumvent these analytical limitations by learning from environmental information and massive datasets generated from complex systems. By shifting most of the computational overhead to an offline training phase, learning-based methods are capable of meeting the stringent real-time latency requirements during online inference.

While AI-driven 6G research remains a highly active frontier, the seamless and comprehensive integration of AI into next-generation wireless networks is still at an early stage, leaving critical open challenges to be investigated. In this thesis, learning-based approaches are explored to address the challenges in CFmMIMO systems.

1.1.5 Cell-Free Massive MIMO (CFmMIMO)

Another significant paradigm shift envisioned for 6G wireless networks is the transition from conventional cellular architectures to cell-free network

deployments [30, 31, 32, 33]. The term “cell-free” indicates no conventional cell boundaries from a user equipment (UE) perspective, as a UE is cooperatively served by a cluster of neighboring APs. This paradigm therefore represents a transition from traditional AP-centric to UE-centric architectures. In traditional cellular networks, the quality of service (QoS) experienced by a UE heavily depends on its proximity to its serving AP. Consequently, cell-edge UEs suffer from severe performance degradation due to large path loss and inter-cell interference. In contrast, CFmMIMO networks deploy a large number of densely distributed APs to cooperatively serve UEs through joint coherent transmission and processing [30]. By eliminating conventional cell boundaries, CFmMIMO systems are able to provide nearly uniform QoS across the coverage area. Furthermore, the distributed deployment of massive APs provides abundant spatial DoFs, enabling highly directional beamforming that enhances spatial multiplexing capability and effectively mitigates inter-user interference.

Compared with conventional cellular architectures, CFmMIMO networks have been demonstrated to achieve higher SE, more uniform QoS, and enhanced robustness to interference [31, 32]. To fully realize these advantages, several critical challenges must be addressed. In CFmMIMO systems, each UE is served collaboratively by a number of APs, requiring the system to jointly handle AP-UE association, power allocation, precoding, and pilot assignment across large-scale deployments [26, 30]. Conventional approaches to these tasks primarily rely on iterative techniques whose computational complexity scales exponentially with the numbers of APs and UEs. Consequently, in large-scale CFmMIMO deployments, this leads to prohibitive computational overhead and latency, rendering conventional approaches inadequate for the ultra-low latency demands of next-generation wireless networks. More recently, advancements in AI have opened new avenues for overcoming scalability bottlenecks and offering powerful new approaches to optimization problems in CFmMIMO systems.

A learning-based method for CFmMIMO power control is presented in Chapter 4, as developed in Publication V.

1.2 Objectives

This thesis investigates the research directions outlined above and pursues the following objectives:

- Propose a slow-time TB waveform to optimize the distribution of transmit energy while maintaining waveform diversity. Building on waveform design, develop a parameter estimation algorithm aimed at enhancing sensing accuracy and mitigating the inherent velocity ambiguity of Doppler division multiple access (DDMA) approaches.

- Develop a resource allocation strategy to balance the S&C performance tradeoff that fits the vehicular ISAC applications. Within this ISAC framework, develop a novel multibeam sensing approach to achieve accurate parameter estimation, outperforming conventional beam-scanning-based methods.
- Design a neural network model to solve the power control optimization problem while explicitly accounting for pilot contamination, with the aim of significantly reducing computational complexity to meet real-time requirements while outperforming both traditional methods and state-of-the-art learning-based approaches.

1.3 Summary of Publications and Their Contributions

- In Publication I, a novel slow-time TB MIMO radar technique is proposed to improve sensing performance. Based on the DDMA technique, the method designs a slow-time phase modulation matrix by partitioning the Doppler spectrum into more subbands than the number of transmit antenna elements. The resulting modulation matrix is constructed from pulse vectors whose beams are steered toward the desired region, enabling transmit energy focusing. Furthermore, a 3-dimensional (3-D) tensor-based signal model is developed to facilitate accurate sensing parameter estimation.
- Publication II extends the slow-time TB technique in Publication I to an L-shaped array radar equipped with uniform linear subarrays for joint 2-dimensional (2-D) directions of arrival (DOAs) and velocity estimation. The performance impact of the virtual transmit array size and beam selection is investigated. Moreover, a rigorous mathematical analysis of the Doppler spectrum characteristics of the proposed waveform design is provided, revealing the limitations of conventional matrix-based methods and motivating the development of a tensor-based model. Extensive simulation results further validate the superior performance of the proposed approach.
- Publication III proposes a multibeam TD approach to balance the simultaneous demands for high sensing accuracy and high communication data rates in vehicular ISAC frameworks. The TD strategy is introduced to flexibly balance the S&C performance tradeoff. Rather than relying on conventional beam-scanning methods that suffer from limited angular resolution, this approach proposes a novel multibeam design. By exploiting inter-beam relationships, the proposed approach achieves wide-area sensing with significantly improved sensing accuracy.

- Building upon the multibeam TD framework in Publication III, Publication IV establishes a new uniqueness condition for the parallel factor (PARAFAC) decomposition featuring two Vandermonde factor matrices in the presence of identical sensing parameters. To practically deploy this framework, a low-complexity algorithm is proposed for efficient PARAFAC implementation. Moreover, this work analyzes the impact of null directions on sensing parameter estimation by characterizing the effect for different virtual array sizes, and subsequently proposes a dedicated velocity estimation method for targets located at null directions.
- Publication V proposes a self-supervised, pilot contamination-aware graph attention network for power control in CFmMIMO systems. The proposed model employs an attention-enhanced graph neural network (GNN) to effectively capture information from neighboring nodes. It distinguishes interactions between APs and UEs by categorizing graph edges into two types and explicitly accounts for pilot contamination by incorporating the cross-correlation of pilot sequences as an input. Furthermore, a padding mechanism is introduced to handle dynamic scenarios with a varying number of UEs.

1.4 Structure of Thesis

The remainder of this thesis is organized as follows. Chapter 2 first presents preliminaries on tensors to provide the necessary mathematical groundwork for subsequent development of this thesis. Next, an L-shaped radar system model is introduced, upon which a TB design technique for MIMO radar is developed. After analyzing the Doppler spectrum characteristics of this design, the chapter constructs a 3-D tensor framework. A PARAFAC-based algorithm is then derived to achieve accurate sensing parameter estimation. Chapter 3 first presents the signal model for a TD vehicular ISAC scenario, together with the performance metrics for sensing and communication. A novel multibeam method is then developed to enable accurate sensing, and a relaxed uniqueness condition for PARAFAC with Vandermonde-structured factor matrices is derived for sensing parameter estimation. Moreover, velocity estimation for targets located at null directions is specifically analyzed with a corresponding solution provided. Chapter 4 first formulates a max–min fairness power control optimization problem in CFmMIMO systems. A graph attention network model is then developed to address this problem, and the details of the proposed model are presented. Numerical experiment results are provided to validate the proposed neural network. Chapter 5 provides conclusions and potential directions for future research.

2. Slow-Time Transmit Beamspace Design for MIMO Radar

To enable the evolution of wireless networks from communication-oriented architectures to environment-aware and intelligent platforms, sensing is expected to be a core functionality in 6G wireless systems. Sensing performance is fundamentally influenced by the transmit beamspace design for array radars, which in turn is critically shaped by the transmitted waveforms, whose coherence and diversity dictate the system's sensing capability. Classical phased-array radars [34, 35] transmit identical waveforms with carefully designed phase shifts across antenna elements, achieving high directivity at the expense of spatial DoFs. In contrast, MIMO radars [36, 37, 38] employ fully orthogonal waveforms to maximize both effective aperture and DoFs; however, this yields an omnidirectional beampattern, resulting in energy inefficiency. To strike a balance between waveform coherence and diversity, various TB design techniques were developed [34, 39, 40, 41]. These methods employ linear combinations of a reduced number of orthogonal waveforms to achieve directional energy focusing while preserving waveform diversity. However, most existing TB designs consider fast-time waveforms and are typically confined to single-pulse formulations, such as frequency division multiple access and code division multiple access. In contrast, slow-time waveform design, particularly DDMA waveform design, has received comparatively little attention.

DDMA waveform design stands out because of its simple implementation and low hardware cost. It achieves Doppler-domain orthogonality by applying phase modulation in the slow-time (pulse) domain. However, the resulting transmit energy distribution of DDMA waveforms is omnidirectional within each pulse period [42, 43]. Moreover, DDMA MIMO radar inherently suffers from velocity ambiguity because the Doppler spectrum division is required for transmit-channel separation prior to parameter estimation [44].

Existing direction estimation algorithms range from nonparametric Fourier-based methods [45] and conventional beamforming techniques, such as the Capon beamformer, to higher-resolution parametric methods, including multiple signal classification (MUSIC) [46, 47] and the estima-

tion of signal parameters via rotational invariance technique (ESPRIT) [48]. However, these algorithms typically operate on a matrix-form representation constructed from the multidimensional received measurements. Consequently, the multi-linear structure inherent in the signal measurements is not fully exploited. It has been verified that tensor-based methods can provide better performance than matrix-based methods, particularly in low signal-to-noise ratio (SNR) scenarios [49, 50, 51, 52]. Motivated by this, tensor-based methods have been increasingly adopted for direction estimation in MIMO radar systems.

Accordingly, this chapter develops a slow-time TB design technique that adopts the modulation concept of DDMA to optimize the transmit beam-pattern, and a tensor-based algorithm is developed for sensing parameter estimation.

2.1 Preliminaries on Tensor

Considering that the sensing parameter estimation algorithms presented in Chapters 2 and 3 are based on tensor models, this section reviews the preliminary concepts of tensor and tensor decomposition before proceeding to the main development of this thesis.

Ranks of matrix: Given a matrix $\mathbf{A} \in \mathbb{C}^{I \times J}$, the rank of $\mathbf{A}$, denoted by $r(\mathbf{A})$, is defined as the maximum number of linearly independent columns (or rows). The Kruskal rank of $\mathbf{A}$, denoted by $k(\mathbf{A})$, is defined as the largest integer l such that every set of l columns (or rows) of $\mathbf{A}$ is linearly independent, i.e., $k(\mathbf{A}) = l$.

Tensor decomposition: An N th-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times \dots \times I_N}$ is said to be rank-1 if it can be expressed as the outer product of N non-zero vectors $\mathbf{a}^{(n)} \in \mathbb{C}^{I_n}$, $n = 1, \dots, N$, i.e., $\mathcal{X} = \mathbf{a}^{(1)} \circ \dots \circ \mathbf{a}^{(N)}$, where $\circ$ denotes the outer product of vectors. A decomposition of a tensor into a sum of rank-1 terms is referred to as polyadic decomposition (PD):

$$\mathcal{X} = \sum_{r=1}^R \mathbf{a}_r^{(1)} \circ \dots \circ \mathbf{a}_r^{(N)}. \quad (2.1)$$

The rank of a tensor $\mathcal{X}$ is defined as the smallest integer R for which the decomposition in (2.1) holds. If the rank of $\mathcal{X}$ is R , then (2.1) is referred to as the canonical PD (CPD) of $\mathcal{X}$, also known as PARAFAC decomposition or analysis.

Stacking the vectors $\{\mathbf{a}_r^{(n)}\}_{r=1}^R$ into a matrix:

$$\mathbf{A}^{(n)} = [\mathbf{a}_1^{(n)}, \dots, \mathbf{a}_R^{(n)}] \in \mathbb{C}^{I_n \times R}, \quad n = 1, \dots, N, \quad (2.2)$$

$\{\mathbf{A}^{(n)}\}_{n=1}^N$ are referred to as the factor matrices of $\mathcal{X}$. If $\mathbf{a}_r^{(n)}$ satisfies

$$\mathbf{a}_r^{(n)} = \left[1, z_r^{(n)}, (z_r^{(n)})^2, \dots, (z_r^{(n)})^{I_n-1} \right]^T, \quad r = 1, \dots, R, \quad (2.3)$$

$\mathbf{A}^{(n)}$ is said to be a Vandermonde matrix, and $\{z_r^{(n)}\}_{r=1}^R$ are called the generators of $\mathbf{A}^{(n)}$. Here, $(\cdot)^T$ denotes the transpose operation.

Unfoldings of tensor: The mode- n unfolding of $\mathcal{X}$, denoted by $[\mathcal{X}]_{(n)}$, arranges the mode- n fibers to be the columns of the resulting matrix. A 3-D tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ is considered in this thesis, and its three modes of unfolding in terms of factor matrices can then be expressed by

$$\begin{aligned} [\mathcal{X}]_{(1)} &= \mathbf{A}^{(1)} (\mathbf{A}^{(3)} * \mathbf{A}^{(2)})^T, \\ [\mathcal{X}]_{(2)} &= \mathbf{A}^{(2)} (\mathbf{A}^{(3)} * \mathbf{A}^{(1)})^T, \\ [\mathcal{X}]_{(3)} &= \mathbf{A}^{(3)} (\mathbf{A}^{(2)} * \mathbf{A}^{(1)})^T, \end{aligned} \quad (2.4)$$

where $*$ denotes the Khatri-Rao product operator. Different authors use different orderings of the columns for the mode- n unfolding. In general, the specific permutation of columns is inconsequential, provided that the same convention is used consistently throughout the related calculations.

Kruskal's condition: For a 3-D tensor $\mathcal{X}$, the PARAFAC decomposition in (2.1) is unique, up to permutation and scaling of the columns, if [53]

$$k(\mathbf{A}^{(1)}) + k(\mathbf{A}^{(2)}) + k(\mathbf{A}^{(3)}) \geq 2R + 2, \quad (2.5)$$

where R is the rank of $\mathcal{X}$. Kruskal's condition is a sufficient but not necessary condition for ensuring the uniqueness of PARAFAC. More relaxed conditions can also guarantee uniqueness, particularly when the factor matrices exhibit specific structures, such as a Vandermonde structure.

Alternating least squares (ALS): ALS is an iterative optimization technique to compute PARAFAC decomposition. The key idea of ALS is to update one factor matrix at a time while keeping the other factor matrices fixed, allowing each update to be computed by solving a least square subproblem with a closed-form solution. For a 3-D tensor $\mathcal{X}$, when $\mathbf{A}^{(2)}$ and $\mathbf{A}^{(3)}$ are fixed, the update for $\mathbf{A}^{(1)}$ can be obtained by solving the following optimization problem:

$$\hat{\mathbf{A}}^{(1)} \leftarrow \arg \min_{\mathbf{A}^{(1)}} \left\| [\mathcal{X}]_{(1)} - \mathbf{A}^{(1)} (\hat{\mathbf{A}}^{(3)} * \hat{\mathbf{A}}^{(2)})^T \right\|_F^2, \quad (2.6)$$

where $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. This process alternates among factor matrices until convergence.

Although the ALS method in (2.6) is widely used for PARAFAC decomposition, its iterative optimization can become computationally demanding for large-scale tensors or high assumed ranks. A range of approaches has therefore been developed to reduce this computational burden. Randomized and sampling-based methods reduce the cost of the ALS updates by approximating the associated least-squares problems, while sketching techniques can further accelerate tensor contractions without explicitly forming large intermediate tensors [54, 55]. Other approaches improve

the efficiency of ALS by exploiting sparsity or optimizing the computation of Khatri–Rao products [56]. Non-iterative approaches provide an efficient alternative by reformulating PARAFAC estimation as matrix diagonalization or subspace-based problems, thereby reducing or avoiding the iterative ALS procedure [51, 57, 58].

Another important direction is to exploit known structures in the factor matrices. Specialized algorithms can incorporate such structures into the decomposition and thereby reduce the computational burden. Such approaches are particularly attractive in array signal processing, where factor matrices often exhibit structures such as Vandermonde or circulant forms [59, 60, 61, 62].

2.2 Signal Model

2.2.1 DDMA MIMO Radar with Transmit ULA

Consider a mono-static pulsed MIMO radar system equipped with a transmit array comprising M_t antenna elements arranged in a uniform linear array (ULA) configuration with an inter-element spacing of d_t . A total of Q pulses are transmitted within a single coherent processing interval (CPI) with pulse repetition interval $T_r = 1/\text{PRF}$, where PRF denotes the pulse repetition frequency (PRF). Assuming that K targets are present within the illuminated area, characterized by direction θ_k , range R_k , and velocity v_k for $k = 1, \dots, K$, the transmit steering vector for the k th target can be written as

$$\mathbf{a}(\theta_k) = \left[1, e^{-j2\pi \frac{d_t}{\lambda} \sin \theta_k}, \dots, e^{-j2\pi \frac{d_t}{\lambda} (M_t-1) \sin \theta_k} \right]^T, \quad (2.7)$$

where λ denotes the wavelength. Since this work focuses on transmit energy distribution optimization, the receive steering vector is generally denoted by $\mathbf{b}(\varphi_k)$ of size $M_r \times 1$.

The radar cross section (RCS) coefficients are assumed to obey the Swerling I model, implying that the returned signal amplitude remains constant across pulses within a CPI. Denoting the phase modulation vector for the q th pulse as $\mathbf{w}_q$, after carrier frequency downconversion and matched filtering, the signal received by the n th antenna element for the q th pulse can then be described by

$$y_{nq} = \sum_{k=1}^K \sigma_k e^{j2\pi f_k (q-1)T_r} \left(\mathbf{a}^T(\theta_k) \mathbf{w}_q \right) b_n(\varphi_k) + z_{nq}, \quad n = 1, \dots, M_r, \quad q = 1, \dots, Q, \quad (2.8)$$

where σ_k denotes the RCS fading coefficient, $f_k = 2v_k/\lambda$ represents the Doppler frequency shift, $b_n(\varphi_k)$ is the n th element of $\mathbf{b}(\varphi_k)$, and z_{nq} is the noise.

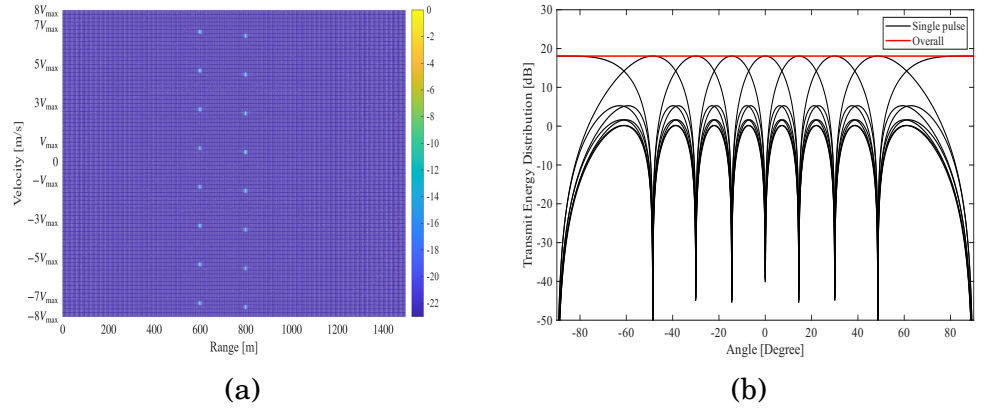


Figure 2.1. DDMA characteristics with $M_t = 8$: (a) Doppler spectrum with $K = 2$ and (b) transmit energy beampattern.

The DDMA technique, as a representative waveform that achieves orthogonality in the slow-time (Doppler) domain, enables the separability of transmit channel signals by modulating each channel signal with a distinct Doppler shift. Specifically, as depicted in Fig. 2.1(a) with $M_t = 8$ and $K = 2$, for each target, the signal energy is accumulated at eight uniformly spaced peaks across the entire Doppler spectrum. This energy distribution is achieved by assigning the m th channel signal the following Doppler shift:

$$f_m = (m - 1) \frac{\text{PRF}}{M_t}, m = 1, \dots, M_t. \quad (2.9)$$

For the q th pulse, the corresponding phase modulation vector is then expressed as

$$\mathbf{w}_q = \left[1, e^{j2\pi \frac{\text{PRF}}{M_t} (q-1) T_r}, \dots, e^{j2\pi (M_t-1) \frac{\text{PRF}}{M_t} (q-1) T_r} \right]^T. \quad (2.10)$$

With DDMA modulation, the whole Doppler spectrum is divided into M_t subbands, and only a subband of width PRF/M_t is allocated to transmit channel individuals. Consequently, the maximum unambiguous velocity for DDMA technique is reduced to

$$v_{\max} = \frac{\lambda \text{PRF}}{4M_t}. \quad (2.11)$$

It can be observed from (2.11) that $v_{\max}$ is only $1/M_t$ of the maximum limit imposed by the pulse sampling rate PRF according to Shannon's sampling theorem, thereby exacerbating the velocity ambiguity.

2.2.2 MIMO Radar with L-Shaped Array

An L-shaped array is a sparse radar array composed of two orthogonal uniform linear subarrays arranged in an “L” configuration, enabling 2-D angle estimation (azimuth angle β_k and elevation angle ϕ_k for the k th

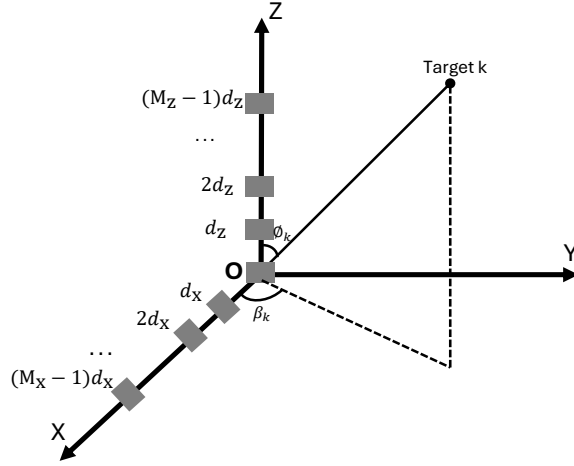


Figure 2.2. L-shaped array configuration.

target) with reduced hardware complexity. To locate targets in the 3-D space by extending a 1-dimensional (1-D) ULA to a 2-D geometry, we consider an L-shaped array composed of two ULAs positioned along x-axis and z-axis, as illustrated in Fig. 2.2. To differentiate them from a 1-D array, the ULAs forming the L-shaped array are referred to as subarrays in this chapter. The subarrays along x-axis and z-axis contain M_x and M_z antenna elements with inter-element spacings d_x and d_z , respectively. The Cartesian coordinates of the m_x th and m_z th antenna elements in the x-axis and z-axis subarrays are respectively given by

$$\begin{aligned}\mathbf{r}_{x,m_x} &= ((m_x - 1)d_x, 0, 0)^T, \quad m_x = 1, 2, \dots, M_x, \\ \mathbf{r}_{z,m_z} &= (0, 0, (m_z - 1)d_z)^T, \quad m_z = 1, 2, \dots, M_z.\end{aligned}\tag{2.12}$$

Array coordinate matrices $\mathbf{R}_x \in \mathbb{R}^{M_x \times 3}$ and $\mathbf{R}_z \in \mathbb{R}^{M_z \times 3}$ are then defined as

$$\begin{aligned}\mathbf{R}_x &= [\mathbf{r}_{x,1}, \mathbf{r}_{x,2}, \dots, \mathbf{r}_{x,M_x}]^T, \\ \mathbf{R}_z &= [\mathbf{r}_{z,1}, \mathbf{r}_{z,2}, \dots, \mathbf{r}_{z,M_z}]^T.\end{aligned}\tag{2.13}$$

Different from the 1-D ULA described in Subsection 2.2.1, the problem of interest becomes the estimation of the angle pairs $\{\beta_k, \phi_k\}_{k=1}^K$.

Define the propagation vector as

$$\mathbf{p}_k = [\sin \phi_k \cos \beta_k, \sin \phi_k \sin \beta_k, \cos \phi_k]^T.\tag{2.14}$$

With $\cos \psi_k = \sin \phi_k \cos \beta_k$, the transmit steering vectors of subarrays are

given by

$$\begin{aligned}
 \mathbf{a}_x(\psi_k) &= e^{-j2\pi \mathbf{R}_x \mathbf{p}_k} \\
 &= \left[1, e^{-j2\pi \frac{d_x}{\lambda} \cos \psi_k}, \dots, e^{-j2\pi (M_x-1) \frac{d_x}{\lambda} \cos \psi_k} \right]^T, \\
 \mathbf{a}_z(\phi_k) &= e^{-j2\pi \mathbf{R}_z \mathbf{p}_k} \\
 &= \left[1, e^{-j2\pi \frac{d_z}{\lambda} \cos \phi_k}, \dots, e^{-j2\pi (M_z-1) \frac{d_z}{\lambda} \cos \phi_k} \right]^T.
 \end{aligned} \tag{2.15}$$

Unless otherwise specified, the azimuth angle is denoted by ψ_k for convenience hereafter. Given the estimated ϕ_k and ψ_k , β_k can be readily determined from the geometric relationship between these angles.

Assuming that the waveforms transmitted by two subarrays are orthogonal, the L-shaped array can be treated as a combination of two separate ULAs. Therefore, from (2.15), ψ_k and ϕ_k can be estimated separately by applying angle estimation methods to the two subarray signals. Accordingly, the TB design can also be performed separately for the two subarrays. Specifically, two modulation matrices, denoted by $\mathbf{W}_{x,TB}$ and $\mathbf{W}_{z,TB}$, can be designed separately according to the respective practical requirements, which will be elaborated in Section 2.3. Assuming ideal signal separation, the signals from two transmit subarrays and received at the n th receive element can be described by

$$\begin{aligned}
 y_{x,nq} &= \sum_{k=1}^K \sigma_k e^{j2\pi f_k(q-1)T_r} \left(\mathbf{a}_x^T(\psi_k) \mathbf{w}_{x,TB,q} \right) b_n(\psi_k, \phi_k) + z_{x,nq}, \\
 y_{z,nq} &= \sum_{k=1}^K \sigma_k e^{j2\pi f_k(q-1)T_r} \left(\mathbf{a}_z^T(\phi_k) \mathbf{w}_{z,TB,q} \right) b_n(\psi_k, \phi_k) + z_{z,nq},
 \end{aligned} \tag{2.16}$$

where $\mathbf{w}_{x,TB,q}$ and $\mathbf{w}_{z,TB,q}$ are respectively the q th column vectors of $\mathbf{W}_{x,TB}$ and $\mathbf{W}_{z,TB}$, $b_n(\psi_k, \phi_k)$ denotes the n th element of the receive steering vector $\mathbf{b}(\psi_k, \phi_k)$, and $z_{x,nq}$ and $z_{z,nq}$ are the noise components.

2.3 Proposed Slow-Time Beamspace Design

Recalling (2.8), the transmit energy beampattern of the DDMA MIMO radar for the q th pulse is given by

$$G(\theta) = \left| \mathbf{a}^T(\theta) \mathbf{w}_q \right|^2, \tag{2.17}$$

where θ denotes the spatial angle and $|\cdot|$ represents the element-wise modulus operation applied to a scalar or a vector.

Rewrite $\mathbf{w}_q$ defined in (2.10) as

$$\mathbf{w}_q = \left[1, e^{j2\pi \frac{q-1}{M_t}}, \dots, e^{j2\pi \frac{(M_t-1)(q-1)}{M_t}} \right]^T. \tag{2.18}$$

Observing the ULA steering vector $\mathbf{a}(\theta_k)$ in (2.7), $\mathbf{w}_q$ can be interpreted as a beamforming vector whose mainlobe is directed toward $\theta_{b,q}$. Assuming $d_t = \lambda/2$, which is commonly adopted in practical radar systems, the beamforming mainlobe direction $\theta_{b,q}$ for the q th pulse satisfies the following condition¹:

$$\sin \theta_{b,q} = 2l + \frac{2(q-1)}{M_t}, \quad (2.19)$$

where l is an integer introduced to ensure that $\sin \theta_{b,q} \in [-1, 1]$. It can be observed that $\theta_{b,q}$ varies from pulse to pulse with a period of M_t pulses, and the mainlobes of these beams within a pulse period sweep across the entire spatial region. Fig. 2.1(b) illustrates both the distributions of the individual beams and the overall transmit beampattern within a pulse cycle. As shown in Fig. 2.1(b), the overall transmit energy is omnidirectionally distributed across the entire spatial region, thereby preventing DDMA from exploiting beamforming gain. If the transmit energy can be focused within a specific region of interest, denoted by Θ , parameter estimation accuracy for targets located within Θ can be enhanced due to the resulting increase in SNR gain. Therefore, in this section, a slow-time TB MIMO radar approach built upon the DDMA technique is developed to optimize the transmit energy distribution.

2.3.1 Slow-Time Transmit Beamspace Design

From (2.19), it can be observed that the mainlobe spacings between adjacent pulse beams depend on the number of subbands into which the entire Doppler spectrum is divided, which corresponds to the value of M_t in the DDMA technique. To differentiate the number of subbands from the number of transmit elements, we denote the number of divided subbands by M_v , also referred to as the number of *virtual* transmit elements. Accordingly, the period of modulation vectors becomes M_v instead of M_t , and the Doppler shifts can be expressed as

$$f_m = (m-1) \frac{\text{PRF}}{M_v}, \quad m = 1, \dots, M_v. \quad (2.20)$$

With $M_v > M_t$, more Doppler shifts are available for modulation than the number of transmit elements. The orthogonality across M_t transmit channels can, in principle, be achieved by any set of M_t distinct Doppler shifts. Moreover, any set of consecutive M_t Doppler shifts yields identical beams except for a different extra phase, thus making no difference to the transmit beampattern. As an illustrative example, the first M_t Doppler shifts are selected for phase modulation, and the corresponding phase

¹This condition is derived for a 1-D ULA and can be readily extended to a 2-D L-shaped array with steering vectors with respect to $\cos \theta$, as in (2.15).

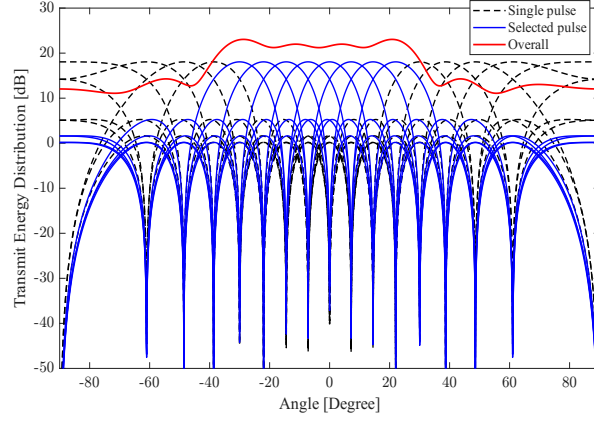


Figure 2.3. Transmit energy beampattern of the proposed approach.

modulation vector for a M_t -ULA transmitter can be expressed as

$$\mathbf{w}_{\text{TB},q} = \left[1, e^{j2\pi \frac{q-1}{M_v}}, \dots, e^{j2\pi \frac{(M_t-1)(q-1)}{M_v}} \right]^T. \quad (2.21)$$

Recalling (2.19), it can be readily observed that a larger M_v results in narrower spacings between adjacent pulse beams. Consequently, more pulse beams can be steered toward a given spatial sector. Assuming that Θ is known *a priori*², there are $M < M_v$ pulses within every M_v pulses whose beam mainlobes fall within Θ . Let $\mathbf{q}_v = [q_1, q_2, \dots, q_M]$ denote the vector containing the indices of these M pulses in the first pulse period. To concentrate transmit energy within Θ , only the modulation vectors corresponding to these M beams are selected to form a new phase modulation matrix $\mathbf{W}_{\text{TB}} \in \mathbb{C}^{M_t \times Q}$ for the proposed slow-time TB design, which is given by

$$\begin{aligned} \mathbf{W}_{\text{TB},0} &\triangleq [\mathbf{w}_{\text{TB},q_1}, \mathbf{w}_{\text{TB},q_2}, \dots, \mathbf{w}_{\text{TB},q_M}] \in \mathbb{C}^{M_t \times M}, \\ \mathbf{W}_{\text{TB}} &\triangleq \left[\underbrace{\mathbf{W}_{\text{TB},0}, \mathbf{W}_{\text{TB},0}, \dots, \mathbf{W}_{\text{TB},0}}_{Q/M} \right], \end{aligned} \quad (2.22)$$

where Q/M is assumed to be an integer, and the q th column vector of $\mathbf{W}_{\text{TB}}$ is applied to the q th pulse. By constructing a new phase modulation matrix that includes only the modulation vectors whose mainlobes fall within Θ , the transmit energy can be effectively concentrated.

Fig. 2.3 illustrates the energy distribution of individual beams as well as the overall beampattern for the proposed TB design, where $M_t = 8$, $M_v = 16$, $M = 8$, and $\Theta = [-30^\circ, 30^\circ]$. Compared to the DDMA approach shown in Fig. 2.1(b), the adjacent beams in the proposed method are positioned closer to one another, resulting in a greater concentration of beams within Θ . By selecting the pulse vectors whose beamforming mainlobes are di-

²For an unknown target scene, a practical approach is to first obtain a coarse estimate of the target region and then determine Θ accordingly [41].

rected towards Θ , the proposed design effectively concentrates the transmit energy within Θ , thereby achieving transmit energy optimization.

The value of M_v is mainly determined by the width of Θ and the number of selected pulses per period M . For a fixed M , a wider Θ generally requires a smaller M_v to ensure sufficient transmit beam coverage, whereas M plays a role in determining the maximum number of detectable targets. Therefore, an appropriate M_v should be selected by jointly considering the desired TB energy distribution and the required number of detectable targets.

2.3.2 Doppler Spectrum of Transmit Beamspace Design

DDMA technique exploits the independence among different channel signals in the Doppler domain to separate channel signals. Specifically, after DDMA modulation, the signal energy from each channel corresponding to a given target is concentrated at a single peak and at a unique Doppler location, distinct from those of other channels, which ensures channel separability. However, this property does not necessarily hold for the proposed slow-time TB design.

As evident from (2.10) and (2.22), over a CPI of Q pulses, the m th transmit channel is modulated by the Q elements of the m th row of the modulation matrix. Denote the m th row of the DDMA modulation matrix $\mathbf{W}$ as

$$\mathbf{v}_m = \left[1, e^{j2\pi \frac{(m-1)\text{PRF}}{M_v} T_r}, \dots, e^{j2\pi \frac{(m-1)\text{PRF}}{M_v} (Q-1)T_r} \right], \quad (2.23)$$

whereas the m th row of the proposed slow-time TB matrix $\mathbf{W}_{\text{TB}}$ as

$$\mathbf{v}_{\text{TB},m} \triangleq \left[\underbrace{\mathbf{v}_0, \mathbf{v}_0, \dots, \mathbf{v}_0}_{Q/M} \right] \in \mathbb{C}^{1 \times Q}, \quad (2.24)$$

$$\mathbf{v}_0 = e^{j2\pi \frac{(m-1)\text{PRF}}{M_v} T_r [q_1-1, q_2-1, \dots, q_M-1]} = e^{j2\pi \frac{(m-1)\text{PRF}}{M_v} T_r (\mathbf{q}_v - \mathbf{1})},$$

where $\mathbf{1}$ denotes an all-ones vector with the same size as $\mathbf{q}_v$. Note that, for ease of the subsequent comparison between DDMA and the proposed TB technique, M_v is used in place of M_t in (2.23). According to (2.23), DDMA technique operates by multiplying each transmit channel signal with a distinct complex sinusoid in the slow-time domain (across pulses). The frequency of the sinusoid for the m th channel is $f_{v,m} = (m-1)\text{PRF}/M_v$, and its sampling rate is $1/T_r = \text{PRF}$. Element-wise multiplication of a signal by a complex sinusoid with frequency $f_{v,m}$ is equivalent to shifting the signal in the Doppler domain by $f_{v,m}$. Consequently, if the original signal energy is concentrated at a single peak, the modulated signal will also appear at a single peak, with its location determined by the Doppler shift. Thus, using DDMA modulation, the M_t transmit channel signals corresponding to the same target are distributed across M_t separate peaks.

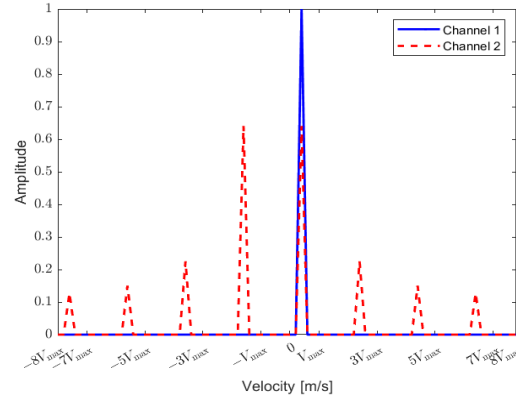


Figure 2.4. Doppler spectrums of two transmit channels with $M_t = 8$ and $M = 8$.

In contrast, the slow-time TB technique described in (2.24) selects only M out of M_v DDMA modulation pulse vectors whose beam mainlobes are directed toward the desired spatial region, discarding the others. As a result, the modulation signal composed of Q sample points is no longer a single complex sinusoid, although remaining M -pulse periodic.

According to the Fourier analysis of a periodic signal $f(t)$, with the Fourier decomposition

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{j2\pi n f_0 t}, \quad (2.25)$$

where t is the time index, a_n is the Fourier coefficient, n is an integer, and $1/f_0$ is the signal period of $f(t)$, the modulation signal for the proposed slow-time TB design can be regarded as a superposition of multiple complex sinusoids with different frequencies. Consequently, under the slow-time TB design, each transmit channel can be considered as being modulated with multiple Doppler shifts simultaneously, with shift frequencies $\{n/(MT_r)\}_{n=-\infty}^{\infty}$. As a result, the signal energy of a transmit channel for a given target is spread over multiple Doppler peaks, and the peak positions coincide with those of other channels. Fig. 2.4 provides the signal distribution of the first two channels as an illustrative example. It can be observed that the signal of Channel 2 is distributed across M peaks with a uniform spacing $2v_{\max} = \lambda/(2MT_r)$ in the velocity scale, but with varying amplitudes, which is consistent with the discussion above. To conclude, signals from different transmit channels overlap in the Doppler domain, making channel separation via Doppler spectrum division impossible.

Nearly all direction estimation algorithms for DDMA-based MIMO radar, including MUSIC, ESPRIT, and conventional CPD methods, are based on the assumption that transmit channel signals can be separated in the Doppler domain. However, as discussed earlier, the transmit signals generated by the proposed slow-time TB modulation cannot be separated in this manner, rendering the aforementioned algorithms inapplicable. To overcome this limitation, we propose a PARAFAC-based parameter

estimation method for slow-time TB MIMO radar that does not require Doppler spectrum separation and enables improved parameter estimation, as detailed in the following section.

2.4 Parameter Estimation Using PARAFAC

As discussed in Subsection 2.2.2, parameter estimation algorithms for the L-shaped radar can be independently applied to each subarray. Therefore, for simplicity of exposition, a ULA signal model is adopted to illustrate the proposed PARAFAC-based parameter estimation algorithm, which can be straightforwardly extended to the L-shaped array counterpart.

To facilitate comparison with the proposed TB approach, the PARAFAC formulation for the conventional DDMA technique is first presented. The application of tensor-based methods to DDMA MIMO radar systems first requires channel signal separation. This is achieved through phase demodulation, where the received signals from M_t transmit elements are shifted back to the Doppler baseband using Doppler shifts of $-(m-1)\text{PRF}/M_t$, $m = 1, \dots, M_t$. Low-pass filtering with a cutoff frequency $\text{PRF}/M_t/2$ is then applied, and $M_t M_r$ virtual transmit-receive channel signals are obtained. After the separation, the 3-D tensor representation for DDMA can be expressed as follows:

$$\mathcal{Y}_{\text{DDMA}} = \sum_{k=1}^K \sigma_k \mathbf{a}(\theta_k) \circ \mathbf{b}(\varphi_k) \circ \mathbf{d}_{\text{DDMA}}(f_k). \quad (2.26)$$

Here,

$$\mathbf{d}_{\text{DDMA}}(f_k) = [d_1(f_k), d_{1+M_t}(f_k), \dots, d_{1+M_t(P_D-1)}(f_k)]^T, \quad (2.27)$$

where $d_q(f_k) = e^{j2\pi f_k(q-1)T_r}$ and $P_D = Q/M_t$ is an integer. For simplicity, the noise term is omitted in the derivations throughout this section.

Next, a tensor model is constructed for the proposed slow-time TB design.³ Storing the received signal in $\mathbf{Y}$ of size $M_r \times Q$, we have

$$\mathbf{Y} = (\mathbf{p} * \mathbf{B}) [(\mathbf{W}_{\text{TB}}^T \mathbf{A}) \odot \mathbf{D}]^T. \quad (2.28)$$

Here, $\mathbf{p} = [\sigma_1, \sigma_2, \dots, \sigma_K]$, $\odot$ denotes the Hadamard product operation, and

$$\begin{aligned} \mathbf{B} &= [\mathbf{b}(\varphi_1), \mathbf{b}(\varphi_2), \dots, \mathbf{b}(\varphi_K)], \\ \mathbf{D} &= [\mathbf{d}(f_1), \mathbf{d}(f_2), \dots, \mathbf{d}(f_K)], \end{aligned} \quad (2.29)$$

$$\mathbf{d}(f_k) = [d_1(f_k), d_2(f_k), \dots, d_Q(f_k)]^T.$$

Taking advantage of the periodicity of $\mathbf{W}_{\text{TB}}$, we have

$$\mathbf{W}_{\text{TB}}^T \mathbf{A} = \mathbf{1}_{M \times K} * \mathbf{W}_{\text{TB},0}^T \mathbf{A}, \quad (2.30)$$

³The tensor-based parameter estimation method developed in this section is also applicable when $M_v = M_t$.

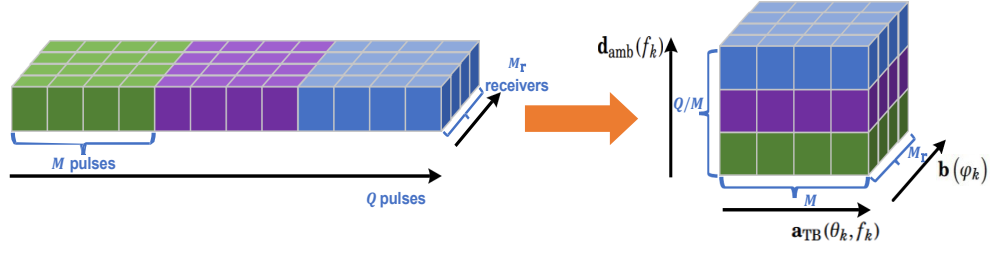


Figure 2.5. Illustration of transforming a matrix signal to a 3-D tensor.

where $\mathbf{1}_{M \times K}$ denotes the $M \times K$ all-one matrix. Define

$$\mathbf{D}_{\text{amb}} = [\mathbf{d}_{\text{amb}}(f_1), \mathbf{d}_{\text{amb}}(f_2), \dots, \mathbf{d}_{\text{amb}}(f_K)], \quad (2.31)$$

where $\mathbf{d}_{\text{amb}}(f_k)$ is defined in a manner analogous to $\mathbf{d}_{\text{DDMA}}(f_k)$ in (2.27), with M_t replaced by M . According to the pulse sampling relationship, $\mathbf{D}$ is expressed as

$$\mathbf{D} = \mathbf{D}_{\text{amb}} * \mathbf{D}_0, \quad (2.32)$$

where $\mathbf{D}_0$ consists of the first M rows of $\mathbf{D}$.

Based on (2.30) and (2.32), we have the following:

$$\mathbf{W}_{\text{TB}}^T \mathbf{A} \odot \mathbf{D} = (\mathbf{1}_{M \times K} * (\mathbf{W}_{\text{TB},0}^T \mathbf{A})) \odot (\mathbf{D}_{\text{amb}} * \mathbf{D}_0) = \mathbf{D}_{\text{amb}} * (\mathbf{W}_{\text{TB},0}^T \mathbf{A} \odot \mathbf{D}_0). \quad (2.33)$$

Then, (2.28) can be rewritten as

$$\mathbf{Y} = (\mathbf{p} * \mathbf{B}) (\mathbf{D}_{\text{amb}} * \mathbf{A}_{\text{TB}})^T, \quad (2.34)$$

where $\mathbf{A}_{\text{TB}}$ is defined as

$$\mathbf{A}_{\text{TB}} = (\mathbf{W}_{\text{TB},0}^T \mathbf{A}) \odot \mathbf{D}_0. \quad (2.35)$$

Transforming $\mathbf{Y}$ into a 3-D tensor of size $M \times M_r \times P_v$, as illustrated in Fig. 2.5, we obtain

$$\mathcal{Y} = \sum_{k=1}^K \sigma_k \mathbf{a}_{\text{TB}}(\theta_k, f_k) \circ \mathbf{b}(\varphi_k) \circ \mathbf{d}_{\text{amb}}(f_k), \quad (2.36)$$

and the associated factor matrices are $\mathbf{B}$, $\mathbf{D}_{\text{amb}}$, and

$$\mathbf{A}_{\text{TB}} = [\mathbf{a}_{\text{TB}}(\theta_1, f_1), \mathbf{a}_{\text{TB}}(\theta_2, f_2), \dots, \mathbf{a}_{\text{TB}}(\theta_K, f_K)]. \quad (2.37)$$

By comparing (2.26) and (2.36), it can be observed that the part of velocity information, which was lost in (2.26) due to Doppler spectrum division, is preserved in (2.36) in the form of $\mathbf{D}_0$ contained in $\mathbf{A}_{\text{TB}}$. Thus, the unambiguous velocity can be recovered by combining $\mathbf{D}_{\text{amb}}$ with $\mathbf{D}_0$, effectively extending the range of unambiguous detectable velocity by a factor of M_t .

With the obtained factor matrices $\mathbf{A}_{\text{TB}}$ and $\mathbf{D}_{\text{amb}}$, for example, via the ALS method, the squared-error minimization algorithm can be applied to estimate $\{\theta_k\}_{k=1}^K$. From (2.35), we can observe that $\mathbf{A}_{\text{TB}}$ is the Hadamard product of $\mathbf{D}_0$ and $\tilde{\mathbf{A}}_{\text{TB}} = \mathbf{W}_{\text{TB},0}^T \mathbf{A}$, and the k th column of $\tilde{\mathbf{A}}_{\text{TB}}$ is

$$\tilde{\mathbf{a}}_{\text{TB},k} = \mathbf{W}_{\text{TB},0}^T \mathbf{a}(\theta_k) = [\mathbf{w}_{\text{TB},q_1}^T \mathbf{a}(\theta_k), \mathbf{w}_{\text{TB},q_2}^T \mathbf{a}(\theta_k), \dots, \mathbf{w}_{\text{TB},q_M}^T \mathbf{a}(\theta_k)]^T. \quad (2.38)$$

Considering that $\mathbf{w}_{\text{TB},q_m}$ obeys the discrete Fourier transform (DFT) structure for different frequencies $f_{q_m} = (q_m - 1)/M_v$, $q_m = q_1, \dots, q_M$, $\tilde{\mathbf{a}}_{\text{TB},k}$ can be regarded as DFT operations at M distinct frequency points. Meanwhile, every element of $\mathbf{D}_0$ is an exponential value with unit amplitude, having no effect on the element amplitudes of $\mathbf{A}_{\text{TB}}$. Therefore, $\{\theta_k\}_{k=1}^K$ can be obtained by solving the following squared-error minimization problem

$$\hat{\theta}_k = \arg \min_{\theta} \|\tilde{\mathbf{a}}_{\text{TB},k} - |\tilde{\mathbf{a}}_{\text{TB}}(\theta)|\|^2, \quad (2.39)$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector and $\tilde{\mathbf{a}}_{\text{TB}}(\theta) = \mathbf{W}_{\text{TB},0}^T \mathbf{a}(\theta)$.

To obtain unambiguous velocity estimates, the estimated factor $\hat{\mathbf{D}}_{\text{amb}}$ is combined with $\mathbf{D}_0$, where $\mathbf{D}_0$ is estimated as follows:

$$\hat{\mathbf{D}}_0 = \mathbf{A}_{\text{TB}} \oslash \hat{\mathbf{A}}_{\text{TB}}, \quad (2.40)$$

where $\hat{\mathbf{A}}_{\text{TB}}$ is the estimate of $\tilde{\mathbf{A}}_{\text{TB}}$ by substituting $\{\hat{\theta}_k\}_{k=1}^K$ in (2.38) and $\oslash$ denotes the element-wise division of matrices. With $\hat{\mathbf{D}}_{\text{amb}}$ and $\hat{\mathbf{D}}_0$, we can have

$$\hat{\mathbf{D}}_{\text{amb}} * \hat{\mathbf{D}}_0 \triangleq [\mathbf{d}(f_1), \mathbf{d}(f_2), \dots, \mathbf{d}(f_K)]. \quad (2.41)$$

From (2.41), the target velocities $\{v_k\}_{k=1}^K$ can be readily obtained.

Regarding the DOA estimation, since this chapter primarily focuses on transmit energy focusing and velocity disambiguation, the receive-array configuration is not considered. The choice of DOA estimation method depends on the structural properties of the receive array and is, therefore, not further discussed here.

As discussed above, 2-D direction estimation for the proposed L-shaped array slow-time TB radar is achieved by separately estimating azimuth and elevation using the factor matrices $\mathbf{A}_{x,\text{TB}}$, $\mathbf{A}_{z,\text{TB}}$. Here, $\mathbf{A}_{x,\text{TB}}$ and $\mathbf{A}_{z,\text{TB}}$ denote the corresponding factor matrices associated with the two subarray signals, defined analogously to $\mathbf{A}_{\text{TB}}$ in (2.35) with $\mathbf{W}_{x,\text{TB},0}$ and $\mathbf{W}_{z,\text{TB},0}$, respectively. Consequently, 2-D direction pair matching is required when multiple targets are present. Recalling (2.36), factor matrices $\mathbf{A}_{x,\text{TB}}$, $\mathbf{A}_{z,\text{TB}}$, and $\mathbf{B}$ are available after tensor decomposition. However, only the matrices associated with transmit arrays, i.e., $\mathbf{A}_{x,\text{TB}}$ and $\mathbf{A}_{z,\text{TB}}$, are used for direction estimation, and the receive-array factor matrix $\mathbf{B}$ has not yet been utilized. According to the PARAFAC uniqueness condition in terms of Kruskal rank in (2.5), these factor matrices are unique up to column permutation and

scaling if the following inequalities are satisfied:

$$\begin{aligned} k(\mathbf{A}_{x,\text{TB}}) + k(\mathbf{D}_{\text{amb}}) + k(\mathbf{B}) &\geq 2K + 2, \\ k(\mathbf{A}_{z,\text{TB}}) + k(\mathbf{D}_{\text{amb}}) + k(\mathbf{B}) &\geq 2K + 2. \end{aligned} \tag{2.42}$$

Therefore, the parameters estimated from these factor matrices are automatically paired on a column-by-column basis if (2.42) holds. Owing to this property, the factor matrix $\mathbf{B}$ can be used as an anchor for pair matching.

2.5 Conclusion and Discussion

In this chapter, a slow-time TB waveform design for L-shaped array radar was developed to enhance parameter estimation performance and resolve velocity ambiguity. A novel phase modulation matrix was designed for the slow-time TB MIMO radar to optimize transmit energy distribution within the spatial region of interest. The proposed slow-time TB modulation scheme relies only on simple phase modulation and beam selection, which facilitates easy implementation in practical systems. Based on this design, a tensor-based signal model was developed, together with a parameter estimation method using PARAFAC decomposition. This formulation preserves the inherent multilinear structure of the received measurements, thereby further improving parameter estimation performance, especially in low-SNR scenarios. Moreover, unlike matrix-based approaches that rely on slow-time downsampling for signal separation and consequently lose part of the velocity information, the proposed method avoids downsampling and thus preserves velocity information, effectively mitigating velocity ambiguity.

The increase in beamforming gain by the proposed TB design has been validated in this chapter. Nevertheless, the issue of null directions remains unaddressed. It is evident that a number of null directions corresponding to zero beamforming gain exist in the beampattern; however, it remains unclear whether these nulls compromise sensing parameter estimation. Moreover, the influence of the choices of M and M_v , as well as beam selection, on the formation of these nulls and on sensing performance, particularly velocity estimation, requires further investigation. These issues will be addressed in Chapter 3 from the perspective of a multibeam sensing scheme within a TD ISAC framework.

3. Novel Transmit Multibeam Design in Time-Division Vehicular ISAC

The rapid evolution of transportation technology over the past decade has seen increasingly deep integration of intelligence into both vehicular and infrastructural layers, establishing the framework for autonomous driving. To ensure reliable autonomous operation, accurate environmental awareness constitutes a foundational requirement. Although modern vehicle-mounted sensors provide environmental perception capability, they are inherently limited to line-of-sight (LoS) observations. This limitation can impair situational awareness in complex traffic scenarios and pose safety risks that challenge the reliability of autonomous driving. Vehicle-to-vehicle and vehicle-to-machine communication enables the sharing of sensor data and other transportation-related information among vehicles and infrastructures, thereby expanding the perception horizon beyond local LoS and providing an effective means to overcome the constraints of LoS sensing. To realize this, vehicular systems must simultaneously support high-throughput data exchange and high-accuracy sensing, a dual requirement that aligns with the core capability of ISAC technologies.

According to the classification of ISAC architecture design based on the dominant functionality in Subsection 1.1.2, vehicular ISAC falls under the joint design category, which necessitates a balanced performance trade-off between S&C functionalities. On the other hand, vehicular systems must provide wide angular coverage to detect unknown obstacles, pedestrians, and road conditions in their surroundings, thereby ensuring driving safety. Therefore, from the perspective of sensing performance, vehicular ISAC requires both wide field-of-view (FoV) coverage and high parameter estimation accuracy.

To address the S&C performance tradeoff, this chapter first introduces a TD ISAC scheme, which enables flexible adjustment between S&C functionalities. As S&C have fundamentally different requirements, performance tradeoff and mutual interference significantly increase the complexity of waveform design. A practical, effective way to alleviate this complexity is to employ separate, dedicated waveforms for S&C, such as those based on TD multiplexing, spatial-division multiplexing, and frequency division

multiplexing. Among them, TD multiplexing eliminates mutual interference by allocating non-overlapping time resources to each functionality. This not only simplifies system implementation and reduces complexity, but also allows dynamic S&C performance tradeoff through simple time allocation.

Several studies have investigated wide-FoV sensing in ISAC systems, but each approach has inherent limitations. Beam scanning during the communication training phase was proposed in [63, 64] to enable radar sensing over a wide area; however, this method suffers from low angular resolution. In [65], adaptive random switching of transmit antennas was introduced to form coherent beams during data transmission, but only a subset of the transmit antennas was utilized, causing transmit energy inefficiency. The *multibeam* concept was then developed in [66], which employs a fixed sub-beam for communications while a time-varying scanning sub-beam for sensing. Despite this integrated design, direction estimation still relies on beam scanning and fails to exploit the correlation between beams. In [67], a transmit beamformer was designed to generate a narrow beam towards the communication UE while distributing the remaining energy uniformly across the other directions for accurate sensing. However, the so-called "uniform" distribution is guaranteed only at DFT grid locations, leaving a number of null directions within the coverage area. Moreover, this method restricts the number of detectable targets to fewer than half the number of transmit antenna elements. These constraints restrict its feasibility in practical applications.

In this chapter, we present a novel multibeam method for wide-FoV, high-accuracy sensing within the TD vehicular ISAC framework. Subsequently, velocity estimation for targets located at null directions is investigated, and a corresponding solution is developed. Moreover, a relaxed uniqueness condition for PARAFAC with Vandermonde-structured factor matrices is derived to support reliable sensing parameter estimation.

3.1 System Model and Performance Metrics

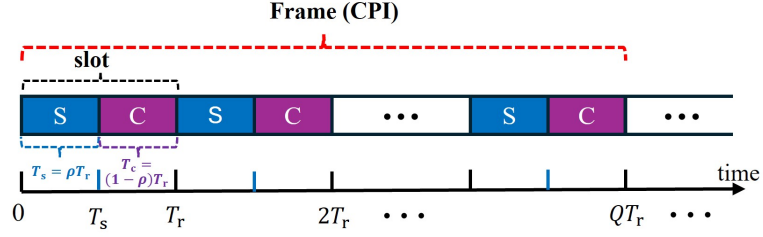


Figure 3.1. Illustration of the TD ISAC waveform frame structure. One frame (CPI) consists of Q slots. For each slot, the first time interval of duration $T_s = \rho T_r$ is allocated for sensing, while the second interval of duration $T_c = (1 - \rho)T_r$ is for communications. For simplicity, inter-slot gaps and boundary symbols are omitted.

Consider a vehicular ISAC MIMO system, where a source vehicle is equipped with two ULAs: a transmit array comprising M_t antenna elements and a receive array consisting of M_r antenna elements. Full-duplex sensing operation is assumed¹, facilitated by advances in isolation and self-interference cancellation [68]. The inter-element spacings of the transmit and receive arrays are denoted by d_t and d_r , respectively. For analytical simplicity, the communication UE is modeled as a single-antenna receiver.

We consider a generic waveform frame structure consisting of Q uniformly spaced slots, within which the positions and relative velocities of UEs and targets are assumed to remain constant, thereby defining a CPI for radar sensing. The duration of each slot is denoted by T_r , which corresponds to the pulse repetition interval in sensing. Within each slot, sensing is performed at the beginning, followed by communications, with durations $T_s = \rho T_r$ and $T_c = (1 - \rho)T_r$ respectively for S&C segments, where $\rho \in (0, 1)$ denotes the proportion of time resource allocated to sensing in the TD ISAC framework. It is assumed that the communication receiver can detect the termination of the sensing phase, which can be realized via boundary symbol detection techniques. For ease of exposition, a simplified frame structure is adopted, where inter-slot gaps and boundary symbols between S&C phases are neglected. Consequently, a CPI is given by $\text{CPI} = Q T_r$. Let the sampling duration be T_f , and thus the lengths of S&C sampling sequences per slot are $N_s = \lfloor T_s / T_f \rfloor$ and $N_c = \lfloor T_c / T_f \rfloor$, respectively. Here, $\lfloor \cdot \rfloor$ represents the rounding down to the nearest integer.

¹Residual self-interference is an important factor in full-duplex sensing performance. As its cancellation and impact are beyond the scope of this work, perfect self-interference cancellation is assumed.

3.1.1 Sensing System Model

Let $s[n]$, $n = 0, 1, \dots, N_f - 1$ denote the sampling sequence employed in the sensing phase. This sequence is typically invariant across slots (pulses) to enable coherent signal processing within a CPI and satisfies the power constraint $\mathbb{E}(|s[n]|^2) = 1$. Moreover, $s[n]$ is designed to exhibit favorable autocorrelation and spectral characteristics for sensing applications. A representative example is $s[n] = \exp(j\pi n^2/N_f)$, corresponding to a classical linear frequency modulation waveform. To optimize the transmit energy distribution for sensing, a beamforming vector $\mathbf{w}_{\text{MB},q} \in \mathbb{C}^{M_t}$ is designed for each slot q to enable multibeam FoV sensing. The beamforming matrix, comprising all the beamforming vectors for Q pulses, is expressed as $\mathbf{W}_{\text{MB}} = [\mathbf{w}_{\text{MB},1}, \dots, \mathbf{w}_{\text{MB},Q}] \in \mathbb{C}^{M_t \times Q}$. The transmitted signal during the sensing phase can then be written as

$$\mathbf{x}_s(t) = \sum_{q=1}^Q \sum_{n=0}^{N_f-1} \sqrt{P_s} \mathbf{w}_{\text{MB},q} s[n] g_{\text{TX}}(t - nT_f - (q-1)T_r), \quad (3.1)$$

where P_s denotes the transmit power of the sensing signal per pulse, and $g_{\text{TX}}(t)$ represents the Nyquist waveform whose sampled autocorrelation satisfies $R_{g_{\text{TX}}}[n] = \delta[n]$, with $\delta[n]$ being the Kronecker delta function.

Suppose that there are K targets within the illuminated area, and the direction of departure (DOD) and DOA for the k th target are denoted by θ_k and φ_k , respectively. For analytical simplicity, LoS propagation is assumed in this chapter. After matched filtering, the received sensing signal by M_r antenna elements within a CPI can be expressed as

$$\mathbf{y}_s = \sum_{k=1}^K \sigma_k N_f \mathbf{b}(\varphi_k) \otimes [\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k) \odot \mathbf{d}(f_k)] + \mathbf{z}_s, \quad (3.2)$$

where σ_k is the complex-valued coefficient that accounts for the transmit power, receiver antenna gain, and radar cross-section of the k th target. The operator $\otimes$ denotes the Kronecker product. The noise vector $\mathbf{z}_s$ has independently and identically distributed (IID) entries following $\mathcal{CN}(0, \sigma_z^2)$, where $\mathcal{CN}(0, \sigma_z^2)$ denotes a complex normal distribution with zero mean and variance σ_z^2 .

It can be observed from (3.2) that the signal processing gain of the received signal is directly proportional to N_f . Therefore, with other parameters held constant, the sensing performance improves as the duration of the sensing phase increases. On the other hand, the beamforming vectors also influence sensing performance by controlling the distribution of transmit energy, which is mathematically characterized by $\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k)$. The design of $\mathbf{W}_{\text{MB}}$ will be presented in Section 3.2.

To measure the sensing performance, CRB is employed as a performance metric. The covariance matrix of $\mathbf{y}_s$ is given by

$$\mathbf{R} = \mathbf{y}_s \mathbf{y}_s^H = N_f^2 \mathbf{C} \mathbf{P} \mathbf{C}^H + \sigma_z^2 \mathbf{I}, \quad (3.3)$$

where $(\cdot)^H$ denotes the conjugate transpose operation, $\mathbf{P}$ denotes a diagonal matrix with diagonal elements $\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2$, $\mathbf{I}$ is an identity matrix, and the k th column of $\mathbf{C}$ is given by

$$\mathbf{c}_k = \mathbf{b}(\varphi_k) \otimes [\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k) \odot \mathbf{d}(f_k)]. \quad (3.4)$$

Denote the unknown parameter vector as

$$\boldsymbol{\rho} = [\theta_1, \dots, \theta_K, \varphi_1, \dots, \varphi_K, f_1, \dots, f_K, \sigma_1^2, \dots, \sigma_K^2, \sigma_z^2]. \quad (3.5)$$

The (m, n) -th element of the Fisher information matrix (FIM) can be expressed as

$$[\mathbf{FIM}]_{m,n} = \text{Tr} \left(\frac{\partial \mathbf{R}}{\partial \rho_m} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \rho_n} \mathbf{R}^{-1} \right). \quad (3.6)$$

Here, $\text{Tr}(\cdot)$ denotes the trace of a matrix, ρ_n is the n th element of $\boldsymbol{\rho}$, $\partial \mathbf{R} / \partial \rho_n$ stands for the partial derivative of $\mathbf{R}$ with respect to ρ_n , and $(\cdot)^{-1}$ denotes the inverse of a matrix. To comprehensively evaluate all estimated parameters, including DOD, DOA, and velocity, we define the following scalar metric as the final measure of sensing performance:

$$\overline{\text{CRB}}(\varphi, \theta, f) = \frac{1}{K} \text{Tr}([\mathbf{FIM}^{-1}]_{1:3K}), \quad (3.7)$$

where $[\mathbf{FIM}^{-1}]_{1:3K}$ denotes the submatrix obtained by extracting the first $3K$ rows and columns of $\mathbf{FIM}^{-1}$. Since $\{\sigma_k^2\}_{k=1}^K$ and σ_z^2 are not parameters of interest, they are excluded when computing $\overline{\text{CRB}}$.

3.1.2 Communication System Model

We consider the case of a single UE, with the channel state information (CSI) between the transmitter and UE assumed to be known and constant within a frame. In automotive scenarios with high mobility, the CSI varies across frames due to channel dynamics; thus, the constant CSI assumption is emphasized to apply only within a single frame here. Consequently, the communication beamforming vector remains unchanged across Q slots and is denoted as $\mathbf{w}_c$, satisfying $\|\mathbf{w}_c\| = M_t$. The transmitted communication signal can then be expressed as

$$\mathbf{x}_c(t) = \sum_{q=1}^Q \sum_{n=0}^{N_c-1} \sqrt{P_c} \mathbf{w}_c c_q[n] g_{\text{TX}}(t - (n + N_f)T_f - (q-1)T_r), \quad (3.8)$$

where P_c denotes the transmit power of the communication signal in each slot and $c_q[n]$ represents the sampled sequence for the communication part in the q th slot, satisfying $\mathbb{E}(|c_q[n]|^2) = 1$.

The received communication signal by the UE for the q th slot is given by

$$y_{c,q}[n] = \sqrt{P_c} \mathbf{h}_c^T \mathbf{w}_c c_q[n] + z_{c,q}, \quad (3.9)$$

where $\mathbf{h}_c$ denotes the complex channel coefficient vector between the M_t -antenna transmitter and the UE receiver, and $z_{c,q}$ represents the additive noise in the q th slot, modeled as $z_{c,q} \sim \mathcal{CN}(0, \sigma_c^2)$. Based on (3.9), the effective communication SE can be expressed as

$$\text{SE} = (1 - \rho) \log_2 \left(1 + \frac{P_c |\mathbf{h}_c^T \mathbf{w}_c|^2}{\sigma_c^2} \right). \quad (3.10)$$

As observed from (3.3) and (3.10), the time allocation ratio, ρ , affects the S&C performance: a larger ρ improves sensing performance but reduces the communication SE. Therefore, a dynamic ρ provides a direct and intuitive means to flexibly balance the tradeoff between S&C performance.

Owing to the separability of the S&C functionalities, more complicated communication scenarios, such as imperfect CSI or multiple UEs, do not affect the sensing beamformer design. Therefore, the multibeam TD-ISAC developed in this chapter can be readily extended to more practical scenarios with nonideal conditions.

3.2 Proposed Multibeam Method for Accurate Sensing

Traditional beam-scanning techniques are limited by their reliance on a single beam for direction estimation, which constrains the achievable angular resolution to the beamwidth. To overcome this limitation, it is essential to exploit the inherent interbeam relationships. This necessitates a beamforming design in which any target, irrespective of its angular position, is illuminated by at least two different beams. Intuitively, such coverage can be realized through two strategies: the deployment of multiple wide beams, each spanning the entire desired region, as exemplified in [67], or the utilization of a set of narrow but partially overlapping beams that collectively span the desired spatial field, such as in [66]. This chapter adopts the latter strategy, leveraging the interbeam amplitude differences for superior direction-finding accuracy. Guided by this design principle, this chapter develops a beamforming design that accommodates both hardware-efficient implementation and low-complexity parameter estimation.

3.2.1 Proposed Multibeam Beamformers

Revisiting the slow-time TB technique proposed in Chapter 2, it was originally motivated by the objective of optimizing energy distribution in DDMA waveform design. In this chapter, the TB technique is recast from a multibeam perspective, upon which a novel multibeam method is developed. Specifically, the slow-time TB technique can be interpreted as generating M distinct narrow, partially overlapping beams within each beam cycle to collectively cover the spatial region of interest, which aligns with the core principle of the second beampattern design strategy discussed

previously. Accordingly, the beampatterns of the M beams with respect to the spatial angle θ and the corresponding mainlobe direction $\theta_{b,m}$ of the m th beam can be expressed as

$$G_m(\theta) = |\mathbf{w}_{\text{MB},m}^T \mathbf{a}(\theta)|^2 = \left| \frac{\sin \left[\frac{\pi}{2} M_t \left(\sin \theta - \frac{2(q_m-1)}{M_v} \right) \right]}{\sin \left[\frac{\pi}{2} \left(\sin \theta - \frac{2(q_m-1)}{M_v} \right) \right]} \right|^2, \quad (3.11)$$

$$\sin \theta_{b,m} = 2l + \frac{2(q_m-1)}{M_v}.$$

Since the transmitted beams exhibit a periodicity of M , only the first cycle of M beams is considered in (3.11) for simplicity without loss of generality, i.e., $m = 1, \dots, M$, $M \leq M_v$, $1 \leq q_m \leq M_v$.

As seen from (3.11), its beampattern follows a *Sinc* function, implying that its mainlobe is directive, and its width is determined by the number of transmit elements M_t , assuming the adjacent spacing is fixed. Two critical characteristics emerge prior to beam selection. Firstly, within a cycle of M_v beams, $\theta_{b,m}$ varies from beam to beam, and the mainlobes of M_v beams sweep across the entire spatial region. Moreover, wide-FoV sensing over a desired region, instead of the whole spatial domain, can be achieved by properly selecting a subset of beams covering Θ , while avoiding high computational complexity by using beamforming vectors drawn from the pre-generated set. Secondly, as M_v increases, the spacings between adjacent mainlobes become smaller because the spacings are inversely proportional to M_v . Given a fixed array configuration—defined by M_t and d_t/λ , among other parameters—the mainlobe beamwidth at a given direction remains constant, regardless of variations in M_v for modulation purposes. Consequently, denser beam placement with constant mainlobe beamwidth results in a higher focusing of transmit energy within the targeted region. This characteristic allows the beamforming gain within Θ to be enhanced just by increasing M_v .

Figure 3.2 presents the performance tradeoff between S&C, in terms of CRB and SE under varying time allocation proportions ρ and different M_v . Two sensing targets are placed at directions $\{\theta_k\}_{k=1}^2 = \{\varphi_k\}_{k=1}^2 = \{-10^\circ, 10^\circ\}$ with identical target ranges. Moreover, $M = 8$ beams centered around 0° are selected. The communication beamformer is $\mathbf{w}_c = \mathbf{a}^*(\theta_c)$, where $(\cdot)^*$ represents the conjugate operation. As shown in Fig. 3.2, the CRB increases as the SE increases, indicating that improved communication performance comes at the cost of reduced sensing accuracy. This tradeoff is governed by the value of ρ , which controls the time allocation between S&C. In terms of the beamforming design, we can find that a larger M_v contributes to improved sensing performance for a given SE, as evidenced by a lower CRB value. Therefore, the performance tradeoff in the proposed TD-ISAC system can be managed by adjusting the time allocation proportion, while a suitable M_v in the proposed multibeam scheme can further improve

sensing performance.

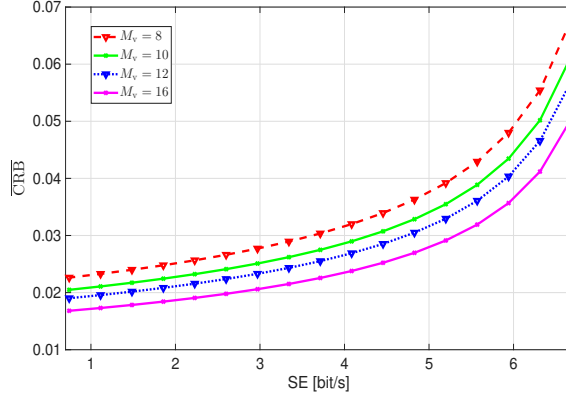


Figure 3.2. Tradeoff between CRB and SE. Lower CRBs are achieved at the cost of reduced SEs, while a larger M_v contributes to lower CRBs.

As established by the foregoing analysis, the selections of $\{q_m\}_{m=1}^M$ and M_v critically shape the transmit energy distribution: $\{q_m\}_{m=1}^M$ determines the extent and positioning of the spatial coverage, while M_v governs the beam density; together, they define the overall beampattern. Consequently, when selecting these parameters to realize a desired beampattern, a natural question arises: *Are arbitrary combinations of $\{q_m\}_{m=1}^M$ and M_v applicable to multibeam sensing to achieve accurate parameter estimation?* A null direction, denoted as θ_{null} , refers to a direction where one or more beams exhibit zero gain, i.e., $\mathbf{w}_{\text{MB},m}^T \mathbf{a}(\theta_{\text{null}}) = 0$, a characteristic inherent in the *Sinc* function. To address this question, this chapter then explores how $\{q_m\}_{m=1}^M$ and M_v influence the occurrence of null directions and the interbeam relationship, and then develops a corresponding solution for parameter estimation in the presence of nulls.

3.3 Parameter Estimation at Null Directions

For the proposed multibeam method within the TD-ISAC framework, the PARAFAC-based parameter estimation algorithm remains applicable, as detailed in Section 2.4. To facilitate the subsequent analysis and avoid redundant formulations, only the factor matrices of the tensor constructed from the received sensing signal in (3.2) are presented in this chapter.

With the multibeam beamformers $\mathbf{W}_{\text{MB}}$ proposed in Section 3.2, the factor matrices $\mathbf{B}$, $\mathbf{D}_{\text{amb}}$, and $\mathbf{A}_{\text{MB}}$ of the tensor $\mathcal{Y}_s$ constructed from the received signal are obtained by stacking the column vectors $\{\mathbf{b}(\varphi_k)\}_{k=1}^K$, $\{\mathbf{d}_{\text{amb}}(f_k)\}_{k=1}^K$, and $\{\mathbf{a}_{\text{MB}}(\theta_k, f_k)\}_{k=1}^K$, respectively. The multibeam steering vector $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$ is given by

$$\mathbf{a}_{\text{MB}}(\theta_k, f_k) = (\mathbf{W}_{\text{MB},0}^T \mathbf{a}(\theta_k)) \odot \mathbf{d}_0(f_k), \quad (3.12)$$

where $\mathbf{W}_{\text{MB},0}$ consists of the first M column vectors.

In principle, provided the PARAFAC decomposition is uniquely determined, sensing parameters, including DODs, DOAs, and velocities, can be recovered using the estimated factor matrices. However, a target located at a null direction may complicate velocity estimation, even leading to velocity ambiguity. As stated in Section 2.4, velocity estimation is realized using $\mathbf{d}_0(f_k)$ and $\mathbf{d}_{\text{amb}}(f_k)$, and $\mathbf{d}_0(f_k)$ enables velocity deambiguation by exploiting the phase relationship between its elements. Observing (3.12), $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$ is the Hadamard product of $\tilde{\mathbf{a}}_{\text{MB}}(\theta_k) = \mathbf{W}_{\text{MB},0}^T \mathbf{a}(\theta_k)$ and $\mathbf{d}_0(f_k)$, while $\mathbf{d}_0(f_k)$ is a normalized phase vector. Consequently, the estimation of $\mathbf{d}_0(f_k)$ relies on the phase information of $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$ and $\tilde{\mathbf{a}}_{\text{MB}}(\theta_k)$. However, when a target is located at or near a null direction of the m th beam, i.e., $\mathbf{w}_{\text{MB},m}^T \mathbf{a}(\theta_k) \approx 0$, the m th element amplitude of $\mathbf{a}_{\text{MB}}(\theta_k)$ approaches zero. In this case, the corresponding phase estimation becomes highly sensitive to noise, which significantly degrades estimation accuracy and may even prevent reliable recovery of $\mathbf{d}_0$. In the worst case where only one element in $\tilde{\mathbf{a}}_{\text{MB},k}$ is nonzero, only a single phase observation is available. Since velocity estimation relies on the phase difference across elements, velocity deambiguation through $\mathbf{d}_0$ cannot be realized, causing the velocity estimation algorithm in Section 2.4 to fail.

To address the above issue, this section analyzes the impact of null directions on velocity estimation. The effect of null directions is first characterized for different values of M_v . Next, a velocity estimation method is proposed to address targets located at null directions.

3.3.1 Beampattern at Null Directions

Recalling (3.11), it can be observed that each beam contains a finite number of null directions, and the null direction for the m th beam, denoted by $\theta_{\text{null},m}$, satisfies

$$\begin{cases} M_t \left(\sin \theta_{\text{null},m} - \frac{2(q_m-1)}{M_v} \right) = 2i_{\text{null}}, \\ \sin \theta_{\text{null},m} - \frac{2(q_m-1)}{M_v} \neq 2j_{\text{null}}. \end{cases} \quad (3.13)$$

Here, i_{null} and j_{null} are arbitrary integers, and i_{null} refers to the index of a null direction for the m th beam. Consider two cases: one with $M_t = M_v$ and another with $M_t < M_v$. For illustration, we set $M_t = 8$ for both cases, while $M_v = 2M_t = 16$ for the second case. The corresponding transmit beam energy distributions of a pulse cycle are shown in Fig. 3.3, where all M_v potential available beams are plotted.

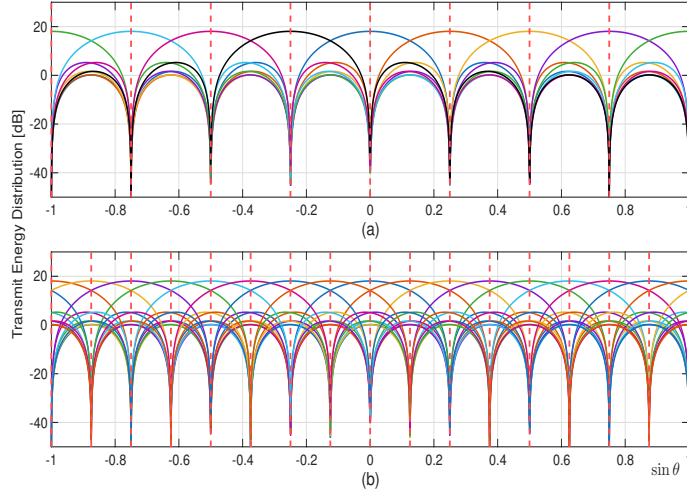


Figure 3.3. Transmit beams and nulls when (a) $M_v = 8$ and (b) $M_v = 16$.

From Fig. 3.3(a), it can be observed that a total of 9 null directions are distributed over the angular region $[-90^\circ, 90^\circ]$ ($\sin(\theta) \in [-1, 1]$). Each beam exhibits 7 or 8 null directions, depending on the location of its mainlobe, and these null directions are shared among different beams. At any given null direction, indicated by vertical red dashed lines, only one beam exhibits a nonzero gain, and this null direction coincides with the mainlobe direction of that beam, while the remaining 7 beams exhibit zero gain. This phenomenon can be mathematically explained by (3.14), which is obtained by substituting M_v with M_t in (3.13) to accommodate the first case.

$$\begin{cases} \sin \theta_{\text{null},m} = \frac{2(i_{\text{null}} + q_m - 1)}{M_t} \in [-1, 1], & (3.14a) \\ \sin \theta_{\text{null},m} \neq \frac{2(q_m - 1)}{M_t} + 2j_{\text{null}} \in [-1, 1]. & (3.14b) \end{cases}$$

For a null direction denoted by θ_{null,m_0} with q_{m_0} , let the corresponding null index i_{null} defined in (3.14a) be $i_{\text{null},0}$. To satisfy the condition in (3.14b), it is required that $i_{\text{null},0} \neq nM_t$ for any integer n . In other words, if $i_{\text{null},0} = nM_t$ holds, a mainlobe instead of a null will be formed at θ_{null,m_0} for the beam m_0 . For another beam indexed by m_1 such that $q_{m_1} - q_{m_0} = \Delta q$, the condition in (3.14a) at θ_{null,m_0} can be readily satisfied by simply setting $i_{\text{null},1} = i_{\text{null},0} - \Delta q$, thereby ensuring a null at θ_{null,m_0} . As a result, only the beams with parameters satisfying $q_{m_1} = q_{m_0} + i_{\text{null},0} - nM_t$ produce a mainlobe at θ_{null,m_0} for the beam m_1 , which corresponds to the condition $i_{\text{null},1} = nM_t$, while all other beams yield null response. Since $1 \leq q_{m_1} \leq M_t$, there exists only one feasible q_{m_1} that satisfies this condition. To conclude, in conventional DDMA systems generating M_t distinct beams, only a single beam exhibits nonzero beamforming gain at a null direction. Consequently, for a target located at such a null direction, only one entry of $\mathbf{d}_0(f_k)$ provides usable phase information, preventing reliable velocity information extraction from $\mathbf{d}_0(f_k)$. As a result, for a target at a null

direction, the maximum unambiguously detectable velocity is reduced by a factor of M_t , since only $\mathbf{D}_{\text{amb}}$ can be effectively utilized for velocity estimation.

For the case $M_v > M_t$, an example with $M_v = 16$ is considered, and the corresponding beampattern is shown in Fig. 3.3(b). Compared to the first case, a larger number of distinct beams and nulls are distributed across the spatial region, with multiple beams exhibiting nonzero gain at each null direction. Under this setting, the expression in (3.13) can be reformulated as

$$\begin{cases} \sin \theta_{\text{null},m} = \frac{2(q_m-1)}{M_v} + \frac{2i_{\text{null}}}{M_t} \in [-1, 1], \\ \sin \theta_{\text{null},m} \neq \frac{2(q_m-1)}{M_v} + 2j_{\text{null}} \in [-1, 1]. \end{cases} \quad (3.15)$$

In this case, a beam, indexed by m_1 , sharing the null direction θ_{null,m_0} with parameter q_{m_0} , must satisfy

$$\frac{i_{\text{null},0} - i_{\text{null},1}}{q_{m_1} - q_{m_0}} = \frac{M_t}{M_v}, \quad i_{\text{null},1} \neq nM_t. \quad (3.16)$$

It can be seen from (3.16) that the number of beams sharing a null direction is determined by the ratio M_t/M_v . Let M_t/M_v be expressed in its reduced form as M_1/M_2 , where M_1 and M_2 are coprime integers. Accordingly, in addition to satisfying the second subcondition in (3.16), the difference $q_{m_1} - q_{m_0}$ must be an integer multiple of M_2 in order to form a null at θ_{null,m_0} . In this case of $M_1/M_2 = 1/2$, $q_{m_1} - q_{m_0}$ must be even to satisfy this condition. Therefore, when $M_v > M_t$, more than one beam exhibits nonzero gain at a given null direction, thereby enabling the phase-based velocity estimation with reduced ambiguity. In contrast to traditional DDMA, the proposed multibeam scheme overcomes the limitation of failing to estimate the velocity of targets located at null directions, thereby achieving more robust performance in practical applications.

As mentioned earlier, flexible wide FoV coverage is required in vehicular ISAC scenarios; therefore, beam selection is performed to accommodate different spatial regions of interest. The impact of this beam selection on the maximum detectable velocity, as well as the corresponding velocity estimation algorithm under such conditions, is discussed in the following.

3.3.2 Velocity Estimation of Null-Direction Targets

As discussed above, velocity estimation relies on phase information, while the accuracy of phase estimation depends on the element amplitudes of $\mathbf{a}_{\text{MB},\mathbf{k}}$, i.e., the beamforming gains. When the beamforming gain is sufficiently low, the corresponding phase estimates become highly sensitive to noise and unreliable; thus, such phase estimates should be excluded from the velocity estimation procedure. Supposing that a target is located at a null direction, the indices of the reliable beams for this null are denoted by $m_1, \dots, m_P$, where $1 \leq m_1 < \dots < m_P \leq M_v$. After discarding the

unreliable phase estimates obtained from low-amplitude entries of $\tilde{\mathbf{a}}_{\text{MB},k}$, $\mathbf{d}_0$ is modified as²

$$\mathbf{d}_{\text{null},0}(f_k) = [e^{j2\pi f_k(m_1-1)T_r}, \dots, e^{j2\pi f_k(m_P-1)T_r}]^T. \quad (3.17)$$

Let $\Delta m_p = m_{p+1} - m_p$. Then, the phase difference vector can be expressed as follows:

$$\Delta\phi_{\text{amb}} = [\Delta m_1, \dots, \Delta m_{P-1}]^T f_k T_r. \quad (3.18)$$

To account for ambiguity when representing a target velocity v , the ambiguity numbers $\{l_{\text{amb},p}\}_{p=1}^{P-1}$ are introduced, allowing the velocity to be expressed as

$$v = \frac{\lambda \text{PRF}}{2\Delta m_p} l_{\text{amb},p} + v_{b,p}, \quad v_{b,p} \in \left[-\frac{\lambda \text{PRF}}{4\Delta m_p}, \frac{\lambda \text{PRF}}{4\Delta m_p}\right). \quad (3.19)$$

It is assumed that the target velocity does not exceed the maximum unambiguous velocity corresponding to the case where $\Delta m_p = 1$ in (3.19). To achieve accurate velocity estimation, the phase difference corresponding to the minimum index spacing, $\Delta m_{\min}$, is first utilized to determine the ambiguity numbers $\{l_{\text{amb},p}\}_{p=1}^{P-1}$.

To fuse the velocity information contained in $\mathbf{d}_{\text{null},0}(f_k)$ and $\mathbf{d}_{\text{amb}}(f_k)$, the phase difference corresponding to $\Delta m_P = M$ is incorporated, which can be obtained by $\Delta\phi_{\text{amb},P} = M f_k T_r = \frac{1}{P_v-1} \sum_{i=1}^{P_v-1} \{\langle \mathbf{d}_{\text{amb}}(f_k) \rangle_{i+1} - \langle \mathbf{d}_{\text{amb}}(f_k) \rangle_i\}$, where $\langle \cdot \rangle_i$ denotes the operation of extracting the phase of the i th element of a vector and then dividing it by 2π . Once the velocity ambiguity numbers $\{l_{\text{amb},p}\}_{p=1}^P$ are determined, the estimated $\{\Delta\phi_{\text{amb},p}\}_{p=1}^P$ can be used to obtain

$$\begin{bmatrix} \Delta m_1 \\ \vdots \\ \Delta m_{P-1} \\ M \end{bmatrix} T_r f_k = \begin{bmatrix} \Delta\phi_{\text{amb},1} + l_{\text{amb},1} \\ \vdots \\ \Delta\phi_{\text{amb},P-1} + l_{\text{amb},P-1} \\ \Delta\phi_{\text{amb},P} + l_{\text{amb},P} \end{bmatrix}. \quad (3.20)$$

Using a least-squares fitting method, f_k can be estimated as

$$\hat{f}_k = \frac{1}{T_r} \frac{\Delta \mathbf{m}^T \Delta \phi}{\Delta \mathbf{m}^T \Delta \mathbf{m}}, \quad (3.21)$$

where $\Delta \mathbf{m}$ and $\Delta \phi$ represent the vectors enclosed in the square brackets on the left- and right-hand sides of (3.20), respectively.

From the above discussion, the key challenge in velocity deambiguation lies in determining the ambiguity numbers $\{l_{\text{amb},p}\}_{p=1}^P$. If $\Delta m_{\min} = 1$, i.e., there exists at least one pair of beams with spacing 1 that allows resolving

²In the worst case, e.g., under low-SNR conditions, only one beam may exceed the reliability threshold; in this case, the proposed sensing parameter estimation method fails since at least two reliable beams are required. This represents a boundary condition of the proposed approach.

the ambiguity, the velocity can initially be estimated using the associated $\phi_{\text{amb},p}$, although this velocity estimate may lack accuracy since it relies on only a single phase difference value. With this preliminary velocity estimate, $\{l_{\text{amb},p}\}_{p=1}^P$ can subsequently be determined, thereby enabling a more accurate velocity estimation.

When $\Delta m_{\min} > 1$, its corresponding ambiguity number must first be determined. For notational simplicity, $\Delta m_1 = \Delta m_{\min}$ is assumed. The corresponding $l_{\text{amb},1}$ can be determined by

$$\hat{l}_{\text{amb},1} = \min_{0 \leq |l| \leq \lceil \frac{\Delta m_1 - 1}{2} \rceil} \sum_{p=2}^P \left| \llbracket T_r \hat{f}_k \Delta m_p - \Delta \phi_{\text{amb},p} \rrbracket \right|, \quad (3.22)$$

where $\hat{f}_k = (\Delta \phi_{\text{amb},1} + l) / \Delta m_1 / T_r$, $\lceil \cdot \rceil$ represents the rounding up to the nearest integer, and $\llbracket \cdot \rrbracket$ denotes the operation of extracting the fractional part of a value. Since Δm_1 is the smallest among $\{\Delta m_p\}_{p=1}^P$, the number of candidates for $l_{\text{amb},1}$ is limited, and thus, only a few loops are required to identify its true value.

Unfortunately, however, even if the condition in (3.22) is satisfied, the resulting estimate of f_k is not necessarily correct. Let the hypothesized value of $l_{\text{amb},1}$ be $l_{\text{amb},1} + \Delta l$, where $\Delta l \neq 0$. If the following condition holds:

$$\frac{\Delta m_p \Delta l}{\Delta m_1} = n, \quad p = 1, \dots, P, \quad (3.23)$$

the estimate $l_{\text{amb},1} + \Delta l$ will be misidentified as the true value of $l_{\text{amb},1}$. Here, $\Delta m_p = M$, and n denotes an arbitrary integer.

Recalling (3.16), it has been concluded that the differences between the values of q_m associated with nonzero-gain beams at a given null direction are determined by the ratio, M_1/M_2 . Consider the following two cases: the first is $M_2 \geq 3$ and the second is $M_2 = 2$. For the first case, it can be found that the values of q_m for zero-gain beams must satisfy $\Delta q = q_{m_1} - q_{m_0} = nM_2$. As a result, there exist at least $M_2 - 1$ consecutive beams exhibiting nonzero gain at a null direction, ensuring $\Delta m_{\min} = 1$. The term ‘‘at least’’ arises here because an additional subcondition, $i_{\text{null}} \neq M_t j_{\text{null}}$, must also be satisfied simultaneously. To investigate the second case, rewrite (3.15) with $M_v = 2M_t$ as

$$\begin{cases} \sin \theta_{\text{null},m} = \frac{2i_{\text{null}} + (q_m - 1)}{M_t} \in [-1, 1], \\ \sin \theta_{\text{null},m} \neq \frac{q_m - 1}{M_t} + 2j_{\text{null}} \in [-1, 1]. \end{cases} \quad (3.24)$$

For a null direction given by $\sin \theta_{\text{null},m_0} = \frac{2i_{\text{null},0} + (q_{m_0} - 1)}{M_t}$ with $i_{\text{null},0} \neq 0$, let $q_{m_1} = 2i_{\text{null},0} + q_{m_0}$ and set $i_{\text{null}} = 0$. It can be observed that the beam m_1 forms a mainlobe at $\theta_{\text{null},q_{m_0}}$, and adjacent beams corresponding to $q_{m_1} - 1$ and $q_{m_1} + 1$ also exhibit nonzero gain at the same direction. This implies the existence of consecutive beams with nonzero gain in the second case. In conclusion, for $M_v > M_t$, it follows that $\Delta m_{\min} = 1$. This result remains valid under flexible beam selection aimed at adapting to a desired spatial region, since the region of interest typically is a continuous angular sector.

3.4 Relaxed Uniqueness Condition for PARAFAC

The feasibility of PARAFAC-based parameter estimation algorithms fundamentally relies on the uniqueness of PARAFAC decomposition. According to the uniqueness condition in (2.5), the factor matrices of $\mathcal{Y}_s$ are uniquely identifiable if $k(\mathbf{B}) + k(\mathbf{D}_{\text{amb}}) + k(\mathbf{A}_{\text{MB}}) \geq 2K + 2$ holds. However, this condition is sufficient but not necessary, and can be relaxed when factor matrices possess specific structural properties.

The configurations of the ULA receiver and uniform slow-time sampling lead to both factor matrices, $\mathbf{B}$ and $\mathbf{D}_{\text{amb}}$, exhibiting a Vandermonde structure with generators $\{z_{\mathbf{B},k}\}_{k=1}^K$ and $\{z_{\mathbf{D}_{\text{amb}},k}\}_{k=1}^K$, respectively. On the other hand, the parameters of the K targets, including DODs, DOAs, and velocities, may not be distinct or may be very close to each other in practice. Consequently, the corresponding factor matrices may not have full-column rank. Motivated by this, a new uniqueness condition is established for tensors with two Vandermonde-structured and potentially rank-deficient factor matrices.

First, the spatial smoothing mapping technique [69] is employed to address rank deficiency caused by repeated sensing parameters. This technique constructs overlapping subarrays or segments along the relevant signal dimension and maps the resulting data into a higher-order tensor, thereby introducing additional dimensions that can increase the ranks of factor matrices. In this chapter, the spatial smoothing mapping technique is applied along both the receive-array and slow-time dimensions. By exploiting the Vandermonde structures of $\mathbf{B}$ and $\mathbf{D}_{\text{amb}}$, $\mathbf{B} \in \mathbb{C}^{M_r \times K}$ and $\mathbf{D}_{\text{amb}} \in \mathbb{C}^{P_v \times K}$ are mapped into $\mathbf{B}^{(T_1)} \mathbf{B}^{(L_1)T} \in \mathbb{C}^{T_1 \times L_1}$ and $\mathbf{D}_{\text{amb}}^{(T_2)} \mathbf{D}_{\text{amb}}^{(L_2)T} \in \mathbb{C}^{T_2 \times L_2}$, respectively. Here, $\mathbf{B}^{(T_1)}$ denotes the matrix formed by selecting the top T_1 rows of $\mathbf{B}$, and the same notation applies to other matrices. The integers T_1 , T_2 , L_1 and L_2 should be chosen such that $T_1 + L_1 = M_r + 1$ and $T_2 + L_2 = P_v + 1$. Consequently, spatial smoothing along the two dimensions introduces two additional dimensions, and a new 5-D tensor, denoted by $\mathcal{Y}$, can be obtained from $\mathcal{Y}_s$ and expressed as

$$[\mathcal{Y}]_{t_1, t_2, l_1, l_2, m} = [\mathcal{Y}_s]_{t_1 + l_1 - 1, t_2 + l_2 - 1, m}, \quad (3.25)$$

where $[\mathcal{Y}]_{t_1, t_2, l_1, l_2, m}$ denotes the (t_1, t_2, l_1, l_2, m) -th entry of $\mathcal{Y}$. The same indexing notation is also used for $\mathcal{Y}_s$. Unfolding $\mathcal{Y} \in \mathbb{C}^{T_1 \times T_2 \times L_1 \times L_2 \times M}$, a new matrix, $\mathbf{Y} \in \mathbb{C}^{T_1 T_2 \times L_1 L_2 M}$, can be constructed as

$$\mathbf{Y} = \left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) \left(\mathbf{B}^{(L_1)} * \mathbf{D}_{\text{amb}}^{(L_2)} * \mathbf{A}_{\text{MB}} \right)^T. \quad (3.26)$$

Since we are primarily concerned with the relationships among elements within each column, column-wise scaling does not affect the parameter estimation in this context. Therefore, for simplicity, column scalings are omitted in (3.26) and the subsequent derivations.

Assume that $\mathbf{B}$ has $G \leq K$ distinct generators with multiplicity $P_1, \dots, P_G$, where $1 \leq P_g \leq K$ for $g \in \{1, \dots, G\}$. Then, $\mathbf{B}$ can be grouped in column blocks according to these multiplicities and represented as

$$\mathbf{B} = [\mathbf{1}_{P_1}^T \otimes \mathbf{b}_1, \mathbf{1}_{P_2}^T \otimes \mathbf{b}_2, \dots, \mathbf{1}_{P_G}^T \otimes \mathbf{b}_G], \quad (3.27)$$

where $\mathbf{1}_{P_g}$ denotes an all-one column vector of length P_g and $\mathbf{b}_g = [1, z_{\mathbf{B},g}, \dots, z_{\mathbf{B},g}^{M_r-1}]^T$ is the Vandermonde vector associated with the g th generator $z_{\mathbf{B},g}$. Since $\mathbf{B}^{(T_1)}$ and $\mathbf{B}^{(L_1)}$ share the same generators as $\mathbf{B}$ and differ only in their row dimensions, they can be expressed in a form analogous to (3.27). Despite the presence of repeated generators, the uniqueness condition can be relaxed as follows:

$$\begin{cases} r(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}) = K & (3.28a) \end{cases}$$

$$\begin{cases} r(\mathbf{B}^{(L_1)} * \mathbf{D}_{\text{amb}}^{(L_2)} * \mathbf{A}_{\text{MB}}) = K & (3.28b) \end{cases}$$

$$\begin{cases} P_g + 1 \leq T_2, \quad g = 1, \dots, G. & (3.28c) \end{cases}$$

Here, $\underline{\mathbf{X}}$ and $\overline{\mathbf{X}}$ are obtained by deleting the bottom and top rows of $\mathbf{X}$, respectively. Under this condition, the Vandermonde-constrained PARAFAC decomposition of $\mathcal{Y}$ is unique, enabling the recovery of the sensing parameters of up to K targets with automatic parameter pairing.

The proof of the condition in (3.28a)-(3.28c) and the procedure for calculating the sensing parameters using PARAFAC in the presence of repeated sensing parameters are elaborated in the appendix, while an outline of the procedure is presented in Algorithm 1. It is worth noting that the effectiveness of the proposed parameter estimation remains unaffected by the phase ambiguity of singular value decomposition (SVD) because the estimation of DODs, DOAs, and velocities relies on the relative phase and amplitude relationships among the elements of the vectors $\mathbf{b}(\varphi_k)$, $\mathbf{d}_{\text{amb}}(f_k)$, and $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$, rather than on individual elements in isolation.

It follows from (3.28a)-(3.28c) that the selection of T_1 , T_2 , L_1 , and L_2 is governed not only by the number of targets to be identified, K , but also by the dimensional constraints imposed by the size of receive array, M_r , and P_v , i.e., $T_1 < M_r$, $L_1 \leq M_r$, $T_2 \leq P_v$, and $L_2 \leq P_v$. Therefore, these parameters should be selected by jointly considering the required number of identifiable targets and the aforementioned dimensional constraints.

To further illustrate the advantage of the proposed uniqueness condition over the conventional Kruskal condition, consider a numerical example with $M = 8$, $M_r = 8$, $P_v = 10$, and $K = 12$. Each of the two Vandermonde-structured factor matrices contains 4 different generators, each with multiplicity 3. Consequently, we have $r(\mathbf{B}) = r(\mathbf{D}_{\text{amb}}) = 4$ and $k(\mathbf{B}) = k(\mathbf{D}_{\text{amb}}) = 1$. Assuming $k(\mathbf{A}_{\text{MB}}) = r(\mathbf{A}_{\text{MB}}) = 8$, the conventional Kruskal condition yields $k(\mathbf{A}_{\text{MB}}) + k(\mathbf{B}) + k(\mathbf{D}_{\text{amb}}) = 8 + 1 + 1 = 10 < 2K + 2 = 26$, and therefore cannot guarantee uniqueness of the PARAFAC decomposition. In contrast, the

proposed uniqueness condition can be satisfied by selecting, for example, $T_1 = 5$, $T_2 = 4$, $L_1 = 4$, and $L_2 = 7$. In particular,

$$(T_1 - 1)T_2 = 16 > K = 12, \quad L_1L_2M = 224 > K = 12, \quad P_g + 1 = 4 = T_2, \quad g = 1, \dots, 4.$$

Therefore, exploiting the spatial smoothing technique and Vandermonde structure yields a more relaxed uniqueness condition than the conventional Kruskal condition.

Finally, even if the uniqueness condition is satisfied under the considered model, the practical feasibility of the proposed approach may still be limited by the stability of the PARAFAC decomposition. Specifically, when multiple targets have closely spaced angular or Doppler parameters, the corresponding factor vectors can become highly correlated, resulting in ill-conditioning and reduced numerical stability of PARAFAC. Techniques such as regularization and informed initialization [70, 71] have been developed to mitigate this issue. Since a detailed investigation of this aspect is beyond the scope of this work, sufficient separation in the angular and Doppler domains is assumed.

Computational Complexity: The most computationally intensive step of Algorithm 1 is the SVD of $\mathbf{Y}$. Consequently, the overall complexity of Algorithm 1 is on the order of $\mathcal{O}(T_1T_2L_1L_2M\min(T_1T_2, L_1L_2M))$. For the ALS-based PARAFAC decomposition, the computational complexity per iteration is $\mathcal{O}(T_1T_2L_1L_2MK)$. According to the uniqueness condition in (3.28), the parameters T_1 , T_2 , L_1 , L_2 , and M are chosen based on the maximum number of identifiable targets K , such that $(T_1 - 1)T_2 \geq K$ and $L_1L_2M \geq K$. As a result, both T_1T_2 and L_1L_2M scale on the order of K , implying that the complexity of Algorithm 1 is comparable to that of a single iteration of the ALS-based method. Since Algorithm 1 does not require multiple iterations to reach convergence, it is therefore more computationally efficient than conventional ALS-based methods. In the practical implementation, the computational burden can be further reduced by employing optimized linear algebra libraries and leveraging parallel computing techniques. Therefore, a PARAFAC-based sensing parameter estimation method can be performed within a practical runtime, making the proposed approach suitable for latency-constrained ISAC applications.

Algorithm 1 Procedure for estimating sensing parameters via PARAFAC under repeated parameters

1: **Input:** $\mathcal{Y}_s$.

Choose integer pairs (T_1, L_1) and (T_2, L_2) subject to $T_1 + L_1 = M_r + 1$ and $T_2 + L_2 = P_v + 1$, and then construct a new matrix according to (3.25):

$$\mathbf{Y} = \left(\mathbf{B}^{\langle T_1 \rangle} * \mathbf{D}_{\text{amb}}^{\langle T_2 \rangle} \right) \left(\mathbf{B}^{\langle L_1 \rangle} * \mathbf{D}_{\text{amb}}^{\langle L_2 \rangle} * \mathbf{A}_{\text{MB}} \right)^T.$$

2: Compute SVD $\mathbf{Y} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H$, and then determine K from $\mathbf{\Lambda}$.

3: Build $\mathbf{U}_1$ and $\mathbf{U}_2$ as

$\mathbf{U}_1 = \mathbf{U}(1 : (T_1 - 1)T_2, :)$, $\mathbf{U}_2 = \mathbf{U}(T_2 + 1 : T_1 T_2, :)$ where $\mathbf{U}(x_1 : x_2, :)$ denotes a submatrix of $\mathbf{U}$ consisting of rows from index x_1 to x_2 ;

Determine $\{z_{\mathbf{B},k}\}_{k=1}^K$ and $\mathbf{M}$ using eigenvalue decomposition (EVD):

$$\mathbf{U}_1^\dagger \mathbf{U}_2 = \mathbf{M} \mathbf{Z}_{\mathbf{B}} \mathbf{M}^{-1},$$

where $(\cdot)^\dagger$ denotes the Moore-Penrose pseudo-inverse;

Build $\mathbf{b}_k^{\langle T_1 \rangle} = \left[1, z_{\mathbf{B},k}, \dots, z_{\mathbf{B},k}^{T_1-1} \right]^T$, $k \in \{1, \dots, K\}$.

4: Compute $\mathbf{d}_{\text{amb},k}^{\langle T_2 \rangle'} = \left(\mathbf{b}_k^{\langle T_1 \rangle H} \otimes \mathbf{I}_{T_2} \right) \mathbf{U} \mathbf{m}_k$, where $\mathbf{m}_k$ is the k th column vector of $\mathbf{M}$;

Reorder the estimated set $\{\mathbf{b}_k^{\langle T_1 \rangle}\}_{k=1}^K$ according to the grouping structure in (3.27), and apply the same permutation to $\{\mathbf{d}_{\text{amb},k}^{\langle T_2 \rangle'}\}_{k=1}^K$ and $\{\mathbf{m}_k\}_{k=1}^K$, resulting in:

$$\mathbf{D}_{\text{amb}}^{\langle T_2 \rangle'} = [\mathbf{D}_{\text{amb},1}^{\langle T_2 \rangle'}, \dots, \mathbf{D}_{\text{amb},G}^{\langle T_2 \rangle'}], \mathbf{M} = [\mathbf{M}_1, \dots, \mathbf{M}_G].$$

5: For $g \in [1, G]$:

If $P_g = 1$: $z_{\mathbf{D}_{\text{amb},g}} = \left(\mathbf{D}_{\text{amb},g}^{\langle T_2 \rangle} \right)^\dagger \overline{\mathbf{D}_{\text{amb},g}^{\langle T_2 \rangle}}$, $\mathbf{N}_g = \mathbf{m}_g$;

Else:

Build $\mathbf{U}_{\mathbf{D},1} = \mathbf{D}_{\text{amb},g}^{\langle T_2 \rangle'}$ and $\mathbf{U}_{\mathbf{D},2} = \overline{\mathbf{D}_{\text{amb},g}^{\langle T_2 \rangle'}}$, and then implement EVD:

$$\mathbf{U}_{\mathbf{D},1}^\dagger \mathbf{U}_{\mathbf{D},2} = \mathbf{M}_{\mathbf{D}_{\text{amb},g}} \mathbf{Z}_{\mathbf{D}_{\text{amb},g}} \mathbf{M}_{\mathbf{D}_{\text{amb},g}}^{-1};$$

Determine $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=k_1}^{k_{P_g}}$ from $\mathbf{Z}_{\mathbf{D}_{\text{amb},g}}$;

$$\mathbf{N}_g = \mathbf{M}_g \mathbf{M}_{\mathbf{D}_{\text{amb},g}}.$$

End if

6: Construct $\{\mathbf{b}_k^{\langle L_1 \rangle}\}_{k=1}^K$, $\{\mathbf{d}_{\text{amb},k}^{\langle L_2 \rangle}\}_{k=1}^K$, and $\mathbf{N} = [\mathbf{N}_1, \dots, \mathbf{N}_G]$, and then compute $\{\mathbf{a}_{\text{MB},k}\}_{k=1}^K$ through [69]

$$\mathbf{a}_{\text{MB},k} = \left(\frac{\mathbf{b}_k^{\langle L_1 \rangle H}}{\mathbf{b}_k^{\langle L_1 \rangle H} \mathbf{b}_k^{\langle L_1 \rangle}} \otimes \frac{\mathbf{d}_{\text{amb},k}^{\langle L_2 \rangle H}}{\mathbf{d}_{\text{amb},k}^{\langle L_2 \rangle H} \mathbf{d}_{\text{amb},k}^{\langle L_2 \rangle}} \otimes \mathbf{I}_M \right) \mathbf{V}^* \mathbf{\Lambda} \mathbf{c}_k, \text{ where } \mathbf{c}_k \text{ is the } k\text{th row vector of matrix } \mathbf{N}^{-1}.$$

7: With the obtained $\{z_{\mathbf{B},k}\}_{k=1}^K$, $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=1}^K$, and $\{\mathbf{a}_{\text{MB},k}\}_{k=1}^K$, the sensing parameters $\{\varphi_k\}_{k=1}^K$, $\{v_k\}_{k=1}^K$, and $\{\theta_k\}_{k=1}^K$ can be recovered accordingly.

8: **Output:** $\{\theta_k, \varphi_k, v_k\}_{k=1}^K$.

3.5 Conclusion and Discussion

This chapter introduced a TD scheme for vehicular MIMO ISAC to flexibly balance S&C performance requirements and proposed a novel multibeam approach for high-accuracy sensing over a wide area. The TD strategy addresses the simultaneous high demands for sensing performance and communication rate by allocating time resources between S&C, while avoiding spectrum and spatial interference. Building on the TD MIMO ISAC framework, a multibeam design employing simple phase modulation was developed, enabling flexible sensing coverage and improved parameter estimation performance. The characteristics of null directions with respect to the parameter M_v and beam selection were investigated, and the issue of velocity ambiguity for targets located at null directions, a critical limitation of conventional DDMA-based methods, was addressed with a corresponding solution provided. Moreover, a newly derived uniqueness condition for PARAFAC involving two Vandermonde factor matrices was developed for sensing parameter estimation based on PARAFAC decomposition.

While the proposed multibeam TD ISAC framework demonstrates promising performance, several issues remain to be addressed to complete this work. First, the multi-UE TD ISAC case is worth further study, particularly regarding user scheduling and beam allocation strategies. Second, the TD scheme leads to reduced SE due to non-overlapping time allocation. Although separate S&C beamformers are designed to optimize their respective performance, sensing beams may potentially be reused to carry communication information, thereby mitigating the SE loss caused by the TD scheme. Spatial sharing may represent a potential means to further enhance SE, provided that it does not degrade sensing performance or introduce significant implementation complexity. Finally, to enhance practical applicability, future research may focus on the adaptability of the proposed method to dynamic channel conditions, as well as its robustness against RCS uncertainty, imperfect CSI, and other forms of imperfect prior knowledge.

4. Pilot Contamination-Aware Graph Attention Network for Power Control in CFmMIMO

As discussed in Subsection 1.1.5, power control remains one of the major challenges in CFmMIMO systems due to the large-scale nature of the networks, and learning-based algorithms have been developed as a promising candidate to meet the stringent real-time demands for 6G wireless systems. Meanwhile, as the number of UEs grows, the limited set of available orthogonal pilot sequences becomes increasingly inadequate, leading to severe pilot contamination and degraded channel estimation accuracy. This degradation subsequently impairs communication performance, highlighting the necessity of explicitly considering pilot contamination when addressing power control in CFmMIMO systems.

Various neural network architectures have been explored for power control, including convolutional neural networks [26, 33], transformer-based networks [27], and graph neural networks (GNNs) [28, 29, 72, 73, 74]. Among them, GNNs have demonstrated particularly strong performance, primarily due to their structural similarity to cell-free wireless networks [72, 73]. However, existing GNN-based methods assume perfectly orthogonal pilot sequences and therefore neglect the effect of pilot contamination, an assumption that is generally unrealistic in CFmMIMO deployments.

To bridge this gap, this chapter presents a pilot contamination-aware graph attention network for power control in CFmMIMO systems. Specifically, the CFmMIMO signal model is first introduced, and then the corresponding power control optimization problem is formulated. A detailed description of the proposed neural network is then provided, including its graph representation and layer-wise architecture design. Finally, numerical results are presented to demonstrate the superior performance of the proposed approach.

4.1 Power Control Optimization in CFmMIMO

4.1.1 Signal Model of CFmMIMO

Consider the downlink power control in a CFmMIMO system, in which UEs are jointly served by APs sharing the same time-frequency resources, and all APs operate in a synchronized manner. Coordinated by a central processing unit (CPU), APs collaboratively perform resource allocation tasks based on aggregated large-scale fading information, including power allocation, pilot assignment, and AP selection. Meanwhile, each AP independently copes with small-scale fading by locally estimating channels and applying its own beamforming scheme. The system operates in TD duplex mode and consists of randomly distributed M APs and K single-antenna UEs, where each AP is equipped with N antennas.

Assuming a block-fading channel model, where the channel states remain constant over a coherence block of T_c symbols, the uplink channel from the k th UE to the m th AP can be expressed as

$$\mathbf{g}_{m,k} = \sqrt{\beta_{m,k}} \mathbf{h}_{m,k}, \quad (4.1)$$

where $\beta_{m,k}$ denotes the large-scale fading coefficient, while $\mathbf{h}_{m,k} \in \mathbb{C}^N$ represents the small-scale fading coefficient vector. The entries of $\mathbf{h}_{m,k}$ are IID complex Gaussian random variables with zero mean and unit variance, i.e., $\mathcal{CN}(0, 1)$.

Pilot sequences of length T_p ($T_p \ll T_c$) are used for channel estimation during the uplink training stage. Assuming channel reciprocity, the downlink channels can be estimated using the uplink pilots. Let the pilot sequence assigned to the k th UE be $\sqrt{T_p} \boldsymbol{\varphi}_k \in \mathbb{C}^{T_p}$ with $\|\boldsymbol{\varphi}_k\|^2 = 1$. Serving a large number of UEs simultaneously results in $T_p < K$, thereby causing pilot contamination. Therefore, one of the primary objectives of this chapter is to address the impact of pilot contamination in power control. During the uplink training phase, the received signal at the m th AP can be described as

$$\mathbf{Y}_m^{\text{up}} = \sqrt{\zeta_p T_p} \sum_{i=1}^K \mathbf{g}_{m,i} \boldsymbol{\varphi}_i^H + \mathbf{W}_m^{\text{up}}, \quad (4.2)$$

where ζ_p represents the normalized transmit SNR per pilot symbol and $\mathbf{W}_m^{\text{up}} \in \mathbb{C}^{N \times T_p}$ is the noise matrix with IID entries following $\mathcal{CN}(0, 1)$. The minimum mean square error channel estimate can be derived as [75]

$$\hat{\mathbf{g}}_{m,k} = \frac{\sqrt{\zeta_p T_p} \beta_{m,k}}{1 + \zeta_p T_p \sum_{i=1}^K \beta_{m,i} |\boldsymbol{\varphi}_i^H \boldsymbol{\varphi}_k|^2} \mathbf{Y}_m^{\text{up}} \boldsymbol{\varphi}_k, \quad (4.3)$$

and the mean square of each entry of $\hat{\mathbf{g}}_{m,k}$ is given by

$$\bar{g}_{m,k} = \mathbb{E} \left[|\hat{\mathbf{g}}_{m,k}[n]|^2 \right] = \frac{\zeta_p T_p \beta_{m,k}^2}{1 + \zeta_p T_p \sum_{i=1}^K \beta_{m,i} |\boldsymbol{\varphi}_i^H \boldsymbol{\varphi}_k|^2}. \quad (4.4)$$

Let the power control coefficient matrix be $\mathbf{M} \in \mathbb{R}^{M \times K}$, where the (m, k) -th element, denoted by $\mu_{m,k}$, represents the power control coefficient for the signal transmitted from the m th AP to the k th UE. A practical constraint on the transmit power per AP is considered, which is given by $\|\boldsymbol{\mu}_m\|^2 \leq \frac{1}{N}$, then the feasible set of $\mathbf{M}$ can be described as

$$\mathcal{S} = \left\{ \mathbf{M} \mid \mu_{m,k} \geq 0, \|\boldsymbol{\mu}_m\|^2 \leq \frac{1}{N}, \forall m, k \right\}, \quad (4.5)$$

where $\boldsymbol{\mu}_m$ denotes the m th row of $\mathbf{M}$. Let s_k be the data symbol transmitted to the k th UE, satisfying $\mathbb{E}[|s_k|^2] = 1$, and let ζ_d represent the maximum downlink transmit power per symbol at each AP normalized by noise power. Then, the downlink signal transmitted from the m th AP using conjugate beamforming can be expressed as

$$\mathbf{x}_m = \sqrt{\zeta_d} \sum_{k=1}^K \frac{\mu_{m,k}}{\sqrt{g_{m,k}}} \hat{\mathbf{g}}_{m,k}^* s_k. \quad (4.6)$$

Assuming IID Gaussian noise with zero mean and unit variance at UEs, and applying the use-and-then-forget bounding technique, the SINR for the k th UE can be derived as [75]

$$\gamma_k = \frac{\zeta_d (\bar{\boldsymbol{\mu}}_k^T \mathbf{v}_{k,k})^2}{\sum_{i=1, i \neq k}^K \zeta_d (\bar{\boldsymbol{\mu}}_i^T \mathbf{v}_{i,k})^2 + \frac{\zeta_d}{N} \sum_{i=1}^K \|\mathbf{B}_{D,k} \bar{\boldsymbol{\mu}}_i\|^2 + \frac{1}{N^2}}, \quad (4.7)$$

where $\bar{\boldsymbol{\mu}}_k$ is the k th column of $\mathbf{M}$, $\mathbf{B}_{D,k}$ is a diagonal matrix with diagonal elements $\sqrt{\beta_{1,k}}, \dots, \sqrt{\beta_{M,k}}$, and the m th element of $\mathbf{v}_{i,k} \in \mathbb{R}^M$ is given by

$$\mathbf{v}_{i,k}[m] = |\boldsymbol{\varphi}_i^H \boldsymbol{\varphi}_k| \sqrt{g_{m,i}} \frac{\beta_{m,k}}{\beta_{m,i}}. \quad (4.8)$$

The downlink network SE of the k th UE is then given by

$$\text{SE}_k = \left(1 - \frac{T_p}{T_c} \right) \log_2 (1 + \gamma_k). \quad (4.9)$$

4.1.2 Optimization Problem Formulation

Let $\mathbf{B} \in \mathbb{R}_{++}^{M \times K}$ denote the large-scale fading coefficient matrix, with $\beta_{m,k}$ as its (m, k) -th entry. Let $\boldsymbol{\Phi} \in \mathbb{R}_+^{K \times K}$ be the pilot allocation matrix, whose (i, j) -th entry is $|\boldsymbol{\varphi}_i^H \boldsymbol{\varphi}_j|$. Here, $\mathbb{R}_{++}^{M \times K}$ and $\mathbb{R}_+^{M \times K}$ denote the sets of $M \times K$ matrices with positive and non-negative real entries, respectively. According to (4.7) and (4.9), the SE is a function of the power control coefficient matrix $\mathbf{M}$ and the pilot allocation matrix $\boldsymbol{\Phi}$ for a given large-scale fading matrix $\mathbf{B}$.

Aiming to achieve uniform SE across all UEs, we adopt a max-min fairness criterion and formulate the objective as

$$\underset{\mathbf{M}}{\text{maximize}} \left\{ \min_{1 \leq k \leq K} \text{SE}_k(\mathbf{M}; \boldsymbol{\Phi}, \mathbf{B}) \mid \mathbf{M} \in \mathcal{S} \right\}. \quad (4.10)$$

Using a smoothing technique with smoothing parameter λ_s , the objective function in (4.10) can be reformulated as [75]

$$u(\mathbf{M}; \Phi, \mathbf{B}) = -\frac{1}{\lambda_s} \log \left(\frac{1}{K} \sum_{k=1}^K \exp(-\lambda_s \text{SE}_k(\mathbf{M}; \Phi, \mathbf{B})) \right). \quad (4.11)$$

To address the optimization problem in (4.11), we develop a learning-based approach in which a deep neural network models the mapping $\mathbf{M} = f(\Phi, \mathbf{B}; \Theta)$, where f maps the pilot allocation matrix Φ and the large-scale fading matrix $\mathbf{B}$ to the power control matrix $\mathbf{M}$. Here, Θ denotes the trainable network weights. Within this framework, the network learns the mapping from its input to the desired power control coefficients $\mathbf{M}$ with the objective of maximizing the function in (4.11), ultimately yielding the optimal network weights Θ_{opt} . Moreover, the network is trained in a self-supervised manner, using unlabeled data as the training input.

4.2 Graph Attention Network for Power Control Optimization

GNNs are a class of deep learning models tailored for graph-structured data, where nodes represent entities and edges depict their interactions. Graph attention networks extend GNNs by assigning learnable attention weights to edges, allowing the model to focus on more relevant connections. By mapping the CFmMIMO network to a graph structure, graph attention networks can efficiently capture the relationship among APs and UEs and naturally adapt to large-scale topologies. Therefore, the graph attention network presents a promising solution to the CFmMIMO power control problem.

4.2.1 Graph Representation

To effectively capture the complex dependencies and interactions among UEs and APs in CFmMIMO networks, we model the network as a heterogeneous graph, denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, and this graph is composed of a set of nodes $\mathcal{V}$ and edges $\mathcal{E}$. Each node represents a unique AP-UE pair, and it is indexed by $v = M(k-1) + m$, where $m \in \{1, \dots, M\}$, $k \in \{1, \dots, K\}$. The bidirectional edge between the i th and j th nodes is denoted as $e_{i,j}$ for $i \neq j$. To capture the different dependencies between nodes, edges are categorized based on whether the connected nodes share the same AP, the same UE, or neither:

- **AP-type edges** ($e_{i,j}^{\text{AP}}$): Nodes share the same AP.
- **UE-type edges** ($e_{i,j}^{\text{UE}}$): Nodes share the same UE.
- **No connection**: Nodes that share neither an AP nor a UE, no edge existing between them.

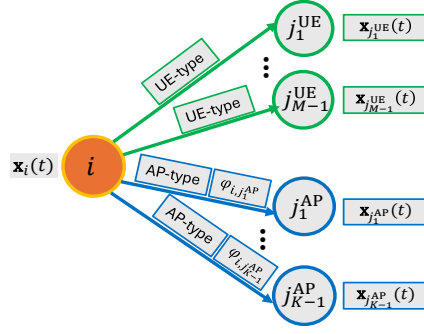


Figure 4.1. Illustration of graph characteristics.

To enable pilot contamination awareness, the pilot allocation matrix Φ is used as an attribute for AP-type edges, capturing the orthogonality of pilot sequences among UEs served by the same AP. Specifically, the (k_1, k_2) -th element of Φ , denoted by $\bar{\varphi}_{k_1, k_2}$, is assigned to AP-type edges connecting nodes associated with the k_1 -th and k_2 -th UEs. Within the GNN framework, we define $\varphi_{i,j} = \bar{\varphi}_{k_1, k_2}$ to represent the pilot contamination information between nodes i and j , which are respectively associated with k_1 -th and k_2 -th UEs.

The characteristics of the proposed graph structure, including the node feature representation $\mathbf{x}_i(t)$, the AP-type edges with $\varphi_{i,j}$, and the UE-type edges, are shown in Fig. 4.1. This design allows the network to effectively capture the relationship between APs and UEs while accounting for pilot contamination.

4.2.2 Proposed Graph Attention Network Model

The proposed graph neural network consists of three blocks, which are elaborated in the following.

1) *Preprocessing Stage*: Considering that the large-scale fading coefficients span multiple orders of magnitude, the natural logarithm operation is first applied to each element of $\mathbf{B}$, yielding $\mathbf{B}^{\log}$. $\mathbf{B}^{\log}$ is then standardized to zero mean and unit variance across all elements, resulting in $\mathbf{B}^{\text{std}}$. Next, the row-wise scaling and shifting are applied to $\mathbf{B}^{\text{std}}$, which can be expressed as

$$\mathbf{B}^{\text{pre}} = (\alpha \mathbf{1}_K^T) \odot \mathbf{B}^{\text{std}} + \beta \mathbf{1}_K^T, \quad (4.12)$$

where α and β respectively denote the trainable parameters for scaling and shifting, and $\mathbf{1}_K$ is an all-ones vector of length K . The resulting matrix $\mathbf{B}^{\text{pre}}$ is then vectorized into $\mathbf{b} \in \mathbb{R}^{MK \times 1}$.

2) *Attention Mechanism Block*: The attention mechanism enables the model to focus on specific parts of the input data by assigning different levels of importance to each component. To capture the inter-node relationship, a single-head attention mechanism is employed. The attention

mechanism block consists of $T = 4$ hidden layers, and input and output features of t th graph layer for the i th node are denoted as $\mathbf{x}_i(t-1)$ and $\mathbf{x}_i(t)$, $t = 1, \dots, T$, with $\mathbf{b}$ as the input to the first layer, i.e., $\mathbf{x}_i(0)$. The respective output sizes of 4 layers are 32, 64, 64, and 64. The attention mechanism structure will be detailed in Subsection 4.2.3.

3) *Postprocessing Stage*: Following the attention mechanism block, post-processing is performed on $\mathbf{x}_i(4)$ to refine the node feature representation and ensure the final network output lies within the feasible solution set. A combination of linear and nonlinear processing is first performed, yielding the following output

$$\mathbf{Y}_1 = \mathcal{L}_2\left(\text{ReLU}(\mathcal{L}_1(\mathbf{X}(4)))\right), \quad (4.13)$$

where the i th row of $\mathbf{X}(4) \in \mathbb{R}^{MK \times 64}$ is $\mathbf{x}_i(4)$, $\mathcal{L}_1$ and $\mathcal{L}_2$ denote two distinct linear transformations, each mapping a 64-dimensional input to a 64-dimensional output, and $\text{ReLU}(\cdot)$ represents the rectified linear unit activation function.

Next, the node representation vectors are passed through another linear transformation to map them into scalar values. Subsequently, additional processing steps, similar to those in [27], are applied to obtain the targeted power control coefficients, $\hat{\mathbf{M}}$. These post-processing procedures are summarized as

$$\begin{aligned} \mathbf{y}_2 &= \exp\left(-\text{SoftPlus}(\mathcal{L}_3(\mathbf{Y}_1) + 6)\right), \\ \hat{\mathbf{M}} &= \text{Proj}_{\mathcal{S}}(\mathbf{Y}_2). \end{aligned} \quad (4.14)$$

Here, $\text{SoftPlus}(\cdot)$ is the Softplus activation function, $\mathcal{L}_3$ is a linear function that maps a vector to a scalar value, and $\mathbf{Y}_2 \in \mathbb{R}^{M \times K}$ is obtained by reshaping $\mathbf{y}_2$. To extract the power control coefficients for the active UEs while satisfying the constraints in (4.5), the projection operation $\text{Proj}_{\mathcal{S}}(\cdot)$ in [75] is applied at the final step.

4.2.3 Attention Mechanism for Graph Neural Network

For the proposed attention mechanism block, all layers share the same structure and differ only in the sizes of their inputs and outputs. Therefore, we use the t th layer as a representative example to illustrate the network structure. The output node representation vector of the t th layer is computed by

$$\mathbf{x}_i(t) = \text{Norm}\left(\text{ReLU}(\mathbf{y}_i^{\text{AP}}(t) + \mathbf{y}_i^{\text{UE}}(t))\right). \quad (4.15)$$

Here, $\text{Norm}(\cdot)$ denotes the normalization operation. $\mathbf{y}_i^{\text{AP}}(t)$ represents the component of the update produced by the attention mechanism based on the information from nodes associated with the same APs as the i th node and the corresponding pilot coherence information $\varphi_{i,j}$, which can be

described by

$$\mathbf{y}_i^{\text{AP}}(t) = \mathcal{L}_1^{\text{AP},t}(\mathbf{x}_i(t-1)) + \mathcal{L}_5^{\text{AP},t} \left(\sum_{j \in \mathcal{V}_i^{\text{AP}}} \alpha_{i,j}^{\text{AP}} \left[\mathcal{L}_2^{\text{AP},t}(\mathbf{x}_j(t-1)) + \mathcal{L}_\varphi^t(\varphi_{i,j}) \right] \right), \quad (4.16)$$

$$\alpha_{i,j}^{\text{AP}} = \frac{\left\langle \mathcal{L}_3^{\text{AP},t}(\mathbf{x}_i(t-1)), \mathcal{L}_4^{\text{AP},t}(\mathbf{x}_j(t-1)) + \mathcal{L}_\varphi^t(\varphi_{i,j}) \right\rangle}{\sum_{u \in \mathcal{V}_i^{\text{AP}}} \left\langle \mathcal{L}_3^{\text{AP},t}(\mathbf{x}_i(t-1)), \mathcal{L}_4^{\text{AP},t}(\mathbf{x}_u(t-1)) + \mathcal{L}_\varphi^t(\varphi_{i,u}) \right\rangle}. \quad (4.17)$$

Here, $\mathcal{L}_q^{\text{AP},t}$ ($q = 1, \dots, 5$) represents 5 different linear transformations used to capture information from AP-type edges, while $\mathcal{L}_\varphi^t$ is a linear transformation that captures pilot contamination information in the t th layer. Moreover, $\mathcal{V}_i^{\text{AP}}$ represents the set of nodes that share the same AP as the i th node, and the similarity function is defined as $\langle \mathbf{a}, \mathbf{b} \rangle = \exp(\mathbf{a}^T \mathbf{b} / \sqrt{D})$, where D is the length of $\mathbf{a}$.

Note that $\mathbf{y}_i^{\text{UE}}(t)$ is obtained in a similar manner but without accounting for pilot contamination. This is because the update relies solely on information from nodes associated with the same UE, where no pilot allocation competition occurs. Specifically, $\mathcal{L}_\varphi^t(\cdot)$ is omitted since pilot contamination only occurs between UEs.

4.2.4 Padding Strategy for Dataset Preparation

To satisfy the requirement in modern deep-learning frameworks such as PyTorch, where all tensors within a batch must share the same dimensions, we adopt a zero-padding strategy to handle training samples with a varying number of UEs. Specifically, the number of UEs K is fixed to a predefined maximum value $K_{\max}$ within the coverage area. When the actual number of UEs, denoted by K_{act} , is smaller than $K_{\max}$, additional $K_{\max} - K_{\text{act}}$ virtual UEs are introduced as padding. During data preparation, zero-padding is applied to $\mathbf{B}$ and Φ , producing padded inputs of sizes $M \times K_{\max}$ and $K_{\max} \times K_{\max}$, respectively. To eliminate the influence of padded UEs, the following modifications are made. In (4.14), $\mathbf{Y}_2$ is replaced with $\mathbf{Y}_2 \Phi_{\text{mask}}$; in (4.16), the attention coefficient $\alpha_{i,j}^{\text{AP}}$ is modified to $\varphi_{j,j} \alpha_{i,j}^{\text{AP}}$. Here, Φ_{mask} is diagonal whose diagonal entries match those of Φ after padding.

4.3 Experiment Results

The performance of the proposed method is evaluated in comparison with two other power allocation methods. The APG method, a conventional optimization-based method as described in [75], is used as a baseline to evaluate the effectiveness of the proposed graph attention network. The transformer-based method in [27], referred to as PAPC, is also included as a

representative learning-based benchmark. Given that existing GNN-based approaches typically assume pilot contamination-free scenarios, while the proposed method explicitly addresses pilot contamination, they are not considered in the comparative evaluation.

The empirical cumulative distribution function (CDF) of the per-UE SE is used to visualize the SE distribution. Besides, we provide the running times of methods to evaluate their feasibility in real-time applications.

Table 4.1. Parameters for 5 scenarios.

Parameters	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
Coverage Area (km ²)	0.16	0.32	0.32	0.32	0.32
M	16	32	32	64	64
$K_{\max}$	8	20	20	40	40
$K_{\min}$	8	20	10	40	20
Pilot Contamination	×	✓	✓	✓	✓
Varying UEs	×	×	✓	×	✓
Training Samples	50K	100K	800K	100K	800K

To evaluate the adaptability and scalability of the proposed method, the performances of the three methods are assessed across five scenarios. The parameter settings for the five scenarios are listed in Table 4.1. For example, in Scenario 1, $K_{\max} = 8$ UEs are served by $M = 16$ APs, with both UEs and APs uniformly randomly distributed within a coverage area of 0.16km² in both the training and test datasets.¹ Since $K_{\max} < T_p$ in this case ($T_p = 18$ is assumed), no pilot contamination occurs. The number of UEs is fixed, so $K = K_{\max} = K_{\min}$ for all samples. If the number of UEs is varying, as in Scenarios 3 and 5, the number of UEs varies within the range $[K_{\min}, K_{\max}]$. In Scenarios 2-5, where $K_{\max} > T_p$, pilot contamination arises. All scenario results are evaluated on 500 test samples, and the number of training samples used for each scenario is provided in Table 4.1 as well. It is emphasized that, for each scenario, the training data preparation and model training are performed offline and need to be conducted only once. The resulting trained model can then be applied to different UE-AP network configurations without requiring retraining.

The CDF results for the five scenarios are provided in Fig. 4.2. In Figs. 4.2 and 4.3, the proposed Graph Attention network method is labeled as GAT. For the max-min fairness objective, a steeper CDF curve that shifts towards the right indicates better performance, reflecting improved fairness among UEs and higher SEs for all UEs. From Fig. 4.2, it is evident that the proposed method consistently outperforms both APG and PAPC

¹The generalizability of the trained model when training and test datasets follow different UE-AP deployment distributions depends on how well the test deployments are represented in the training dataset. A systematic investigation of this generalizability is important, but it is out of the scope of this work.

across all scenarios, demonstrating the proposed method's adaptability to different network settings, including variations in network size, pilot contamination, and whether the number of UEs is fixed or varying. In Scenarios 3 and 5, the CDF curves rise less sharply than in other scenarios. This is due to the varying number of active UEs in each test sample, which results in a wider distribution of SE per UE.

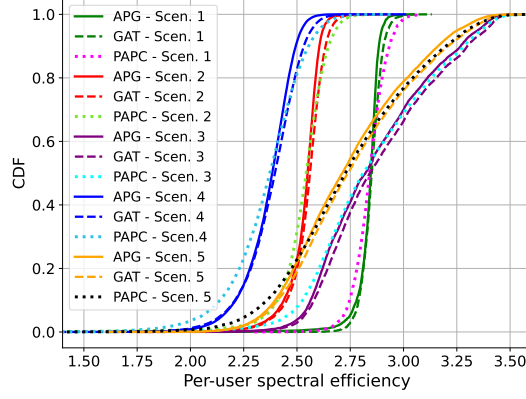


Figure 4.2. Result comparison between 3 methods across 5 scenarios.

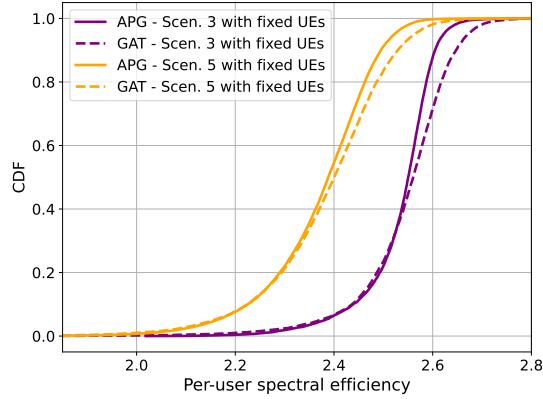


Figure 4.3. Performance comparison for the proposed method trained with a varying number of UEs but tested with fixed UEs.

Next, to evaluate the flexibility of the proposed method, the models trained in Scenarios 3 and 5 with varying numbers of UEs are tested under fixed UE settings, specifically $K = 20$ for Scenario 3 and $K = 40$ for Scenario 5. As shown in Fig. 4.3, the models trained on datasets with varying numbers of UEs still perform well, even outperforming the APG method.

The runtime for each method across different scenarios is presented in Table 4.2. CPU measurements were obtained on a 64-bit Windows system with 16 GB RAM and an Intel Core i5 (2.60 GHz), while GPU measure-

ments were obtained using an NVIDIA A40. As shown, in Scenario 5 with 64 APs and 40 UEs, the APG method requires 5.5185[s] of GPU processing time, whereas the proposed method achieves only 2.2[ms] on the GPU and 592[ms] on the CPU. This highlights the real-time capability of the proposed method and its potential for power control in real-time applications. While the PAPC method exhibits a slightly lower runtime than the proposed model, the improved performance of the proposed approach justifies this negligible difference.

Table 4.2. Runtime comparison between methods (in seconds).

	Scen. 1		Scen. 3		Scen. 5	
	CPU	GPU	CPU	GPU	CPU	GPU
APG	1.5626	1.1391	3.5581	2.4226	11.2485	5.5185
PAPC	0.0022	0.0012	0.0134	0.0013	0.0167	0.0014
Proposed	0.0083	0.0020	0.0613	0.0021	0.5920	0.0022

4.4 Conclusion and Discussion

This chapter presented an attention mechanism enhanced GNN model for power control in CFmMIMO systems. The developed model effectively captures information from neighboring nodes through an attention mechanism and explicitly accounts for pilot contamination by incorporating the cross-correlation of pilot sequences as an input. Moreover, it can adapt to dynamic scenarios with a varying number of UEs using a padding mechanism. In addition, the network is trained in a self-supervised manner, thereby reducing the computational burden associated with computing the power control coefficients. Experimental results show that the proposed GNN model achieves performance comparable to that of APG while significantly reducing the computational complexity, and it outperforms the pilot contamination-aware transformer-based neural network, highlighting the suitability of GNNs for CFmMIMO applications. The real-time capability of the proposed approach further underscores its potential to meet the ultra-low-latency requirements expected in future 6G applications.

5. Conclusion and Future Research Directions

5.1 Conclusion

This thesis addressed critical research challenges in future 6G wireless systems, focusing specifically on waveform design for vehicular ISAC and power control in CFmMIMO systems. Driven by the stringent performance demands of 6G networks, a set of methodologies was developed, as outlined below.

First, this thesis addressed the problem of TB design in MIMO sensing systems. A slow-time TB design was developed to improve transmit energy distribution while preserving waveform diversity. Based on the design, a new tensor-based signal model was formulated, and PARAFAC decomposition was employed, enabling accurate parameter estimation and resolving velocity ambiguity.

Second, this thesis investigated ISAC in vehicular scenarios. A TD ISAC framework was first introduced to flexibly balance the requirements of S&C. Within this framework, a multibeam sensing strategy was developed to achieve wide-area coverage and improved direction estimation accuracy compared to conventional beam-scanning approaches. The impact of beamforming-induced null directions was analyzed, and an effective solution was provided. Moreover, a new identifiability condition for Vandermonde-structured factor matrices was established for PARAFAC-based parameter estimation.

Third, power control in CFmMIMO systems under pilot contamination was studied. A GNN model enhanced with an attention mechanism was proposed to solve the max-min fairness power allocation problem. By exploiting graph-structured representations, the proposed model significantly reduces the computational complexity compared to conventional optimization-based approaches while exhibiting superior performance. Furthermore, the model demonstrates scalability with respect to network size and adaptability to a dynamic number of UEs. In addition, a self-

supervised training strategy was adopted, eliminating the need for labeled data and thereby enhancing practical deployability.

To conclude, this thesis presented a comprehensive and effective framework for tackling key challenges in 6G systems, delivering promising solutions that advance the development of future wireless networks.

5.2 Future Research Directions

Following the research topics addressed in this thesis, multiple promising directions for further investigation remain.

The proposed TB design in Chapter 2 was developed under the assumption of a ULA transmitter. Although ULAs are widely adopted in practical radar systems, alternative array configurations are receiving increasing attention as cost-effective and hardware-efficient solutions. Accordingly, how transmit array configurations affect the transmit energy distribution under the proposed TB design, and whether the framework can be adapted to non-ULA transmitters or a fundamentally different TB design is required, remain important issues for future investigation.

Chapter 3 focuses on wide-coverage sensing within a TD vehicular ISAC framework, while key communication protocol components such as inter-slot guard intervals, frame boundary design, and synchronization overhead are not explicitly modeled. Moreover, sensing requirements such as the unambiguous detectable range, as well as the utilization of sensing results to enhance communication performance, introduce additional coupling in determining time allocation. Therefore, a more integrated design that jointly considers protocol-level implementation under practical constraints represents an important direction for future research.

Another further potential research direction is the extension of the proposed attention-based GNN model to a hierarchical federated learning framework for scalable power control in CFmMIMO systems. The GNN model presented in Chapter 4 adopts a fully centralized architecture, which incurs substantial communication overhead between the APs and the CPU. In contrast, a hierarchical framework with adaptive clustering can be adopted to enable localized training and aggregation within groups of APs, thereby reducing the fronthaul signaling overhead and improving robustness under statistically heterogeneous data distributions.

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Appendix: Derivation of (3.28)

Let the SVD of $\mathbf{Y}$ be $\mathbf{Y} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H$, where $\mathbf{U} \in \mathbb{C}^{T_1 T_2 \times K}$, $\mathbf{\Lambda} \in \mathbb{C}^{K \times K}$, and $\mathbf{V} \in \mathbb{C}^{L_1 L_2 M \times K}$. Under condition (3.28a), which guarantees that $\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}$ has full column rank, there exists a nonsingular matrix $\mathbf{M} \in \mathbb{C}^{K \times K}$ such that $\mathbf{U}\mathbf{M} = \mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}$. By exploiting the Vandermonde structure of $\mathbf{B}$, we have

$$\left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) \mathbf{Z}_{\mathbf{B}} = \overline{\mathbf{B}^{(T_1)}} * \mathbf{D}_{\text{amb}}^{(T_2)}, \quad (1)$$

where $\mathbf{Z}_{\mathbf{B}}$ denotes a diagonal matrix with diagonal entries given by the generators of $\mathbf{B}$, namely $z_{\mathbf{B},1}, \dots, z_{\mathbf{B},K}$. Then we have

$$\begin{aligned} \mathbf{U}_1 \mathbf{M} \mathbf{Z}_{\mathbf{B}} &= \left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) \mathbf{Z}_{\mathbf{B}} = \overline{\mathbf{B}^{(T_1)}} * \mathbf{D}_{\text{amb}}^{(T_2)} = \mathbf{U}_2 \mathbf{M}, \\ \mathbf{U}_1 &= \mathbf{U}(1:(T_1-1)T_2, :), \quad \mathbf{U}_2 = \mathbf{U}(T_2+1:T_1 T_2, :). \end{aligned} \quad (2)$$

Since $\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}$ has full column rank, $\mathbf{U}_1$ and $\mathbf{U}_2$ also have full column rank. Hence, $\mathbf{Z}_{\mathbf{B}}$ can be obtained via EVD as $\mathbf{U}_1^\dagger \mathbf{U}_2 = \mathbf{M} \mathbf{Z}_{\mathbf{B}} \mathbf{M}^{-1}$. The set $\{z_{\mathbf{B},k}\}_{k=1}^K$ can then be obtained by extracting the diagonal entries of $\mathbf{Z}_{\mathbf{B}}$.

Next, we rearrange the estimated matrix $\mathbf{B}$ according to the grouping structure specified in (3.27), and then apply the same ordering to $\mathbf{M}$, yielding $\mathbf{M} = [\mathbf{M}_1, \dots, \mathbf{M}_G]$.

When $P_g = 1$, $\mathbf{m}_g$ is used instead. If an eigenvalue, $z_{\mathbf{B},g}$, is isolated, its corresponding eigenvector is uniquely determined up to a scaling factor, and thus, $\mathbf{d}_{\text{amb},g}^{(T_2)}$ can be directly obtained by

$$\mathbf{d}_{\text{amb},g}^{(T_2)} = (\mathbf{b}_g^{(T_1)H} \otimes \mathbf{I}_{T_2}) \mathbf{U} \mathbf{m}_g, \quad (3)$$

where $\mathbf{b}_g^{(T_1)}$ is constructed using the estimated $z_{\mathbf{B},g}$.

When an eigenvalue is repeated with multiplicity $P_g > 1$, its corresponding eigenvectors span the associated eigenspace. Although these eigenvectors are mutually orthogonal, they are not unique, as any basis spanning the same eigenspace is valid. Consequently, $\{\mathbf{d}_{\text{amb},k}^{(T_2)}\}_{k=k_1}^{k_{P_g}}$ cannot be uniquely determined using (3). However, by exploiting the Vandermonde structure of $\mathbf{D}_{\text{amb}}$, the set $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=k_1}^{k_{P_g}}$ can still be recovered from $\mathbf{M}_g$ as follows.

Given a repeated eigenvalue $z_{\mathbf{B},g}$ with multiplicity P_g and its associated eigenvectors $\{\mathbf{m}_k\}_{k=k_1}^{k_{P_g}}$, which correspond to the column vectors of $\mathbf{M}_g$, we apply the same procedure as in (3) to compute the estimates, but denote the computed results as $\{\mathbf{d}_{\text{amb},k}^{(T_2)'}\}_{k=k_1}^{k_{P_g}}$ to distinguish them from $\{\mathbf{d}_{\text{amb},k}^{(T_2)}\}_{k=k_1}^{k_{P_g}}$, due to the non-uniqueness of the eigenvectors. Then we can have

$$\mathbf{D}_{\text{amb},g}^{(T_2)'} = [\mathbf{d}_{\text{amb},k_1}^{(T_2)'}, \mathbf{d}_{\text{amb},k_2}^{(T_2)'}, \dots, \mathbf{d}_{\text{amb},k_{P_g}}^{(T_2)'}] \in \mathbb{C}^{T_2 \times P_g}, \quad (4)$$

and it satisfies

$$\left(\mathbf{1}_{P_g}^T \otimes \mathbf{b}_g^{(T_1)}\right) * \mathbf{D}_{\text{amb},g}^{(T_2)'} = \mathbf{U} \mathbf{M}_g. \quad (5)$$

Construct $\mathbf{U}_{\mathbf{D},1} = \mathbf{D}_{\text{amb},g}^{(T_2)'}$ and $\mathbf{U}_{\mathbf{D},2} = \overline{\mathbf{D}_{\text{amb},g}^{(T_2)'}}$. Then, the set $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=k_1}^{k_{P_g}}$ can be obtained from $\overline{\mathbf{Z}_{\mathbf{D}_{\text{amb},g}}}$ via EVD as follows:

$$\mathbf{U}_{\mathbf{D},1}^\dagger \mathbf{U}_{\mathbf{D},2} = \mathbf{M}_{\mathbf{D}_{\text{amb},g}} \mathbf{Z}_{\mathbf{D}_{\text{amb},g}} \mathbf{M}_{\mathbf{D}_{\text{amb},g}}^{-1}. \quad (6)$$

Note that this EVD-based approach holds when both $\mathbf{U}_{\mathbf{D},1}$ and $\mathbf{U}_{\mathbf{D},2}$ are full column rank, which explains why condition in (3.28c) is required to guarantee the uniqueness of PARAFAC decomposition.

From (3.28a), it follows that $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=k_1}^{k_{P_g}}$ are distinct for $g = 1, \dots, G$. Using the EVD in (6), we can then obtain

$$\mathbf{D}_{\text{amb},g}^{(T_2)} = \mathbf{D}_{\text{amb},g}^{(T_2)'} \mathbf{M}_{\mathbf{D}_{\text{amb},g}}. \quad (7)$$

From (5) and (7), one can have

$$\left(\mathbf{1}_{P_g}^T \otimes \mathbf{b}_g^{(T_1)}\right) * \left(\mathbf{D}_{\text{amb},g}^{(T_2)} \mathbf{M}_{\mathbf{D}_{\text{amb},g}}^{-1}\right) = \mathbf{U} \mathbf{M}_g, \quad (8)$$

which yields to the equivalent formulation:

$$\left(\mathbf{1}_{P_g}^T \otimes \mathbf{b}_g^{(T_1)}\right) * \mathbf{D}_{\text{amb},g}^{(T_2)} = \mathbf{U} \mathbf{M}_g \mathbf{M}_{\mathbf{D}_{\text{amb},g}}. \quad (9)$$

Introduce the matrix $\mathbf{N}_g = \mathbf{M}_g \mathbf{M}_{\mathbf{D}_{\text{amb},g}}$, $g = 1, \dots, G$. Note that, for an isolated eigenvalue $z_{\mathbf{B},g}$, we have $\mathbf{N}_g = \mathbf{m}_g$. Consequently, (9) can be written in a compact form as

$$\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} = \mathbf{U} \mathbf{N}, \quad (10)$$

where $\mathbf{N} = [\mathbf{N}_1, \dots, \mathbf{N}_G]$.

Based on the above, the SVD of $\mathbf{Y}$ can be rewritten as

$$\mathbf{Y} = (\mathbf{U} \mathbf{N}) (\mathbf{V} \Lambda^* \mathbf{N}^{-H})^H = \left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}\right) (\mathbf{V}^* \Lambda \mathbf{N}^{-T})^T. \quad (11)$$

With $\{z_{\mathbf{B},k}\}_{k=1}^K$, $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=1}^K$, and $\mathbf{N}$, the third factor matrix $\mathbf{A}_{\text{MB}}$ can then be obtained by

$$\mathbf{a}_{\text{MB},k} = \left(\frac{\mathbf{b}_k^{(L_1)H}}{\mathbf{b}_k^{(L_1)H} \mathbf{b}_k^{(L_1)}} \otimes \frac{\mathbf{d}_{\text{amb},k}^{(L_2)H}}{\mathbf{d}_{\text{amb},k}^{(L_2)H} \mathbf{d}_{\text{amb},k}^{(L_2)}} \otimes \mathbf{I}_M \right) \mathbf{V}^* \Lambda \mathbf{c}_k, \quad (12)$$

where $\mathbf{c}_k$ is the k th row vector of matrix $\mathbf{N}^{-1}$.

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TRANSMIT ENERGY FOCUSING FOR PARAMETER ESTIMATION IN TRANSMIT BEAMSPACE SLOW-TIME MIMO RADAR

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ABSTRACT

Recently, Parallel Factor-Direct (PARAFAC-Direct) method has been proposed for parameter estimation including velocity disambiguation for Doppler Division Multiple Access (DDMA) Multiple-Input Multiple-Output (MIMO) radar. However, DDMA MIMO radar spreads the overall transmit energy into the entire spatial region, and therefore, suffers from beam-shape loss that can limit the performance of PARAFAC-Direct method. To solve this problem, a Transmit Beam-space (TB) Slow-Time MIMO (ST-MIMO) approach is proposed that focuses the transmit energy within a desired spatial region. Unlike traditional DDMA MIMO radars, the Doppler spectrum is divided into more subbands than the number of transmit elements to reduce the mainlobe intervals between adjacent beams formed by DDMA modulation vectors. Then, the TB ST-MIMO beam set can be directed to the spatial region of interest via a proper selection of DDMA modulation vectors. Parameter estimation performance of TB ST-MIMO is improved as compared to conventional DDMA MIMO techniques. Simulations are conducted to validate the proposed method.

Index Terms— PARAFAC, Parameter estimation, TB ST-MIMO, Velocity disambiguation

1. INTRODUCTION

In the last two decades, intensive studies have been conducted on Multiple-Input Multiple-Output (MIMO) radar [1–3] due to its excellent multiple targets detection and parameter estimation performance. Depending on the application scenarios, the design of orthogonal waveforms is critical to MIMO radar systems. Many waveforms have been studied, including Frequency Division Multiple Access (FDMA) [4], Code Division Multiple Access (CDMA) [5], Time Division Multiple Access (TDMA) [6] and Doppler Division Multiple Access (DDMA) [5, 7] waveforms. Each of them has its own strengths and weaknesses. Among those waveforms, TDMA waveform stands out due to its low hardware cost and ease of implementation.

However, transmission capability of TDMA waveform is underutilized when compared to other waveforms whose

transmit elements emit signals simultaneously. In other words, TDMA waveforms suffer a shorter target detectable range due to the loss of average transmit power. Moreover, more transmit elements will be required for a higher demand on the system performance in the future, further limiting the performance of TDMA waveforms. To tackle this problem, DDMA technique has been proposed [8, 9]. Inheriting the transmitter configuration of TDMA waveform, DDMA achieves waveform diversity in Doppler domain by modulating each transmit channel with a different Doppler shift. The transmission capability is fully utilized and transmit channel separation can be achieved through Doppler spectrum division.

Direction of arrival (DOA) estimation is the key parameter estimation problem in sensor array signal processing. Sensor array processing tools range from non-parametric Fourier-based methods [10] to parametric high-resolution techniques such as Multiple Signal Classification (MUSIC) [11, 12] and Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [13]. To maintain the multi-linear structure in the measurement data, tensor representation has also been introduced. Algorithms including Parallel Factor (PARAFAC) [14, 15] analysis techniques, Higher Order Singular Value Decomposition (HOSVD) [16] and constrained tensor decomposition [17], have been proposed to achieve better parameter estimation performance in low signal-to-noise-ratio (SNR) cases. Recently, a PARAFAC-Direct approach [18] has been proposed to exploit the benefits of both DDMA technique and tensor representation for slow-time MIMO radar parameter estimation. Nevertheless, the mainlobes shaped by DDMA modulation vectors are directed across the whole spatial region rather than a certain limited region of interest [8, 19], which causes serious beam-shape loss problem that can degrade the parameter estimation performance.

To solve this problem, a method called Transmit Beam-space (TB) Slow-Time MIMO (ST-MIMO) is proposed to optimize the distribution of the transmit energy. Different from the traditional DDMA MIMO technique, Doppler spectrum is divided into more subbands than the number of transmit elements. The mainlobes of two adjacent beams shaped by DDMA modulation vectors get closer, which enables the TB

ST-MIMO beam set focused in a given spatial region. The proposed method can also be regarded as achieving waveform orthogonality by phase coding in slow time [20], and thus, it is referred to as TB ST-MIMO. Then, PARAFAC approach is used to improve the parameter estimation performance including velocity disambiguation. The effectiveness of the proposed method is validated by simulation experiments.

2. SIGNAL MODEL AND PROBLEM FORMULATION

2.1. DDMA MIMO Radar

Consider a bistatic MIMO radar system equipped with M_t transmit antennas and N receive antennas. Both of the transmit and receive arrays are uniform linear arrays (ULAs), and the spacings of two adjacent transmitters and receivers are d_t and d_r measured in the propagation wavelength λ , respectively. Assuming that there are K targets with θ_k , φ_k and f_k being the Direction of Departure (DOD), DOA and Doppler frequency of k th target, respectively, the steering vectors of transmit and receive arrays are

$$\begin{aligned} \mathbf{a}(\theta_k) &= [1, e^{-j2\pi d_t \sin \theta_k}, \dots, e^{-j2\pi d_t (M_t-1) \sin \theta_k}]^T, \\ \mathbf{b}(\varphi_k) &= [1, e^{-j2\pi d_r \sin \varphi_k}, \dots, e^{-j2\pi d_r (N-1) \sin \varphi_k}]^T, \end{aligned} \quad (1)$$

where $(\cdot)^T$ denotes the transpose operation. We also assume that Q pulses with pulse repetition interval (PRI) T_r are transmitted in a single coherent processing interval (CPI) for each transmit element.

Different from conventional MIMO transmission schemes that achieve waveform diversity in fast time, DDMA MIMO approach obtains orthogonality in slow time by phase-coding the signals of different transmit channels with a predesigned phase modulation matrix $\mathbf{W} \triangleq [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_Q]_{M_t \times Q}$. Signals from different transmit elements are modulated with different Doppler shifts. To divide the entire Doppler frequency band into M_t subbands uniformly, the corresponding Doppler shift of each transmit element is

$$f_m = (m-1) \frac{\text{PRF}}{M_t}, m = 1, 2, \dots, M_t, \quad (3)$$

where $\text{PRF} = 1/T_r$ denotes the pulse repetition frequency. For the q th pulse, the phase modulation vector $\mathbf{w}_q$ for a transmit array can be written as

$$\mathbf{w}_q = e^{j2\pi \mathbf{f}(q-1)T_r}, q = 1, 2, \dots, Q, \quad (4)$$

where $\mathbf{f} = [f_1, f_2, \dots, f_{M_t}]^T$. After pulse compression and range gating, the signal received by the n th receive element for the q th pulse is

$$x_{nq} = \sum_{k=1}^K \sigma_k^2 e^{j2\pi f_k q T_r} (\mathbf{a}^T(\theta_k) \mathbf{w}_q) b_n(\varphi_k) + z_{nq}, \quad (5)$$

where σ_k^2 is the complex reflection coefficient of the k th target, $b_n(\varphi_k)$ denotes the n th element of $\mathbf{b}(\varphi_k)$, and z_{nq} is the residual noise after pulse compression processing.

From (5), it can be seen that the transmit beampattern of each pulse is determined by the DDMA modulation vector $\mathbf{w}_q$. If $\mathbf{w}_q$ is designed properly, the overall transmit power distribution can be optimized and the parameter estimation performance can be improved accordingly.

2.2. Parameter Estimation using PARAFAC

For the n th receive element, the received signal in a single CPI can be constructed as a vector $\mathbf{x}_n = [x_{n1}, x_{n2}, \dots, x_{nQ}]^T$. Applying the fast Fourier transform to $\mathbf{x}_n$ in slow time, M_t peaks with an adjacent distance $\Delta f = \text{PRF}/M_t$ are uniformly distributed along the target range slice for each target.

To separate the signal of each transmit channel, phase demodulation is applied to shift the signal of each transmit channel to baseband in the Doppler domain separately. By lowpass filtering with cutoff frequency $\Delta f/2$ and range gating, signals of $N \times M_t$ virtual transmit-receive channels can be obtained. Note that the uniform division of Doppler frequency spectrum declines the sampling rate to PRF/M_t , the number of pulse samples is decreased by M_t times.

Storing the separated signal in a three-dimensional (3-D) tensor χ of size $M_t \times N \times (Q/M_t)$, we have

$$\chi = \sum_{k=1}^K \sigma_k^2 \mathbf{a}(\theta_k) \circ \mathbf{b}(\varphi_k) \circ \tilde{\mathbf{d}}(f_k) + \mathcal{Z}, \quad (6)$$

where $\circ$ denotes the outer product, $\mathcal{Z}$ is the noise tensor after spectrum division, and the k th column of the third factor matrix is given by

$$\tilde{\mathbf{d}}(f_k) \triangleq [d_1(f_k), d_{1+M_t}(f_k), \dots, d_{1+M_t(P-1)}(f_k)]^T, \quad (7)$$

where $d_q(f_k) = e^{j2\pi f_k(q-1)T_r}$, $q = 1, 2, \dots, Q$ and $P = Q/M_t$ is assumed to be an integer. The factor matrices $\mathbf{A}$, $\mathbf{B}$ and $\tilde{\mathbf{D}}$ of the constructed tensor are denoted by

$$\begin{aligned} \mathbf{A} &\triangleq [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_K)] \\ \mathbf{B} &\triangleq [\mathbf{b}(\varphi_1), \mathbf{b}(\varphi_2), \dots, \mathbf{b}(\varphi_K)] \\ \tilde{\mathbf{D}} &\triangleq [\tilde{\mathbf{d}}(f_1), \tilde{\mathbf{d}}(f_2), \dots, \tilde{\mathbf{d}}(f_K)]. \end{aligned} \quad (8)$$

To estimate the factor matrices, PARAFAC decomposition via Alternating Least Squares (ALS) can be used [9, 15]. Taking the first factor matrix for example, the problem of estimating $\mathbf{A}$ boils down to solving the following optimization problem

$$\hat{\mathbf{A}} \leftarrow \min_{\hat{\mathbf{A}}} \left\| \chi_{(1)} - [(\mathbf{B} \odot \tilde{\mathbf{D}}) \hat{\mathbf{A}}^T] \right\|_F^2, \quad (9)$$

where $\chi_{(1)}$ is the mode-1 unfolding of tensor χ , $\odot$ stands for the Khatri-Rao product of two matrices, and $\|\cdot\|_F$ denotes the

Frobenius norm of a matrix. As a result, parameter estimation including target DOD, DOA and velocity can be achieved for DDMA MIMO radar.

3. PROPOSED TB ST-MIMO METHOD

3.1. Slow-time Transmit Beamspace Design

Rewriting (4) as

$$\mathbf{w}_q = \left[1, e^{j2\pi \frac{(q-1)}{M_t}}, \dots, e^{j2\pi \frac{(M_t-1)(q-1)}{M_t}} \right]^T, \quad (10)$$

and comparing it with the transmit steering vector in (1), the DDMA modulation vector $\mathbf{w}_q$ can be regarded as a beamforming vector of the transmit ULA. Given that $\mathbf{w}_q$ varies from pulse to pulse periodically, the beamforming direction of (10) sweeps across the entire spatial region with a cycle of M_t pulses. The corresponding mainlobe directions θ_b for the first M_t pulses can be denoted by

$$\sin \theta_b = \frac{q-1}{M_t d_t}, q = 1, 2, \dots, M_t. \quad (11)$$

If some certain pulses are selected from each period, the transmit energy can be focused within a spatial region of interest. Meanwhile, it can be found that the value of M_t influences the mainlobe interval of two adjacent beams. If the whole Doppler spectrum is divided into more than M_t pieces, the mainlobe interval between two adjacent beams can be narrowed down, and more mainlobes can be focused within a desired spatial region. For clarity of description, we use M_v to denote the total number of subbands after dividing the Doppler spectrum.¹ Then, the phase modulation vector $\mathbf{w}_{\text{TB},q}$ can be expressed as

$$\mathbf{w}_{\text{TB},q} = \left[1, e^{j2\pi \frac{(q-1)}{M_v}}, \dots, e^{j2\pi \frac{(M_v-1)(q-1)}{M_v}} \right]^T. \quad (12)$$

Knowing the value of M_v and the desired region, there are $M < M_v$ pulses whose beamforming mainlobes are directed to the desired region in every M_v pulses. The modulation vectors for these M pulses are selected to construct a new phase modulation matrix $\mathbf{W}_{\text{TB}}$ for TB ST-MIMO radar, given by

$$\mathbf{W}_{\text{TB}} \triangleq \left[\underbrace{\mathbf{W}_0, \mathbf{W}_0, \dots, \mathbf{W}_0}_{Q/M} \right]_{M_t \times Q}, \quad (13)$$

where $\mathbf{W}_0 \triangleq [\mathbf{w}_{\text{TB},q_1}, \mathbf{w}_{\text{TB},q_2}, \dots, \mathbf{w}_{\text{TB},q_M}]_{M_t \times M}$ and $\{q_1, q_2, \dots, q_M\}$ are the indices of the selected pulses from the first period. Here Q/M is assumed to be an integer. Note that any set of consecutive M_t Doppler shifts can be chosen for the M_t transmit channels.

¹For traditional DDMA MIMO technique, $M_v = M_t$.

With a larger M_v , the mainlobes of two adjacent beams become closer, and thus, more beams in each period can be focused within a certain spatial region. By a proper selection of DDMA modulation vectors, the transmit energy can be focused within a desired region and the optimization of energy distribution can be realized.

3.2. PARAFAC for TB ST-MIMO Radar

Although PARAFAC decomposition can be applied to DDMA MIMO radars after separating all transmit-receive channels in slow-time, the useful information contained in the data measurement for velocity estimation is partly lost due to pulse data downsampling. As a result, velocity ambiguity will arise when the factor matrix $\tilde{\mathbf{D}}$ in (8) is used for target velocity estimation. To tackle this problem, the PARAFAC method is used in this paper. Velocity information contained between pulses can be preserved, and thus, velocity ambiguity can be resolved.

Restoring the received signal of TB ST-MIMO radar in a matrix $\mathbf{X}$ of size $N \times Q$, we have

$$\mathbf{X} = (\mathbf{p} \odot \mathbf{B}) [(\mathbf{W}_{\text{TB}}^T \mathbf{A}) * \mathbf{D}]^T + \mathbf{Z}, \quad (14)$$

where $\mathbf{p} \triangleq [\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2]$, $*$ denotes Hadamard product, and

$$\begin{aligned} \mathbf{D} &\triangleq [\mathbf{d}(f_1), \mathbf{d}(f_2), \dots, \mathbf{d}(f_K)] \\ \mathbf{d}(f_k) &\triangleq [d_1(f_k), d_2(f_k), \dots, d_Q(f_k)]^T. \end{aligned} \quad (15)$$

The (n, q) -th element of matrix $\mathbf{Z}$ is z_{nq} .

Taking advantage of the periodicity of phase modulation matrix $\mathbf{W}_{\text{TB}}$, $\mathbf{X}$ can be re-expressed as

$$\mathbf{X} = (\mathbf{p} \odot \mathbf{B}) [\tilde{\mathbf{H}} \odot \mathbf{A}_{\text{TB}}]^T + \mathbf{Z}, \quad (16)$$

where $\tilde{\mathbf{H}} \triangleq [\tilde{\mathbf{h}}(f_1), \tilde{\mathbf{h}}(f_2), \dots, \tilde{\mathbf{h}}(f_K)]$, and $\tilde{\mathbf{h}}(f_k)$ can be obtained simply by substituting M_t with M in $\tilde{\mathbf{d}}(f_k)$ defined in (7). Matrix $\mathbf{A}_{\text{TB}}$ can be further denoted by

$$\mathbf{A}_{\text{TB}} = (\mathbf{W}_0^T \mathbf{A}) * \mathbf{D}_0, \quad (17)$$

where $\mathbf{D}_0$ consists of the first M columns of $\mathbf{D}$. Stacking $\mathbf{X}$ in (16) into a 3-D tensor of size $M \times N \times (Q/M)$, we have

$$\chi_{\text{D}} = \sum_{k=1}^K \sigma_k^2 \mathbf{a}_{\text{TB}}(\theta_k) \circ \mathbf{b}(\varphi_k) \circ \tilde{\mathbf{h}}(f_k) + \mathcal{Z}_{\text{D}}. \quad (18)$$

where $\mathcal{Z}_{\text{D}}$ is the noise tensor of size $M \times N \times (Q/M)$.

Comparing (6) with (18), the lost velocity information is preserved in $\mathbf{A}_{\text{TB}} \triangleq [\mathbf{a}_{\text{TB}}(\theta_1), \mathbf{a}_{\text{TB}}(\theta_2), \dots, \mathbf{a}_{\text{TB}}(\theta_K)]$ in (18). The unambiguous velocity can be restored by combining $\tilde{\mathbf{H}}$ and $\mathbf{D}_0$. With the obtained factor matrices $\mathbf{B}$, $\mathbf{A}_{\text{TB}}$ and $\tilde{\mathbf{H}}$ after tensor decomposition, ESPRIT is used to estimate DOA, and the minimum mean squared error estimation

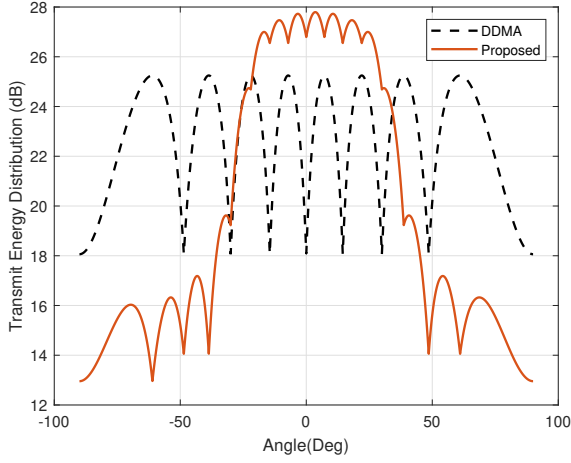


Fig. 1. Comparison of transmit energy distribution.

algorithm and least squares method are employed to estimate DOD and velocity, respectively. Details of parameter estimation can be also found in [18].

By exploiting the PARAFAC algorithm and the proposed TB ST-MIMO approach, we can improve the parameter estimation performance and achieve velocity disambiguation simultaneously.

4. SIMULATION RESULTS

In this section, we compare the parameter estimation performance of the proposed method with three other existing methods including original PARAFAC-Direct [18], PARAFAC [9] and ESPRIT [13] for bistatic DDMA MIMO radars.

In our simulations, we let $M_t = 8$ and $N = 12$ with d_t and d_r being half the working wavelength. For each CPI, we consider $Q = 80$ pulses with $T_r = 12\mu s$. Assuming there are $K = 2$ targets following Swerling I model, the DODs and DOAs are set as $\{-4^\circ, 13^\circ\}$ and $\{10^\circ, -17^\circ\}$ with complex reflections $\{\sigma_k^2\}$ chosen from the standard Gaussian distribution randomly. The number of Monte Carlo trials is 400.

Assuming that the desired spatial region $[-30^\circ, 30^\circ]$ is known a priori, the entire Doppler spectrum is divided into $M_v = 16$ subbands for the proposed TB ST-MIMO approach. Specifically, there are a total of $M = 8$ different beams directing to the desired spatial region. The indices of the selected pulses among the first 16 pulses and the corresponding beam directions $\{\sin \theta_b\}$ are $\{1, 2, 3, 4, 13, 14, 15, 16\}$ and $\{0, 1/8, 1/4, 3/8, -1/2, -3/8, -1/4, -1/8\}$, respectively. The corresponding modulation vectors are selected to build the new modulation matrix $\mathbf{W}_0$.

As a result, the transmit energy can be focused within the desired region. The comparison of transmit energy distribution between the proposed method and traditional DDMA MIMO technique can be found in Fig. 1. It can be seen that

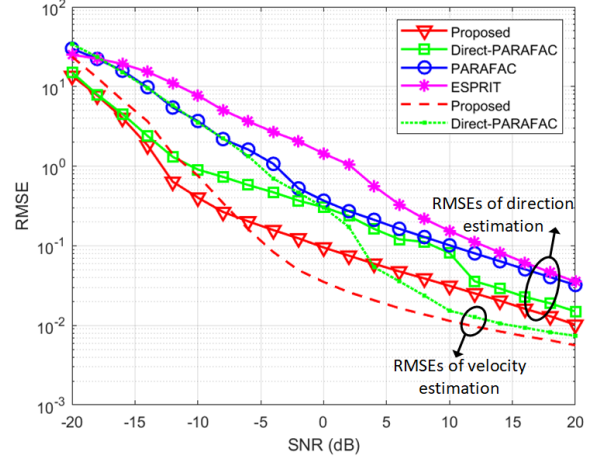


Fig. 2. RMSEs versus SNR for different parameters estimation approaches including DOD, DOA and velocity.

the transmit energy is mostly distributed within the desired spatial region. A higher SNR gain can be obtained as compared to traditional DDMA techniques, which can improve the parameter estimation performance.

Finally, let us compare the parameter estimation performance by evaluating the results of root mean square errors (RMSEs) versus SNR for DOA, DOD and velocity estimates. It should be noted that RMSEs of direction estimation are obtained by averaging those of DODs and DOAs which are computed separately. Let $\Delta\theta_k$ and $\Delta\varphi_k$ denote the RMSEs of DOD and DOA for the k th target, the RMSEs of direction estimation depicted in Fig. 2 are calculated by

$$\text{RMSE} = \frac{1}{2} \left(\sqrt{\frac{1}{K} \sum_{k=1}^K \Delta\theta_k^2} + \sqrt{\frac{1}{K} \sum_{k=1}^K \Delta\varphi_k^2} \right) \quad (19)$$

It can be found in Fig. 2 that the proposed method surpasses all the other methods by achieving a lower RMSE. Particularly, the proposed method outperforms Direct-PARAFAC algorithm especially at low SNRs due to the optimization of transmit energy.

5. CONCLUSION

We have proposed a TB ST-MIMO approach to improve the parameter estimation performance while maintaining velocity disambiguation. Unlike conventional DDMA MIMO techniques, the proposed TB ST-MIMO method enables transmit energy focusing via a specially designed phase modulation matrix. Then, the PARAFAC algorithm has been used to improve the parameter estimation performance especially in low SNR region. Simulation results have verified that the proposed TB ST-MIMO method has better parameter estimation accuracy.

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Publication II

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Transmit Energy Focusing for Parameter Estimation in Slow-Time Transmit Beamspace L-Shaped MIMO Radar

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Abstract—We present a novel slow-time transmit beamspace (TB) multiple-input multiple-output (MIMO) technique for L-shaped array radar with uniform linear subarrays to estimate target parameters including 2-dimensional (2-D) directions of arrival (DOA) and unambiguous velocity. Doppler division multiple access (DDMA) approach, as a type of slow-time waveform achieving waveform orthogonality across multiple pulses within a coherent processing interval, disperses the transmit energy over the entire spatial region, suffering from beam-shape loss. Moreover, Doppler spectrum division, which is necessary for transmit channel separation prior to parameter estimation, leads to the loss of crucial information for velocity disambiguation. To optimize transmit energy distribution, slow-time TB technique is proposed to focus the energy within a desired spatial region. Unlike DDMA approach, slow-time TB technique divides the entire Doppler spectrum into more subbands than the number of transmit antenna elements to narrow down the beam mainlobe intervals between adjacent beams formed by DDMA modulation vectors. As a result, more beams are incorporated into the region of interest, and slow-time TB radar can direct transmit energy to the region of interest by properly selecting the DDMA modulation vectors whose beams are directed there. To resolve velocity ambiguity, tensor signal modeling, by storing measurements in a tensor without Doppler spectrum division, is used. Parameter estimation is then addressed using canonical polyadic decomposition (CPD), and the performance of slow-time TB L-shaped MIMO radar is shown to be improved as compared to DDMA MIMO techniques. Simulations are conducted to validate the proposed method.

Index Terms—Slow-time transmit beamspace, 2-D DOA estimation, CPD, L-shaped array, velocity disambiguation.

I. INTRODUCTION

IN the last two decades, multiple-input multiple-output (MIMO) radars [1], [2], [3], [4], [5] have received a great deal of attention and seen prosperous development. More degrees of freedom and a larger virtual radar aperture, which amount to more detectable targets with higher spatial resolution, can be achieved by the transmission of multiple independent waveforms from different transmit channels, which is what differentiates MIMO radars from phased-array radars [6], [7]. To ensure the benefits listed above, waveform diversity is one of the research focuses in the MIMO radar field.

Waveforms of various types, including frequency division multiple access (FDMA) [8], code division multiple access (CDMA) [9], time division multiple access (TDMA) [10], and Doppler division multiple access (DDMA) [9], [11], have been developed. Among them, TDMA stood out due to its easy implementation and low hardware cost. The most basic and intuitive way to achieve orthogonality between transmit channels is to alternately activate transmit antenna elements without overlapping, which is exactly the core concept of TDMA. However, TDMA's transmission capability is seriously underutilized and its average transmit power is significantly lower when compared to other waveforms whose transmit antennas radiate signals simultaneously, resulting in a reduced detectable range.

More transmit antennas will be required in pursuit of higher system performance in the future, further reducing the average transmit power of TDMA. Meanwhile, a higher pulse repetition frequency (PRF) is required to avoid Doppler ambiguity in the presence of high-speed vehicles, whereas range ambiguity arises as a result of the higher PRF. TDMA exacerbates this conflict because of its alternate transmission. Given the foregoing, TDMA is no longer as appealing as it once was, and DDMA technique is capturing the research attention. DDMA technique, inheriting the TDMA's transmitter configuration, achieves waveform diversity in the slow time [8], [12] by modulating each transmit channel with a different Doppler shift. For DDMA technique, transmission capability is fully utilized, and transmit channel divisibility is accomplished as well.

Estimating 2-dimensional (2-D) directions of arrival (DOA), including azimuth and elevation angles, is a topic of considerable interest for real-world applications. Among these applications, radar systems for detection and localization are

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integral components of modern advanced driver assistance systems and autonomous vehicles, aimed at reducing human error and enhancing vehicle safety [13], [14]. To achieve reliable functionality, the performance of 2-D DOA estimation is crucial for automotive radar systems. Several 2-D array structures such as uniform rectangular array (URA), circular array, and L-shaped array have been intensively employed. Although all of these array structures find their applications in practice, the L-shaped array has been found to be superior to URA due to a significantly lower (37%) Cramér-Rao bound (CRB) on DOA estimation [15]. Moreover, L-shaped arrays are easy to implement in comparison to some other array structures among 2-D planar arrays for 2-D DOA estimation that have been developed, leading to L-shaped arrays being at the forefront of array processing research [15], [16], [17], [18], [19], [20], [21], [22].

This paper considers an L-shaped array with two subarrays shaped as uniform linear arrays (ULAs) positioned along the x-axis and z-axis, respectively, where azimuth and elevation angles can be estimated by processing the signals transmitted from subarrays along the x-axis and z-axis separately. In other words, 2-D DOA estimation can be converted into 1-dimensional (1-D) angle estimation with a two-fold operation. Thus, the majority of 1-D DOA estimation algorithms can be used in L-shaped arrays with few adaptations. Considering the benefits of both the DDMA waveform and the L-shaped array configuration, DDMA technique is proposed for L-shaped array MIMO radar. Even though the entire Doppler spectrum can be uniformly divided into the number of transmit elements of two subarrays to achieve channel separation among all the transmit channels of an L-shaped array, the number of pulse samples will inevitably be reduced as many times as the number of total transmit elements. As a result, parameter estimation performance deteriorates, particularly at a low signal-to-noise ratio (SNR). As a compromise, additional two waveforms orthogonal in the fast time are used. As a result, independence between subarrays is realized in the fast time, while waveform diversity within subarray elements is achieved in the slow time, e.g., DDMA.

Existing 2-D DOA estimation algorithms range from adoption of nonparametric Fourier-based methods [23] and traditional beamforming techniques such as Capon beamformer to higher-resolution parametric methods like multiple signal classification (MUSIC) [24], [25] and estimation of signal parameters via rotational invariance technique (ESPRIT) [26]. However, these algorithms work with the multidimensional received measurements collected into a matrix form. As a consequence, the multi-linear structure inherent in the signal is not exploited. It has been validated that tensor-based methods outperform matrix-based methods, especially in low SNR cases [27], [28], [29], [30]. Therefore, tensor representation has been used for DOA estimation. Existing tensor-based methods can be mainly categorized into higher order singular value decomposition [29], [31], canonical polyadic decomposition (CPD), also named as parallel factor (PARAFAC) analysis [32], [33], constrained tensor decomposition [34], [35], and more recently learning-based techniques [36], [37]. The first two

classes of methods are applicable for tensors with arbitrary factors, whereas the third one is intended for tensors with factor matrices that obey a certain structure, for example, Vandermonde structure. The methods from the last class help to significantly reduce the computational complexity at the testing phase, and show more efficiency also when the known tensor structure is exploited.

To apply tensor representation to DDMA MIMO radar, [38] has proposed a tensor model and developed a modified alternating least squares (ALS) tensor decomposition algorithm for DOA estimation. However, Doppler spectrum division, which inevitably downsamples the pulse data and leads to the loss of velocity information that originally was present between adjacent pulses, is still demanded for transmit channel separation in [38]. As a result, the unambiguous velocity is reduced as many times as the number of transmit elements, and velocity ambiguity tends to arise for high-speed targets. To address this issue, a method called PARAFAC-Direct in [39] designs a tensor model for DDMA radar measurements, obviating the requirement for Doppler spectrum division. Thus, velocity information is well preserved, and unambiguous velocity can be restored.

Although MIMO radar gains more degrees of freedom and a larger virtual aperture thanks to waveform diversity compared to phased-array radar, its transmit energy is distributed omnidirectionally, resulting in transmit energy waste and lower SNRs. It is acknowledged that a higher SNR contributes to better radar system performance [2]. Hence, coherent transmit beamspace (TB) design techniques have been developed in [7], [28], [35], [40] in the last decade to improve parameter estimation performance even further. With prior knowledge of the desired spatial region, transmit beams of orthogonal waveforms can be designed to direct energy toward the desired region, and transmit energy distribution can be optimized.

In this paper, slow-time TB L-shaped MIMO technique is proposed to improve parameter estimation performance.¹ Slow-time MIMO radar, as its name suggests, is a waveform technique that achieves orthogonality in the slow time rather than the fast time, with DDMA technique being a typical example. For traditional DDMA MIMO radar [9], [39], the number of pulses within a pulse cycle equals the number of transmit antenna elements, and each pulse within a cycle is modulated with a different vector. These vectors form distinct beams from one another, and mainlobes of these beams are directed across the entire spatial region. That is to say, transmit energy can be regarded as being distributed over the entire region rather than being focused within a specific limited region of interest. For DDMA technique, the intervals of adjacent beam mainlobes shaped by DDMA modulation vectors for pulses are correlated to the number of transmit elements. The more transmit elements, the closer the adjacent beams get. Therefore, if the entire Doppler spectrum is uniformly divided into more subbands than the number of transmit elements, a larger number of different beams can be formed, and adjacent beam mainlobes get closer

¹We have recently reported some preliminary concept verification type of results only in [41].

[42]. Consequently, more beams are comprised within a specific sector. With prior knowledge of 2-D spatial sector of interest, those pulses whose beamforming vectors direct their beams toward the desired region are selected for each subarray, and the corresponding phase modulation vectors for those pulses constitute new phase modulation matrices for the proposed slow-time TB MIMO radar. As a result, the transmit energy is successfully focused within the desired region. The novel contributions of this paper can be summarized as follows.

- **Optimization of transmit energy distribution:** Inheriting DDMA's advantages, we propose the slow-time TB L-shaped MIMO radar technique which achieves transmit energy distribution optimization. The latter fact contributes to the performance improvement of the introduced radar system, especially at low SNRs.
- **Enhancing parameter estimation using tensor-based signal model:** A new tensor-based (instead of matrix-based) signal model without Doppler spectrum division is developed in slow-time TB L-shaped MIMO radar for parameter estimation. It preserves the multi-linear structure inherent in the signal measurements to further improve the parameter estimation performance. It resolves the velocity ambiguity as well.
- **Benchmarking against existing algorithms:** The performance of the proposed slow-time TB L-shaped MIMO technique is demonstrated and validated through simulation experiments, benchmarking it against existing parameter estimation algorithms including ESPRIT, CPD, and PARAFAC-Direct. The SNR threshold [40] for the resolution of closely spaced targets, for example, is demonstrated to be approximately 10–15 dB lower than that of the aforementioned algorithms, highlighting the superior efficacy of the proposed approach.

The rest of the paper is organized as follows. In Section II, signal models of DDMA MIMO radar with transmit 1-D ULA and 2-D L-shaped array are introduced. In Section III, the proposed slow-time TB L-shaped array radar is presented. In Section IV, parameter estimation using CPD for L-shaped array radar is developed. Simulation based experiments in Section V validate the effectiveness of the proposed method for energy distribution optimization and parameter estimation, and the conclusion is given in Section VI.

II. SIGNAL MODEL

A. DDMA MIMO Radar With Transmit ULA

Consider a mono-static pulsed MIMO radar with M_t transmit antennas and M_r receive antennas.² The transmit and receive arrays are assumed to be close enough to each other so that a far-field target can be observed from the same spatial angle by both arrays. Altogether Q pulses are emitted with pulse repetition interval $T_r = 1/\text{PRF}$ within a single coherent processing interval (CPI), and the duration of each pulse is T_p . Transmit antenna elements are arranged in the shape of a ULA where the

distance between adjacent elements is d_t . As a transmit array, d_t should be no larger than half a wavelength λ to meet the Nyquist sampling criterion.

Omnidirectional antennas theoretically have the ability to provide 360-degree coverage. However, typically in practice, we first roughly specify the potential spatial area where targets are located, termed the region of interest Θ in this paper. With Θ known *a priori*, advanced techniques can be applied to achieve higher accuracy of detection and location. Assuming that there are K targets within the region of interest Θ with directions³ θ_k , ranges R_k , and velocities v_k , $k = 1, 2, \dots, K$, the steering vector of the transmit ULA for the k th target can be expressed as

$$\mathbf{a}(\theta_k) = \left[1, e^{-j2\pi \frac{d_t}{\lambda} \cos \theta_k}, \dots, e^{-j2\pi \frac{d_t}{\lambda} (M_t-1) \cos \theta_k} \right]^T \quad (1)$$

where $(\cdot)^T$ stands for the transpose operation. Since this work focuses on the transmit energy distribution optimization, we leave the shape of the receive array flexible, and we use $\mathbf{b}(\theta_k)$ to denote the receive steering vector in general.

To achieve waveform orthogonality, with a predesigned phase modulation matrix $\mathbf{W}$ of size $M_t \times Q$, DDMA MIMO radar modulates each transmit channel with a different Doppler shift to achieve the divisibility of transmit signals in the slow time (Doppler) domain. Dividing the whole Doppler frequency spectrum uniformly into M_t subbands, the Doppler shift for the m th transmit channel is given as

$$f_m = (m-1) \frac{\text{PRF}}{M_t}, m = 1, 2, \dots, M_t. \quad (2)$$

For the q th pulse, its phase modulation vector $\mathbf{w}_q$, which is also the q th column of $\mathbf{W}$, can be written as

$$\mathbf{w}_q = \left[1, e^{-j2\pi \frac{\text{PRF}}{M_t} (q-1)T_r}, \dots, e^{-j2\pi (M_t-1) \frac{\text{PRF}}{M_t} (q-1)T_r} \right]^T, \quad q = 1, 2, \dots, Q. \quad (3)$$

With DDMA modulation, only a Doppler spectrum subband of width PRF/M_t is available for each transmit channel. Consequently, the maximum unambiguous velocity is

$$v_{\max} = \frac{\text{PRF}\lambda}{4M_t}. \quad (4)$$

It can be seen from (4) that $v_{\max}$ is reduced by a factor of M_t compared to, for example, the case of FDMA where each transmit channel is accessible to the whole Doppler spectrum, which exacerbates the velocity (Doppler) ambiguity.

We assume that the target reflection coefficient obeys Swerling I model [43]. With modulation matrix $\mathbf{W}$ depicted in (3), the received signal by the n th receive antenna element for the q th pulse can be expressed as

$$y_{nq}(t) = \sum_{k=1}^K \sum_{m=1}^{M_t} \sigma_k w_{mq} s(t - \tau_{mnq,k}) + z_{nq}(t), \quad n = 1, 2, \dots, M_r \quad (5)$$

³To be consistent with the subsequent expressions of L-shaped array, θ_k , the complementary angle of the target incident angle, is used instead.

²All antennas are assumed to be omnidirectional in this paper.

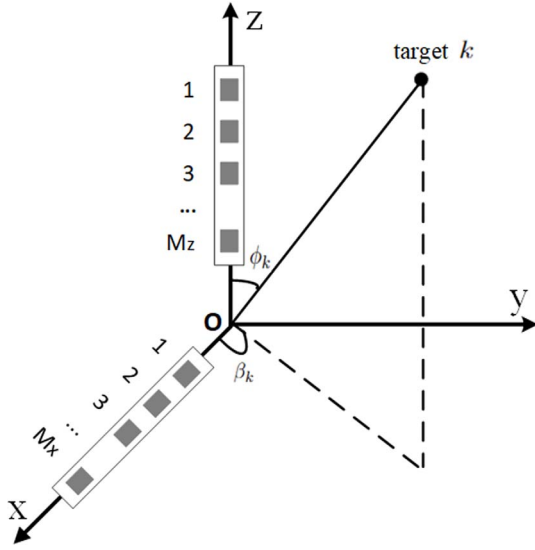


Fig. 1. L-shaped array configuration.

where t is the fast-time index, σ_k is the complex-valued reflection coefficient of the k th target, w_{mq} is the (m, q) -th element of $\mathbf{W}$, $s(t)$ is the transmitted waveform, and $z_{nq}(t)$ is the noise component in the n th receive antenna element for the q th pulse. The time delay $\tau_{mnq,k}$, caused by the wave traveling distance from the m th transmit element to the k th target and then back to the n th receive element for the q th pulse, can be expressed as

$$\tau_{mnq,k} = \frac{2(R_k - (q-1)T_r v_k) + (m-1)d_t \cos \theta_k + d_{n,k}}{c} \quad (6)$$

where $d_{n,k}$ is the difference in traveling distance from the k th target to the n th element compared to the reference element within the receive array, and c is the light speed. After carrier frequency downconversion and matched filtering, the received signal in (5) can be rewritten as

$$y_{nq} = \sum_{k=1}^K \sigma_k e^{j2\pi f_k (q-1)T_r} (\mathbf{a}^T(\theta_k) \mathbf{w}_q) b_n(\theta_k) + z_{nq} \quad (7)$$

where factor σ_k incorporates other constant factors, $f_k = 2v_k/\lambda$ is the Doppler shift caused by target velocity, $b_n(\theta_k)$ is the n th element of $\mathbf{b}(\theta_k)$, and z_{nq} is the residual noise after filtering.

B. DDMA MIMO Radar With Transmit L-Shaped Array

Extending the 1-D ULA array to a 2-D array for 2-D DOA estimation (azimuth and elevation angles), consider an L-shaped array with two ULAs positioned along the x-axis and z-axis, respectively, as shown in Fig. 1. To distinguish from 1-D array, each ULA within an L-shaped array is referred to as subarray in this paper. Subarrays along the x-axis and z-axis consist of M_x and M_z antenna elements spaced at distances d_x and d_z from each other, respectively. The space Cartesian coordinate of the m_x th and m_z th array elements within the x-axis and z-axis subarrays can be denoted as, respectively,

$$\mathbf{r}_{x,m_x} = ((m_x - 1)d_x, 0, 0)^T, m_x = 1, 2, \dots, M_x$$

$$\mathbf{r}_{z,m_z} = (0, 0, (m_z - 1)d_z)^T, m_z = 1, 2, \dots, M_z. \quad (8)$$

In addition, array coordinate matrices $\mathbf{R}_x \in \mathbb{R}^{M_x \times 3}$ and $\mathbf{R}_z \in \mathbb{R}^{M_z \times 3}$ are defined as follows

$$\begin{aligned} \mathbf{R}_x &\triangleq [\mathbf{r}_{x,1}, \mathbf{r}_{x,2}, \dots, \mathbf{r}_{x,M_x}]^T \\ \mathbf{R}_z &\triangleq [\mathbf{r}_{z,1}, \mathbf{r}_{z,2}, \dots, \mathbf{r}_{z,M_z}]^T. \end{aligned} \quad (9)$$

The 2-D DOAs of the k th far-field target include azimuth angle β_k and elevation angle ϕ_k . Different from the 1-D ULA described in Section II-A, the problem of interest becomes, in this case, the estimation of the DOA pairs $\{\beta_k, \phi_k\}_{k=1}^K$.

Define the propagation vector as

$$\mathbf{p}_k = [\sin \phi_k \cos \beta_k, \sin \phi_k \sin \beta_k, \cos \phi_k]^T. \quad (10)$$

With $\cos \varphi_k = \sin \phi_k \cos \beta_k$, the transmit steering vectors of two subarrays can be expressed as

$$\begin{aligned} \mathbf{a}_x(\varphi_k) &= e^{-j2\pi \mathbf{R}_x \mathbf{p}_k} \\ &= \left[1, e^{-j2\pi \frac{d_x}{\lambda} \cos \varphi_k}, \dots, e^{-j2\pi (M_x-1) \frac{d_x}{\lambda} \cos \varphi_k} \right]^T \\ \mathbf{a}_z(\phi_k) &= e^{-j2\pi \mathbf{R}_z \mathbf{p}_k} \\ &= \left[1, e^{-j2\pi \frac{d_z}{\lambda} \cos \phi_k}, \dots, e^{-j2\pi (M_z-1) \frac{d_z}{\lambda} \cos \phi_k} \right]^T. \end{aligned} \quad (11)$$

Unless otherwise specified, azimuth angle refers to φ_k for convenience from now on. Observing (11), it can be found that azimuth φ_k and elevation ϕ_k are independently contained in two transmit subarrays without coupling, which means that 2-D DOAs can be estimated just by applying existing DOA estimation algorithms to the signals transmitted by the two subarrays separately.

With the orthogonality between two waveforms transmitted by subarrays, a 2-D L-shaped array can be approximately regarded as a combination of two independent ULAs. On this basis, DDMA is applicable to L-shaped array as well except that two DDMA modulation matrices are demanded. To apply the DDMA approach, two phase modulation matrices $\mathbf{W}_x$ and $\mathbf{W}_z$ are designed to modulate transmit channels with different Doppler shifts for each independent subarray. Owing to the independence, $\mathbf{W}_x$ and $\mathbf{W}_z$ can be designed separately based on the practical requirements, which will be covered in depth in Section III. With two orthogonal waveforms $u_1(t)$ and $u_2(t)$, the signals transmitted from $M_x + M_z$ antenna elements can be expressed as

$$\mathbf{h}_q(t) = \begin{bmatrix} \mathbf{h}_{x,q}(t) \\ \mathbf{h}_{z,q}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{w}_{x,q} u_1(t) \\ \mathbf{w}_{z,q} u_2(t) \end{bmatrix} \quad (12)$$

where $\mathbf{h}_{x,q}(t)$ and $\mathbf{h}_{z,q}(t)$ are the transmitted signals from subarrays along the x-axis and z-axis, and $\mathbf{w}_{x,q}$ and $\mathbf{w}_{z,q}$ are the q th column vectors of $\mathbf{W}_x$ and $\mathbf{W}_z$ for the q th pulse modulation, respectively.

To separate the signals from different subarrays, range compression is performed twice on the received signals with reference signals $u_i(t)$, $i = 1, 2$, then the separated signals from

two transmit subarrays to the n th receive element can be expressed as

$$\begin{aligned} y_{x,nq} &= \sum_{k=1}^K \sigma_k e^{j2\pi f_k(q-1)T_r} (\mathbf{a}_x^T(\varphi_k) \mathbf{w}_{x,q}) b_n(\varphi_k, \phi_k) + z_{x,nq} \\ y_{z,nq} &= \sum_{k=1}^K \sigma_k e^{j2\pi f_k(q-1)T_r} (\mathbf{a}_z^T(\phi_k) \mathbf{w}_{z,q}) b_n(\varphi_k, \phi_k) + z_{z,nq} \end{aligned} \quad (13)$$

where $b_n(\varphi_k, \phi_k)$ denotes the n th element of the receive steering vector $\mathbf{b}(\varphi_k, \phi_k)$, and $z_{x,nq}$ and $z_{z,nq}$ are the noise components after range compression. After obtaining the signals from two subarrays as is depicted in (13), parameter estimation algorithms for DDMA MIMO radars can be applied to each subarray, respectively, and thus, target azimuth angles $\{\varphi_k\}_{k=1}^K$, elevation angles $\{\phi_k\}_{k=1}^K$, and velocities $\{v_k\}_{k=1}^K$ can be estimated.

III. PROPOSED SLOW-TIME TRANSMIT BEAMSPACE L-SHAPED ARRAY RADAR

A. Slow-Time Transmit Beamspace Design

Recalling (7), the transmit energy beampattern of DDMA MIMO radar for the q th pulse can be expressed as

$$G(\theta) = |\mathbf{a}^T(\theta) \mathbf{w}_q|^2 \quad (14)$$

where θ denotes the spatial angle of a target, $|\cdot|$ denotes the element-wise modulus operation on a number or a vector (here a number). It is noteworthy to mention that 1-D ULA and 2-D L-shaped array share the similar property of transmit beamspace, and thus, here the former one is taken as an example to discuss for the ease of description. It can be found from (14) that the energy beampattern for a fixed transmit array configuration depends on the design of DDMA modulation matrix $\mathbf{W}$ consisting of phase modulation vectors $\mathbf{w}_q, q = 1, 2, \dots, Q$.

Rewrite the vector $\mathbf{w}_q$ defined in (3) as

$$\mathbf{w}_q = \left[1, e^{-j2\pi \frac{q-1}{M_t}}, \dots, e^{-j2\pi \frac{(M_t-1)(q-1)}{M_t}} \right]^T. \quad (15)$$

Observing the ULA steering vector $\mathbf{a}(\theta)$ in (1), $\mathbf{w}_q$ can be regarded as a beamforming vector with mainlobe direction $\theta_{b,q}$. Taking $d_t = \lambda/2$ as an illustration, which is commonly used in practical radar applications, the mainlobe direction $\theta_{b,q}$ for the q th pulse satisfies

$$\cos \theta_{b,q} = 2l - \frac{2(q-1)}{M_t}, q = 1, 2, \dots, Q \quad (16)$$

where l is a nonnegative integer just to guarantee that $\cos \theta_{b,q} \in [-1, 1]$. It can be found that $\theta_{b,q}$ varies from pulse to pulse with a cycle of M_t pulses, and mainlobes of the formed beams in a pulse period sweep across the entire spatial region with specific intervals. As a result, the overall transmit energy within a pulse period is distributed across the whole spatial region rather than focused within a certain spatial sector. If the transmit

energy can be focused within the region of interest Θ , the performance of parameter estimation for targets in this region can be significantly improved due to a higher SNR. In this paper, therefore, a slow-time TB MIMO radar is proposed to optimize the transmit energy distribution and improve the performance of parameter estimation.

Observing (16), we can find that the mainlobe distance between adjacent pulse beams depends on the value of M_t , which is also the number of divided subbands in the Doppler domain for traditional DDMA. To distinguish the number of subbands and the number of transmit elements, we use M_v to denote the total number of divided subbands, which is also referred to as the number of virtual transmit elements [42]. Meanwhile, the pulse period of modulation vectors becomes M_v instead of M_t . The Doppler shifts and phase modulation vectors can then be rewritten as

$$f_m = (m-1) \frac{\text{PRF}}{M_v}, m = 1, 2, \dots, M_v \quad (17)$$

$$\mathbf{w}_q = \left[1, e^{-j2\pi \frac{q-1}{M_v}}, \dots, e^{-j2\pi \frac{(M_v-1)(q-1)}{M_v}} \right]^T, \quad q = 1, 2, \dots, Q. \quad (18)$$

With a larger M_v than M_t , M_v Doppler shifts for phase modulation are available for transmit elements to choose from. The orthogonality across M_t transmit channels can theoretically be guaranteed by any set of M_t Doppler shifts. What is more, any set of consecutive M_t Doppler shifts results in the same beamformed mainlobe directions except for a different extra phase, thus making no difference to the transmit beampattern. As an example for discussion, the first M_t Doppler shifts are chosen for phase modulation by default in this paper. In this case, phase modulation vector for a transmitter of M_t elements can be expressed as the following $M_t \times 1$ vector

$$\mathbf{w}_{\text{TB},q} = \left[1, e^{-j2\pi \frac{q-1}{M_v}}, \dots, e^{-j2\pi \frac{(M_t-1)(q-1)}{M_v}} \right]^T. \quad (19)$$

Equations (16) and (19) make it clear that a larger M_v leads to smaller mainlobe distances between adjacent pulse beams. As a result, more pulse beams are directed toward a fixed spatial sector. Assuming that Θ is known *a priori*, there are $M < M_v$ pulses whose beam mainlobes are directed toward Θ in every M_v pulses, and the index vector of these M pulses from the first pulse period is denoted as $\mathbf{q}_v = [q_1, q_2, \dots, q_M]$. To focus the transmit energy within Θ , only the modulation vectors of these M pulses are selected to constitute a new phase modulation matrix $\mathbf{W}_{\text{TB}} \in \mathbb{C}^{M_t \times Q}$ for slow-time TB MIMO radar, which is given by

$$\begin{aligned} \mathbf{W}_{\text{TB},0} &\triangleq [\mathbf{w}_{\text{TB},q_1}, \mathbf{w}_{\text{TB},q_2}, \dots, \mathbf{w}_{\text{TB},q_M}] \in \mathbb{C}^{M_t \times M} \\ \mathbf{W}_{\text{TB}} &\triangleq \left[\underbrace{\mathbf{W}_{\text{TB},0}, \mathbf{W}_{\text{TB},0}, \dots, \mathbf{W}_{\text{TB},0}}_{Q/M} \right] \end{aligned} \quad (20)$$

where Q/M is assumed to be an integer, and the q th column vector of $\mathbf{W}_{\text{TB}}$ is used for the q th pulse. Building a new phase modulation matrix that only includes modulation vectors whose mainlobes are directed toward the desired region allows to focus

the transmit energy within Θ , resulting in energy distribution optimization.

As it is stated in Section II-B, after the separation of subarray signals, signals from different subarrays can be treated as independent signals, and azimuth and elevation can be estimated separately. Therefore, based on the spatial region of interest, slow-time phase modulation matrices $\mathbf{W}_{x,TB}$ for x-subarray and $\mathbf{W}_{z,TB}$ for z-subarray can be designed independently for beamspace optimization.

B. Doppler Spectrum for Slow-Time Transmit Beamspace MIMO Radar

DDMA technique takes advantage of the divisibility between different channel signals in the Doppler domain to achieve waveform diversity. With a DDMA phase modulation matrix, each transmit channel is modulated with a distinct shift in the Doppler domain. To be more specific, after DDMA modulation, the signal energy of each channel for a target is focused on a single peak and at a different location from the other channels in the Doppler domain, which is the basis for channel separation. However, this condition does not hold for the case of slow-time TB technique.

Observing (3) and (20), we can see that the m th transmit channel in a CPI of Q pulses is modulated with Q elements in the m th row of a modulation matrix. Denote the m th row of DDMA modulation matrix $\mathbf{W}$ as

$$\mathbf{v}_m = \left[1, e^{-j2\pi \frac{(m-1)\text{PRF}}{M_v} T_r}, \dots, e^{-j2\pi \frac{(m-1)\text{PRF}}{M_v} (Q-1)T_r} \right], \quad (21)$$

and the m th row of the proposed slow-time TB matrix $\mathbf{W}_{TB}$ as

$$\mathbf{v}_{TB,m} \triangleq \left[\underbrace{\mathbf{v}_0, \mathbf{v}_0, \dots, \mathbf{v}_0}_{Q/M} \right] \in \mathbb{C}^{1 \times Q}. \quad (22)$$

Here

$$\begin{aligned} \mathbf{v}_0 &= e^{-j2\pi \frac{(m-1)\text{PRF}}{M_v} T_r} [q_1-1, q_2-1, \dots, q_M-1] \\ &= e^{-j2\pi \frac{(m-1)\text{PRF}}{M_v} T_r} (\mathbf{q}_v-1) \in \mathbb{C}^{1 \times M} \end{aligned} \quad (23)$$

where $\mathbf{1}$ is an all-ones vector of the same size as $\mathbf{q}_v$. According to (21), DDMA technique works by multiplying each transmit channel signal with a distinct complex sinusoid [44] in slow time domain (across pulses), whose frequency is $f_{v,m} = (m-1)\text{PRF}/M_v$, $m = 1, 2, \dots, M_t$, and its sampling rate is $1/T_r = \text{PRF}$. A signal, multiplication (element-wise product) by a complex sinusoid of frequency $f_{v,m}$ is equivalent to shifting it in the Doppler domain by $f_{v,m}$. As a result of such a shift, if the original signal energy focuses on a single peak, the modulated signal will also spread on only one peak, and its location depends on the amount of Doppler shift. Therefore, M_t transmit channel signals for the same target will be distributed on M_t separate peaks when using DDMA technique.

In contrast, for every period of M_v pulse vectors, the slow-time TB technique described in (22) and (23) only uses those M DDMA modulation pulse vectors whose beam mainlobes

are directed toward the desired spatial region, discarding the remaining pulse vectors. As a result, the signal composed of Q sample points used for slow-time TB modulation is no longer a complex sinusoid, albeit remaining periodical with a period of M pulses.

According to the Fourier analysis of a periodical signal $f(t)$, with the Fourier decomposition

$$f(t) = \sum_{n=-\infty}^{\infty} a_n e^{j2\pi n f_0 t} \quad (24)$$

where t denotes the time index, a_n is the Fourier coefficient, n is an integer, and $1/f_0$ is the period of signal $f(t)$, the slow-time TB modulation signal can be viewed as a superposition of multiple complex sinusoids with different frequencies. Therefore, using slow-time TB technique, a transmit channel can be considered to be modulated with multiple different Doppler shifts simultaneously, and these Doppler shift values are $\{n/(MT_r)\}_{n=-\infty}^{\infty}$. As a result of slow-time TB modulation, the signal energy of one transmit channel for a target is scattered on more than one Doppler peak, and its peak positions coincide with those of other channels. Signals from distinct transmit channels are mixed up and overlap with each other in the Doppler domain, making it impossible to separate transmit channels using Doppler spectrum division.

Nearly all DOA estimation algorithms, including MUSIC, ESPRIT, and conventional CPD methods [38], are based on the assumption that M_t separated transmit channel signals can be obtained by signal processing procedures before DOA estimation, such as Doppler spectrum division for DDMA. As previously concluded, transmit signals employing slow-time TB technique are no longer separable in the Doppler domain, rendering the algorithms mentioned above invalid. To handle this issue and take advantage of the tensor-based model as well, a method using CPD without Doppler spectrum division is proposed for slow-time TB MIMO radar to achieve parameter estimation, which is described in detail in the next section.

IV. PARAMETER ESTIMATION USING CPD FOR L-SHAPED ARRAY

In this section, we present a new tensor-based signal model and a new parameter estimation method using this model for the proposed slow-time TB L-shaped MIMO radar. This method avoids transmit channel separation. Moreover, it fully leverages the multi-linear structure inherent in the signal to improve estimation performance and resolve velocity ambiguity issue. Section IV-A explains the CPD algorithm for the traditional DDMA modulation technique as a basis for comparison, while Section IV-B presents the proposed tensor-based signal model and parameter estimation algorithm.

A. CPD for DDMA L-Shaped Array

To achieve the objective of separating the DDMA signal from each transmit channel within a subarray, phase demodulation is implemented by first shifting the received signal from M_L

transmit elements back to the baseband in the Doppler domain, which is done with Doppler shifts $-(m-1)\text{PRF}/M_L$, $m = 1, 2, \dots, M_L$, respectively, where $M_x = M_z = M_L$ is assumed for simplicity in this section. After lowpass filtering with cut-off frequency $\text{PRF}/M_L/2$, signals of $M_L M_r$ virtual transmit-receive channels for each ULA subarray are obtained. After separation, the signals from the (m, n) -th transmit-receive channels of x-axis and z-axis subarrays for the $\bar{q}$ th pulse can be written as, respectively,

$$\begin{aligned} y_{x,mn\bar{q}} &= \sum_{k=1}^K \sigma_k e^{j2\pi f_k (\bar{q}-1)T_r} a_{x,m}(\varphi_k) b_n(\varphi_k, \phi_k) + z_{x,mn\bar{q}} \\ y_{z,mn\bar{q}} &= \sum_{k=1}^K \sigma_k e^{j2\pi f_k (\bar{q}-1)T_r} a_{z,m}(\phi_k) b_n(\varphi_k, \phi_k) + z_{z,mn\bar{q}} \end{aligned} \quad (25)$$

where $a_{x,m}(\varphi_k)$ and $a_{z,m}(\phi_k)$ are the m th elements of $\mathbf{a}_x(\varphi_k)$ and $\mathbf{a}_z(\phi_k)$, respectively, and $z_{x,mn\bar{q}}$ and $z_{z,mn\bar{q}}$ are the noise components of two subarray channels. Considering that the number of pulse samples is decreased by M_L times after Doppler spectrum division, $\bar{q}$ instead of q is used in (25), and $\bar{q} = 1, 1 + M_L, \dots, 1 + M_L(P-1)$, where $P = Q/M_L$ is assumed to be an integer.

Storing the separated signals in (25) in 3-dimensional (3-D) tensors, we have

$$\begin{aligned} \mathcal{Y}_x &= \sum_{k=1}^K \sigma_k \mathbf{a}_x(\varphi_k) \circ \mathbf{b}(\varphi_k, \phi_k) \circ \bar{\mathbf{d}}(f_k) + \mathcal{Z}_x \\ \mathcal{Y}_z &= \sum_{k=1}^K \sigma_k \mathbf{a}_z(\phi_k) \circ \mathbf{b}(\varphi_k, \phi_k) \circ \bar{\mathbf{d}}(f_k) + \mathcal{Z}_z \end{aligned} \quad (26)$$

where $\circ$ denotes the outer product of matrices, $\mathcal{Z}_x$ and $\mathcal{Z}_z$ are the noise tensors, and $\bar{\mathbf{d}}(f_k)$ is given by

$$\bar{\mathbf{d}}(f_k) \triangleq [d_1(f_k), d_{1+M_L}(f_k), \dots, d_{1+M_L(P-1)}(f_k)]^T \quad (27)$$

where

$$d_q(f_k) = e^{j2\pi f_k (q-1)T_r}, q = 1, 2, \dots, Q. \quad (28)$$

Factor matrices $\mathbf{A}_x$, $\mathbf{A}_z$, $\mathbf{B}$, and $\bar{\mathbf{D}}$ of the constructed tensors can be defined as

$$\begin{aligned} \mathbf{A}_x &\triangleq [\mathbf{a}_x(\varphi_1), \mathbf{a}_x(\varphi_2), \dots, \mathbf{a}_x(\varphi_K)] \\ \mathbf{A}_z &\triangleq [\mathbf{a}_z(\phi_1), \mathbf{a}_z(\phi_2), \dots, \mathbf{a}_z(\phi_K)] \\ \mathbf{B} &\triangleq [\mathbf{b}(\varphi_1, \phi_1), \mathbf{b}(\varphi_2, \phi_2), \dots, \mathbf{b}(\varphi_K, \phi_K)] \\ \bar{\mathbf{D}} &\triangleq [\bar{\mathbf{d}}(f_1), \bar{\mathbf{d}}(f_2), \dots, \bar{\mathbf{d}}(f_K)]. \end{aligned} \quad (29)$$

To obtain the factor matrices in (29), the CPD approach via ALS is employed. Taking the factor matrix $\mathbf{A}_x$ as an example, the problem of estimating $\mathbf{A}_x$ boils down to solving the following optimization (data fitting) problem

$$\hat{\mathbf{A}}_x \leftarrow \arg \min_{\mathbf{A}_x} \left\| [\mathcal{Y}_x]_{(1)} - [(\mathbf{B} \circ \bar{\mathbf{D}}) \mathbf{A}_x^T] \right\|_F^2 \quad (30)$$

where $[\cdot]_{(1)}$ is the mode-1 unfolding of a tensor, $\circ$ stands for the Khatri-Rao product (or column-wise Kronecker product) of

matrices, and $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. As a result, 2-D DOAs can be estimated using traditional ULA DOA estimation methods, such as ESPRIT, with the obtained $\hat{\mathbf{A}}_x$ and $\hat{\mathbf{A}}_z$.

B. CPD for Slow-Time Transmit Beamspace L-Shaped Array

As shown in Section IV-A, the CPD method can give DOA estimates for DDMA L-shaped array radar. However, Doppler spectrum division operation, which is required prior to CPD operation, results in pulse signal downsampling. As a result, the useful velocity information contained in the pulse signal measurements is lost partly, and velocity ambiguity arises when the factor matrix $\bar{\mathbf{D}}$ in (29) is used to estimate target velocity. On the other hand, DDMA phase modulation matrix $\mathbf{W}$, whose column vectors function as beamformers, distributes the transmit energy over the whole spatial region and does not allow for energy focusing. Considering the above issues, a new method called slow-time TB L-shaped array based on CPD is proposed, which enables velocity disambiguation and performance improvement of parameter estimation.

To focus transmit energy within the region of interest Θ , slow-time phase modulation matrices $\mathbf{W}_{x,\text{TB}}$ and $\mathbf{W}_{z,\text{TB}}$ are designed first. Storing the received signals of subarrays in matrices $\mathbf{Y}_x$ and $\mathbf{Y}_z$ both of size $M_r \times Q$, we have

$$\begin{aligned} \mathbf{Y}_x &= (\mathbf{p} \odot \mathbf{B}) [(\mathbf{W}_{x,\text{TB}}^T \mathbf{A}_x) * \mathbf{D}]^T + \mathbf{Z}_x \\ \mathbf{Y}_z &= (\mathbf{p} \odot \mathbf{B}) [(\mathbf{W}_{z,\text{TB}}^T \mathbf{A}_z) * \mathbf{D}]^T + \mathbf{Z}_z \end{aligned} \quad (31)$$

where $\mathbf{p} = [\sigma_1, \sigma_2, \dots, \sigma_K]$, $*$ denotes the Hadamard product of matrices, and

$$\begin{aligned} \mathbf{D} &\triangleq [\mathbf{d}(f_1), \mathbf{d}(f_2), \dots, \mathbf{d}(f_K)] \\ \mathbf{d}(f_k) &\triangleq [d_1(f_k), d_2(f_k), \dots, d_Q(f_k)]^T. \end{aligned} \quad (32)$$

The (n, q) -th elements of matrices $\mathbf{Z}_x$ and $\mathbf{Z}_z$ are $z_{x,nq}$ and $z_{z,nq}$ in (13), respectively. Taking advantage of the periodicity of the phase modulation matrices $\mathbf{W}_{x,\text{TB}}$ and $\mathbf{W}_{z,\text{TB}}$, (31) can be re-expressed as

$$\begin{aligned} \mathbf{Y}_x &= (\mathbf{p} \odot \mathbf{B}) [\bar{\mathbf{H}} \odot \mathbf{A}_{x,\text{TB}}]^T + \mathbf{Z}_x \\ \mathbf{Y}_z &= (\mathbf{p} \odot \mathbf{B}) [\bar{\mathbf{H}} \odot \mathbf{A}_{z,\text{TB}}]^T + \mathbf{Z}_z \end{aligned} \quad (33)$$

where $\bar{\mathbf{H}} \triangleq [\bar{\mathbf{h}}(f_1), \bar{\mathbf{h}}(f_2), \dots, \bar{\mathbf{h}}(f_K)]$, and $\bar{\mathbf{h}}(f_k)$ is given by

$$\bar{\mathbf{h}}(f_k) \triangleq [d_1(f_k), d_{1+M}(f_k), \dots, d_{1+M(P_v-1)}(f_k)]^T \quad (34)$$

where $P_v = Q/M$ is an integer. Matrices $\mathbf{A}_{x,\text{TB}}$ and $\mathbf{A}_{z,\text{TB}}$ can be further expressed as

$$\begin{aligned} \mathbf{A}_{x,\text{TB}} &= (\mathbf{W}_{x,\text{TB},0}^T \mathbf{A}_x) * \mathbf{D}_0 \\ \mathbf{A}_{z,\text{TB}} &= (\mathbf{W}_{z,\text{TB},0}^T \mathbf{A}_z) * \mathbf{D}_0 \end{aligned} \quad (35)$$

where $\mathbf{W}_{x,\text{TB},0}$ and $\mathbf{W}_{z,\text{TB},0}$ are defined in the same way as $\mathbf{W}_{\text{TB},0}$ in (20), and $\mathbf{D}_0$ consists of the first M rows of $\mathbf{D}$.

Taking the signal $\mathbf{Y}_x$ in (33) as an example, the signal transformation from a matrix into a 3-D tensor of size $M \times M_r \times P_v$ is illustrated in Fig. 2. Then we have

$$\mathcal{Y}_{x,D} = \sum_{k=1}^K \sigma_k \mathbf{a}_{x,\text{TB}}(\varphi_k) \circ \mathbf{b}(\varphi_k, \phi_k) \circ \bar{\mathbf{h}}(f_k) + \mathcal{Z}_{x,D}$$

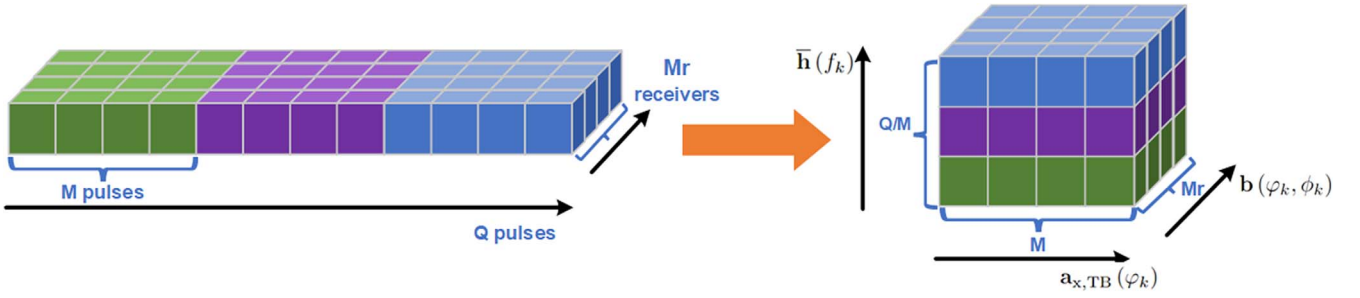


Fig. 2. Illustration of transforming a matrix signal into a 3-D tensor.

$$\mathcal{Y}_{z,D} = \sum_{k=1}^K \sigma_k \mathbf{a}_{z,TB}(\phi_k) \circ \mathbf{b}(\varphi_k, \phi_k) \circ \bar{\mathbf{h}}(f_k) + \mathcal{Z}_{z,D} \quad (36)$$

where $\mathcal{Z}_{x,D}$ and $\mathcal{Z}_{z,D}$ are noise tensors.

Comparing (26) with (36), the part of velocity information that is lost in (26) is preserved in (36), that is,

$$\begin{aligned} \mathbf{A}_{x,TB} &\triangleq [\mathbf{a}_{x,TB}(\varphi_1), \mathbf{a}_{x,TB}(\varphi_2), \dots, \mathbf{a}_{x,TB}(\varphi_K)] \\ \mathbf{A}_{z,TB} &\triangleq [\mathbf{a}_{z,TB}(\phi_1), \mathbf{a}_{z,TB}(\phi_2), \dots, \mathbf{a}_{z,TB}(\phi_K)]. \end{aligned} \quad (37)$$

Thus, the unambiguous velocity can be restored by combining $\bar{\mathbf{H}}$ with $\mathbf{D}_0$ contained in $\mathbf{A}_{x,TB}$ or $\mathbf{A}_{z,TB}$.

With factor matrices $\mathbf{A}_{x,TB}$, $\mathbf{A}_{z,TB}$ and $\bar{\mathbf{H}}$ obtained by CPD, the minimum mean square error (MMSE) estimation algorithm can be employed to estimate DOA pairs $\{\varphi_k, \phi_k\}_{k=1}^K$. Looking into (35), we can find that $\mathbf{A}_{x,TB}$ is the Hadamard product of $\mathbf{D}_0$ and $\mathbf{E}_x = \mathbf{W}_{x,TB,0}^T \mathbf{A}_x$. For the k th column of $\mathbf{E}_x$, we can write

$$\begin{aligned} \mathbf{e}_{x,k} &= \mathbf{W}_{x,TB,0}^T \mathbf{a}_x(\varphi_k) \\ &= [\mathbf{w}_{x,TB,q_1}^T \mathbf{a}_x(\varphi_k), \mathbf{w}_{x,TB,q_2}^T \mathbf{a}_x(\varphi_k), \dots, \mathbf{w}_{x,TB,q_M}^T \mathbf{a}_x(\varphi_k)]^T \end{aligned} \quad (38)$$

where $\mathbf{w}_{x,TB,q_m}$ is the m th column of $\mathbf{W}_{x,TB,0}$. Considering that each TB phase modulation vector $\mathbf{w}_{x,TB,q_m}$ obeys the discrete Fourier transform (DFT) structure for different frequencies $f_{q_m} = (q_m - 1)/M_v$, $q_m = q_1, q_2, \dots, q_M$, $\mathbf{e}_{x,k}$ can be regarded as DFT operations at M different frequency points. Meanwhile, every element of $\mathbf{D}_0$ is an exponential value with unit amplitude, having no effect on the element amplitudes of $\mathbf{A}_{x,TB}$. Therefore, $\{\varphi_k\}_{k=1}^K$ can be estimated by solving the following MMSE problem

$$\hat{\varphi}_k = \arg \min_{\varphi} \|\mathbf{e}_{x,k} - \mathbf{e}(\varphi)\|_F^2 \quad (39)$$

where $\mathbf{e}(\varphi) = \mathbf{W}_{x,TB,0}^T \mathbf{a}_x(\varphi)$. In a similar way, $\{\phi_k\}_{k=1}^K$ can be estimated as well.

To obtain unambiguous velocity, the estimated factor $\hat{\bar{\mathbf{H}}}$ is combined with $\mathbf{D}_0$, and $\hat{\mathbf{D}}_0$ is estimated by

$$\hat{\mathbf{D}}_0 = \mathbf{A}_{x,TB} \oslash \hat{\mathbf{E}}_x \quad (40)$$

where $\oslash$ denotes the element-wise division of two matrices. $\hat{\mathbf{E}}_x$ is the estimate of $\mathbf{E}_x$ by substituting $\{\hat{\varphi}_k\}_{k=1}^K$ in (38). With $\hat{\bar{\mathbf{H}}}$ and $\hat{\mathbf{D}}_0$, we can find

$$\hat{\bar{\mathbf{H}}} \odot \hat{\mathbf{D}}_0 \triangleq [\mathbf{d}(f_1), \mathbf{d}(f_2), \dots, \mathbf{d}(f_K)]. \quad (41)$$

From (41), target velocities $\{v_k\}_{k=1}^K$ can then be easily obtained.

As discussed above, 2-D DOA estimation for the proposed L-shaped array slow-time MIMO TB radar is achieved by estimating azimuth and elevation separately. Consequently, 2-D DOA pair-matching is required in the presence of more than one target. Recalling (36), factor matrices $\mathbf{A}_{x,TB}$, $\mathbf{A}_{z,TB}$, and $\mathbf{B}$ are obtained after tensor decomposition via ALS. However, only the matrices associated with transmit arrays, i.e., $\mathbf{A}_{x,TB}$ and $\mathbf{A}_{z,TB}$, are utilized for DOA estimation, while factor matrix $\mathbf{B}$ associated with the receive array is not utilized.⁴ According to the PARAFAC uniqueness conditions given in terms of Kruskal rank [45], these factor matrices are unique up to permutation and scaling of columns if the following inequalities are satisfied

$$\begin{aligned} r_{\mathbf{A}_{x,TB}} + r_{\bar{\mathbf{H}}} + r_{\mathbf{B}} &\geq 2K + 2 \\ r_{\mathbf{A}_{z,TB}} + r_{\bar{\mathbf{H}}} + r_{\mathbf{B}} &\geq 2K + 2 \end{aligned} \quad (42)$$

where $r_{\{\cdot\}}$ denotes the Kruskal rank of a matrix. Therefore, the parameters estimated based on these factor matrices are automatically paired column-by-column if (42) holds. Thanks to this property, factor matrix $\mathbf{B}$ can be utilized for pair-matching. Given that two transmit subarrays in the proposed L-shaped array arrangement can already give 2-D DOA estimation except for pair-matching, the receive array's shape has no effect on pair-matching capability. A concrete example is given and discussed in Section V.

Finally, the DOA pairs $\{\varphi_k, \phi_k\}_{k=1}^K$ can be found with performance improvement as will be validated in the next section, and velocity ambiguity can be resolved at the same time.

V. SIMULATION RESULTS

In this section, simulation experiments are conducted to demonstrate the effectiveness of the proposed slow-time TB method for pulsed L-shaped array MIMO radar with frequency-modulated continuous waves in the scenario of automotive radar applications. Throughout all the simulations, we assume an L-shaped transmit array where two ULA subarrays are positioned along the x-axis and z-axis, and each subarray is made up of $M_L = M_x = M_z = 8$ antenna elements spaced half a wavelength apart from each other. The receiver, which is collocated with the transmitter, is set as a ULA along the x-axis with

⁴It is because the estimate based on the factor matrices $\mathbf{A}_{x,TB}$ consistently outperforms the estimate based on $\mathbf{B}$ in all our simulations.

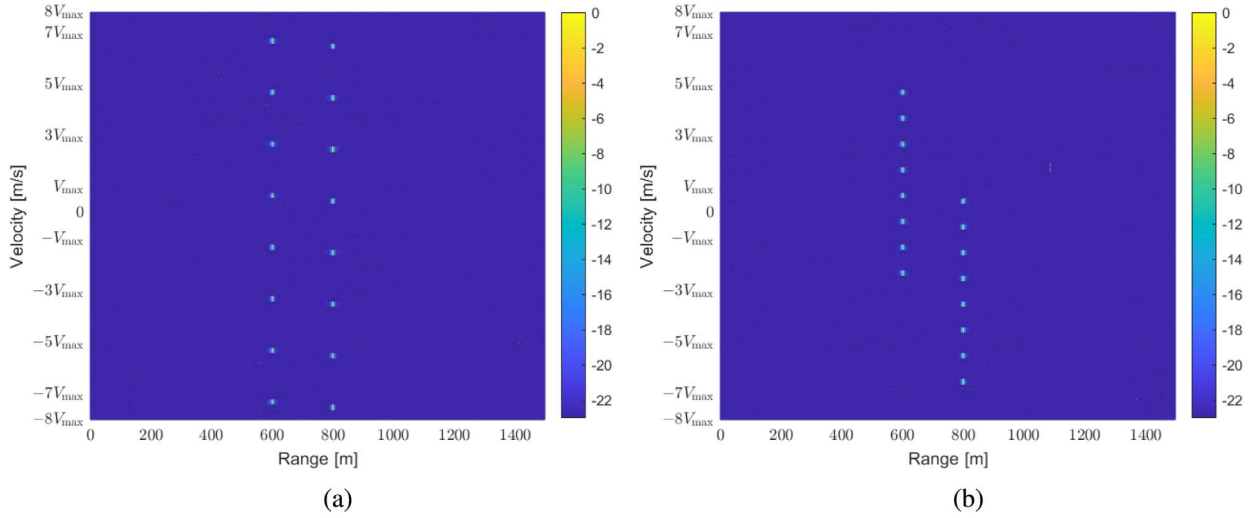


Fig. 3. Range-Doppler map of DDMA approach with (a) $M_v = M_L$ and (b) $M_v > M_L$.

$M_t = 8$ elements and the spacing between adjacent elements $d_r = \lambda/2$ in simulations for demonstration.⁵ The frequency of carrier waveform is $f_c = 79$ GHz, and the transmit signal is sampled with sampling frequency $f_s = 70$ MHz. We also set $Q = 80$ pulses with $T_r = 10$ μ s in a single CPI, and each pulse duration is $T_p = 3$ μ s. It is assumed that source targets follow the Swerling I model, and the additive noise is Gaussian zero-mean unit-variance spatially and temporally white.

A. Example 1: Comparison Between DDMA and Slow-Time Transmit Beamspace Approaches

In this example, there are 2 moving targets with ranges $\mathbf{R} = \{800 \text{ m}, 600 \text{ m}\}$ and velocities $\mathbf{v} = \{0.4v_{\max}, 4.6v_{\max}\}$, where $v_{\max} = \lambda \text{PRF}/4/M_L = 11.87$ m/s is the maximum unambiguous velocity for DDMA. The value of $v_{\max}$ remains constant throughout this section without any special remarks. To demonstrate the range-Doppler map of the received signal, down & up chirp signals (see (43)) are used for two subarrays as baseband transmit waveforms

$$u_i(t) = \text{rect}\left(\frac{t - T_p/2}{T_p}\right) e^{j\pi K_i t^2}, \quad i = 1, 2 \quad (43)$$

where $K_1 = -B/T_p$ and $K_2 = B/T_p$ are the chirp rates for down-chirp and up-chirp signals, respectively, and the transmit signal bandwidth is $B = 60$ MHz throughout this section.

The phase modulation matrix for DDMA approach is described in (3) with $M_t = M_L = 8$. After 2-D DFT, the range-Doppler map of the received signal is depicted in Fig. 3(a). With DDMA phase modulation, the signal from each transmit element is modulated with a distinct Doppler shift. Therefore, as it can be seen in Fig. 3(a), 8 separate peaks are uniformly distributed over the entire Doppler spectrum along an identical range cell for each target. Each peak can be regarded as attributed to the signal from only one transmit element, which ensures that waveform diversity can be achieved by Doppler spectrum division.

To enhance the performance of MIMO radar further, as discussed in Section III, a new phase modulation matrix is proposed that optimizes the transmit energy distribution. The region of interest is set as $\Theta = [60^\circ, 120^\circ]$ throughout this section. First, the entire spectrum is divided into $M_v = 16$ subbands in this example, which narrows down the mainlobe distance of adjacent beams formed by phase modulation vectors compared to conventional DDMA with $M_v = 8$, and its range-Doppler map is given in Fig. 3(b). Observing two figures in Fig. 3, we can see that peaks are distributed uniformly for both of them, while the distance of adjacent peaks gets smaller, and there exists an extra Doppler spectrum band for the DDMA with $M_v = 16$ in Fig. 3(b) due to the fact that $M_v > M_t$. Next, those modulation vectors of pulses $\{1, 2, 3, 4, 13, 14, 15, 16\}$ in every period of 16 pulses are selected to construct a new phase modulation for transmit energy optimization. The transmit energy distribution of conventional DDMA and the proposed approach are demonstrated in Fig. 4. Compared to DDMA approach in Fig. 4(a), adjacent beams of the proposed approach in Fig. 4(b) get closer to each other, resulting in more beams directing toward Θ . By selecting those pulse vectors whose beamforming mainlobes direct toward Θ , the overall transmit energy of the proposed approach in a pulse period is no longer spread across the whole spatial region as it is in the conventional DDMA technique, and instead the proposed method achieves transmit energy optimization: its transmit energy accumulates within Θ .

Different from DDMA technique, as it is discussed in Section III-B, the separation of transmit channels for the proposed slow-time TB technique cannot be achieved by Doppler spectrum division because the energy of a single transmit channel for each target can be distributed over multiple peaks in the Doppler domain rather than one single cluster of range-Doppler bins and mixed up with those peaks of other channels. Fig. 5(a) depicts the received signal with the proposed TB modulation matrix $\mathbf{W}_{\text{TB}}$ in the Doppler domain. Compared to DDMA in Fig. 3(a),

⁵The receiver does not have to be an ULA with $\lambda/2$ spacing.

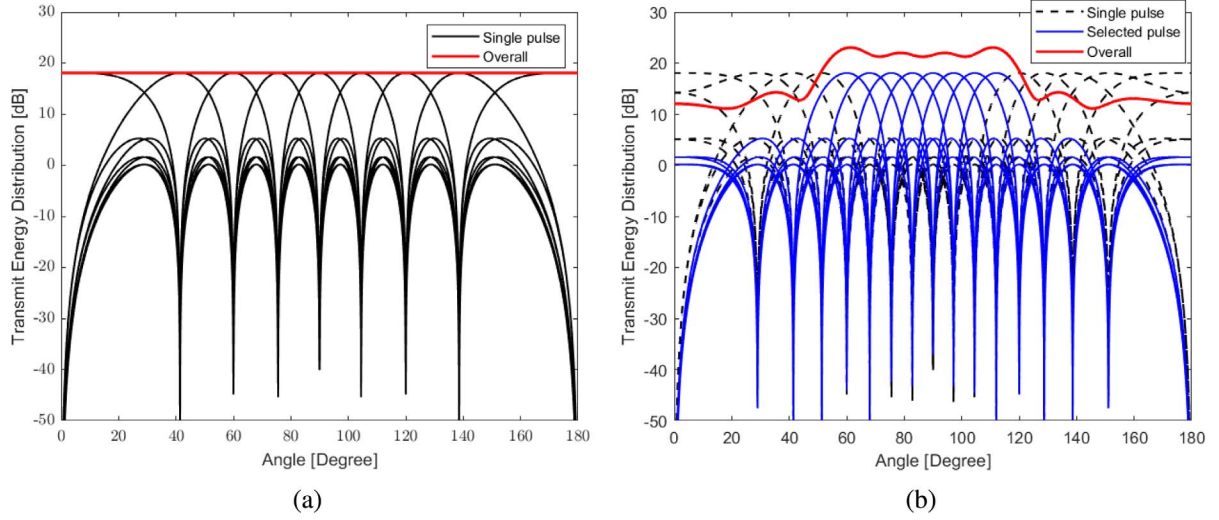


Fig. 4. Transmit energy beampatterns of (a) DDMA and (b) the proposed approach.

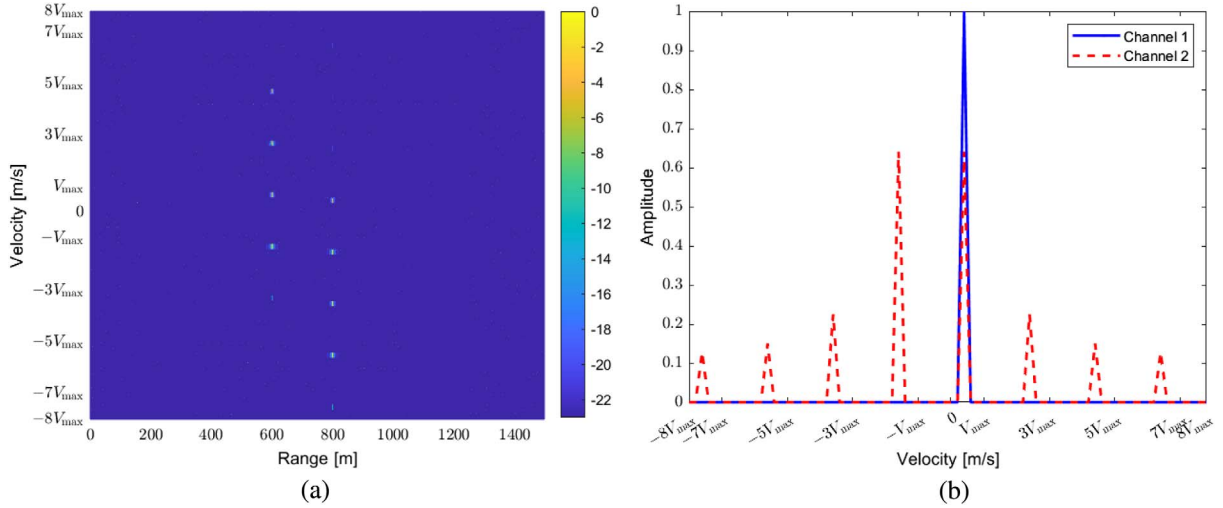


Fig. 5. (a) Range-Doppler map of the proposed approach and (b) Doppler spectrums of two transmit channels.

there are the same number of peaks as DDMA for a target in the Doppler domain, but amplitudes of these peaks differ a lot from each other. Taking the first and second transmit channels of the first target as an example, their Doppler spectrums are shown in Fig. 5(b). For the first transmit channel, its energy only accumulates over one peak due to zero Doppler shift, and the energy dispersed over other regions is so low as to be ignored. In contrast, the signal of the second channel is spread over 8 peaks with uniform distances but different amplitudes, which verifies the analysis in Section III-B. As a consequence, waveform diversity cannot be achieved by Doppler spectrum division. Therefore, conventional parameter estimation algorithms including MUSIC, ESPRIT, and CPD in [38] no longer work. To obtain both transmit energy distribution optimization and parameter estimation, a new parameter estimation method based on CPD is proposed, and its performance is evaluated in Section V-C.

B. Example 2: Discussion on M_v

A larger M_v narrows down the mainlobe interval between adjacent beams, which consequently directs more beams within the region of interest Θ . With a predetermined Θ , any $M_v > M_t$ can achieve transmit energy focusing by proper pulse selection, and different M_v leads to a different transmit energy beampattern. To evaluate the performance of transmit energy distribution optimization, here we define the average energy per pulse coming from within Θ as the transmit beamspace energy P , which can be expressed as

$$P = \frac{1}{M} \sum_{q \in \mathcal{I}} \int_{\Theta} |\mathbf{a}^T(\theta) \mathbf{w}_q|^2 d\theta \quad (44)$$

where M is the number of selected pulses of each period for slow-time TB MIMO radar, and $\mathcal{I}$ is the index set of the selected pulses.

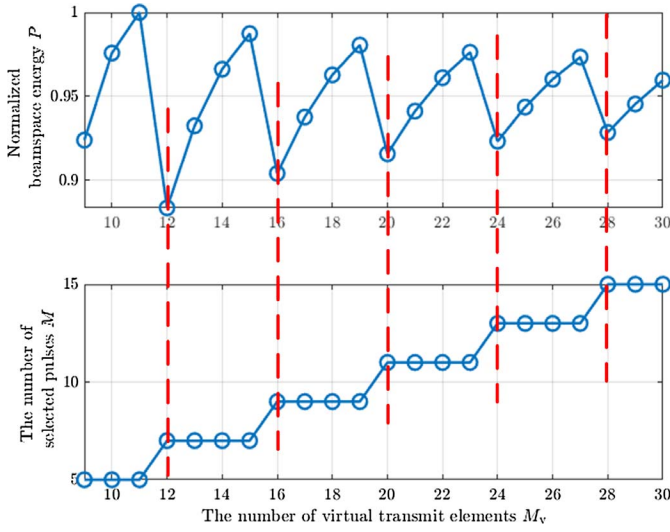


Fig. 6. Beamspace energy P and the number of selected pulses M within each pulse period versus M_v .

To illustrate the different beamspace beampatterns for different M_v , consider the following example. If the mainlobe direction formed by a pulse vector satisfies $\theta_{b,q} \in \Theta$, its phase modulation vector is selected to construct a new phase modulation matrix for slow-time TB MIMO radar. Taking $M_v = 16$ as an example, 9 modulation pulse vectors within a period of 16 pulses are selected, and their pulse index set is $\mathcal{I} = \{1, 2, 3, 4, 5, 13, 14, 15, 16\}$, and the corresponding mainlobe directions $\{\cos \theta_{b,q}\}_{q \in \mathcal{I}}$, defined in (16) with $M_t = 16$, are $\{0, -1/8, -1/4, -3/8, -1/2, 1/2, 3/8, 1/4, 1/8\}$. Transmit beamspace energy P and the total number of the selected pulses within each period for $M_v \in [9, 30]$ are calculated, and results are shown in Fig. 6. Observing this figure, transmit beamspace energy P varies with M_v , and does not always decrease or increase as M_v increases. While the number of selected pulses remains unchanged, the transmit beamspace energy P increases as M_v gets larger. This is because a larger M_v leads the selected beams to approach the center of Θ , resulting in more transmit energy of sidelobes distributed within Θ . However, as long as more pulses are added, P falls sharply, and then gradually climbs until the number of selected pulses increases again. More selected pulses lead to more energy of sidelobes and even of mainlobes dispersing out of Θ , which explains why P falls sharply when more beams are comprised.

The mainlobe direction $\theta_{b,q}$ is used as an indicator to determine which modulation pulse vectors are selected for transmit energy focusing, but the selection criterion is flexible and should be adjusted to meet other requirements. In the following example, the number of selected pulses is required to be no smaller than M_t in order to ensure the maximum number of detectable targets. In such a case, $M_v = 16$ is set, and 8 instead of 9 pulses are selected as an example to compare its performance to traditional DDMA MIMO radars with $M_t = 8$. A higher beamspace energy P can be obtained by choosing a higher M_v to some extent, but not always, as is shown in Fig. 6. Meanwhile, the maximum number of detectable targets

of parameter estimation also influences the value of M_v selected. Therefore, both the beamspace energy distribution and the required number of detectable targets should be considered when selecting a proper M_v for slow-time TB MIMO radars.

C. Example 3: Performance Comparison of Parameter Estimation

In this example, parameter estimation performance of the proposed method is assessed in two scenarios: targets that are distantly spaced and targets that are closely spaced. The performance of the proposed method is also compared to that of three other algorithms, including ESPRIT [26], CPD [38], and PARAFAC-Direct [39]. CRBs for both PARAFAC-Direct and the proposed methods are given as well. The detailed derivation of CRBs can be found in the Appendix. According to the configuration of the proposed L-shaped array radar, one transmit subarray is arranged along z-axis, while the receive array is arranged along x-axis. Thus, transmitter matrix $\mathbf{A}_{z,TB}$ is associated with elevation ϕ , while receiver matrix $\mathbf{B}$ is associated with azimuth φ . By performing the proposed method on the separated signal from the z-axis transmit subarray, $\{\phi_k\}_{k=1}^K$ using $\mathbf{A}_{z,TB}$ and $\{\varphi_k\}_{k=1}^K$ using $\mathbf{B}$ can be easily paired.

Assuming that Θ is known, $M_v = 16$ and the selected pulse set is $\mathcal{I} = \{1, 2, 3, 4, 13, 14, 15, 16\}$, then the corresponding beam mainlobe directions $\{\cos \theta_{b,q}\}_{q \in \mathcal{I}}$ for the selected pulses are $\{0, -1/8, -1/4, -3/8, 1/2, 3/8, 1/4, 1/8\}$. The root mean square error (RMSE) and the probability of source resolution versus SNR for azimuth φ , elevation ϕ , and velocity v are evaluated. RMSE is a standard metric used to measure the estimation error, and it is defined as

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x - \hat{x}_i)^2} \quad (45)$$

where N is the number of observations, x represents the true value of a parameter to be estimated, and $\hat{x}_i$ is the estimate for the i th trial. $N = 300$ Monte Carlo trials are performed throughout this section. The probability of resolution will be defined later. Considering that this example focuses on parameter estimation after transmit subarray signals separation, to ensure the ideal independence of the subarrays, two orthogonal waveforms (see (46)) are used as transmit baseband waveforms in this example

$$u_i(t) = \text{rect}\left(\frac{t - T_p/2}{T_p}\right) e^{j2\pi \frac{i}{T_p} t}, \quad i = 1, 2. \quad (46)$$

First, two distant targets are set with $\varphi = \{80^\circ, 88^\circ\}$, $\phi = \{92^\circ, 102^\circ\}$, and $\mathbf{v} = \{0.4v_{\max}, 0.6v_{\max}\}$, and their estimation results are shown in Fig. 7. The first two figures show the angle estimation results of azimuth φ using the x-subarray transmitter and elevation ϕ using z-subarray transmitter, respectively, and it can be found that they almost share the same results for each algorithm. To assess the performance of the proposed method precisely, only the related information in the estimated transmitter matrices $\mathbf{A}_{x,TB}$ and $\mathbf{A}_{z,TB}$ defined in (35) are utilized

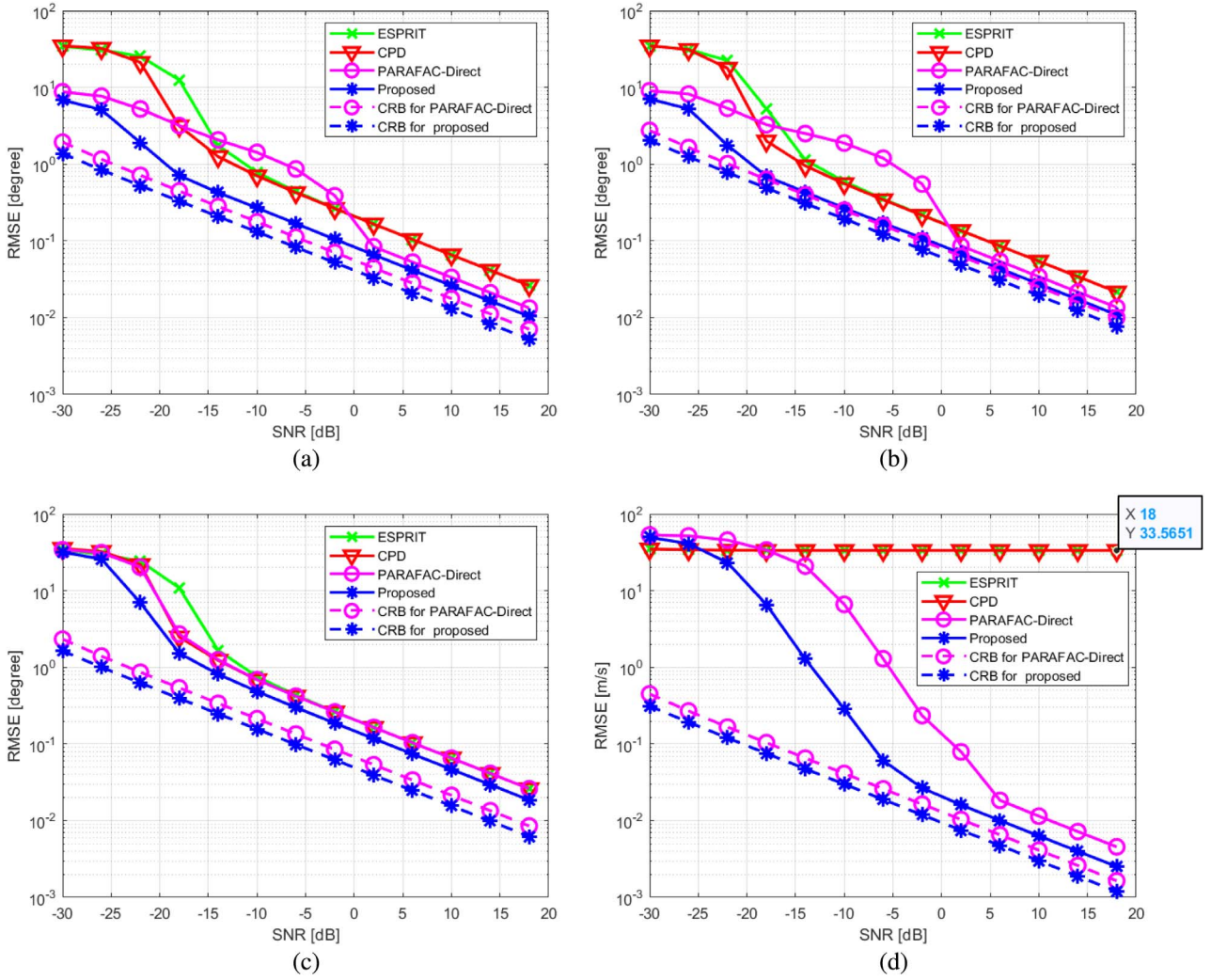


Fig. 7. RMSE results of 2 distinctly spaced targets. (a) Azimuth φ by transmit x-subarray. (b) Elevation ϕ by transmit z-subarray. (c) Azimuth φ by receive array. (d) Velocity.

for angle estimation here.⁶ ESPRIT and CPD achieve nearly the same performance except that CPD gains lower/better RMSE results than ESPRIT for lower SNR cases, which confirms that the multi-linear structure preserved in tensor-based signal models leads to better performance in lower SNR cases. The performance of the PARAFAC-Direct method is worse than ESPRIT and CPD when SNR is roughly in the range of -20 dB to 0 dB, while it starts to outperform them when SNR is larger than 0 dB. The proposed slow-time TB MIMO technique outperforms all aforementioned methods, particularly when SNRs are low, which is mainly attributed to the optimization of transmit energy distribution.

The results of azimuth estimation only using the receive array are shown in Fig. 7(c). They are obtained by averaging the RMSE results using the signals of x-subarray and z-subarray,

⁶Note that the receive array and the transmit subarray along z-axis can form a virtual URA that can also be used for DOA estimation, but to keep the focus on transmit energy focusing, the estimates based on matrices $\mathbf{A}_{x, \text{TB}}$ and $\mathbf{A}_{z, \text{TB}}$ are sufficient to demonstrate the benefits of the proposed approach.

which can be described as

$$\overline{\text{RMSE}} = \frac{1}{2} \left(\sqrt{\frac{1}{K} \sum_{k=1}^K \text{RMSE}_{x,k}^2} + \sqrt{\frac{1}{K} \sum_{k=1}^K \text{RMSE}_{z,k}^2} \right) \quad (47)$$

where $\text{RMSE}_{x,k}$ and $\text{RMSE}_{z,k}$ are the RMSE results of the k th target for x-subarray and z-subarray, respectively. For the azimuth estimation using the receive array, ESPRIT, CPD, and PARAFAC-Direct produce comparable results, while the proposed method outperforms the other three methods tested. However, the improvement of azimuth angle estimation using the receive array is smaller than the improvement by using transmit arrays in Fig. 7(a), confirming once again that transmit energy optimization contributes to performance improvement.

To validate the proposed method's ability to resolve velocity ambiguity, the velocity of the second target is changed to $4.6v_{\text{max}}$, while other parameters remain the same as in the previous experiment of this example. As it is illustrated in Fig. 7(d), only the results of PARAFAC-Direct and the proposed method

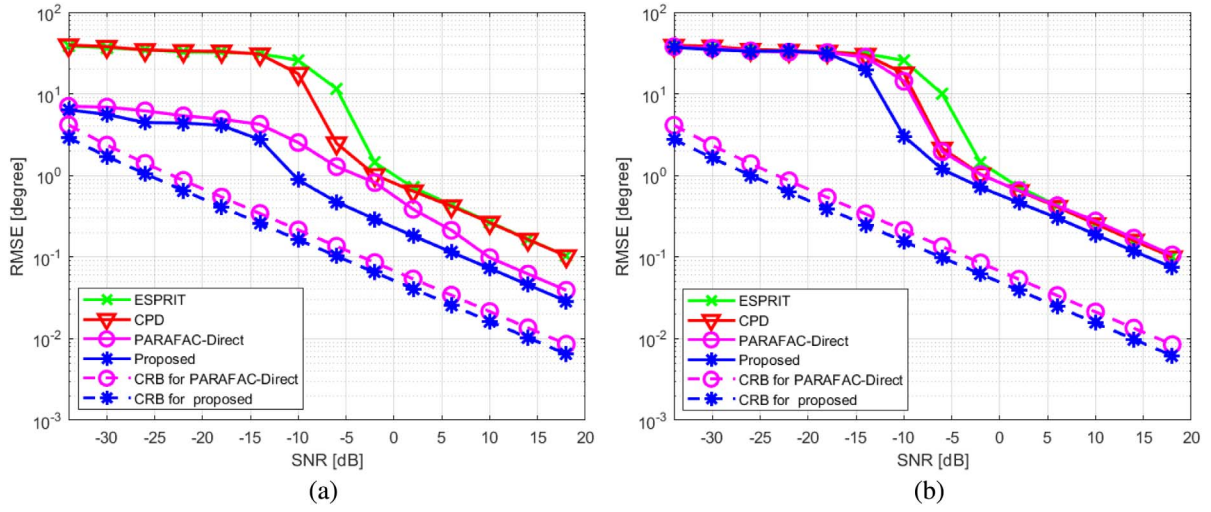


Fig. 8. RMSE results of 2 closely spaced targets. (a) Angle estimation by transmit array. (b) Azimuth φ estimation by receive array.

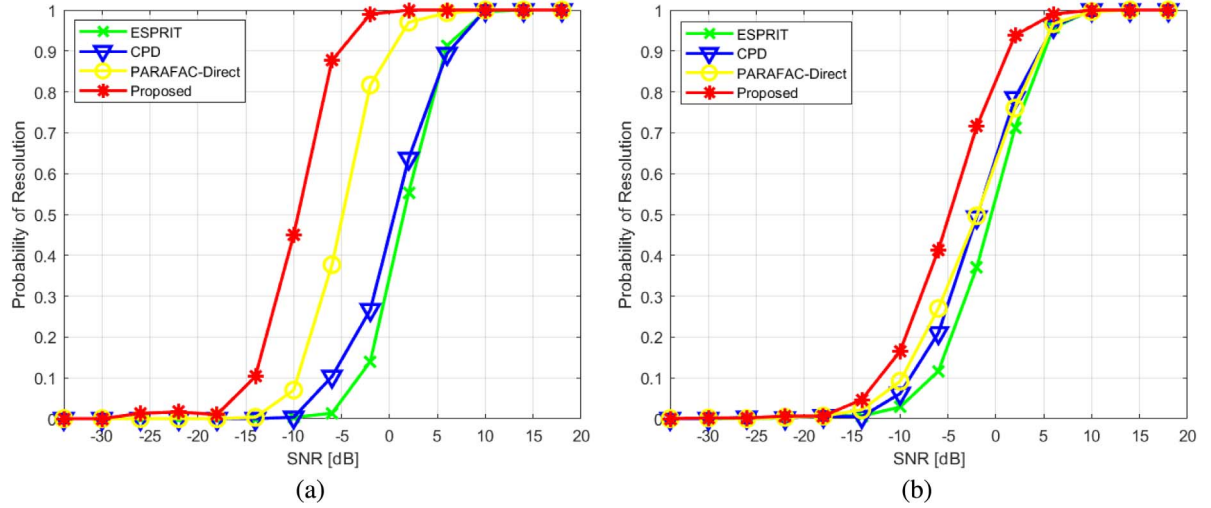


Fig. 9. Probability of target resolution versus SNR of 2 closely spaced targets. (a) Angle estimation by transmit array. (b) Azimuth φ estimation by receive array.

follow the descending trends of RMSE versus SNR, while RMSEs of ESPRIT and CPD methods are high (around 33.57 m/s) even at high SNRs. It is because ESPRIT and CPD methods are unable to resolve velocity ambiguity. Besides the ability to resolve Doppler ambiguity, the proposed method achieves lower RMSEs than the PARAFAC-Direct method, especially for the SNRs lower than 0 dB. It is also noteworthy that Doppler ambiguity makes it difficult for DDMA-based ESPRIT and CPD methods to determine which transmit elements that the signals come from, even after signal separation, rendering these two methods invalid for DOA estimation. It demonstrates once again that the proposed method may achieve velocity disambiguation and improved performance of parameter estimation at the same time.

Next, to assess the angle resolution of the proposed method, two closely-spaced targets with $\varphi = \{86^\circ, 88^\circ\}$, $\phi = \{100^\circ, 102^\circ\}$, and $\mathbf{v} = \{0.4v_{\max}, 0.6v_{\max}\}$ are used in the following experiment. Given that the estimation results of φ and ϕ are similar as illustrated in Fig. 7(a) and 7(b), the

angle estimation results given in Fig. 8(a) are obtained by averaging the results of φ and ϕ using two transmit matrices, and its calculation method can refer to (47). The RMSE results for azimuth angles estimated by receive array are shown in Fig. 8(b). Similar to the previous experiment, the proposed method outperforms the other three methods, especially for the estimation results using transmit array in Fig. 8(a).

The probability of target resolution, which refers to the likelihood that an algorithm can resolve two closely spaced targets, is assessed among four methods. Two targets are considered to be resolved if the angle estimation error satisfies

$$|\hat{\theta} - \theta| < \frac{\Delta\theta}{2} \quad (48)$$

where $\hat{\theta}$ and θ are the estimated and real angle values, respectively, and $\Delta\theta = |\theta_1 - \theta_2|$ is the absolute angle difference between two targets. The angle resolution results by transmit and receive array are shown in Fig. 9(a) and 9(b), respectively. Transmit matrices can be used to determine azimuth φ and

elevation ϕ . To locate source targets in 3-D geometry, two closely-spaced targets are considered to be resolved if their azimuth φ and elevation ϕ fulfill (48) simultaneously. Thus, only when φ and ϕ are resolvable simultaneously, the result is recognized as resolvable in Fig. 9(a). Observing Fig. 9, the probability of target resolution starts ramping up for each method tested as SNR value increases until it eventually reaches 100% correct resolution, and the SNR value at which this transition happens is called as SNR threshold [40]. It can be clearly seen from Fig. 9(a) that transitions of ESPRIT, CPD, and PARAFAC-Direct techniques happen at nearly the same SNR value, while the SNR threshold of the proposed method is around 10–15 dB lower than the other three methods tested. When compared to angle estimation by the transmit array, the proposed method also outperforms all other methods in angle estimation by the receive array, as is depicted in Fig. 9(b), but its improvement is less evident, same as in the scenario with targets placed significantly apart from one another.

From two experiments conducted in this example, it can be found that the proposed method achieves an evident performance improvement of parameter estimation compared to ESPRIT, CPD, and PARAFAC-Direct in terms of both estimation accuracy and closely spaced targets resolution.

VI. CONCLUSION

In this paper, a technique called slow-time TB MIMO for L-shaped array radar is proposed to improve parameter estimation performance and resolve velocity ambiguity. A new phase modulation matrix for slow-time TB MIMO radar is designed to achieve transmit energy optimization. By dividing the entire spectrum into more subbands than the number of transmit antenna elements, adjacent beams formed by pulse vectors get closer, and more transmit beams can be comprised within the region of interest. The new modulation matrix for the proposed slow-time TB MIMO radar is constructed from those pulse vectors whose beams are directed toward the desired region. A new tensor-based signal model is designed for parameter estimation, which preserves the multi-linear structure of signal measurements. Moreover, it also preserves the velocity information for velocity disambiguation which is missing in the matrix-based methods with Doppler spectrum division. The parameter estimation performance of the proposed method is enhanced, especially for lower SNR cases, and velocity ambiguity is alleviated. Simulation results have verified that the proposed slow-time TB MIMO radar technique has more effective parameter estimation performance than the other existing methods.

APPENDIX CRBs DERIVATION

First, the CRBs are derived for the signal of z-subarray. Vectorizing $\mathbf{Y}_z$ in (31) into $\mathbf{y}_z = \text{vec}(\mathbf{Y}_z) \in \mathbb{C}^{QM_r \times 1}$, the received signal can be re-expressed as

$$\mathbf{y}_z = \sum_{k=1}^K \sigma_k \mathbf{b}(\varphi_k) \odot [\mathbf{W}_{z,\text{TB}}^T \mathbf{a}_z(\phi_k) * \mathbf{d}(f_k)] + \text{vec}(\mathbf{Z}_z) \quad (49)$$

where $\text{vec}(\cdot)$ denotes the vectorization operation. Given that the receive array is set as a ULA along the x-axis in the simulation experiments, $\mathbf{b}(\varphi_k)$ is used here. The corresponding covariance matrix is then

$$\mathbf{R} = \mathbf{y}_z \mathbf{y}_z^H = \mathbf{C} \mathbf{P} \mathbf{C}^H + \sigma_n^2 \mathbf{I} \quad (50)$$

where $\mathbf{P}$ is a diagonal matrix with diagonal elements $\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2, \sigma_n^2$ is the noise variance, $\mathbf{I}$ is an identity matrix of size $QM_r \times QM_r$, and the k th column of $\mathbf{C}$ is given as

$$\mathbf{c}_k = \mathbf{b}(\varphi_k) \odot [\mathbf{W}_{z,\text{TB}}^T \mathbf{a}_z(\phi_k) * \mathbf{d}(f_k)]. \quad (51)$$

Denote the unknown parameter vector as

$$\boldsymbol{\rho} = [\varphi_1, \dots, \varphi_K, \phi_1, \dots, \phi_K, f_1, \dots, f_K, \sigma_1^2, \dots, \sigma_K^2, \sigma_n^2]^T. \quad (52)$$

According to (4) in [46] the (m, n) -th element of the Fisher information matrix (FIM) can be found as

$$[\mathbf{FIM}_z]_{m,n} = N_s \text{Tr} \left(\frac{\partial \mathbf{R}}{\partial \rho_m} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \rho_n} \mathbf{R}^{-1} \right) \quad m, n = 1, 2, \dots, 4K + 1 \quad (53)$$

where $\text{Tr}(\cdot)$ denotes the trace of a matrix, ρ_n is the n th element of $\boldsymbol{\rho}$, $\partial \mathbf{R} / \partial \rho_n$ stands for the partial derivative of $\mathbf{R}$ with respect to ρ_n , $(\cdot)^{-1}$ denotes the inverse of a matrix, and N_s is the number of snapshots. In the simulation experiments, $N_s = 1$, i.e., a single snapshot is used.

The partial derivatives of $\mathbf{R}$ with respect to $\{\varphi_k\}_{k=1}^K$, $\{\phi_k\}_{k=1}^K$, $\{f_k\}_{k=1}^K$, $\{\sigma_k^2\}_{k=1}^K$, and σ_n^2 in the case of z-subarray can be calculated by

$$\begin{aligned} \frac{\partial \mathbf{R}}{\partial \varphi_k} &= \sigma_k^2 \left(\frac{\partial \mathbf{c}_k}{\partial \varphi_k} \mathbf{c}_k^H + \mathbf{c}_k \frac{\partial \mathbf{c}_k^H}{\partial \varphi_k} \right) \\ \frac{\partial \mathbf{c}_k}{\partial \varphi_k} &= \frac{\partial \mathbf{b}(\varphi_k)}{\partial \varphi_k} \odot [\mathbf{W}_{z,\text{TB}}^T \mathbf{a}_z(\phi_k) * \mathbf{d}(f_k)] \\ \frac{\partial \mathbf{R}}{\partial \phi_k} &= \sigma_k^2 \left(\frac{\partial \mathbf{c}_k}{\partial \phi_k} \mathbf{c}_k^H + \mathbf{c}_k \frac{\partial \mathbf{c}_k^H}{\partial \phi_k} \right) \\ \frac{\partial \mathbf{c}_k}{\partial \phi_k} &= \mathbf{b}(\varphi_k) \odot \left[\mathbf{W}_{z,\text{TB}}^T \frac{\partial \mathbf{a}_z(\phi_k)}{\partial \phi_k} * \mathbf{d}(f_k) \right] \\ \frac{\partial \mathbf{R}}{\partial f_k} &= \sigma_k^2 \left(\frac{\partial \mathbf{c}_k}{\partial f_k} \mathbf{c}_k^H + \mathbf{c}_k \frac{\partial \mathbf{c}_k^H}{\partial f_k} \right) \\ \frac{\partial \mathbf{c}_k}{\partial f_k} &= \mathbf{b}(\varphi_k) \odot \left[\mathbf{W}_{z,\text{TB}}^T \mathbf{a}_z(\phi_k) * \frac{\partial \mathbf{d}(f_k)}{\partial f_k} \right] \\ \frac{\partial \mathbf{R}}{\partial \sigma_k^2} &= \mathbf{c}_k \mathbf{c}_k^H, \quad \frac{\partial \mathbf{R}}{\partial \sigma_n^2} = \mathbf{I}. \end{aligned} \quad (54)$$

Using (53), FIM for the signal of z-subarray can be obtained, and CRBs for azimuth φ , elevation ϕ , and Doppler shift f can be respectively computed as

$$\varphi_{\text{CRB}} = \sqrt{\frac{1}{K} \sum_{i=1}^K [\mathbf{FIM}_z^{-1}]_{i,i}}$$

$$\phi_{\text{CRB}} = \sqrt{\frac{1}{K} \sum_{i=K+1}^{2K} [\mathbf{FIM}_z^{-1}]_{i,i}}$$

$$f_{\text{CRB}} = \sqrt{\frac{1}{K} \sum_{i=2K+1}^{3K} [\mathbf{FIM}_z^{-1}]_{i,i}}. \quad (55)$$

The CRBs for the case of x-subarray can be derived in a similar manner. Considering that there is no information about $\{\phi_k\}_{k=1}^K$ contained in the signal transmitted from x-subarray, the unknown parameter vector becomes

$$\boldsymbol{\rho} = [\varphi_1, \dots, \varphi_K, f_1, \dots, f_K, \sigma_1^2, \dots, \sigma_K^2, \sigma_n^2]^T. \quad (56)$$

The derivative of $\mathbf{c}_k$ with respect to φ_k in this case becomes

$$\frac{\partial \mathbf{c}_k}{\partial \varphi_k} = \frac{\partial \mathbf{b}(\varphi_k)}{\partial \varphi_k} \odot [\mathbf{W}_{x,\text{TB}}^T \mathbf{a}_x(\varphi_k) * \mathbf{d}(f_k)]$$

$$+ \mathbf{b}(\varphi_k) \odot \left[\mathbf{W}_{x,\text{TB}}^T \frac{\partial \mathbf{a}_x(\varphi_k)}{\partial \varphi_k} * \mathbf{d}(f_k) \right]. \quad (57)$$

The corresponding CRBs of φ and f can be calculated by

$$\varphi_{\text{CRB}} = \sqrt{\frac{1}{K} \sum_{i=1}^K [\mathbf{FIM}_x^{-1}]_{i,i}}$$

$$f_{\text{CRB}} = \sqrt{\frac{1}{K} \sum_{i=K+1}^{2K} [\mathbf{FIM}_x^{-1}]_{i,i}} \quad (58)$$

where $\mathbf{FIM}_x$ is the FIM for the signal of x-subarray.

The FIMs for the PARAFAC-Direct model have the same form, and the only difference is that matrices $\mathbf{W}_{x,\text{TB}}$ and $\mathbf{W}_{z,\text{TB}}$ are substituted by $\mathbf{W}_x$ and $\mathbf{W}_z$, respectively.

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Publication III

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A NOVEL MULTIBEAM TIME-DIVISION ISAC APPROACH WITH ACCURATE SENSING

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ABSTRACT

We propose a novel multibeam time-division (TD) integrated sensing and communication approach. The TD strategy is introduced to address the simultaneous high requirements for sensing performance and communication rate, while the new multibeam design is proposed to achieve wide-area sensing with improved accuracy even for null-direction targets. Unlike most multibeam methods that typically rely on beam scanning and thus suffer from limited direction estimation accuracy, the proposed method exploits inter-beam relationships to enhance estimation accuracy. Moreover, the proposed multibeam method achieves a significantly higher maximum unambiguous velocity for null-direction targets, mitigating the velocity ambiguity inherent in the conventional Doppler division multiple access approach. Simulation results validate the effectiveness of the proposed approach.

Index Terms— Multibeam, integrated sensing and communication (ISAC), time-division (TD), wide-area sensing, velocity deambiguity.

1. INTRODUCTION

Integrated sensing and communication (ISAC), which unifies wireless sensing and communication (S&C) using shared hardware and spectrum, has become a fundamental requirement for sixth-generation intelligent networks. Among its extensive applications, automotive systems, particularly those enabling advanced autonomous driving, stand out as one of the most prominent [1–3].

While vehicle-mounted sensor systems are well developed and provide high sensing accuracy, their reliance on line-of-sight paths imposes fundamental constraints, limiting their ability to support fully informed decision-making. Sharing sensing data and other transportation-related information among vehicles becomes a crucial step towards reliable vehicle automation [4]. However, this continuous data sharing requires high data-rate communication to

ensure operational effectiveness. In light of these requirements, automotive systems need to simultaneously provide high-throughput communication and accurate sensing with a wide field-of-view (FoV). As a result, sensing-centric approaches, which prioritize sensing but may suffer from low data rates, and communication-centric approaches, which rely on fine directional beams for high-throughput links but lack wide sensing coverage, both fall short in automotive applications.

To tackle the aforementioned challenge, a novel multibeam time-division (TD) ISAC approach is proposed, enabling both flexible tuning of the sensing-communication performance tradeoff and high-accuracy sensing with a wide FoV. The TD scheme is introduced to allocate non-overlapping time resources for each functionality, which not only simplifies implementation and system complexity, but also enables dynamic performance tradeoff between S&C via time allocation adjustment [4–10].

Previous research has investigated the wide-FoV sensing in ISAC systems. Beam scanning during the communication training phase was proposed in [11, 12] to achieve radar sensing across a wide area. A concept of *multibeam* was proposed in [13], using a fixed sub-beam for communication while a time-varying scanning sub-beam for radar sensing. However, these approaches rely on beam scanning for direction estimation, failing to exploit the inter-beam relationships. To achieve accurate direction estimation, [14] proposed to generate multiple beams with identical beampatterns, realized by circulant shifted transmit beamformers, and beams yield a uniform spatial energy distribution for sensing. A compressed sensing method was then employed to achieve accurate direction estimation. However, the “uniform” energy distribution is ensured only at discrete Fourier transform grid locations, and multiple null directions appear within the beam coverage, limiting the approach’s practicality.

Inspired by the Doppler division multiple access (DDMA) approach [15], which uses phase modulation to generate as many distinct beams as transmit antennas, M_t , we propose to generate more beams by dividing the Doppler spectrum into a larger number of subbands, denoted as M_v , satisfying $M_v > M_t$. By exploiting inter-beam relationships, accurate direction estimation can be achieved by adopting the parallel

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factor analysis (PARAFAC)-based method [16, 17]. For velocity estimation at null directions, a larger M_v allows more than one beam within a pulse cycle to attain nonzero gain. This allows inter-beam phase differences for velocity deambiguity, thereby overcoming DDMA's inherent limitation—the reduction of the maximum detectable unambiguous velocity by a factor of M_t . The proposed method is validated through simulation experiments.

2. SYSTEM MODEL

2.1. Signal Model

Consider an automotive ISAC system where the source vehicle employs two uniform linear arrays (ULAs): an M_t -element transmit array and an M_r -element receive array, with inter-element spacings d_t and d_r , respectively. For simplicity, the communication user equipment (UE) is modeled as a single-antenna receiver. A waveform with Q equispaced frames is considered. During Q frames, the states of UEs and targets remain constant, defining a coherent processing interval (CPI) of $\text{CPI} = QT_f$, where T_f is the duration per frame. Each frame begins with sensing followed by communication, with respective durations $T_s = \rho T_f$ and $T_c = (1 - \rho)T_f$, where $\rho \in (0, 1)$. Let T_f be the signal sampling interval. The numbers of samples per frame for S&C sequences are $N_s = T_s/T_f$ and $N_c = T_c/T_f$, respectively.

Let $s[n]$ denote the sensing-slot sequence, satisfying $\mathbb{E}(|s[n]|^2) = 1$, where $|\cdot|$ denotes absolute value operation. To optimize the transmit energy distribution for sensing, a beamforming vector $\mathbf{w}_{\text{MB},q}$ is designed for the q th frame, forming the multibeam matrix $\mathbf{W}_{\text{MB}} = [\mathbf{w}_{\text{MB},0}, \dots, \mathbf{w}_{\text{MB},Q-1}]$ for Q frames. The transmitted signal during the sensing part can then be expressed as

$$\mathbf{x}_s(t) = \sum_{q=0}^{Q-1} \sum_{n=0}^{N_s-1} \sqrt{P_s} \mathbf{w}_{\text{MB},q} s[n] g_{\text{TX}}(t - nT_f - qT_f) \quad (1)$$

where P_s denotes the transmit signal power for sensing, and $g_{\text{TX}}(t)$ is the sampled waveform.

Consider K targets with directions of departure (DODs) θ_k , directions of arrival (DOAs) φ_k , and Doppler frequency shifts f_k , $k \in [1, K]$, with the corresponding transmit and receive steering vectors $\mathbf{a}(\theta_k)$ and $\mathbf{b}(\varphi_k)$. After matched filtering, the received sensing signal over Q pulses can be stacked into

$$\mathbf{y}_s = \sum_{k=1}^K \sigma_k N_s \mathbf{b}(\varphi_k) \otimes [\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k) \odot \mathbf{d}(f_k)] + \mathbf{z}_s \quad (2)$$

where σ_k is the complex coefficient incorporating the transmit power, the receive antenna gain and the target reflection, $(\cdot)^T$ denotes the transpose, and the q th entry of the Doppler frequency shift vector $\mathbf{d}(f_k)$ is $d_q(f_k) = e^{j2\pi f_k q T_f}$. Symbols $\otimes$ and $\odot$ stand for Kronecker and Hadamard products,

respectively. $\mathbf{z}_s$ is the additive white Gaussian noise with entries following $\mathcal{CN}(0, \sigma_s^2)$.

Equation (2) shows that the signal processing gain is directly proportional to N_s , indicating that sensing performance improves with a larger time resource. On the other hand, the beamforming vectors also affect performance through the transmit energy distribution, mathematically characterized by $\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k)$.

Next we consider the communication with a single UE, where the channel state information between transmitter and UE is assumed known and constant over a CPI. Therefore, the communication beamforming vector, denoted by $\mathbf{w}_c$, is fixed across Q frames. The transmitted communication signal can then be expressed as

$$x_c(t) = \sum_{q=0}^{Q-1} \sum_{n=0}^{N_c-1} \sqrt{P_c} \mathbf{w}_c c_q[n] g_{\text{TX}}(t - (n + N_s)T_f - qT_f) \quad (3)$$

where P_c is the transmit signal power for communication and $c_q[n]$ is the sampling sequence in the q th frame, with $\mathbb{E}(|c_q[n]|^2) = 1$.

The received communication signal at the UE during the q th frame is

$$y_{c,q}[n] = \sqrt{P_c} \mathbf{h}_c^T \mathbf{w}_c c_q[n] + z_{c,q} \quad (4)$$

where $\mathbf{h}_c$ is the complex channel coefficient vector and $z_{c,q} \sim \mathcal{CN}(0, \sigma_c^2)$ is the additive noise.

2.2. Performance Metrics

To measure the ISAC system performance, the Cramér–Rao Bound (CRB) and the communication spectrum efficiency (SE) are used as performance metrics.

To comprehensively evaluate the estimation of all sensing parameters, including DOD, DOA, and velocity, the overall CRB is found as

$$\overline{\text{CRB}}(\varphi, \theta, f) = \text{CRB}_\varphi + \text{CRB}_\theta + \text{CRB}_f \quad (5)$$

where CRB_φ , CRB_θ , and CRB_f represent the CRBs of φ , θ , and f , respectively.

According to (4), the effective communication SE is

$$\text{SE} = (1 - \rho) \log_2 \left(1 + \frac{P_c |\mathbf{h}_c^T \mathbf{w}_c|^2}{\sigma_c^2} \right). \quad (6)$$

3. PROPOSED MULTIBEAM SCHEME

3.1. Beamformers for Multibeam Sensing

Inspired by the DDMA approach, which uses only phase modulation to generate as many distinct beams as the number of transmit elements, M_t , we propose to generate a larger number of beams, denoted as M_v , such that $M_v > M_t$.

To realize it, the Doppler spectrum is divided into M_v uniform subbands, and the corresponding Doppler shifts are $f_m = m\text{PRF}/M_v$, where $m \in [0, M_v - 1]$ and $\text{PRF} = 1/T_r$. The phase modulation vector for the q th pulse can then be described as $\mathbf{w}_q = [1, e^{-j2\pi \frac{\text{PRF}}{M_v} q T_r}, \dots, e^{-j2\pi (M_t - 1) \frac{\text{PRF}}{M_v} q T_r}]^T \in \mathbb{C}^{M_t}$. Here we set $d_t = \lambda/2$, a common configuration for multiple-element array radar systems. The transmit energy beampattern for the q th pulse as a function of the spatial angle θ can then be derived as

$$G_q(\theta) = |\mathbf{w}_q^T \mathbf{a}(\theta)|^2 = \left| \frac{\sin \left[\frac{\pi}{2} M_t \left(\sin \theta - \frac{2q}{M_v} \right) \right]}{\sin \left[\frac{\pi}{2} \left(\sin \theta - \frac{2q}{M_v} \right) \right]} \right|^2, \quad (7)$$

and the mainlobe direction of the beam formed by the q th pulse, denoted as $\theta_{b,q}$, satisfies $\sin \theta_{b,q} = 2l + 2q/M_v$, where l is an integer chosen such that $\sin \theta_{b,q}$ remains within the physical bounds of $[-1, 1]$. In this paper, each beams is identified by the index of its associated pulse, q , with $q \in [0, M_v - 1]$, corresponding to the first pulse cycle.

From (7), two critical characteristics emerge. Firstly, $\theta_{b,q}$ changes with each pulse, exhibiting a periodicity of M_v , causing the mainlobes of M_v distinct beams to sweep across the entire spatial region. Therefore, for a specific region of interest (ROI), denoted by Θ , wide-FoV sensing can be accomplished by properly selecting a subset of beams that cover Θ , avoiding high computational complexity. Secondly, as M_v increases, the spacings between adjacent mainlobes decrease inversely proportional to M_v . The resulting denser beams concentrate more transmit energy within a targeted region, thereby enhancing the beamforming gain simply by increasing M_v .

Drawing from the insights above, we propose a novel multibeam sensing scheme. For a given ROI Θ , only these beamforming vectors whose mainlobes fall within this region are selected to generate beams for sensing, thereby avoiding energy waste. To further enhance the beamforming gain, M_v is increased. As noted in the second characteristic, this allows more beams to be incorporated within Θ , which consequently improves sensing performance. With predetermined Θ and M_v , the number of selected beams within a period is denoted as M , and their respective pulse indices are $q_0, \dots, q_{M-1}$. The corresponding phase modulation vectors then form the phase modulation matrix $\mathbf{W}_{\text{MB}}$ for our proposed multibeam ISAC system, defined as

$$\mathbf{W}_{\text{MB},0} = [\mathbf{w}_{q_0}, \mathbf{w}_{q_1}, \dots, \mathbf{w}_{q_{M-1}}]$$

$$\mathbf{W}_{\text{MB}} = \underbrace{\begin{bmatrix} \mathbf{W}_{\text{MB},0} & \mathbf{W}_{\text{MB},0} & \dots & \mathbf{W}_{\text{MB},0} \end{bmatrix}}_{P_v = Q/M \text{ times}} \in \mathbb{C}^{M_t \times Q}. \quad (8)$$

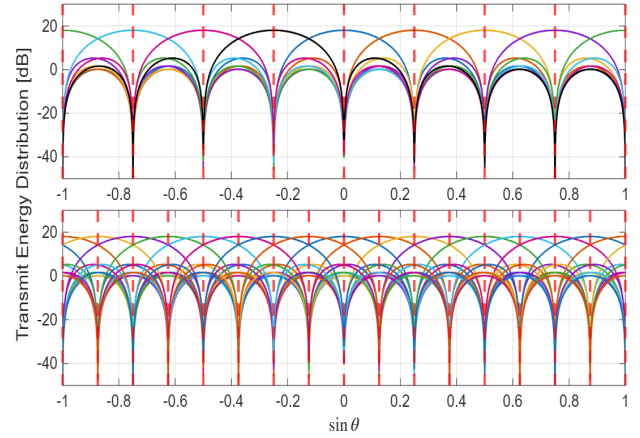


Fig. 1. Transmit beams and nulls when $M_v = 8$ (top) and $M_v = 16$ (below).

3.2. Velocity Estimation at Null Directions

Using the multibeam beamformers, the received signal in (2) can be reshaped into a tensor with dimensions $M_r \times M \times P_v$:

$$\mathcal{Y}_s = \sum_{k=1}^K \sigma_k N_s \mathbf{b}(\varphi_k) \circ \mathbf{a}_{\text{MB}}(\theta_k) \circ \mathbf{d}_{\text{amb}}(f_k) + \mathcal{Z}_s \quad (9)$$

where $\mathcal{Z}_s$ represents the noise tensor, and the velocity-related vectors, $\mathbf{d}_{\text{amb}}(f_k)$ and $\mathbf{d}_0(f_k)$, along with the multibeam steering vector $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$, are given as

$$\mathbf{d}_{\text{amb}}(f_k) = [d_0(f_k), d_M(f_k), \dots, d_{M(P_v-1)}(f_k)]^T$$

$$\mathbf{d}_0(f_k) = [d_0(f_k), d_1(f_k), \dots, d_{M-1}(f_k)]^T \quad (10)$$

$$\mathbf{a}_{\text{MB}}(\theta_k, f_k) = (\mathbf{W}_{\text{MB},0}^T \mathbf{a}(\theta_k)) \odot \mathbf{d}_0(f_k).$$

From (10), the maximum detectable unambiguous velocity is determined by the phase differences among the elements of $\mathbf{d}_0(f_k)$, while the phase estimation of $\mathbf{d}_0(f_k)$ depends on the beam gain. Specifically, if $\mathbf{w}_{q_m}^T \mathbf{a}(\theta_k) = 0$, no phase can be extracted from this beam, and thus $d_m(f_k)$ cannot be estimated. In this paper, such θ_k is referred to as a *null direction*. In the worst case, where only one beam has nonzero gain at θ_k , no phase difference is available, reducing the maximum unambiguous velocity by a factor of M .

To investigate the impact of M_v on velocity ambiguity, two cases are considered: $M_t = M_v$ and $M_t \neq M_v$. For example, let $M_t = 8$ for both cases, while $M_v = 2M_t = 16$ for the second case. The corresponding transmit beam energy distributions of the first M_v beams are shown in Fig. 1. Observing the top plot in Fig. 1, a total of 9 null directions span the whole angular region. At any null direction (marked by vertical red dashed lines), only one beam exhibits nonzero gain, and the null direction coincides with the mainlobe direction of this beam. As a result, for a null-direction target, only

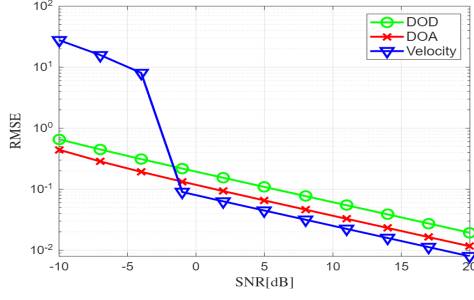


Fig. 2. RMSE results for null-direction targets.

one entry of $\mathbf{d}_0(f_k)$ provides useful phase information, making it impossible to extract velocity information from $\mathbf{d}_0(f_k)$. Consequently, in this case, i.e., DDMA, velocity estimation for null-direction targets suffers a reduction in maximum detectable velocity by a factor of M_t .

The corresponding beam pattern for the second case is depicted in the lower plot of Fig. 1. In contrast to the first case, more distinct beams and nulls are distributed across the spatial region, and more than one beam has nonzero gain at each null. Recalling (7), the null directions for the q th beam, $\theta_{\text{null},q}$, can be derived as

$$\begin{cases} \sin \theta_{\text{null},q} = \frac{2q}{M_v} + \frac{2i_{\text{null}}}{M_t} \in [-1, 1] \\ \sin \theta_{\text{null},q} \neq \frac{2q}{M_v} + 2j_{\text{null}} \in [-1, 1] \end{cases} \quad (11)$$

where i_{null} and j_{null} are integers, and i_{null} refers to the index of a null direction for the q th beam. Equation (11) mathematically explains the beam patterns depicted in Fig. 1. According to (11), beam q , sharing the null direction θ_{null,q_0} with beam q_0 , must satisfy

$$\frac{i_{\text{null},0} - i_{\text{null}}}{q - q_0} = \frac{M_t}{M_v} = \frac{M_1}{M_2}, \quad \frac{i_{\text{null}}}{M_t} \neq j_{\text{null}} \quad (12)$$

where $i_{\text{null},0}$ is the null index of θ_{null,q_0} for the beam q_0 , and M_1 and M_2 are coprime. It can be seen from (12) that the number of beams sharing a null direction is determined by M_2 : $q - q_0$ must be integer multiple of M_2 to form a null at θ_{null,q_0} . When $M_v > M_t$, $M_2 > 1$ is guaranteed. This ensures that multiple beams do not satisfy the condition in (12), and consequently, more than one beam exhibits nonzero gain, enabling velocity estimation with lower ambiguity.

Although it has been proven that more than one entry of $\mathbf{d}_0$ can be estimated when $M_v > M_t$, the impact of these estimated entries on the actual maximum unambiguous velocity is left for future work due to page limitations.

4. SIMULATION RESULTS

This section presents simulation results to validate the proposed method. For all experiments, both the transmit and receive arrays are ULAs, each consisting of 8 elements. Each CPI consists of $Q = 48$ frames.

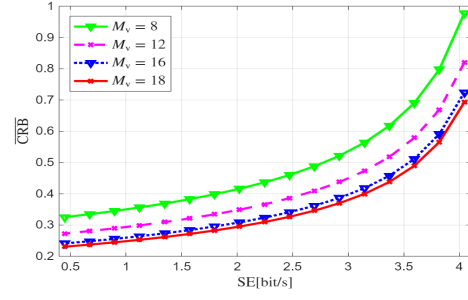


Fig. 3. Tradeoff between CRB and SE.

Let $M_v = 2M_t = 16$. With Θ defined as $[-30^\circ, 30^\circ]$, the indices of the selected beams are 0, 1, 2, 3, 12, 13, 14, 15. To validate the proposed algorithm at null directions, an experiment considers 4 targets located at $\sin(\theta) = \{-1/2, -1/4, 0, 1/4\}$, all at nulls, with respective velocities $\{-3.85, -1.28, 1.28, 3.85\} \lambda \text{PRF}/16$. The root mean square error (RMSE) is used to quantify estimation error, and is computed based on 300 Monte Carlo trials. Fig. 2 presents the parameter estimation results in terms of RMSE versus signal-to-noise ratio (SNR), obtained using the PARAFAC-based method described in [17]. The results show that the proposed method achieves accurate parameter estimation for null-direction targets and effectively resolves velocity ambiguity.

Two sensing targets are positioned at $\{\theta_k\}_{k=1}^2 = \{-10^\circ, 10^\circ\}$ with identical ranges. Moreover, $M = 8$ beams centered around 0° are selected. For communication, $\mathbf{w}_c = \mathbf{a}^*(\theta_c)$ is set, where θ_c is the UE direction and $(\cdot)^*$ denotes conjugation. Fig. 3 illustrates the overall CRB and SE results (as defined in (5) and (6)) under varying time allocation proportions ρ and M_v values to evaluate the performance tradeoff between S&C. As shown in Fig. 3, the overall CRB increases with SE, indicating that enhancing communication performance leads to reduced sensing accuracy. This tradeoff is managed by ρ , which dictates the time allocation between S&C. Regarding beamforming, a larger M_v enhances sensing performance for a given SE, as indicated by lower CRBs. Therefore, the performance tradeoff for the proposed TD ISAC system can be managed by adjusting the time allocation proportion, with a proper M_v further improving sensing performance.

5. CONCLUSION

We have presented a TD scheme for ISAC to balance S&C requirements, and proposed a novel multibeam method for high-accuracy, wide-area sensing. Building on the proposed TD ISAC framework, we have also developed a multibeam design that utilizes simple phase modulation. This approach allows for flexible sensing coverage and enhanced parameter estimation. Moreover, the proposed method effectively addresses the limitation of velocity ambiguity for null-direction targets, a common issue in traditional DDMA approaches.

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Publication IV

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A Novel Multibeam Time-Division ISAC Approach for Accurate Sensing Parameter Estimation

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Abstract—A novel multibeam time-division (TD) multiple-input multiple-output (MIMO) integrated sensing and communications (ISAC) approach is proposed to achieve a balanced tradeoff between sensing and communication functionalities and accurate sensing parameter estimation with a wide field-of-view. Firstly, the TD strategy is introduced to address the simultaneous high demands for sensing performance and communication rate. By allocating time resources between sensing and communications, this approach can reach a desired balance between them while avoiding spectrum and spatial interference, as well as competition in power allocation. Next, a new multibeam method is developed to achieve wide-area sensing for TD MIMO ISAC. Conventional multibeam methods typically rely on beam scanning for direction estimation, suffering from limited accuracy. Inspired by Doppler division multiple access (DDMA) approach, the proposed method divides the Doppler spectrum into more subbands, generating more beams than the number of transmit antenna elements using only phase modulation. Beyond enabling flexible control over the sensing coverage location through beam selection, the proposed method also improves parameter estimation accuracy by fully leveraging the inter-beam relationships, particularly for targets located at null directions. Specifically, for such targets, the proposed method achieves a significantly higher maximum unambiguous velocity, mitigating the velocity ambiguity inherent in conventional DDMA. Simulation results validate the effectiveness of the proposed approach in enhancing both the performance tradeoff between sensing and communication, and the accuracy of sensing parameter estimation.

Index Terms—Multibeam, ISAC, time-division (TD), wide-area sensing parameter estimation, velocity deambiguity.

I. INTRODUCTION

THE advent of sixth-generation (6G) wireless networks defines a new era in mobile communications, where the integration of sensing and communications (S&C)

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using shared hardware and spectrum resources (known as integrated sensing and communications (ISAC)), fundamentally transforms how networks perceive and interact with their environments. This integration offers the potential for simultaneous high-speed data transmission as well as real-time sensing, localization, and tracking of user equipments (UEs). By leveraging information about UEs and their surrounding environments obtained via both S&C, 6G networks aim to intelligently adapt to diverse and dynamic mobility patterns, delivering context-aware and intelligent services across various emerging applications [1]. Among these, automotive systems, particularly those supporting advanced autonomous driving, represent one of the most prominent applications [2].

Although vehicle-equipped sensor systems have been highly developed and capable of accurately sensing their surroundings, they are inherently limited to line-of-sight paths, which is insufficient for fully informed decision-making. To address this challenge, machine-to-machine (M2M) communication plays a pivotal role. It was introduced in the fifth-generation networks to provide connected services in regions without broadband access, and is also expected to be more interactive and autonomous in 6G [3]. Enabled by M2M communication, the exchange of sensing data and other transportation-related information among vehicles is a crucial step towards achieving reliable automation in transportation systems [4]. However, this continuous data sharing calls for high data-rate communication to maintain operational effectiveness. On the other hand, unlike communication-centric approaches—where sensing relies on narrow beams that are directed toward known UEs for data transmission, leaving targets located outside the communication beam undetected—automotive systems must scan a wide angular field to detect unknown obstacles, pedestrians, and road conditions in their surroundings to ensure driving safety. Given these requirements, automotive systems should simultaneously support high-throughput communication and high-accuracy sensing with a wide field-of-view (FoV). As a result, both sensing-centric approaches, which prioritize sensing but suffer from low data rates, and communication-centric approaches, which rely on fine directional beams for high-throughput links but suffer from narrow spatial coverage, fall short in automotive applications. The real challenge, then, is to simultaneously guarantee sensing performance and communication quality while striking a performance balance between them to satisfy the practical needs. To address the challenge above, we first introduce the time-division (TD) ISAC approach to enable flexible tuning of the sensing-communication performance tradeoff.

Since S&C have fundamentally different requirements, performance tradeoffs and mutual interferences between two functionalities lead to increased complexity in waveform design. One way to reduce this complexity is to employ separate, dedicated waveforms for S&C, such as TD multiplexing [4], [5], [6], [7], [8], [9], [10], space division multiplexing [11], [12], and code division multiplexing [13]. Among them, the TD scheme eliminates interference by allocating non-overlapping time resources for each functionality. This not only simplifies implementation and system complexity, but also enables the dynamic performance tradeoff between S&C through flexible time allocation adjustment. Several studies have explored TD-based ISAC systems, and validated their effectiveness and superiority to other ISAC approaches. The works [8], [9] study the performance tradeoff for a TD scheme in integrated bi-static radar and communication systems, and the closed-form expressions for the achievable rate and radar detection accuracy are provided. In [4], an mmWave-enabled connected automated vehicle cooperation algorithm is designed based on the TD ISAC system for raw sensing data sharing among vehicles, and hardware experiments are conducted to verify its feasibility. The full-duplex capability of the ISAC node is emphasized in [10] to enable TD multiplexing of dedicated S&C waveforms. However, to the best of our knowledge, the investigation of TD ISAC in multiple-input multiple-output (MIMO) systems remains largely unexplored in the existing literature.

Several studies have explored wide-FoV sensing in ISAC systems, but each approach presents limitations. Beam scanning during the communication training phase was proposed in [14] and [15] to enable radar sensing across a wide area; however, this method inherently suffers from low angular resolution in direction estimation. In [16], adaptive random switching of transmit antennas during data transmission was introduced to form a coherent beam, but only subsets of the transmit antennas were utilized, resulting in transmit energy inefficiency. The concept of *multibeam* was developed in [17], where a fixed sub-beam was designed for communications while a time-varying scanning sub-beam was used for radar sensing. Despite this integrated design, direction estimation still relies on beam scanning, failing to exploit the relationship between different beam signals for direction estimation.

In [18], a transmit beamformer was proposed to generate a narrow beam directed towards the communication UE while distributing the remaining energy uniformly across the other directions for radar sensing. To enable accurate estimation of sensing parameters, the transmit beamformer was circularly shifted to produce a number of different beamformers sharing an identical beam pattern, and then a compressed sensing algorithm was applied to achieve improved angle estimation performance compared to beam-scanning based methods. However, the “uniform” energy distribution is only ensured at discrete Fourier transform grid locations, and a number of null directions exist within the wide beam coverage, making the approach inadequate for real-world applications. Moreover, the method in [18] is limited in terms of the number of detectable targets. Specifically, it is restricted to fewer than half the number of transmit antenna elements. These constraints pose challenges for practical application scenarios.

Inspired by the Doppler division multiple access (DDMA) approach [19] (which employs phase modulation to shift the signals from different transmit antennas onto a set of equal-width, nonoverlapping Doppler subbands, with the number of subbands being equal to the number of transmit antenna elements, M_t , thereby forming M_t distinct directional beams), our proposed multibeam method designs a set of similar phase modulation vectors to generate more distinct beams by dividing the Doppler spectrum into a larger number of subbands. The number of Doppler subbands is denoted by M_v in this paper, and $M_v = M_t$ corresponds to the case of DDMA. Clearly, DDMA is a special case of the multibeam method, in which the number of distinct beams and the transmit energy distribution of individual beams are constrained by the number of transmit antenna elements, lacking flexibility in the beam pattern design. Moreover, an inherent bottleneck of DDMA is the severe velocity ambiguity that arises for a target located at a null direction, where a null direction refers to a direction at which a beam exhibits zero gain. According to the Nyquist sampling theorem, the maximum unambiguous velocity is determined by the pulse sampling frequency. In the DDMA scheme, at any null direction, only one of the M_t pulses per period forms a beam that contributes nonzero gain, resulting in a downsampling of the received signal by M_t times in the slow-time domain. Consequently, the maximum unambiguous velocity is reduced by the same factor M_t . A detailed analysis of this phenomenon is provided in Section V.

To address these issues, we propose dividing the Doppler spectrum into a larger number of subbands than the number of transmit antennas, i.e., $M_v > M_t$. The most immediate consequence is the generation of more beams. In practical sensing scenarios, a limited spatial region, rather than the entire angular domain, is typically of interest. To accommodate this, the flexible selection of beamformers can dynamically tailor the sensing coverage, allowing for a reconfigurable FoV and improved energy efficiency. In addition, the beam density can be adjusted by tuning the parameter M_v , offering greater design flexibility in beam pattern design. It is noteworthy that this beam selection causes the signal in the slow-time domain to no longer exhibit a complex sinusoidal form [21]. As a result, orthogonality in the slow-time domain is lost, rendering Doppler spectrum-based separation, as in classical DDMA, infeasible. Nevertheless, channel separability is preserved and can be achieved by using tensor-based methods. In this work, the tensor decomposition-based approach of [20] and [21] is used for high-accuracy angle estimation by exploiting the transmit channel diversity. In terms of velocity estimation, with a larger M_v , more than one beam within a pulse cycle achieves nonzero gain at a null direction, enabling the use of phase differences between beams for velocity deambiguity.

The contributions of this paper are summarized as follows.

- **TD MIMO ISAC:** A TD MIMO ISAC framework is introduced to efficiently manage the dynamic performance tradeoff between S&C with low system and implementation complexity. The Cramér–Rao Bound (CRB) and the communication spectral efficiency (SE) are employed as metrics to analyze the tradeoff between S&C.

- **Novel multibeam method for wide FoV sensing:** A novel multibeam scheme is proposed for wide-area sensing. Using only phase control, the method can flexibly adjust both the coverage area and the beamforming gain over the region of interest.
- **High-accuracy parameter estimation:** Unlike existing multibeam methods that rely on beam scanning for target direction estimation, the proposed approach exploits the inter-beam relationship for accurate target direction estimation. A parallel factor analysis (PARAFAC)-based method is employed for parameter estimation, accompanied by a new uniqueness condition for PARAFAC involving two Vandermonde-structured factor matrices.
- **Velocity deambiguity for null-direction targets:** The feasibility of estimating velocity for targets located at null directions, which is a bottleneck for traditional DDMA methods, is analyzed, and a corresponding velocity estimation technique is developed.

The rest of the paper is organized as follows. In Section II, the signal model of a TD ISAC MIMO system is introduced. Next, the proposed beamformer design for the multibeam sensing scheme is introduced in Section III, and then its beam pattern characteristics are analyzed. In Section IV, a PARAFAC-based algorithm is introduced to estimate sensing parameters, and the uniqueness condition of PARAFAC with two Vandermonde factor matrices is presented. In Section V, the beam pattern of the proposed multibeam scheme at null directions is investigated, and a velocity estimation algorithm for null-direction targets is provided. Section VI conducts simulation experiments to validate the effectiveness of the proposed method.

Notations: Scalars, vectors, matrices, and tensors are denoted by lower-case, boldface lower-case, boldface upper-case, and calligraphic letters, e.g., x , $\mathbf{x}$, $\mathbf{X}$, and $\mathcal{X}$, respectively. The $(i_1, i_2, \dots, i_N)$ -th entry of $\mathcal{X}$ is represented by $\mathcal{X}_{i_1 \dots i_N}$, while the i -th element of $\mathbf{x}$ is x_i . The transpose, conjugate, Hermitian transpose, and inversion operations are respectively represented by $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, and $(\cdot)^{-1}$. Column-wise Kronecker product, Hadamard product, outer product, Kronecker product, and element-wise division are denoted by $*$, $\odot$, $\otimes$, $\oslash$, and $\oslash$, respectively. Moreover, $\text{Tr}(\cdot)$ denotes the trace of a matrix, $\partial \mathbf{X} / \partial x$ stands for the partial derivative of $\mathbf{X}$ with respect to x , $\mathbf{X}^{(T)}$ denotes the matrix formed by selecting the top T rows of $\mathbf{X}$, and $\underline{\mathbf{X}}$ and $\overline{\mathbf{X}}$ are obtained by deleting the bottom and top rows of $\mathbf{X}$, respectively. The (conventional) rank and Kruskal rank of a matrix are denoted by $r(\cdot)$ and $k(\cdot)$, respectively. Finally, $|\cdot|$ denotes the element-wise absolute value operation, $\|\cdot\|_2$ denotes the Euclidean norm of a vector, $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ represent respectively rounding down and rounding up to the nearest integer, and $\llbracket \cdot \rrbracket$ denotes the operation of extracting the fractional part of a value.

II. SYSTEM MODEL

Consider an automotive ISAC MIMO system in which a source vehicle is equipped with two uniform linear arrays (ULAs): one with M_t antenna elements for transmission and the other with M_r elements for reception. Full-duplex sensing operation is assumed, enabled by the advances in isolation and self-interference cancellation [22]. The distances between

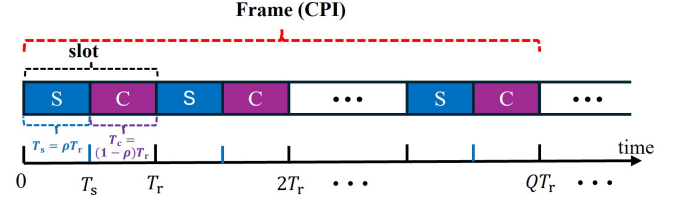


Fig. 1. Illustration of the TD ISAC waveform frame structure. One frame (CPI) consists of Q slots. For each slot, the first time interval of duration $T_s = \rho T_r$ is allocated for sensing, while the second interval of duration $T_c = (1 - \rho) T_r$ is for communications. For simplicity, inter-slot gaps and boundary symbols are omitted.

adjacent elements in transmit and receive arrays are denoted by d_t and d_r , respectively. For simplicity, the communication UE is assumed to be a single-antenna receiver.

We consider a generic waveform frame structure with Q equispaced slots, over which the locations and relative velocities of UEs and targets are assumed to remain constant, thereby defining a coherent processing interval (CPI) for sensing. The duration of each slot is denoted as T_r , which is also known as the pulse repetition interval in radar sensing. Sensing is operated at the beginning of each slot, followed by communication, with durations $T_s = \rho T_r$ and $T_c = (1 - \rho) T_r$ for S&C segments, where $\rho \in (0, 1)$ is the proportion of time resource allocated to sensing in the TD ISAC framework. It is assumed that the communication receiver can detect the end of the sensing signal segment, which can be implemented using boundary symbol detection algorithms. For ease of exposition, a simplified frame structure is employed in this paper, omitting inter-slot gaps and boundary symbols between S&C portions. Consequently, $\text{CPI} = QT_r$. Let the signal bandwidth be B , then the sampling duration is $T_f = 1/B$. Define $N_f = \lfloor T_s/T_f \rfloor$ and $N_c = \lfloor T_c/T_f \rfloor$ as the lengths of S&C sampling sequences per slot, respectively.

A. Signal Model for Sensing

Denote the sampling sequence used in the sensing phase as $s[n]$, $n = 0, 1, \dots, N_f - 1$. This sequence typically remains the same across slots (pulses) for coherent signal processing within a CPI and satisfies the power constraint $\mathbb{E}(|s[n]|^2) = 1$. The sequence $s[n]$ is carefully designed to possess favorable autocorrelation and spectral characteristics for sensing applications. An example is $s[n] = \exp(j\pi n^2/N_f)$, which corresponds to a classical linear frequency modulation waveform. To optimize the transmit energy distribution for sensing, a beamforming vector $\mathbf{w}_{\text{MB},q} \in \mathbb{C}^{M_t}$ for the q th slot is designed for multibeam FoV sensing. The beamforming matrix, which consists of Q beamforming vectors for Q pulses, is represented by $\mathbf{W}_{\text{MB}} = [\mathbf{w}_{\text{MB},1}, \dots, \mathbf{w}_{\text{MB},Q}] \in \mathbb{C}^{M_t \times Q}$. The transmitted signal during the sensing part can then be described as

$$\mathbf{x}_s(t) = \sum_{q=1}^Q \sum_{n=0}^{N_f-1} \sqrt{P_s} \mathbf{w}_{\text{MB},q} s[n] g_{\text{TX}}(t - nT_f - (q-1)T_r), \quad (1)$$

where P_s denotes the transmit sensing signal power for each pulse and $g_{\text{TX}}(t)$ is the Nyquist waveform with bandwidth

B , whose sampled autocorrelation satisfies $R_{g_{\text{TX}}}[n] = \delta[n]$, where $\delta[n]$ denotes the Kronecker delta function.

Suppose that there are K targets in the illuminated area, and the direction of departure (DOD) and direction of arrival (DOA) for the k th target are respectively denoted as θ_k and φ_k , $k = 1, \dots, K$. For simplicity, this paper assumes line-of-sight (LoS) propagation in the theoretical analysis.¹ The steering vectors of the transmit and receive ULAs for the k th target can be expressed as, respectively,

$$\begin{aligned} \mathbf{a}(\theta_k) &= \left[1, e^{j2\pi \frac{d_t}{\lambda} \sin \theta_k}, \dots, e^{j2\pi \frac{d_t}{\lambda} (M_t-1) \sin \theta_k} \right]^T, \\ \mathbf{b}(\varphi_k) &= \left[1, e^{j2\pi \frac{d_r}{\lambda} \sin \varphi_k}, \dots, e^{j2\pi \frac{d_r}{\lambda} (M_r-1) \sin \varphi_k} \right]^T, \end{aligned} \quad (2)$$

where λ is the carrier wavelength. After matched filtering, the received sensing signal for Q pulses in a CPI can be expressed as

$$\mathbf{y}_s = \sum_{k=1}^K \sigma_k N_f \mathbf{b}(\varphi_k) \otimes \left[\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k) \odot \mathbf{d}(f_k) \right] + \mathbf{z}_s, \quad (3)$$

where σ_k is the complex-valued coefficient incorporating the transmit power, receiver antenna gain, and the reflection coefficient of the k th target. $\mathbf{z}_s$ is the additive white Gaussian noise, with elements drawn from $\mathcal{CN}(0, \sigma_z^2)$, where σ_z^2 is the noise power after matched filtering. The q th entry of the Doppler frequency shift vector $\mathbf{d}(f_k)$ is given by

$$d_q(f_k) = e^{j2\pi f_k (q-1)T_r}, \quad q = 1, \dots, Q, \quad (4)$$

where $f_k = 2v_k/\lambda$ is the Doppler frequency shift, and v_k is the velocity of the k th target. It can be seen from (4) that the maximum detectable unambiguous velocity is determined by the slow-time sampling frequency, i.e., pulse repetition frequency (PRF), and is given by $v_{\text{max}} = \lambda \text{PRF}/4$ with $\text{PRF} = 1/T_r$.

It can be observed from (3) that the signal processing gain of the received signal is directly proportional to N_f . Therefore, with other parameters held constant, the sensing performance is positively correlated with the duration of the sensing slot. On the other hand, the beamforming vectors also influence sensing performance by determining the transmit energy distribution, which is mathematically characterized by $\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k)$. The design of $\mathbf{W}_{\text{MB}}$ will be presented in detail in Section III.

To measure the sensing performance, CRB is employed as a benchmark. The covariance matrix of $\mathbf{y}_s$ is

$$\mathbf{R} = \mathbf{y}_s \mathbf{y}_s^H = N_f^2 \mathbf{C} \mathbf{P} \mathbf{C}^H + \sigma_z^2 \mathbf{I}, \quad (5)$$

where $\mathbf{P}$ is a diagonal matrix with diagonal elements $\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2$, $\mathbf{I}$ is an identity matrix (here of size $QM_r \times QM_r$), and the k th column of $\mathbf{C}$ is given by

$$\mathbf{c}_k = \mathbf{b}(\varphi_k) \otimes \left[\mathbf{W}_{\text{MB}}^T \mathbf{a}(\theta_k) \odot \mathbf{d}(f_k) \right]. \quad (6)$$

Denote the unknown parameter vector as

$$\boldsymbol{\rho} = [\varphi_1, \dots, \varphi_K, \theta_1, \dots, \theta_K, f_1, \dots, f_K, \sigma_1^2, \dots, \sigma_K^2, \sigma_z^2]. \quad (7)$$

The (m, n) -th element of the Fisher information matrix (FIM) can be found as

$$[\mathbf{FIM}]_{m,n} = \text{Tr} \left(\frac{\partial \mathbf{R}}{\partial \rho_m} \mathbf{R}^{-1} \frac{\partial \mathbf{R}}{\partial \rho_n} \mathbf{R}^{-1} \right). \quad (8)$$

To comprehensively evaluate all estimated parameters, including DOD, DOA, and velocity, we use the following operation as the final metric for sensing performance, given by

$$\overline{\text{CRB}}(\varphi, \theta, f) = \frac{1}{K} \text{Tr}([\mathbf{FIM}^{-1}]_{1:3K}), \quad (9)$$

where $[\mathbf{FIM}^{-1}]_{1:3K}$ denotes the submatrix obtained by extracting the first $3K$ rows and columns of $\mathbf{FIM}^{-1}$. Since $\{\sigma_k^2\}_{k=1}^K$ and σ_z^2 are not the parameters of interest, they are excluded when calculating $\overline{\text{CRB}}$.

B. Signal Model for Communications

We consider the case with a single UE, and the channel state information (CSI) between the transmitter and UE is assumed to be known and constant within a frame. For automotive scenarios with high mobility, the CSI varies across frames due to channel dynamics; therefore, the constant CSI assumption is emphasized to apply only within a single frame. Considering that this paper focuses on sensing parameter estimation in the TD ISAC framework, and that communication is addressed in terms of the performance tradeoff introduced by the TD scheme, we assume for clearness a single UE and perfect, fixed CSI within a frame for communication. Therefore, the communication beamforming vector remains unchanged across Q slots, and is denoted as $\mathbf{w}_c$, satisfying $\|\mathbf{w}_c\|_2 = M_t$. The transmitted signal for communications can then be described as

$$x_c(t) = \sum_{q=1}^Q \sum_{n=0}^{N_c-1} \sqrt{P_c} \mathbf{w}_c c_q[n] g_{\text{TX}}(t - (n + N_f)T_f - (q-1)T_r), \quad (10)$$

where P_c denotes the transmit signal power for communication in each slot, and $c_q[n]$ denotes the sampled sequence for the communication part in the q th slot, satisfying $\mathbb{E}(|c_q[n]|^2) = 1$.

The received communication signal by the UE for the q th slot is

$$y_{c,q}[n] = \sqrt{P_c} \mathbf{h}_c^T \mathbf{w}_c c_q[n] + z_{c,q}, \quad (11)$$

where $\mathbf{h}_c$ is the complex channel coefficient vector between the M_t antennas of the transmitter and the UE receiver, and $z_{c,q}$ is the additive noise following $\mathcal{CN}(0, \sigma_c^2)$. Based on (11), the effective communication SE can be derived as

$$\text{SE} = (1 - \rho) \log_2 \left(1 + \frac{P_c |\mathbf{h}_c^T \mathbf{w}_c|^2}{\sigma_c^2} \right). \quad (12)$$

As observed from (5) and (12), the time allocation proportion, ρ , affects S&C performance: a larger ρ improves sensing performance but results in lower communication SE. Therefore, ρ serves as a direct and intuitive parameter in balancing the performance tradeoff between S&C.

¹The non-LoS (NLoS) case is presented in Section VI.

III. PROPOSED MULTIBEAM SCHEME FOR TD ISAC

A. Beamformers for Multibeam Sensing

Before presenting the proposed multibeam sensing scheme, we first introduce the fundamental principle of the DDMA approach. Dividing the entire Doppler spectrum into M_t uniform, non-overlapping subbands, and shifting the signals transmitted from different antennas onto distinct Doppler subbands, Doppler-domain orthogonality among the signals transmitted from different channels can be achieved. This can be achieved through DDMA phase modulation with M_t Doppler shifts. With DDMA approach, M_t distinct beams can be formed and cover the whole spatial region. Thanks to the orthogonality among transmit channels, DDMA facilitates accurate direction estimation.

Motivated by the DDMA approach, we propose to generate more beams than the number of transmit antennas, M_t , and the number of beams for the proposed method is denoted as M_v , satisfying $M_v > M_t$. To realize it, the Doppler spectrum is divided into M_v uniform subbands, and the resulting Doppler shifts are

$$f_m = (m-1) \frac{\text{PRF}}{M_v}, \quad m = 1, \dots, M_v. \quad (13)$$

By selecting the first M_t Doppler shifts assigned to the M_t antenna elements, the phase modulation vector for the q th pulse can be expressed as

$$\mathbf{w}_q = \left[1, e^{-j2\pi(q-1)T_r \frac{\text{PRF}}{M_v}}, \dots, e^{-j2\pi(q-1)T_r \frac{\text{PRF}}{M_v} (M_t-1)} \right]^T. \quad (14)$$

Note that any set of M_t consecutive Doppler shifts produces an identical beam pattern except for a different extra phase.

In this work, $d_t = \lambda/2$ is set, which is a common practice for multiple-element array radar systems. The transmit energy beam pattern with respect to the spatial angle θ for the q th pulse can then be expressed as

$$G_q(\theta) = |\mathbf{w}_q^T \mathbf{a}(\theta)|^2 = \left| \frac{\sin \left[\frac{\pi}{2} M_t \left(\sin \theta - \frac{2(q-1)}{M_v} \right) \right]}{\sin \left[\frac{\pi}{2} \left(\sin \theta - \frac{2(q-1)}{M_v} \right) \right]} \right|^2, \quad (15)$$

and the mainlobe direction $\theta_{b,q}$ for the q th pulse satisfies

$$\sin \theta_{b,q} = 2l + \frac{2(q-1)}{M_v}, \quad q = 1, 2, \dots, Q, \quad (16)$$

where l is an integer to guarantee that $\sin \theta_{b,q} \in [-1, 1]$.

From (15) and (16), two critical characteristics emerge. Firstly, $\theta_{b,q}$ varies from pulse to pulse with a cycle of M_v pulses. As a result, the mainlobes of M_v distinct beams sweep across the entire spatial region. Considering the emphasis on the multibeam design, each generated beam is identified by the index of its associated pulse, q , and the pulse vector defined in (14) is also referred to as the beamforming vector of the q th beam. Due to the periodicity of individual beam patterns, the beam indices are confined to a single pulse cycle, i.e., $q \in [1, M_v]$. From the multibeam perspective, DDMA can be interpreted as a special case of multibeam transmission that emits $M_v = M_t$ distinct directional beams per cycle. Moreover, wide-FoV sensing over a desired region, denoted by Θ , instead of the whole spatial domain, can be

achieved by properly selecting a subset of beams covering Θ , while avoiding high computational complexity by using beamforming vectors drawn from the pre-generated set in (14). Secondly, as M_v increases, the spacings between adjacent mainlobes become narrower because the spacings are inversely proportional to M_v . Given a fixed array configuration, defined by M_t and d_t/λ , among other parameters, the mainlobe beamwidth at a given direction remains constant, regardless of variations in M_v for modulation purposes. Consequently, denser beam spacings result in a higher concentration of transmit energy within the targeted region. This characteristic allows the beamforming gain within Θ to be enhanced just by increasing M_v .

Motivated by the insights above, we propose a novel multibeam sensing scheme for TD ISAC systems. Given a spatial region of interest to be sensed, only those beamforming vectors whose mainlobes fall within this region are selected to generate sensing beams, avoiding energy waste. To further improve sensing performance, higher beamforming gain is pursued. This is facilitated by increasing M_v , which reduces the beam spacings as discussed in the second characteristic, thereby allowing more beams to fall within Θ . Given predetermined Θ and M_v , $M \leq M_v$ pulses in every pulse cycle have their beam mainlobes steered towards Θ . The indices of these M beams are $q_1, \dots, q_M$, and their corresponding phase modulation vectors constitute a phase modulation matrix $\mathbf{W}_{\text{MB}}$ for the proposed multibeam ISAC MIMO system, which is given by

$$\mathbf{W}_{\text{MB},0} \triangleq [\mathbf{w}_{q_1}, \mathbf{w}_{q_2}, \dots, \mathbf{w}_{q_M}],$$

$$\mathbf{W}_{\text{MB}} \triangleq \underbrace{\begin{bmatrix} \mathbf{W}_{\text{MB},0} & \mathbf{W}_{\text{MB},0} & \dots & \mathbf{W}_{\text{MB},0} \end{bmatrix}}_{Q/M} \in \mathbb{C}^{M_t \times Q}, \quad (17)$$

where Q/M is assumed to be an integer, and the q th column vector of $\mathbf{W}_{\text{MB}}$ is used for the q th pulse. It is noteworthy to mention that, due to the use of beam selection for adapting to dynamically varying desired regions, only a subset of pulse vectors is utilized, resulting in an actual pulse cycle of M pulses. Therefore, to avoid ambiguity, we clarify that the q th beam is used to identify the M_v different beams, where $q \in [1, M_v]$, even though some beams may not be formed.

As discussed above, M and M_v critically shape the spatial transmit energy distribution. Specifically, M influences the spatial coverage extent, while M_v controls beam density; together, they jointly determine the overall beamforming gain. The quantitative impact of M and M_v on transmit energy distribution and sensing performance will be demonstrated through simulation experiments in Section VI.

In addition to optimizing the transmit energy distribution, the proposed multibeam beamformer can significantly mitigate velocity ambiguity at null directions, yielding a maximum unambiguous velocity that is M_t times larger than that of the DDMA approach, as detailed in Section V. Moreover, with respect to direction finding, the proposed multibeam method gains higher accuracy (by exploiting the inter-beam relationships) compared to traditional beam-scanning techniques. The comprehensive procedure for sensing parameter estimation is present in Section IV.

B. Beamformer for Communications

We now design the beamformer dedicated to the communication UEs. The objective of the communication beamformer is to maximize the transmit beamforming gain at the UEs based on the available channel state information, thereby enhancing the SE. Unlike most existing ISAC systems, which require joint beamformer design to balance S&C performance, the proposed TD ISAC approach eliminates the need for such tradeoffs, except for time allocation between the two functionalities. Specifically, since S&C operate in orthogonal time slots, there is no spectrum or power resource competition between them, nor does the spatial interference from sensing signals affect communication performance. Therefore, the S&C beamformers can be designed independently.

Various types of beamformers have been proposed in wireless communications to meet different performance objectives, including equal gain transmission [23], maximum ratio transmission [24], and other adaptive beamformer algorithms [25]. Since this work primarily focuses on the sensing beamformer design, the communication model is simplified by considering a single UE at a known direction θ_c . The maximum ratio transmission approach is adopted, where the communication beamforming vector is $\mathbf{w}_c = \mathbf{h}_c^*$, with $\mathbf{h}_c = \mathbf{a}(\theta_c)$. This configuration is consistent with the simulation setup in Section VI.

Finally, it is noteworthy that the proposed multibeam TD ISAC can be readily extended to the multi-UE case, although this introduces additional challenges, such as UE scheduling and beam allocation among UEs. Thanks to the separability of S&C functionalities, these challenges do not compromise the effectiveness of the proposed multibeam TD ISAC approach. The TD framework can be readily adapted to the multi-UE case by appropriately adjusting the time allocation ratio, ρ , to satisfy communication performance for all UEs.

IV. PARAFAC FOR SENSING PARAMETER ESTIMATION

In this section, a three-dimensional (3-D) tensor signal model is derived for the proposed multibeam approach, and then a corresponding parameter estimation algorithm based on PARAFAC is presented. Furthermore, a new uniqueness condition for PARAFAC with two Vandermonde factor matrices is established, and a low-complexity algorithm is proposed to efficiently implement the PARAFAC.

A. Preliminaries on Tensor and PARAFAC

Before presenting the proposed tensor signal model, we first review some preliminary concepts on tensors and PARAFAC for clarity. An N th-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times \dots \times I_N}$ is said to be rank-1 if it can be expressed as the outer product of non-zero vectors $\mathbf{a}^{(n)} \in \mathbb{C}^{I_n}$, $n = 1, \dots, N$, such that $\mathcal{X}_{i_1 \dots i_N}$ satisfies $\mathcal{X}_{i_1 \dots i_N} = \prod_{n=1}^N a_{i_n}^{(n)}$. A decomposition of a tensor into a sum of rank-1 terms is referred to as Polyadic Decomposition (PD):

$$\mathcal{X} = \sum_{r=1}^R \mathbf{a}_r^{(1)} \circ \dots \circ \mathbf{a}_r^{(N)}. \quad (18)$$

The rank of a tensor $\mathcal{X}$ is defined as the minimal number of rank-1 tensors whose linear combination yields $\mathcal{X}$. If the rank of $\mathcal{X}$ is R , then (18) is called the Canonical PD of $\mathcal{X}$, also known as PARAFAC decomposition.

Stacking the vectors $\{\mathbf{a}_r^{(n)}\}_{r=1}^R$ into a matrix, we define

$$\mathbf{A}^{(n)} = [\mathbf{a}_1^{(n)}, \dots, \mathbf{a}_R^{(n)}] \in \mathbb{C}^{I_n \times R}, n = 1, \dots, N, \quad (19)$$

where $\{\mathbf{A}^{(n)}\}_{n=1}^N$ are referred to as the factor matrices of $\mathcal{X}$. Moreover, $\mathbf{A}^{(n)}$ is said to be a Vandermonde matrix if

$$\mathbf{a}_r^{(n)} = \left[1, z_r^{(n)}, \left(z_r^{(n)} \right)^2, \dots, \left(z_r^{(n)} \right)^{I_n-1} \right]^T, \quad (20)$$

where $\{z_r^{(n)}\}_{r=1}^R$ are called the generators of $\mathbf{A}^{(n)}$.

Consider a 3-D tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$, as used in this paper. The PARAFAC decomposition of $\mathcal{X}$ in (18) is unique, up to permutation and scaling of the columns, if [28]

$$k \left(\mathbf{A}^{(1)} \right) + k \left(\mathbf{A}^{(2)} \right) + k \left(\mathbf{A}^{(3)} \right) \geq 2R + 2. \quad (21)$$

B. Tensor Model for Multibeam Scheme

With the proposed multibeam beamformers in (17), the received vector-form signal in (3) can be reshaped into a tensor of size $M_r \times P_v \times M$ as follows

$$\mathcal{Y}_s = \sum_{k=1}^K \sigma_k N_f \mathbf{b}(\varphi_k) \circ \mathbf{d}_{\text{amb}}(f_k) \circ \mathbf{a}_{\text{MB}}(\theta_k, f_k) + \mathcal{Z}_s, \quad (22)$$

where $P_v = Q/M$ is assumed to be an integer, $\mathcal{Z}_s$ is a noise tensor, and the velocity-related vectors $\mathbf{d}_{\text{amb}}(f_k)$ and $\mathbf{d}_0(f_k)$, as well as the multibeam steering vector $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$, are given by, respectively,

$$\begin{aligned} \mathbf{d}_{\text{amb}}(f_k) &= [d_1(f_k), d_{1+M}(f_k), \dots, d_{1+M(P_v-1)}(f_k)]^T, \\ & \quad (23) \end{aligned}$$

$$\mathbf{d}_0(f_k) = [d_1(f_k), d_2(f_k), \dots, d_M(f_k)]^T, \quad (24)$$

$$\mathbf{a}_{\text{MB}}(\theta_k, f_k) = \left(\mathbf{W}_{\text{MB},0}^T \mathbf{a}(\theta_k) \right) \odot \mathbf{d}_0(f_k). \quad (25)$$

Here, the factor matrices $\mathbf{B}$, $\mathbf{D}_{\text{amb}}$, and $\mathbf{A}_{\text{MB}}$ of $\mathcal{Y}_s$ are constructed by stacking the vectors $\mathbf{b}(\varphi_k)$, $\mathbf{d}_{\text{amb}}(f_k)$, and $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$ column-wise for $k = 1, \dots, K$, respectively. Similarly, $\mathbf{A}$ and $\mathbf{D}_0$ are formed by stacking $\mathbf{a}(\theta_k)$ and $\mathbf{d}_0(f_k)$, respectively. $\mathbf{B}$, $\mathbf{D}_{\text{amb}}$, and $\mathbf{A}_{\text{MB}}$ can be obtained using PARAFAC, for example, through alternating least squares [26] or singular value decomposition (SVD) algorithms [27].

With the estimated $\mathbf{A}_{\text{MB}}$, denoted by $\hat{\mathbf{A}}_{\text{MB}}$, the DODs $\{\theta_k\}_{k=1}^K$ are estimated via the minimum mean square error (MMSE) estimation algorithm. From (25), it can be seen that $\mathbf{A}_{\text{MB}}$ is the Hadamard product of $\tilde{\mathbf{A}}_{\text{MB}} = \mathbf{W}_{\text{MB},0}^T \mathbf{A}$ and $\mathbf{D}_0$. Considering that $\mathbf{D}_0$ has unit-amplitude exponential elements and does not affect the element amplitudes in $\mathbf{A}_{\text{MB}}$, $\{\theta_k\}_{k=1}^K$ can be estimated by solving the MMSE problem

$$\hat{\theta}_k = \arg \min_{\theta} \left\| \hat{\mathbf{a}}_{\text{MB},k} - |\tilde{\mathbf{a}}_{\text{MB}}(\theta)| \right\|_2^2, \quad (26)$$

where $\hat{\mathbf{a}}_{\text{MB},k}$ is the k th column vector of $\hat{\mathbf{A}}_{\text{MB}}$, and $\tilde{\mathbf{a}}_{\text{MB}}(\theta) = \mathbf{W}_{\text{MB},0}^T \mathbf{a}(\theta)$.

Compared to $\mathbf{d}(f_k)$ defined in (4), we can find that $\mathbf{d}_{\text{amb}}(f_k)$ in (23) can be viewed, from a sampling perspective, as an undersampled version of $\mathbf{d}(f_k)$ with a sampling factor of M . As a result, the maximum unambiguous velocity decreases

by a factor of M if relying solely on $\mathbf{d}_{\text{amb}}(f_k)$ for velocity estimation, leading to serious velocity ambiguity. On the other hand, despite sharing the same sampling rate as $\mathbf{d}(f_k)$, $\mathbf{d}_0(f_k)$ comprises only the first M elements of $\mathbf{d}(f_k)$, which leads to low velocity estimation accuracy when used alone. Therefore, $\mathbf{D}_{\text{amb}}$ should be combined with $\mathbf{D}_0$ to obtain unambiguous, accurate velocity estimation. Let $\hat{\mathbf{D}}_{\text{amb}}$ denote the estimate of $\mathbf{D}_{\text{amb}}$. Then, $\mathbf{D}_0$ can be estimated by

$$\hat{\mathbf{D}}_0 = \hat{\mathbf{A}}_{\text{MB}} \odot \hat{\tilde{\mathbf{A}}}_{\text{MB}}, \quad (27)$$

where $\hat{\tilde{\mathbf{A}}}_{\text{MB}}$ is the estimate of $\tilde{\mathbf{A}}_{\text{MB}}$ using the estimated $\{\hat{\theta}_k\}_{k=1}^K$. With $\hat{\mathbf{D}}_{\text{amb}}$ and $\hat{\mathbf{D}}_0$, we can build

$$\hat{\mathbf{D}}_{\text{amb}} * \hat{\mathbf{D}}_0 \triangleq [\mathbf{d}(f_1), \mathbf{d}(f_2), \dots, \mathbf{d}(f_K)]. \quad (28)$$

From (28), $\{v_k\}_{k=1}^K$ can be recovered without ambiguity.

C. Uniqueness Condition of PARAFAC

The feasibility of PARAFAC-based parameter estimation algorithms essentially depends on the uniqueness of PARAFAC. According to the uniqueness condition in (21), factor matrices of $\mathcal{Y}_s$ can be uniquely determined if $k(\mathbf{B}) + k(\mathbf{D}_{\text{amb}}) + k(\mathbf{A}_{\text{MB}}) \geq 2K + 2$ holds. However, this condition is sufficient but not necessary, and the condition can be relaxed when factor matrices follow some special structures. It can be observed that two factor matrices, $\mathbf{B}$ and $\mathbf{D}_{\text{amb}}$, exhibit Vandermonde structure, with their generators $\{z_{\mathbf{B},k}\}_{k=1}^K$ and $\{z_{\mathbf{D}_{\text{amb}},k}\}_{k=1}^K$, respectively. In practice, the parameters of K targets, including DODs, DOAs, and velocities, may not be distinct, so the factor matrices are not necessarily full-column rank, i.e., they can be rank-deficient. In light of this, a new uniqueness condition is proposed for tensors with two Vandermonde-structured and potentially rank-deficient factor matrices.

First, we construct the matrix

$$\mathbf{Y} = \left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) \left(\mathbf{B}^{(L_1)} * \mathbf{D}_{\text{amb}}^{(L_2)} * \mathbf{A}_{\text{MB}} \right)^T$$

$$T_1 + L_1 = M_r + 1, \quad T_2 + L_2 = P_v + 1, \quad (29)$$

where $\mathbf{Y}$ is the matrix obtained by unfolding $\mathcal{Y} \in \mathbb{C}^{T_1 \times T_2 \times L_1 \times L_2 \times M}$ along a specific mode. Since we are primarily concerned with the relationships among elements within a column, column scalings have no impact on the parameter estimation here. Therefore, for simplicity, column scalings are omitted in (29) and subsequent mathematical expressions. $\mathcal{Y}$ is obtained by applying the spatial smoothing technique [27], given by

$$[\mathcal{Y}]_{t_1, t_2, l_1, l_2, m} = [\mathcal{Y}_s]_{t_1 + l_1 - 1, t_2 + l_2 - 1, m}. \quad (30)$$

Assume that $\mathbf{B}$ has $G \leq K$ distinct generators with multiplicity $P_1, \dots, P_G$, where $1 \leq P_g \leq K$ for $g \in [1, G]$. Then, $\mathbf{B}$ can be grouped in column blocks accordingly and represented as

$$\mathbf{B} \triangleq [\mathbf{1}_{P_1}^T \otimes \mathbf{b}_1, \mathbf{1}_{P_2}^T \otimes \mathbf{b}_2, \dots, \mathbf{1}_{P_G}^T \otimes \mathbf{b}_G], \quad (31)$$

where $\mathbf{1}_{P_g}$ denotes an all-one column vector of length P_g , and $\mathbf{b}_g = [1, z_{\mathbf{B},g}, \dots, z_{\mathbf{B},g}^{M_r-1}]^T$ is the Vandermonde vector associated with the g th distinct generator, $z_{\mathbf{B},g}$. Despite the

multiplicities of generators, the uniqueness condition can be relaxed as

$$r \left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) = K \quad (32a)$$

$$r \left(\mathbf{B}^{(L_1)} * \mathbf{D}_{\text{amb}}^{(L_2)} * \mathbf{A}_{\text{MB}} \right) = K \quad (32b)$$

$$P_g + 1 \leq T_2, \quad g = 1, \dots, G. \quad (32c)$$

Then, the Vandermonde-constrained PARAFAC of $\mathcal{Y}_s$ is unique, enabling the sensing parameter recovery of up to K targets with automatic parameter pairing.

The derivation of conditions (32a)-(32c) is given in the appendix, and an outline of the procedure for calculating the sensing parameters using PARAFAC in the presence of repeated sensing parameters is presented in Algorithm 1. It is worth clarifying that the effectiveness of the proposed parameter estimation remains valid despite the presence of SVD phase ambiguity, because the estimation of parameters, including DODs, DOAs, and velocities, relies on the relative phase or amplitude relationship among the elements of vectors $\mathbf{b}(\varphi_k)$, $\mathbf{d}_{\text{amb}}(f_k)$, and $\mathbf{a}_{\text{MB}}(\theta_k, f_k)$, rather than on any individual element alone.

Finally, although the uniqueness condition is satisfied under the considered model, the feasibility of the proposed approach may still be challenged by the stability of the PARAFAC decomposition. Specifically, when multiple targets exhibit closely spaced angular or Doppler parameters, the associated factor matrices may become highly correlated, leading to ill-conditioning and degraded numerical stability of PARAFAC. Techniques such as regularization and informed initialization [29], [30] have been developed to mitigate this issue. In this work, sufficient angular and Doppler separation is assumed, as a detailed investigation is beyond the scope.

Computational Complexity: The most computationally intensive step of Algorithm 1 is the SVD of $\mathbf{Y}$. Consequently, the overall complexity of Algorithm 1 is on the order of $\mathcal{O}(T_1 T_2 L_1 L_2 M \min(T_1 T_2, L_1 L_2 M))$. For the ALS-based PARAFAC decomposition, the computational complexity of each iteration is $\mathcal{O}(T_1 T_2 L_1 L_2 M K)$. According to the uniqueness condition in (32), T_1, T_2, L_1, L_2 , and M are selected based on the maximum number of detectable targets K , satisfying $(T_1 - 1)T_2 \geq K$ and $L_1 L_2 M \geq K$. As a result, both $T_1 T_2$ and $L_1 L_2 M$ scale on the order of K , implying that the complexity of Algorithm 1 is comparable to that of a single iteration of the ALS-based algorithm. Since Algorithm 1 does not require multiple ALS iterations to achieve convergence, it is therefore more computationally efficient than traditional ALS-based methods. In the practical implementation, the computational burden can be further reduced by employing optimized linear algebra libraries and leveraging parallel computing. Therefore, the sensing parameter estimation method using PARAFAC can be achieved within viable runtime, making the proposed approach feasible for latency-constrained ISAC applications.

V. VELOCITY ESTIMATION AT NULL DIRECTIONS

Theoretically, as long as the uniqueness condition for PARAFAC holds, parameters, DODs, DOAs, and velocities, can be recovered following the method in Section IV. However, a target located at a null direction may lead to velocity

Algorithm 1 fProcedure for calculating sensing parameters using PARAFAC in the presence of repeated parameters

- 1: **Input:** $\mathcal{Y}_s$.
Choose pairs (T_1, L_1) and (T_2, L_2) subject to $T_1 + L_1 = M_r + 1$ and $T_2 + L_2 = P_v + 1$, and then build a new matrix using (30):

$$\mathbf{Y} = \left(\mathbf{B}^{(T_1)} * \mathbf{D}^{(T_2)} \right) \left(\mathbf{B}^{(L_1)} * \mathbf{D}^{(L_2)} * \mathbf{A}_{\text{MB}} \right)^T.$$
- 2: Compute SVD $\mathbf{Y} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, and determine K from $\mathbf{\Sigma}$.
- 3: Build $\mathbf{U}_1$ and $\mathbf{U}_2$ as

$$\mathbf{U}_1 = \mathbf{U}(1 : (T_1 - 1)T_2, :), \mathbf{U}_2 = \mathbf{U}(T_2 + 1 : T_1 T_2, :)$$
where $\mathbf{U}(x_1 : x_2, :)$ denotes a submatrix of $\mathbf{U}$ consisting of rows from index x_1 to x_2 ;
Determine $\{\mathbf{z}_{\mathbf{B},k}\}_{k=1}^K$ and $\mathbf{M}$ using eigenvalue decomposition (EVD):

$$\mathbf{U}_1^\dagger \mathbf{U}_2 = \mathbf{M} \mathbf{Z}_{\mathbf{B}} \mathbf{M}^{-1},$$
where $(\cdot)^\dagger$ denotes the Moore-Penrose pseudo-inverse;
Build $\mathbf{b}_k^{(T_1)} = [1, \mathbf{z}_{\mathbf{B},k}, \dots, \mathbf{z}_{\mathbf{B},k}^{T_1-1}]^T$, $k \in [1, K]$.
- 4: Compute $\mathbf{d}_{\text{amb},k}^{(T_2)'} = \left(\mathbf{b}_k^{(T_1)H} \otimes \mathbf{I}_{T_2} \right) \mathbf{U} \mathbf{m}_k$, where $\mathbf{m}_k$ is the k th column vector of $\mathbf{M}$;
Sort the estimated set $\{\mathbf{b}_k^{(T_1)}\}_{k=1}^K$ according to the grouping structure in (31), and then the same ordering is applied to $\{\mathbf{d}_{\text{amb},k}^{(T_2)'}\}_{k=1}^K$ and $\{\mathbf{m}_k\}_{k=1}^K$, resulting in:

$$\mathbf{D}_{\text{amb}}^{(T_2)'} = [\mathbf{D}_{\text{amb},1}^{(T_2)'}, \dots, \mathbf{D}_{\text{amb},G}^{(T_2)'}], \mathbf{M} = [\mathbf{M}_1, \dots, \mathbf{M}_G].$$
- 5: For $g \in [1, G]$:
If $P_g = 1$: $\mathbf{z}_{\mathbf{D}_{\text{amb},g}} = \left(\mathbf{D}_{\text{amb},g}^{(T_2)'} \right)^\dagger \overline{\mathbf{D}_{\text{amb},g}^{(T_2)'}}$; $\mathbf{N}_g = \mathbf{m}_g$;
Else:
Build $\mathbf{U}_{\mathbf{D},1} = \mathbf{D}_{\text{amb},g}^{(T_2)'}$ and $\mathbf{U}_{\mathbf{D},2} = \overline{\mathbf{D}_{\text{amb},g}^{(T_2)'}}$, and then implement EVD:

$$\mathbf{U}_{\mathbf{D},1}^\dagger \mathbf{U}_{\mathbf{D},2} = \mathbf{M}_{\mathbf{D}_{\text{amb},g}} \mathbf{Z}_{\mathbf{D}_{\text{amb},g}} \mathbf{M}_{\mathbf{D}_{\text{amb},g}}^{-1};$$
Determine $\{\mathbf{z}_{\mathbf{D}_{\text{amb},k}}\}_{k=k_1}^{k_{P_g}}$ from $\mathbf{Z}_{\mathbf{D}_{\text{amb},g}}$;
 $\mathbf{N}_g = \mathbf{M}_g \mathbf{M}_{\mathbf{D}_{\text{amb},g}}$.
End if
- 6: Construct $\{\mathbf{b}_k^{(L_1)}\}_{k=1}^K$, $\{\mathbf{d}_{\text{amb},k}^{(L_2)}\}_{k=1}^K$, and $\mathbf{N} = [\mathbf{N}_1, \dots, \mathbf{N}_G]$, and then compute $\{\mathbf{a}_{\text{MB},k}\}_{k=1}^K$ through [27]

$$\mathbf{a}_{\text{MB},k} = \left(\frac{\mathbf{b}_k^{(L_1)H}}{\mathbf{b}_k^{(L_1)H} \mathbf{b}_k^{(L_1)}} \otimes \frac{\mathbf{d}_{\text{amb},k}^{(L_2)H}}{\mathbf{d}_{\text{amb},k}^{(L_2)H} \mathbf{d}_{\text{amb},k}^{(L_2)}} \otimes \mathbf{I}_M \right) \mathbf{V}^* \mathbf{\Sigma} \mathbf{c}_k,$$
where $\mathbf{c}_k$ is the k th row vector of matrix $\mathbf{N}^{-1}$.
- 7: With the obtained $\{\mathbf{z}_{\mathbf{B},k}\}_{k=1}^K$, $\{\mathbf{z}_{\mathbf{D}_{\text{amb},k}}\}_{k=1}^K$, and $\{\mathbf{a}_{\text{MB},k}\}_{k=1}^K$, the sensing parameters $\{\varphi_k\}_{k=1}^K$, $\{v_k\}_{k=1}^K$, and $\{\theta_k\}_{k=1}^K$ can be estimated accordingly.
- 8: **Output:** $\{\theta_k, \varphi_k, v_k\}_{k=1}^K$.

ambiguity, making velocity estimation difficult. A null direction, denoted as θ_{null} , refers to a direction where one or more beams exhibit zero gain, i.e., $\mathbf{w}_q^T \mathbf{a}(\theta_{\text{null}}) = 0$. Recalling (27), the estimation of $\mathbf{D}_0$ depends on $\mathbf{A}_{\text{MB}}$ and $\tilde{\mathbf{A}}_{\text{MB}}$, and the unambiguous velocity estimation is obtained from the phase relationship between elements within each column of $\mathbf{D}_0$, i.e., $\mathbf{d}_0(f_k)$. When a target is located at a null direction, the amplitudes of certain elements in the corresponding column approach zero. As a result, the corresponding phase information extracted through $\mathbf{A}_{\text{MB}}$ and $\tilde{\mathbf{A}}_{\text{MB}}$ becomes dominated by noise, severely degrading the velocity estimation accuracy

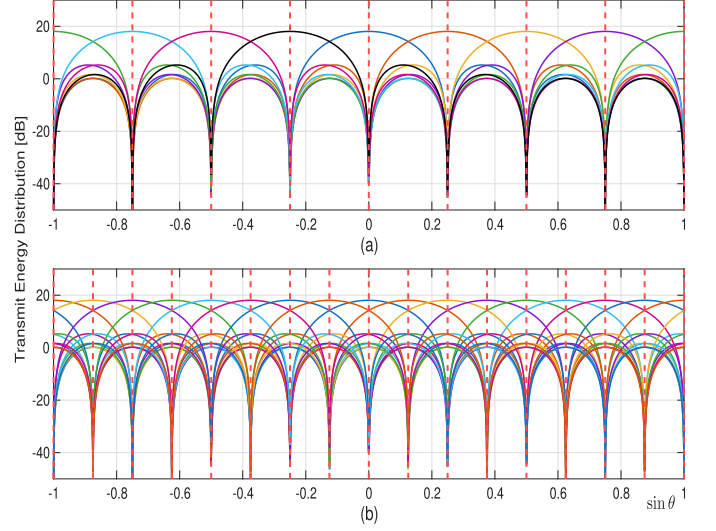


Fig. 2. Transmit beams and nulls when (a) $M_v = 8$ and (b) $M_v = 16$.

and even potentially causing the failure in estimating phase components of $\mathbf{D}_0$. In the worst case where only one element of $\tilde{\mathbf{a}}_{\text{MB},k}$ is nonzero, only one phase value is available. Since velocity estimation relies on the phase difference between at least two elements, velocity deambiguity is disabled, causing the velocity estimation algorithm in (28) to fail.

Thus, we aim to analyze the impact of null directions on the sensing parameter estimation here. We will firstly characterize the effect of null directions for varying M_v . Next, we propose a velocity estimation method for targets at null directions.

A. Beampattern at Null Directions

Unlike most existing multibeam methods that enhance direction estimation by transmitting more directive beams, the proposed scheme exploits inter-beam relationships to achieve accurate estimation. Given the presence of numerous null directions in the angular domain, the energy distribution of different beams at these nulls is worth careful investigation. Considering that the transmit energy beampattern follows a cycle of M_v beams—this is exactly the motivation behind the construction of $\mathbf{D}_0$ and the subsequent generation of the 3-D tensor received signal—without loss of generality, our discussion is restricted to the first M_v beams as a representative example, and thus beam indices satisfy $1 \leq q \leq M_v$.

Recalling (15), it can be found that each beam possesses a certain number of null directions, and the null direction $\theta_{\text{null},q}$ for the q th beam satisfies

$$\begin{cases} M_t \left(\sin \theta_{\text{null},q} - \frac{2(q-1)}{M_v} \right) = 2i_{\text{null}}, \\ \sin \theta_{\text{null},q} - \frac{2(q-1)}{M_v} \neq 2j_{\text{null}}. \end{cases} \quad (33)$$

Here, i_{null} and j_{null} are arbitrary integers, and i_{null} refers to the index of a null direction for the q th beam. Consider two cases: one with $M_t = M_v$ and another with $M_t \neq M_v$. For illustration, we set $M_t = 8$ for both cases, while $M_v = 2M_t = 16$ for the second case. The corresponding transmit beam energy distributions of a pulse cycle are shown in Fig. 2.

Observing the plot in Fig. 2(a), a total of 9 null directions span the angular region $[-90^\circ, 90^\circ]$ ($\sin(\theta) = [-1, 1]$). Each beam has 7 or 8 null directions depending on its mainlobe direction, and these nulls are shared across beams. At any given null direction, which is indicated by a vertical red dashed line, only one beam exhibits nonzero gain, and the null direction coincides with the mainlobe direction of this beam, while all the other 7 beams have zero gain. This phenomenon can be explained mathematically by (34), which is derived by substituting M_v with M_t in (33) to adapt to the first case.

$$\begin{cases} \sin \theta_{\text{null},q} = \frac{2(i_{\text{null}} + q - 1)}{M_t} \in [-1, 1], & (34a) \\ \sin \theta_{\text{null},q} \neq \frac{2(q-1)}{M_t} + 2j_{\text{null}} \in [-1, 1]. & (34b) \end{cases}$$

For a null direction denoted by θ_{null,q_0} corresponding to the q_0 -th beam, let the associated null index i_{null} , defined in (34a), be $i_{\text{null},0}$. To satisfy the condition in (34b), it is required that $i_{\text{null},0} \neq nM_t$ for any integer n . For any other beam indexed by q , such that $q - q_0 = \Delta q$, the setting of $i_{\text{null}} = i_{\text{null},0} - \Delta q$ ensures that the q th beam satisfies the condition at θ_{null,q_0} in (34c). As a result, only these beams with indices satisfying $q = q_0 + i_{\text{null},0} - nM_t$, which corresponds to $i_{\text{null}} = nM_t$, will produce a mainlobe at $\theta_{\text{null},0}$ for the q th beam, while all other beams result in a null. Since $1 \leq q \leq M_t$, there exists only one feasible q satisfying this condition. To conclude, in traditional DDMA systems, which generate as many distinct beams as the transmit elements, only one beam yields nonzero beamforming gain at a null direction. For a null-direction target, only one entry of $\mathbf{d}_0(f_k)$ contributes usable phase information, making it impossible to extract velocity information from $\mathbf{d}_0(f_k)$. Consequently, DDMA MIMO velocity estimation suffers a reduction in maximum detectable velocity by a factor of M_t when a target is located at a null direction due to the fact that only $\mathbf{D}_{\text{amb}}$ can be used for velocity estimation.

When $M_v = 16$ is used, the corresponding beampattern is shown in Fig. 2(b). Compared to the first case, there are more distinct beams and nulls across the spatial region, with more than one beam exhibiting nonzero gain at each null. The expression in (33) can be reformed as

$$\begin{cases} \sin \theta_{\text{null},q} = \frac{2(q-1)}{M_v} + \frac{2i_{\text{null}}}{M_t} \in [-1, 1], & (35) \\ \sin \theta_{\text{null},q} \neq \frac{2(q-1)}{M_v} + 2j_{\text{null}} \in [-1, 1]. \end{cases}$$

In this case, a beam, indexed by q , sharing the null direction θ_{null,q_0} with a beam specified by q_0 , must satisfy

$$\frac{i_{\text{null},0} - i_{\text{null}}}{q - q_0} = \frac{M_t}{M_v}, \quad \frac{i_{\text{null}}}{M_t} \neq j_{\text{null}}. \quad (36)$$

It can be seen from (36) that the number of beams sharing a null direction is determined by the value of M_t/M_v . For $M_t/M_v = 1/2$ in this case, in addition to satisfying the second subcondition, $q - q_0$ must be even to form a null at θ_{null,q_0} . Therefore, when $M_v > M_t$, not only one beam exhibits nonzero gain, enabling the phase-based velocity estimation with lower ambiguity. In contrast to traditional DDMA, the proposed multibeam scheme overcomes the limitation of being unable to estimate the velocity of null-direction targets, which is crucial for robust performance in practical applications.

It can be found that multiple beams exhibit zero gain at a null direction. The impact of this phenomenon on the maximum detectable velocity, as well as the corresponding velocity estimation algorithm under such conditions, is discussed next.

B. Velocity Estimation of Null-Direction Targets

As discussed above, velocity estimation is based on the phase information, while the accuracy of phase estimation depends on the element amplitudes of $\tilde{\mathbf{a}}_{\text{MB},k}$, i.e., the beamforming gains of the beams. If the beamforming gain is very low, the corresponding phase estimation result becomes highly susceptible to noise and not reliable, and thus the associated phase estimates should be excluded from the velocity estimation algorithm. Supposing that a target is located at a null direction, the indices of these reliable beams at this null are denoted by $m_1, \dots, m_P$, where $1 \leq m_1 < \dots < m_P \leq M_v$. After discarding the unreliable phase estimates obtained from low-amplitude elements in $\tilde{\mathbf{a}}_{\text{MB},k}$,² $\mathbf{d}_0$ can be modified as

$$\mathbf{d}_{\text{null},0}(f_k) = [e^{j2\pi f_k(m_1-1)T_r}, \dots, e^{j2\pi f_k(m_P-1)T_r}]^T. \quad (37)$$

Since a constant coefficient is incorporated in the received signal, velocity is estimated based on the phase differences between the elements of $\mathbf{d}_{\text{null},0}$. Let $\Delta m_p = m_{p+1} - m_p$. Then, the phase difference vector can be denoted as

$$\Delta \phi_{\text{amb}} = [\Delta m_1, \dots, \Delta m_{P-1}]^T f_k T_r. \quad (38)$$

To represent the target velocity in the presence of ambiguity, the ambiguity numbers $\{l_{\text{amb},p}\}_{p=1}^{P-1}$ are introduced, and then the velocity can be described as

$$v = \frac{\lambda \text{PRF}}{2\Delta m_p} l_{\text{amb},p} + v_{b,p}, \quad v_{b,p} \in \left[-\frac{\lambda \text{PRF}}{4\Delta m_p}, \frac{\lambda \text{PRF}}{4\Delta m_p}\right]. \quad (39)$$

To achieve accurate velocity estimation, firstly, the phase difference associated with the minimum index spacing, $\Delta m_{\min}$, is employed to determine the ambiguity numbers $\{l_{\text{amb},p}\}_{p=1}^{P-1}$. It is assumed that the target velocity does not exceed the maximum unambiguous velocity when $\Delta m_p = 1$ in (39).

To combine the velocity information in $\mathbf{d}_0(f_k)$ and $\mathbf{d}_{\text{amb}}(f_k)$, the phase difference associated with $\Delta m_P = M$ is included, and it can be obtained by $\Delta \phi_{\text{amb},P} = M f_k T_r = \frac{1}{P_v-1} \sum_{i=1}^{P_v-1} \{\langle \mathbf{d}_{\text{amb}}(f_k) \rangle_{i+1} - \langle \mathbf{d}_{\text{amb}}(f_k) \rangle_i\}$, where $\langle \cdot \rangle_i$ denotes the operation of extracting the phase of the i th element of a vector, and then dividing it by 2π . If $\{l_{\text{amb},p}\}_{p=1}^P$ is determined, with the obtained $\{\Delta \phi_{\text{amb},p}\}_{p=1}^P$, we can have

$$\begin{bmatrix} \Delta m_1 \\ \vdots \\ \Delta m_{P-1} \\ M \end{bmatrix} T_r f_k = \begin{bmatrix} \Delta \phi_{\text{amb},1} + l_{\text{amb},1} \\ \vdots \\ \Delta \phi_{\text{amb},P-1} + l_{\text{amb},P-1} \\ \Delta \phi_{\text{amb},P} + l_{\text{amb},P} \end{bmatrix}. \quad (40)$$

Using the least square fitting, f_k can be estimated by

$$\hat{f}_k = \frac{1}{T_r} \frac{\Delta \mathbf{m}^T \Delta \phi}{\Delta \mathbf{m}^T \Delta \mathbf{m}}, \quad (41)$$

²In the worst case, such as under low SNR conditions, only one beam exceeds the reliable detection threshold; in this case, the proposed sensing parameter estimation method fails because at least two reliable beams are required. This represents a boundary condition of the proposed approach.

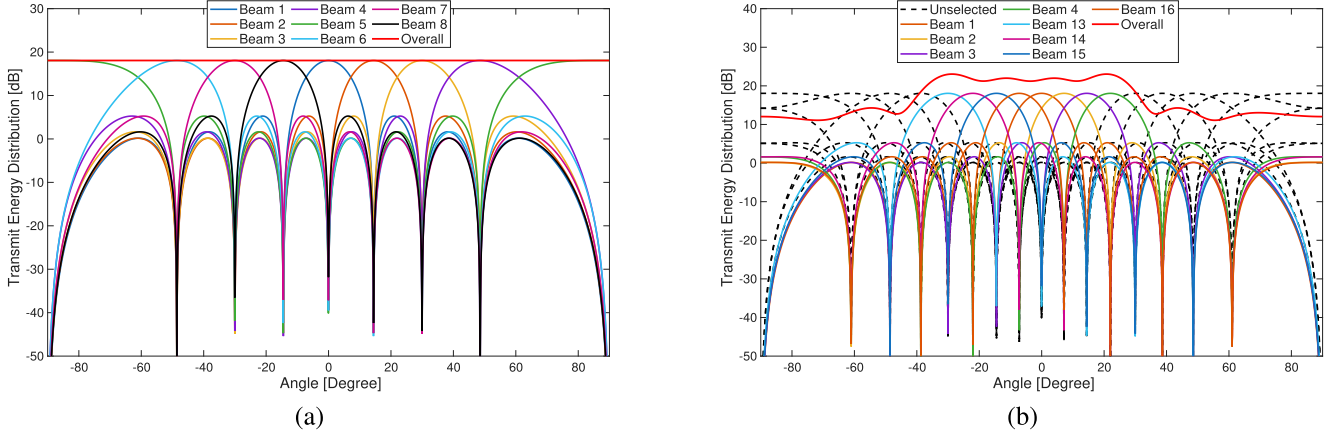


Fig. 3. Beam patterns of different multibeam methods: (a) DDMA ($M_v = 8$) and (b) the proposed ($M_v = 16$).

where $\Delta \mathbf{m}$ and $\Delta \phi$ represent the vectors appearing on the left and right sides of the (40), respectively.

From the discussion above, the key challenge of velocity deambiguity lies in determining the ambiguity numbers $\{l_{\text{amb},p}\}_{p=1}^P$. If $\Delta m_{\min} = 1$, i.e., there exists a pair of beams with spacing 1 that allows resolving the ambiguity, the velocity can initially be estimated using the associated $\phi_{\text{amb},p}$, although the estimation may lack accuracy since it relies on only a single phase difference value. With the estimated velocity, $\{l_{\text{amb},p}\}_{p=1}^P$ can then be specified for high-accuracy velocity estimation.

When $\Delta m_{\min} > 1$, its corresponding ambiguity number needs to be determined first. For simplicity of notations, we assume $\Delta m_1 = \Delta m_{\min}$. The corresponding $l_{\text{amb},1}$ can be determined by

$$\hat{l}_{\text{amb},1} = \min_{0 \leq |l| \leq \lceil \frac{\Delta m_1 - 1}{2} \rceil} \sum_{p=2}^P \left| \llbracket T_r \hat{f}_k \Delta m_p - \Delta \phi_{\text{amb},p} \rrbracket \right|, \quad (42)$$

where $\hat{f}_k = (\Delta \phi_{\text{amb},1} + l) / \Delta m_1 / T_r$. Since Δm_1 is the smallest among $\{\Delta m_p\}_{p=1}^P$, the number of candidates for $l_{\text{amb},1}$ is small, and thus, only a few loops are required to identify its actual value.

Unfortunately, however, even if the condition in (42) is satisfied, the resulting estimate of f_k is not necessarily correct. Denoting the hypothesized value of $l_{\text{amb},1}$ as $l_{\text{amb},1} + \Delta l$, and $\Delta l \neq 0$, if the following condition is satisfied:

$$\frac{\Delta m_p \Delta l}{\Delta m_1} = n, p = 1, \dots, P, \quad (43)$$

the estimate $l_{\text{amb},1} + \Delta l$ will be mistaken as the value of $l_{\text{amb},1}$. Here, $\Delta m_p = M$, and n is an arbitrary integer.

Recalling (36), it can be observed that the differences between the indices of nonzero-gain beams for a given null direction are determined by the ratio, M_t/M_v , and it can be simplified to M_1/M_2 such that M_1 and M_2 are coprime. It can be analyzed by addressing the following two different cases: the first is $M_2 \geq 3$, and the second is $M_2 = 2$. For the first case, it can be found that the indices for zero-gain beams must satisfy $\Delta q = q - q_0 = nM_2$, where n is an integer. As a result, there will be $M_2 - 1$ consecutive beams exhibiting nonzero

gain at a null direction, ensuring $\Delta m_{\min} = 1$. To investigate the second case, rewrite (35) with $M_v = 2M_t$ as

$$\begin{cases} \sin \theta_{\text{null},q} = \frac{2i_{\text{null}} + (q-1)}{M_t} \in [-1, 1], \\ \sin \theta_{\text{null},q} \neq \frac{q-1}{M_t} + 2j_{\text{null}} \in [-1, 1]. \end{cases} \quad (44)$$

For a null direction given by $\sin \theta_{\text{null},q_0} = \frac{2i_{\text{null},0} + (q_0-1)}{M_t}$, where $i_{\text{null},0} \neq 0$, let $q = 2i_{\text{null},0} + q_0$ and set $i_{\text{null}} = 0$. It can be seen that the beam q forms a mainlobe at θ_{null,q_0} , and adjacent beams $q-1$ and $q+1$ also exhibit nonzero gain there. Similar to the first case, consecutive beams with nonzero gain exist in the second case as well. In conclusion, as long as $M_v > M_t$, $\Delta m_{\min} = 1$ holds. This still holds when beam selection is implemented to flexibly adapt to a desired spatial region, as the region of interest is typically a continuous angular sector.

VI. SIMULATION RESULTS

This section presents simulation results to validate the effectiveness of the proposed method in wide-coverage, high-accuracy sensing parameter estimation, and flexible performance tradeoff between S&C functionalities. The transmit and receive antenna arrays are configured as ULAs, each consisting of $M_t = M_r = 8$ elements with an inter-element spacing of $d = \lambda/2$. The UE is assumed to be equipped with a single receive antenna at $\theta_c = 0^\circ$. The carrier frequency is $f_c = 28$ GHz. In this simulation, 3GPP TS 38.211 for 5G NR is adopted, with the frame structure defined as follows. Each frame consists of $Q = 160$ slots with numerology $\mu = 4$, and thus the duration of each slot is $T_r = 62.5 \mu\text{s}$. This configuration yields a maximum unambiguous velocity of $v_{\max} = 42.86$ m/s (154 km/h) and velocity resolution is $v_{\text{res}} = 0.54$ m/s, which is well suited for automotive ISAC scenarios. Unless otherwise specified, all parameter settings are kept constant throughout the experiments.

A. Beam pattern of the Proposed Scheme

We compare the transmit energy beam patterns when different numbers of beams are generated for sensing. First, we simulate two cases as illustrative examples: $M_v = 8$ representing the traditional DDMA approach, and $M_v = 16$

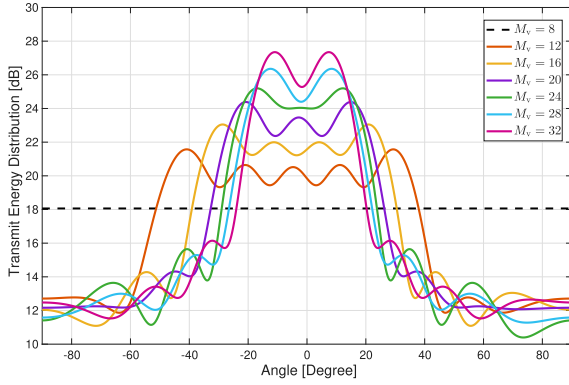


Fig. 4. Overall transmit energy beampatterns for varying M_v : a larger M_v focuses transmit energy within a smaller angular region with higher gain.

corresponding to the proposed multibeam method. For both cases, $M = 8$ beams are selected within each pulse cycle. The resulting beampatterns are shown in Fig. 3, the left one for $M_v = 8$, and the right for $M_v = 16$. Observing Fig. 3(a), 8 beams with distinct mainlobes spread across the whole spatial region, resulting in an omnidirectional overall beampattern over one beam cycle. In contrast, Fig. 3(b) illustrates that there are 16 different beams across the entire region, and they are more densely spaced. This denser beam distribution enables improved energy focusing in a specific spatial region. Let the desired region Θ be specified as $[-30^\circ, 30^\circ]$. In this case, only the beams whose mainlobes are directed towards Θ are selected for actual signal transmission. As illustrated in Fig. 3(b), the indices of these beams (plotted with solid lines) are 1, 2, 3, 4, 13, 14, 15, 16, and the corresponding overall beampattern of 8 beams, labeled as “Overall”, is also presented. It can be seen that, with a larger M_v and proper beam selection, the proposed multibeam design not only allows flexible adjustment of the sensing coverage, but also achieves higher beamforming gain using the same number of pulses.

Next, to assess the impact of M_v on the beamforming performance, various values of M_v are considered, and the corresponding overall beampatterns are illustrated in Fig. 4, where $M_v = 8, 12, 16, 20, 24, 28, 32$ and $M = 8$. Here the selected 8 beams are centered around 0° for all cases. It can be observed that with a higher M_v and a fixed number of the selected pulses M , the transmit energy is concentrated within a smaller angular region, resulting in the increased beamforming gain within this region. This gain enhancement arises from the improved spatial energy concentration, indicating a tradeoff between beamforming gain and coverage area. Therefore, by tuning M_v , one can flexibly control the size of the sensing coverage area when keeping M constant. Conversely, for a fixed M_v , increasing M expands the coverage area.

To demonstrate the joint impact of M_v and M on the sensing performance, CRB results are presented in Fig. 5 as a performance metric under various combinations of M_v and M . The target angles are fixed at $\theta = \{-10^\circ, 10^\circ\}$ for all configurations, the signal-to-noise ratio (SNR) is set to 0dB, and CRB results are calculated based on the received signal within a CPI. Note that $Q = 192$ is chosen in this experiment to ensure that P_v is an integer for all considered values of M . As shown in Fig. 5, before a certain threshold value of M_v

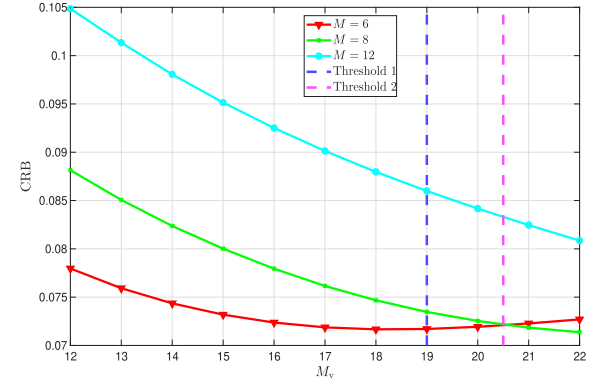


Fig. 5. CRBs for different M and M_v . Before a certain threshold of M_v , a larger M_v and a smaller M result in a lower RMSE; beyond this threshold, this trend no longer holds, as the sensing target falls outside the coverage.

(indicated by the dashed blue vertical line), the CRBs decrease with increasing M_v for a fixed total number of beams M , indicating that a denser beam distribution enhances sensing accuracy. However, beyond this threshold value, the CRBs begin to increase with a larger M_v , as illustrated when $M_v > 19$ for $M = 6$. This degradation occurs because excessively large M_v values reduce the angular spacing between beams when the number of selected beams is fixed, causing the mainlobes of the multibeam to concentrate within a narrower spatial region, which fails to encompass the sensing target.

As a result, the transmit energy is poorly concentrated at the target locations, leading to degraded estimation performance. To maintain sufficient coverage over a given angular range, more beams should be selected. On the other hand, as shown in Fig. 5, increasing M degrades sensing performance. This is because a fixed amount of transmit energy is directed over a broader region, lowering the average gain across this region. However, when $M_v > 20$, as indicated by the dashed pink vertical line, $M = 8$ outperforms $M = 6$. This is because the mainlobes of the beams for $M = 6$ fail to cover the sensing target, whereas $M = 8$, with its broader coverage area, includes it. These results highlight the tradeoff between coverage area and beamforming gain.

B. Parameter Estimation at Null Directions

As discussed in Section V, M_v impacts the maximum detectable unambiguous velocity. To illustrate this, we consider two cases: (1) $M_v = 2M_t = 16$, and (2) $M_v = 12$, $M_t = 8$. The corresponding values of M_2 , as defined in Subsection V-B, are then $M_2 = 2$ and $M_2 = 3$, respectively.

For the first case, the indices of the selected beams are 1, 2, 3, 4, 13, 14, 15, 16, consistent with the beampattern shown in Fig. 3(b). The patterns of these 8 beams are depicted in Fig. 6(a). Consider a beam and one of its null directions, marked by black and red dashed lines, respectively. The beam marked in black corresponds to beam index $q = 1$, and the red dashed line indicates the null direction corresponding to $i_{\text{null}} = 1$ in (35). From the figure, we observe that beams 2, 3, 4, 14, 16 exhibit nonzero gain, while beams 1, 13, 15 have zero gain. Note that the unselected pulses are not considered here. As a result, despite 8 beams are selected in total, only 5 beams can contribute to velocity estimation. For these beams

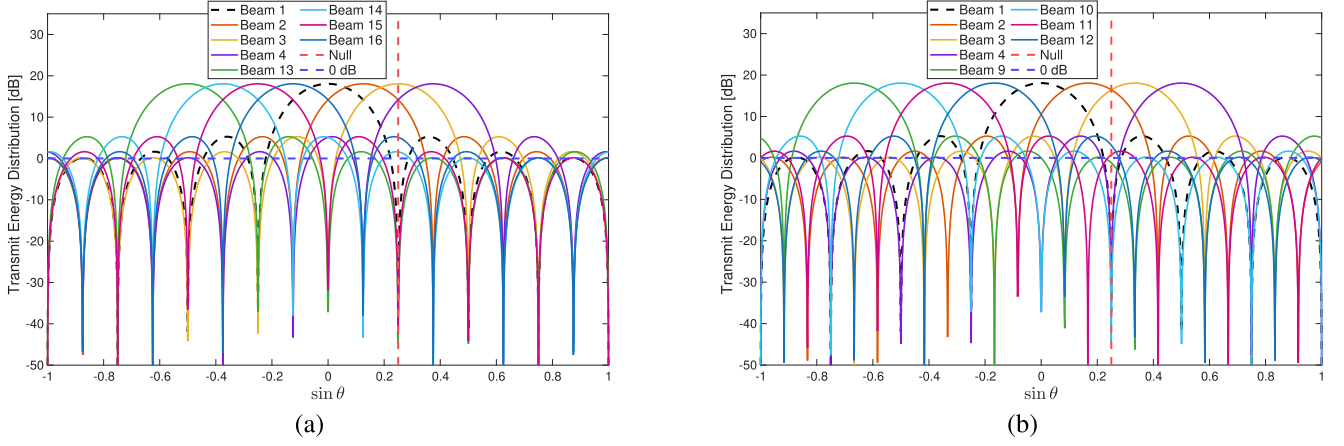


Fig. 6. Beam pattern characteristics at null directions for (a) $M_v = 16$, $M_2 = 2$ and (b) $M_v = 12$, $M_2 = 3$.

with nonzero beamforming gain, the phase difference vector in (38) can be constructed with $\Delta m_1 = 1$, $\Delta m_2 = 1$, $\Delta m_3 = 10$, and $\Delta m_4 = 2$. Moreover, with $\Delta m_1 = 1$, the unambiguous velocity can then be estimated.

Next, we consider the case where $M_v = 12$, resulting in $M_2 = 3$, which corresponds to the first case discussed in Subsection V-B. Beams 1, 2, 3, 4, 9, 10, 11, 12 are selected, and the beam pattern is shown in Fig. 6(b). It can be observed that the beams corresponding to beams 2, 3, 9, 11, 12 exhibit nonzero gain, whereas beams 1, 4, 10 show zero gain. This aligns with the discussion in Subsection V-B: when $M_2 = 3$, the beam indices follow $\Delta q = 3n$, two beams adjacent to a given beam (beams 2, 3 and 11, 12 in this case) exhibit nonzero gain. Although 5 beams achieve nonzero gain at this null, some of them are too weak to be used, particularly those beams associated with $\Delta m = 1$, which are crucial for determining the velocity ambiguity numbers. Therefore, a threshold is typically applied to identify which beams are qualified for velocity estimation. For instance, with a threshold of 0 dB, as indicated by the blue horizontal dashed line in Fig. 6(b), beam 9 is excluded from the consequent velocity estimation, leaving $\Delta m_1 = 1$, $\Delta m_2 = 8$, and $\Delta m_4 = 1$ to resolve velocity ambiguity. When multiple candidates satisfy $\Delta m = 1$, the one associated with the highest combined beamforming gain is selected for determining the ambiguity numbers. Therefore, beams 2 and 3, which are associated with Δm_1 , are selected here.

To validate the proposed velocity estimation algorithm at null directions, two experiments are conducted to assess its performance under ideal propagation conditions with only LoS paths and nonideal propagation conditions comprising both LoS and NLoS paths, respectively. First, in the ideal LoS scenario, an experiment considers the first case discussed in this subsection with 4 targets at $\sin(\theta) = \{-1/2, -1/4, 0, 1/4\}$, all at nulls, with velocities $\{-3.85, -1.28, 1.28, 3.85\}\lambda\text{PRF}/16$. DOAs are set the same as the DODs, and the subsequent experiments in Section VI follow this practice. The root mean square error (RMSE) is used to quantify estimation error. Throughout this section, RMSE is computed based on $N = 300$ Monte Carlo trials. The parameter estimation performance in terms of RMSE versus SNR is presented in Fig. 7, showing that the proposed method

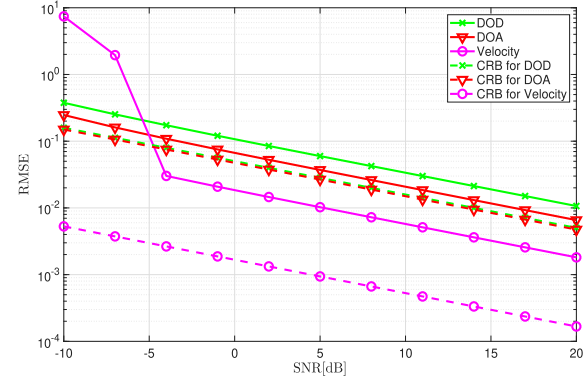


Fig. 7. RMSE results for null-direction targets in the ideal LoS condition.

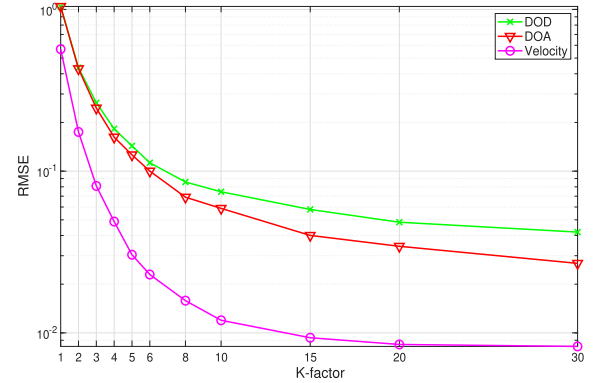


Fig. 8. RMSE results for null-direction targets under the condition with both LoS and NLoS propagations.

successfully estimates the DODs, DOAs, and velocities for targets located at null directions, and effectively resolves velocity ambiguity.

Another experiment is conducted under the nonideal LoS propagation condition to further assess the robustness of the proposed parameter estimation algorithm. In this setup, two targets are positioned at null directions, at $\sin(\theta) = \{-1/2, 1/8\}$, with velocities $\{-1.85, 1.85\}\lambda\text{PRF}/16$. For each target, in addition to the LoS path between the transmitter and receiver, two locally scattered NLoS components are present, where the biases of DODs, DOAs, and velocities are

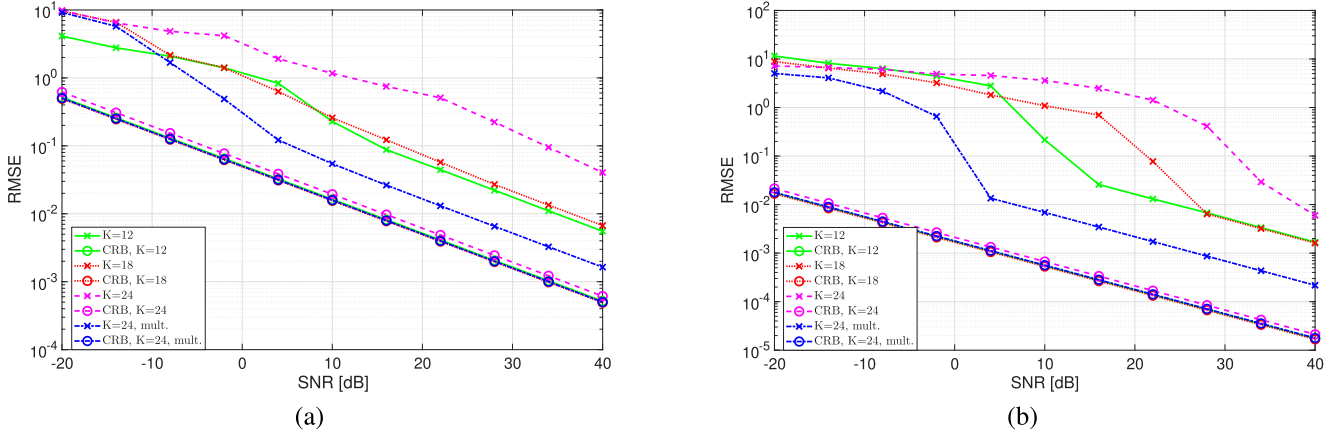


Fig. 9. Parameter estimation results for (a) angle and (b) velocity.

drawn from a uniform distribution in the intervals $[-5, 5]^\circ$, $[-5, 5]^\circ$, and $[-3, 3]$ m/s, respectively (the range dimension is not considered in this work). The SNR is set to $\text{SNR} = 10$ dB, and all other system parameters are kept identical to those used in the first experiment of this subsection. Under these conditions, the overall propagation channel can be expressed as

$$h(t) = \sum_{k=1}^K \left(h_k^{\text{LoS}}(t) + \sum_{l=1}^2 h_{k,l}^{\text{NLoS}}(t) \right), \quad (45)$$

where $h_k^{\text{LoS}}(t)$ denotes the propagation channel of the LoS component for the k th target, while $h_{k,l}^{\text{NLoS}}(t)$ is the propagation channel of the l th NLoS component from the k th target. The ratio between the LoS power and the total NLoS power is characterized by the *Rician* K -factor, given by

$$K_{\text{Rician}} = \frac{P_{\text{LoS}}}{P_{\text{NLoS}}} = \frac{|A_0|^2}{\sum_{l=1}^2 |A_l|^2}, \quad (46)$$

where A_0 is the complex amplitude of the LoS component, and A_l denotes the complex amplitude of the l th NLoS path, $l = 1, 2$. The parameter estimation results, presented in Fig. 8, show that the proposed estimation algorithm still performs well even under non-ideal NLoS conditions, demonstrating its effectiveness and robustness.

C. Uniqueness of PARAFAC

We evaluate the performance of the proposed algorithm under varying numbers of targets to validate the proposed uniqueness condition for PARAFAC here. $M_v = 2M_t = 16$ and $M = 8$ are set, yielding $P_v = 20$. $T_1 = 4$ and $T_2 = 8$ is chosen, resulting in $L_1 = 5$ and $L_2 = 13$. First, the uniqueness condition is validated in the absence of parameter multiplicity. According to the uniqueness condition in (32), the maximum number of identifiable targets is $K = 24$. Experiments are conducted with $K = 12, 18, 24$, and the corresponding estimation results are shown in Fig. 9. A total of 12 DOD and velocity parameters are generated in ascending order with uniform spacing. The first $K = 12, 18, 24$ elements from these parameter sets are used as the DOD (DOA) and velocity values for the three experiments, respectively.

As observed in Fig. 9, where the angle performance is evaluated by averaging the RMSEs of DOD and DOA estimates, the estimation results for $K = 12$ and $K = 18$ clearly outperform those for $K = 24$, since a larger number of targets introduces additional unknown parameters, thereby degrading the estimation accuracy. On the other hand, it is noteworthy that $K = 12$ and $K = 18$ achieved very similar performance despite the different number of targets. This can be attributed to the non-uniform beamforming gains across targets. Despite the impact of K , particularly in the case of $K = 24$, which is the theoretical maximum number of identifiable targets, the estimation algorithm still achieves satisfactory accuracy under a sufficient SNR. These results support the validity of the proposed PARAFAC uniqueness condition.

Next, to validate the feasibility of the proposed PARAFAC method in the presence of parameter multiplicity, another experiment with 24 targets is conducted. The DODs (DOAs) are $\{\theta_k\}_{k=1}^{24} = \{-26 \otimes \mathbf{1}_6^T, -13 \otimes \mathbf{1}_6^T, 4 \otimes \mathbf{1}_6^T, 20 \otimes \mathbf{1}_6^T\}$, and all 24 elements of the generated velocity parameter set correspond to velocity. The corresponding parameter estimation results are also shown in Fig. 9, labeled as “mult.” It can be observed that the parameter estimation performs well despite the multiplicity, demonstrating the effectiveness of the proposed method. Note that the improved performance compared to the other cases can be explained by the sparser DOD (DOA) parameter configuration.

D. Sensing-Communication Performance Tradeoff

To evaluate the performance tradeoff between S&C, the CRB and SE results under varying time allocation proportions ρ and different M_v are presented in Fig. 10. Two sensing targets are placed at directions $\{\theta_k\}_{k=1}^2 = \{\varphi_k\}_{k=1}^2 = \{-10^\circ, 10^\circ\}$ with identical ranges. Moreover, $M = 8$ beams centered around 0° are selected per cycle. The communication beamformer is $\mathbf{w}_c = \mathbf{a}^*(\theta_c)$. As shown in Fig. 10, the CRB increases as the SE increases, indicating that improved communication performance comes at the cost of reduced sensing accuracy. This tradeoff is governed by the value of ρ , which controls the time allocation between S&C. In terms of beamforming, we can find that a larger M_v contributes to improved sensing performance for a given SE, as evidenced by

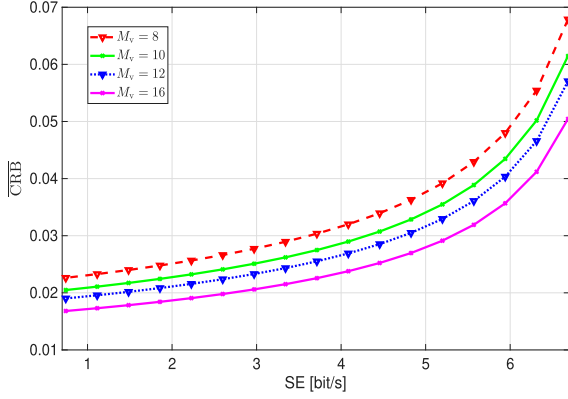


Fig. 10. Tradeoff between CRB and SE. Lower CRB values are achieved at the cost of reduced SE, while a larger M_v contributes to lower CRBs.

a lower CRB value. Therefore, the performance tradeoff for the proposed TD-ISAC system can be managed by adjusting the time allocation proportion, while a suitable M_v in the proposed multibeam scheme can further improve sensing performance.

VII. CONCLUSION AND FUTURE WORK

A. Conclusion

This paper introduces a TD scheme for MIMO ISAC to flexibly balance S&C demands, and proposes a novel multibeam method for high-accuracy sensing over a wide area. Building on the proposed TD MIMO ISAC framework, a multi-beam design employing simple phase modulation has been developed, enabling flexible sensing coverage and improved parameter estimation. A PARAFAC-based algorithm has then been developed for estimating sensing parameters, along with a newly derived uniqueness condition for PARAFAC involving two Vandermonde factor matrices. Moreover, the issue of velocity ambiguity for targets located at null directions, which is a critical limitation for conventional DDMA-based methods, has been addressed. Simulation results have demonstrated the effectiveness of the proposed approach.

B. Future Work

While the proposed multibeam TD ISAC framework demonstrates promising performance, several directions remain for future work. First, the multi-UE TD ISAC case is worth further study, particularly regarding user scheduling and beam allocation strategies. Second, the TD scheme leads to reduced SE due to orthogonal time allocation. Although S&C beamformers are designed to optimize their respective performance, the beams intended for sensing can potentially be leveraged to carry communication information, thereby mitigating the SE loss caused by the TD scheme. Finally, to enhance the practical applicability, the proposed method's adaptability to dynamic channel conditions and its robustness against radar cross section uncertainty, imperfect CSI, and other imperfect knowledge are all relevant topics of future research.

APPENDIX DERIVATION OF (32)

Let the SVD of $\mathbf{Y}$ be $\mathbf{Y} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, where $\mathbf{U} \in \mathbb{C}^{T_1 T_2 \times K}$, $\mathbf{\Sigma} \in \mathbb{C}^{K \times K}$, and $\mathbf{V} \in \mathbb{C}^{L_1 L_2 M \times K}$. Under condition (32a),

which ensures that $\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}$ has full column rank, there exists a nonsingular matrix $\mathbf{M} \in \mathbb{C}^{K \times K}$ such that $\mathbf{U}\mathbf{M} = \mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}$. By exploiting the Vandermonde structure of $\mathbf{B}$, we have

$$\left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) \mathbf{Z}_{\mathbf{B}} = \overline{\mathbf{B}^{(T_1)}} * \mathbf{D}_{\text{amb}}^{(T_2)} \quad (47)$$

where $\mathbf{Z}_{\mathbf{B}}$ is a diagonal matrix whose diagonal entries are the generators of $\mathbf{B}$, $z_{\mathbf{B},1}, \dots, z_{\mathbf{B},K}$. Then we have

$$\begin{aligned} \mathbf{U}_1 \mathbf{M} \mathbf{Z}_{\mathbf{B}} &= \left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) \mathbf{Z}_{\mathbf{B}} = \overline{\mathbf{B}^{(T_1)}} * \mathbf{D}_{\text{amb}}^{(T_2)} = \mathbf{U}_2 \mathbf{M} \\ \mathbf{U}_1 &= \mathbf{U}(1 : (T_1 - 1)T_2, :), \\ \mathbf{U}_2 &= \mathbf{U}(T_2 + 1 : T_1 T_2, :). \end{aligned} \quad (48)$$

Since $\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)}$ has full column rank, $\mathbf{U}_1$ and $\mathbf{U}_2$ have full column rank as well. Thus, $\mathbf{Z}_{\mathbf{B}}$ can be obtained through EVD: $\mathbf{U}_1^\dagger \mathbf{U}_2 = \mathbf{M} \mathbf{Z}_{\mathbf{B}} \mathbf{M}^{-1}$. $\{z_{\mathbf{B},k}\}_{k=1}^K$ can then be determined by extracting the diagonal elements of $\mathbf{Z}_{\mathbf{B}}$.

Next, we rearrange the estimated matrices $\mathbf{B}$ following the grouping structure specified in (31), and then apply the same ordering to $\mathbf{M}$, which is given by $\mathbf{M} = [\mathbf{M}_1, \dots, \mathbf{M}_G]$.

When $P_g = 1$, $\mathbf{m}_g$ is used instead. If an eigenvalue, $z_{\mathbf{B},g}$, is isolated, its eigenvector is uniquely determined up to a scaling factor, and thus, $\mathbf{d}_{\text{amb},g}^{(T_2)}$ can be derived directly by

$$\mathbf{d}_{\text{amb},g}^{(T_2)} = \left(\mathbf{b}_g^{(T_1)H} \otimes \mathbf{I}_{T_2} \right) \mathbf{U} \mathbf{m}_g, \quad (49)$$

where $\mathbf{b}_g^{(T_1)}$ is built using the estimated $z_{\mathbf{B},g}$.

When an eigenvalue is repeated with multiplicity $P_g > 1$, its corresponding eigenvectors span the eigenspace. Although these eigenvectors are mutually orthogonal, they are not unique since any basis spanning the same eigenspace is valid. Consequently, $\{\mathbf{d}_{\text{amb},k}^{(T_2)}\}_{k=k_1}^{k_{P_g}}$ cannot be determined using (49). However, by exploiting the Vandermonde structure of $\mathbf{D}_{\text{amb}}$, $\{\mathbf{d}_{\text{amb},k}^{(T_2)}\}_{k=k_1}^{k_{P_g}}$ can still be derived from $\mathbf{M}_g$ as follows.

Given a repeated eigenvalue $z_{\mathbf{B},g}$ with multiplicity P_g , and its corresponding eigenvectors $\{\mathbf{m}_k\}_{k=k_1}^{k_{P_g}}$, which are given by the column vectors of $\mathbf{M}_g$, we apply the same approach as in (49) to perform computation, but denote the computed result as $\{\mathbf{d}_{\text{amb},k}^{(T_2)'}\}_{k=k_1}^{k_{P_g}}$ to distinguish them from $\{\mathbf{d}_{\text{amb},k}^{(T_2)}\}_{k=k_1}^{k_{P_g}}$ due to the non-uniqueness of the eigenvectors. Then we can have

$$\mathbf{D}_{\text{amb},g}^{(T_2)'} = [\mathbf{d}_{\text{amb},k_1}^{(T_2)'}, \mathbf{d}_{\text{amb},k_2}^{(T_2)'}, \dots, \mathbf{d}_{\text{amb},k_{P_g}}^{(T_2)'}] \in \mathbb{C}^{T_2 \times P_g}, \quad (50)$$

and it satisfies

$$\left(\mathbf{1}_{P_g}^T \otimes \mathbf{b}_g^{(T_1)} \right) * \mathbf{D}_{\text{amb},g}^{(T_2)'} = \mathbf{U} \mathbf{M}_g. \quad (51)$$

Build $\mathbf{U}_{\mathbf{D},1} = \mathbf{D}_{\text{amb},g}^{(T_2)'}$ and $\mathbf{U}_{\mathbf{D},2} = \overline{\mathbf{D}_{\text{amb},g}^{(T_2)'}}$. Then $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=k_1}^{k_{P_g}}$ can be determined from $\mathbf{Z}_{\mathbf{D}_{\text{amb},g}}$ using EVD:

$$\mathbf{U}_{\mathbf{D},1}^\dagger \mathbf{U}_{\mathbf{D},2} = \mathbf{M}_{\mathbf{D}_{\text{amb},g}} \mathbf{Z}_{\mathbf{D}_{\text{amb},g}} \mathbf{M}_{\mathbf{D}_{\text{amb},g}}^{-1}. \quad (52)$$

Note that this EVD-based solution holds when both $\mathbf{U}_{\mathbf{D},1}$ and $\mathbf{U}_{\mathbf{D},2}$ are full column rank, which explains why the condition in (32c) is necessary to ensure the uniqueness of PARAFAC.

From (32a), it follows that $\{z_{\mathbf{D}_{\text{amb},k}}\}_{k=k_1}^{k_{P_g}}$ are distinct for $g = 1, \dots, G$. Using the EVD in (52), we can then derive

$$\mathbf{D}_{\text{amb},g}^{(T_2)} = \mathbf{D}_{\text{amb},g}^{(T_2)'} / \mathbf{M}_{\mathbf{D}_{\text{amb},g}}. \quad (53)$$

Based on (51) and (53), one can have

$$\left(\mathbf{1}_{P_g}^T \otimes \mathbf{b}_g^{(T_1)} \right) * \left(\mathbf{D}_{\text{amb},g}^{(T_2)} \mathbf{M}_{\text{D}_{\text{amb},g}}^{-1} \right) = \mathbf{U} \mathbf{M}_g, \quad (54)$$

which leads to the equivalent formulation:

$$\left(\mathbf{1}_{P_g}^T \otimes \mathbf{b}_g^{(T_1)} \right) * \mathbf{D}_{\text{amb},g}^{(T_2)} = \mathbf{U} \mathbf{M}_g \mathbf{M}_{\text{D}_{\text{amb},g}}. \quad (55)$$

Introduce a new matrix $\mathbf{N}_g = \mathbf{M}_g \mathbf{M}_{\text{D}_{\text{amb},g}}$, $g = 1, \dots, G$. Note that for an isolated eigenvalue $z_{\text{B},g}$, we have $\mathbf{N}_g = \mathbf{m}_g$. Then (55) can be compactly written as

$$\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} = \mathbf{U} \mathbf{N} \quad (56)$$

where $\mathbf{N} = [\mathbf{N}_1, \dots, \mathbf{N}_G]$.

Based on the above, the SVD of $\mathbf{Y}$ can be rewritten as

$$\begin{aligned} \mathbf{Y} &= (\mathbf{U} \mathbf{N}) \left(\mathbf{V} \mathbf{\Sigma}^* \mathbf{N}^{-\text{H}} \right)^{\text{H}} \\ &= \left(\mathbf{B}^{(T_1)} * \mathbf{D}_{\text{amb}}^{(T_2)} \right) \left(\mathbf{V}^* \mathbf{\Sigma} \mathbf{N}^{-\text{T}} \right)^{\text{T}}. \end{aligned} \quad (57)$$

With $\{z_{\text{B},k}\}_{k=1}^K$, $\{z_{\text{D}_{\text{amb},k}}\}_{k=1}^K$, and $\mathbf{N}$, the third factor matrix $\mathbf{A}_{\text{MB}}$ can then be determined by

$$\mathbf{a}_{\text{MB},k} = \left(\frac{\mathbf{b}_k^{(L_1)\text{H}}}{\mathbf{b}_k^{(L_1)\text{H}} \mathbf{b}_k^{(L_1)}} \otimes \frac{\mathbf{d}_{\text{amb},k}^{(L_2)\text{H}}}{\mathbf{d}_{\text{amb},k}^{(L_2)\text{H}} \mathbf{d}_{\text{amb},k}^{(L_2)}} \otimes \mathbf{I}_M \right) \mathbf{V}^* \mathbf{\Sigma} \mathbf{c}_k \quad (58)$$

where $\mathbf{c}_k$ is the k th row vector of matrix $\mathbf{N}^{-1}$.

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Pilot Contamination-Aware Graph Attention Network for Power Control in CFmMIMO

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Abstract—Optimization-based power control algorithms are predominantly iterative with high computational complexity, making them impractical for real-time applications in cell-free massive multiple-input multiple-output (CFmMIMO) systems. Learning-based methods have emerged as a promising alternative, and among them, graph neural networks (GNNs) have demonstrated their excellent performance in solving power control problems. However, all existing GNN-based approaches assume ideal orthogonality among pilot sequences for user equipments (UEs), which is unrealistic given that the number of UEs exceeds the available orthogonal pilot sequences in CFmMIMO schemes. Additionally, supervised training necessitates costly computational resources for computing the target power control solutions for a large volume of training samples. To address these issues, we propose a graph attention network for downlink power control in CFmMIMO systems that operates in a self-supervised manner while effectively handling pilot contamination and adapting to a dynamic number of UEs. Experimental results show its effectiveness, even in comparison to the optimal accelerated projected gradient method as a baseline.

Index Terms—Graph attention network, power control, pilot contamination, cell-free massive MIMO.

I. INTRODUCTION

CELLULAR massive multiple-input multiple-output (MIMO), usually enabled by multi-antenna panel arrays, is the backbone of today's 5G communication technology. However, a persistent problem with cellular massive MIMO deployments is the poor service quality experienced by the cell-edge user equipments (UEs). Cell-free massive MIMO (CFmMIMO) has been proposed as a promising architecture for next-generation wireless communications. CFmMIMO eliminates the cellular boundaries and instead deploys massive distributed access points (APs) across a large coverage area [1]. These APs cooperatively serve UEs

within the coverage area without the constraints of cell boundaries, ensuring fairness in service quality across UEs at different locations and reducing interference through efficient beamforming enabled by the coordination of a massive number of distributed antennas. As a result, CFmMIMO can achieve a higher network spectral efficiency (SE) compared to traditional cellular schemes [2], [3].

To fully realize the benefits of CFmMIMO, several challenges must be addressed. In particular, power control¹ and pilot allocation are critical. With an increasing number of UEs, especially in CFmMIMO systems, the limited number of orthogonal pilot sequences becomes insufficient, leading to pilot contamination. Pilot contamination significantly degrades the communication quality, and thus, its impact cannot be overlooked when addressing power control problems. Traditional power control algorithms are predominantly optimization-based, with computational complexity increasing sharply as the numbers of APs and UEs grow. For CFmMIMO systems with a large number of APs and UEs, existing algorithms lead to high computational costs and processing time, which runs counter to the demands for high efficiency and low latency.

To address this challenge, learning-based algorithms have been proposed to compute power allocation coefficients with significantly improved efficiency. Various neural network models, including convolutional neural networks [4], [5], transformer neural networks [6], [7], and graph neural networks (GNNs) [8], [9], [10], [11], [12] have been explored. GNNs have demonstrated excellent performance in solving power control problems thanks to their structural similarity to wireless communication networks [8], [9]. As mentioned earlier, pilot contamination cannot be ignored when addressing power control. However, to the best of our knowledge, all existing GNN-based methods assume the use of ideally orthogonal pilot sequences, which is a big stretch, especially in CFmMIMO systems. To bridge this gap, we propose a pilot contamination-aware graph attention network to handle power control in CFmMIMO systems. The proposed network model employs an attention mechanism [6], [7] enhanced GNN to effectively capture information from neighboring nodes. The design explicitly accounts for pilot contamination by incorporating the cross-correlation of pilot sequences as an input and categorizes graph edges into two types based on the connected nodes to distinguish interactions between APs and UEs. Moreover, it can adapt to dynamic scenarios with a varying number of UEs using a padding mechanism. In addition, the network is trained in a self-supervised

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¹We focus on downlink power control in a centralized fashion in this letter.

manner, reducing the workload of computing power control coefficients. Experimental results are presented to compare the proposed method against the first-order accelerated projected gradient (APG) method described in [13] and the transformer in [7].

II. SYSTEM MODEL

A. Cell-Free Massive MIMO

Consider downlink power control in a CFmMIMO system where M synchronized APs, each with N antennas, jointly serve K single-antenna UEs using common time-frequency resources.² Operating in time-division duplex mode with block-fading channels constant over T_c symbols, the uplink channel from the k th UE to the m th AP is given by

$$\mathbf{g}_{m,k} = \sqrt{\beta_{m,k}} \mathbf{h}_{m,k}, \quad (1)$$

where $\beta_{m,k}$ denotes the large-scale fading coefficient, and $\mathbf{h}_{m,k} \in \mathbb{C}^N$ denotes the small-scale coefficient vector for an N -antenna array. The entries of $\mathbf{h}_{m,k}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, i.e., $\mathcal{CN}(0, 1)$.

Pilot sequences consisting of T_p ($T_p \ll T_c$) symbols are used for channel estimation during the uplink training stage. Channel reciprocity is assumed, enabling downlink channel estimation using uplink pilot sequences. Let the pilot sequence for the k th UE be $\sqrt{T_p} \boldsymbol{\phi}_k \in \mathbb{C}^{T_p}$ with $\|\boldsymbol{\phi}_k\|^2 = 1$, where $\|\cdot\|$ represents the Euclidean norm of a vector. In CFmMIMO systems, where a large number of UEs are served simultaneously, the condition $T_p < K$ often arises due to the limited length of pilot sequences, resulting in pilot contamination.

In the uplink training phase, the received signal at the m th AP can be expressed as

$$\mathbf{Y}_m^{\text{up}} = \sqrt{\zeta_p T_p} \sum_{i=1}^K \mathbf{g}_{m,i} \boldsymbol{\phi}_i^H + \mathbf{W}_m^{\text{up}}, \quad (2)$$

where ζ_p denotes the normalized transmit signal-to-noise ratio per pilot symbol, $(\cdot)^H$ represents the conjugate transpose operation, and $\mathbf{W}_m^{\text{up}} \in \mathbb{C}^{N \times T_p}$ is the noise matrix whose entries follow i.i.d. $\mathcal{CN}(0, 1)$. The minimum mean square error (MMSE) estimate of the channel between the m th AP and the k th UE can be derived as [13]

$$\hat{\mathbf{g}}_{m,k} = \frac{\sqrt{\zeta_p T_p} \beta_{m,k}}{1 + \zeta_p T_p \sum_{i=1}^K \beta_{m,i} |\boldsymbol{\phi}_i^H \boldsymbol{\phi}_k|^2} \mathbf{Y}_m^{\text{up}} \boldsymbol{\phi}_k, \quad (3)$$

where $|\cdot|$ denotes the absolute value. The mean square of each entry of $\hat{\mathbf{g}}_{m,k}$ is

$$\bar{g}_{m,k} = \mathbb{E} \left[\left| \hat{g}_{m,k}[n] \right|^2 \right] = \frac{\zeta_p T_p \beta_{m,k}^2}{1 + \zeta_p T_p \sum_{i=1}^K \beta_{m,i} |\boldsymbol{\phi}_i^H \boldsymbol{\phi}_k|^2}. \quad (4)$$

APs cooperate to deal with power control using large-scale fading information, which is assumed to be available through the central processing unit.² Denote the power control coefficient matrix as $\mathbf{M} \in \mathbb{R}^{M \times K}$, and the (m, k) -th matrix element, $\mu_{m,k}$, is the power control coefficient for the signal transmitted from the m th AP to the k th UE. According to the

practical constraint on the total transmit power per AP, the feasible set of power control coefficients can be expressed as

$$\mathcal{S} = \left\{ \mathbf{M} \mid \mu_{m,k} \geq 0; \|\boldsymbol{\mu}_m\|^2 \leq \frac{1}{N}, \forall m, k \right\}, \quad (5)$$

where $\boldsymbol{\mu}_m$ is the m th row of $\mathbf{M}$. Denoting the data symbol transmitted to the k th UE as s_k with $\mathbb{E}[|s_k|^2] = 1$, and ζ_d as the maximum downlink transmit power per symbol normalized to noise power at each AP, the downlink signal from the m th AP using conjugate beamforming can be expressed as

$$\mathbf{x}_m = \sqrt{\zeta_d} \sum_{k=1}^K \frac{\mu_{m,k}}{\sqrt{\bar{g}_{m,k}}} \hat{\mathbf{g}}_{m,k}^* s_k, \quad (6)$$

where $(\cdot)^*$ denotes conjugate operation. Using the use-and-then-forget bounding technique, the signal-to-interference-plus-noise ratio for the k th UE can be derived as [13]

$$\gamma_k = \frac{\zeta_d (\bar{\boldsymbol{\mu}}_k^T \mathbf{v}_{k,k})^2}{\sum_{i=1, i \neq k}^K \zeta_d (\bar{\boldsymbol{\mu}}_i^T \mathbf{v}_{i,k})^2 + \frac{\zeta_d}{N} \sum_{i=1}^K \|\bar{\mathbf{B}}_k \bar{\boldsymbol{\mu}}_i\|^2 + \frac{1}{N^2}}, \quad (7)$$

where $\bar{\boldsymbol{\mu}}_k$ denotes the k th column of $\mathbf{M}$, $(\cdot)^T$ is the transpose operator, $\bar{\mathbf{B}}_k$ is a diagonal matrix with diagonal elements $\sqrt{\beta_{1,k}}, \dots, \sqrt{\beta_{M,k}}$, and the m th element of $\mathbf{v}_{i,k} \in \mathbb{R}^M$ is

$$\mathbf{v}_{i,k}[m] = \left| \boldsymbol{\phi}_i^H \boldsymbol{\phi}_k \right| \sqrt{\bar{g}_{m,i}} \frac{\beta_{m,k}}{\beta_{m,i}}. \quad (8)$$

The downlink network SE of the k th UE is then

$$\text{SE}_k = \left(1 - \frac{T_p}{T_c} \right) \log_2(1 + \gamma_k). \quad (9)$$

B. Optimization Problem Formulation

Define $\mathbf{B} \in \mathbb{R}_+^{M \times K}$ as the large-scale fading coefficient matrix with $\beta_{m,k}$ as the (m, k) -th element, and let $\boldsymbol{\Phi} \in \mathbb{R}^{K \times K}$ denote the pilot allocation matrix, where the (i, j) -th element is given by $|\boldsymbol{\phi}_i^H \boldsymbol{\phi}_j|$. Here $\mathbb{R}_+^{M \times K}$ represents the set of matrices of dimension $M \times K$ with non-negative real entries. From (7) and (9), SE depends on both the power control coefficients $\mathbf{M}$ and pilot allocation matrix $\boldsymbol{\Phi}$, given a large-scale fading matrix $\mathbf{B}$. When $T_p \geq K$, pilot sequences are orthogonal across all UEs, making their impact on SE negligible. However, in massive MIMO scenarios, $T_p < K$ often occurs. Therefore, incorporating the information of pilot assignment is essential for optimizing power control under pilot contamination.

Max-min fairness maximization is the objective given by

$$\underset{\mathbf{M}}{\text{maximize}} \left\{ \min_{1 \leq k \leq K} \text{SE}_k(\mathbf{M}; \boldsymbol{\Phi}, \mathbf{B}) \right\}. \quad (10)$$

By adopting a smoothing technique with smoothing parameter λ , the objective function in (10) can be reformulated as [13]

$$u(\mathbf{M}; \boldsymbol{\Phi}, \mathbf{B}) = -\frac{1}{\lambda} \log \left(\frac{1}{K} \sum_{k=1}^K \exp(-\lambda \text{SE}_k(\mathbf{M}; \boldsymbol{\Phi}, \mathbf{B})) \right). \quad (11)$$

Define a deep neural network as a map, $\mathbf{M} = f(\boldsymbol{\Phi}, \mathbf{B}; \boldsymbol{\Theta})$. Here f maps $\boldsymbol{\Phi}$ and $\mathbf{B}$ to the power control matrix $\mathbf{M}$, and $\boldsymbol{\Theta}$ represents the trainable weights of the network. Our goal is to determine the optimal weights $\boldsymbol{\Theta}_{\text{opt}}$ using a self-supervised learning method, in which the neural network is trained on unlabeled data, thus eliminating the computational burden of

²See [14] for justification.

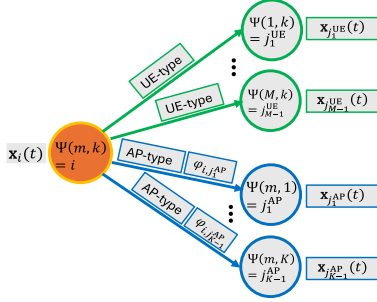


Fig. 1. Illustration of graph characteristics.

generating target power control solutions, for example, via conventional optimization algorithms. In this framework, the network learns the mapping between its input and power control coefficients $\mathbf{M}$ with the aim of maximizing the objective function in (11).

III. GRAPH ATTENTION NETWORK FOR POWER CONTROL

A. Graph Representation

Consider a heterogeneous graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ composed of a set of nodes $\mathcal{V}$ and edges $\mathcal{E}$. Each node v represents an AP-UE pair and is mapped by $\psi(m, k) = v$, where ψ is the mapping function from an AP-UE pair to its corresponding node, and $v = M(k-1) + m$. The bidirectional edge between the i th and j th nodes is denoted as $e_{i,j}$ for $i \neq j$. Edges are categorized based on whether the connected nodes share the same AP or UE, or neither:

- *AP-type edges* ($e_{i,j}^{AP}$): Nodes share the same AP.
- *UE-type edges* ($e_{i,j}^{UE}$): Nodes share the same UE.
- *No connection*: If two nodes share neither an AP nor a UE, no edge exists between them.

To incorporate pilot contamination awareness, the pilot allocation matrix Φ is included as an attribute for AP-type edges, capturing the orthogonality of allocated pilot sequences among UEs that share the same APs. Specifically, the (k_1, k_2) -th element of Φ , denoted by $\bar{\varphi}_{k_1, k_2}$, is assigned to those AP-type edges connecting nodes corresponding to the k_1 -th and k_2 -th UEs. To align with the GNN framework, $\varphi_{i,j} = \bar{\varphi}_{k_1, k_2}$ is introduced to represent the contamination information between nodes i and j , which correspond to k_1 -th and k_2 -th UEs, respectively. Suppose there are T attention layers in the graph structural network, and the input and output features of the i th node at the t th graph attention layer are represented by $\mathbf{x}_i(t-1)$ and $\mathbf{x}_i(t)$, respectively, where $t = 1, \dots, T$. The characteristics of the proposed graph structure, including the node representation $\mathbf{x}_i(t)$, the AP-type edges with $\varphi_{i,j}$, and the UE-type edges, are illustrated in Fig. 1. This structure enables the network to effectively model the relationship between APs and UEs while accounting for pilot contamination.

B. The Proposed Network Model

1) *Preprocessing Stage*: Given that the values of large-scale fading coefficients span different orders of magnitude, the natural logarithm operation is applied to each element of $\mathbf{B}$, resulting in $\mathbf{B}^{\log}$. Following this, $\mathbf{B}^{\log}$ is transformed to have

zero mean and unit variance across all elements, resulting in $\mathbf{B}^{\text{std}}$. Next, scaling and shifting are performed on $\mathbf{B}^{\text{std}}$ along each column, which can be expressed as

$$\mathbf{B}^{\text{pre}} = (\alpha \mathbf{1}_K^T) * \mathbf{B}^{\text{std}} + \beta \mathbf{1}_K^T, \quad (12)$$

where α and β represent the trainable parameters for scaling and shifting, respectively, $\mathbf{1}_K$ is an all-ones vector of length K , and $*$ denotes the Hadamard product of matrices. Finally, $\mathbf{B}^{\text{pre}}$ is vectorized into $\mathbf{b} \in \mathbb{R}^{MK \times 1}$. The i th element of $\mathbf{b}$ is taken as the initial representation of the i th node, also denoted as $\mathbf{x}_i(0)$ in the context of the attention mechanism block.

2) *Attention Mechanism Block*: The attention mechanism allows the model to focus on specific parts of the input data by assigning varying importance to different components. To capture the interrelationship among nodes, a single-head attention mechanism is introduced. The attention mechanism block comprises $T = 4$ hidden layers, with $\mathbf{x}_i(0)$ as the input to the first attention layer, the output sizes of 4 layers are 32, 64, 64, 64, respectively. A detailed explanation of the attention mechanism block is provided in Section III-C.

3) *Postprocessing Stage*: Given the attention mechanism block's output $\mathbf{X}(4) \in \mathbb{R}^{MK \times 64}$, whose i th row corresponds to $\mathbf{x}_i(4)$, a combination of linear and nonlinear processing is first applied. The resulting output can be described as

$$\mathbf{Y}_1 = \mathcal{L}_2(\text{ReLU}(\mathcal{L}_1(\mathbf{X}(4)))), \quad (13)$$

where $\mathcal{L}_1$ and $\mathcal{L}_2$ represent two distinct linear transformations, each with input size of 64 and output size of 64, and $\text{ReLU}(\cdot)$ denotes the rectified linear unit activation function.

Next, the node representation vectors undergo another linear transformation to map them into values. Following this, additional processing steps—similar to those in [7]—are applied, including the operations described in (14) and (15), giving

$$\mathbf{y}_2 = \exp(-\text{SoftPlus}(\mathcal{L}_3(\mathbf{Y}_1) + 6)), \quad (14)$$

where $\text{SoftPlus}(\cdot)$ denotes the Softplus activation function, and $\mathcal{L}_3$ is a linear function that maps a vector to a scalar value. To ensure that the results satisfy the constraints in (5), projection operation $\text{Proj}_{\mathcal{S}}(\cdot)$ in [13] are applied, yielding the power control coefficients as

$$\hat{\mathbf{M}} = \text{Proj}_{\mathcal{S}}(\mathbf{Y}_2), \quad (15)$$

where $\mathbf{Y}_2 \in \mathbb{R}^{M \times K}$ is obtained by reshaping the vector $\mathbf{y}_2$.

C. Attention Mechanism for Graph Neural Network

Considering all layers of the attention mechanism share the same structure, differing only in the sizes of their input and output, the following description uses the t th layer as a representative example. Denoting the representation vector of the i th node as input in the t th layer as $\mathbf{x}_i(t-1)$, the output node representation vector is calculated by

$$\mathbf{x}_i(t) = \text{Norm}\left(\text{ReLU}(\mathbf{y}_i^{\text{AP}}(t) + \mathbf{y}_i^{\text{UE}}(t))\right), \quad (16)$$

where $\text{Norm}(\cdot)$ denotes the normalization operation. Here $\mathbf{y}_i^{\text{AP}}(t)$ represents the component of the update through the attention mechanism, based on the information of nodes that are associated with the same APs as the i th node, and it can be expressed as

$$\mathbf{y}_i^{\text{AP}}(t) = \mathcal{L}_1^{\text{AP}, t}(\mathbf{x}_i(t-1)) +$$

$$\mathcal{L}_5^{\text{AP},t} \left(\sum_{j \in \mathcal{V}_i^{\text{AP}}} \alpha_{i,j}^{\text{AP}} [\mathcal{L}_2^{\text{AP},t}(\mathbf{x}_j(t-1)) + \mathcal{L}_\varphi^t(\varphi_{i,j})] \right), \quad (17)$$

$$\alpha_{i,j}^{\text{AP}} = \frac{\langle \mathcal{L}_3^{\text{AP},t}(\mathbf{x}_i(t-1)), \mathcal{L}_4^{\text{AP},t}(\mathbf{x}_j(t-1)) + \mathcal{L}_\varphi^t(\varphi_{i,j}) \rangle}{\sum_{u \in \mathcal{V}_i^{\text{AP}}} \langle \mathcal{L}_3^{\text{AP},t}(\mathbf{x}_i(t-1)), \mathcal{L}_4^{\text{AP},t}(\mathbf{x}_u(t-1)) + \mathcal{L}_\varphi^t(\varphi_{i,u}) \rangle}, \quad (18)$$

where $\mathcal{L}_q^{\text{AP},t}$ ($q = 1, \dots, 5$) denotes 5 linear transformations for capturing the information from AP-type edges in the t th layer, while $\mathcal{L}_\varphi^t$ is a linear transformation used to capture pilot contamination information in the t th layer. Moreover, $\mathcal{V}_i^{\text{AP}}$ denotes the set of nodes that share the same AP as the i th node, $\langle \mathbf{a}, \mathbf{b} \rangle = \exp(\mathbf{a}^T \mathbf{b} / \sqrt{D})$, and D is the length of $\mathbf{a}$.

Note that $\mathbf{y}_i^{\text{UE}}(t)$ is obtained through a similar process, but without considering pilot contamination. Specifically, $\mathcal{L}_\varphi^t(\cdot)$ is removed, as pilot contamination only arises between users.

IV. NUMERICAL RESULTS

A. Simulation Setup

In this simulation, the large-scale fading coefficient $\beta_{m,k}$ in dB is modeled as

$$\beta_{m,k} = \text{PL}_{m,k} + z_{m,k}, \quad (19)$$

where $\text{PL}_{m,k}$ denotes the path loss in dB, modeled using a three-slope model, and $z_{m,k}$ accounts for shadow fading, following $\mathcal{CN}(0, \sigma_{\text{sh}}^2)$. The same large-scale fading parameters as in [2] are used to characterize the propagation path. The noise signal is simulated using the same parameters as in [7].

The lengths for the coherence block and pilot sequence are set to $T_c = 200$ and $T_p = 18$, respectively. A total of T_p orthogonal pilot sequences are allocated among UEs. The first K sequences (corresponding to the actual number of UEs) are assigned, while for cases where $K > T_p$, the remaining $K - T_p$ sequences are randomly selected from the T_p available sequences. Simulation experiments are conducted in a CFmMIMO system with MMSE channel estimation in (3) and conjugate beamforming in (6).

B. Dataset Preparation and Padding Strategy

To address the requirement that all tensors within a batch must have the same shape in modern deep-learning frameworks such as PyTorch, which is used in our implementation, we employ a zero-padding strategy to handle training datasets with a varying number of UEs. Specifically, K is set as the maximum number of UEs, K_{max} , within the coverage area. If the actual number of UEs, K_{act} , is smaller than K_{max} , an extra $K_{\text{max}} - K_{\text{act}}$ UEs are added as padding. During training data preparation, zero-padding is applied to $\mathbf{B}$ and Φ , producing a new $\mathbf{B}$ of size $M \times K_{\text{max}}$ and a new Φ of size $K_{\text{max}} \times K_{\text{max}}$ as actual inputs, ensuring uniform input dimensions across training batches. Accordingly, the following modifications are made to eliminate the influence of padded UEs: in (15), $\mathbf{Y}_2$ is replaced with $\mathbf{Y}_2 \Phi_{\text{mask}}$; in (17), $\alpha_{i,j}^{\text{AP}}$ is substituted with $\varphi_{j,j} \alpha_{i,j}^{\text{AP}}$. Here, the diagonal matrix Φ_{mask} shares the same diagonal elements as the padded matrix Φ .

TABLE I
PARAMETERS FOR 5 SCENARIOS

Parameters	Scen. 1	Scen. 2	Scen. 3	Scen. 4	Scen. 5
Coverage Area (km ²)	0.16	0.32	0.32	0.32	0.32
M	16	32	32	64	64
K_{max}	8	20	20	40	40
K_{min}	8	20	10	40	20
Pilot Contamination	×	✓	✓	✓	✓
Varying UEs	×	×	✓	×	✓
Training Samples	50K	100K	800K	100K	800K

C. Methods Compared and Performance Metrics

We compared the performance of the proposed method with two other power allocation methods: the APG method and the transformer-based method.³ The APG method, a traditional optimization-based method as described in [13], serves as a baseline to evaluate the performance of the proposed graph attention network, with the smoothness parameter $\lambda = 3$ in (11). The transformer-based method of [7], a learning-based method also referred to as PAPC, is included for comparison as well. Considering that all existing GNN-based methods thus far are under the assumption of free pilot contamination, whereas our proposed method highlights its ability to handle pilot contamination, GNNs are not included for comparison.

The empirical cumulative distribution function (CDF) of the per-UE SE serves as a tool to visualize the SE distribution. Besides, we provide the running times of methods to assess their feasibility in real-time applications.

D. Performance Comparison in Different Scenarios

In this section, we compare the performance of 3 methods across 5 scenarios, demonstrating the adaptability and scalability of the proposed method. The parameter settings for 5 scenarios are listed in Table I. For the first scenario, as an example, $K_{\text{max}} = 8$ UEs are served by $M = 16$ APs, with both UEs and APs uniformly randomly distributed within a coverage area of 0.16 km². Since $K < T_p$ in this case, pilot contamination does not occur. The number of UEs is not varying, thus $K = K_{\text{max}} = K_{\text{min}}$ for all sample data. If the number of UEs is varying, as in Scenarios 3 and 5, the number of UEs changes within $[K_{\text{min}}, K_{\text{max}}]$. In Scenarios 2-5 with $K_{\text{max}} > T_p$, pilot contamination arises accordingly. All scenario results are tested on 500 samples, and the number of training samples for each scenario is provided in Table I.

The CDF results for 5 scenarios are presented in Fig. 2. Note that the proposed Graph ATtention network method is labeled as GAT in Figs. 2 and 3. For the max-min fairness objective, a steeper CDF curve that shifts towards the right indicates a more desirable result, as it suggests improved fairness across UEs, as well as higher SEs for all UEs. It can be concluded from Fig. 2 that the proposed method outperforms both APG and PAPC in all scenarios, demonstrating the proposed method's adaptability across different application scenarios, including variations in communication network size, pilot contamination, and whether the number of UEs is fixed or not. Note that in Scenarios 3 and 5, the curves do not ascend

³The proposed network and transformer-based network were trained on the NVIDIA H200 GPU.

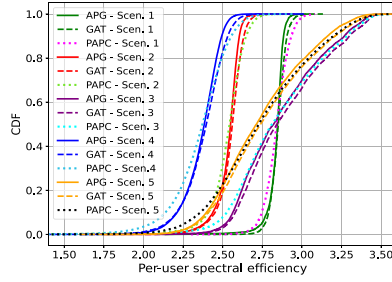


Fig. 2. Result comparison between 3 methods across 5 scenarios.

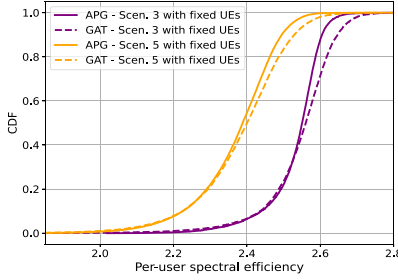


Fig. 3. Performance comparison for the proposed method trained with a varying number of UEs but tested with fixed UEs.

TABLE II
RUNTIME COMPARISON BETWEEN METHODS (IN SECONDS)

	Scen. 1		Scen. 3		Scen. 5	
	CPU	GPU	CPU	GPU	CPU	GPU
APG	1.5626	1.1391	3.5581	2.4226	11.2485	5.5185
PAPC	0.0022	0.0012	0.0134	0.0013	0.0167	0.0014
Proposed	0.0083	0.0020	0.0613	0.0021	0.5920	0.0022

as sharply as those of the other three scenarios. This is because the number of active UEs varies for each test sample, leading to a wider range of SE per UE.

To evaluate the flexibility of the proposed method, the trained models from Scenarios 3 and 5, which were trained with varying numbers of UEs, are tested on fixed UE settings, i.e., Scenario 3 with $K = 20$ and Scenario 5 with $K = 40$. The results, presented in Fig. 3, show that the models trained on datasets with varying numbers of UEs still demonstrate strong performance, even surpassing the APG method.

Table II presents the runtime for each method across different scenarios. The CPU measurements are obtained on a 64-bit Windows system with 16 GB RAM and an Intel Core i5 (2.60 GHz), while the GPU measurements are obtained using an NVIDIA A40. As shown, compared to 5.5185 [s] GPU processing time for APG in Scenario 5 with 64 APs and 40 UEs, the proposed method achieves a processing time of 2.2 [ms] on the GPU and 592 [ms] on the CPU, highlighting the real-time performance of the learning-based method and demonstrating its potential for power control in real-time applications. Note that although the PAPC's runtime is slightly less than the proposed model, the performance improvement justifies the negligible time difference.

In conclusion, the proposed method achieves performance comparable to APG. Moreover, it significantly reduces

computational complexity, highlighting the superior suitability of GNNs for CFmMIMO applications.

The impact of imperfect large-scale fading information and an ablation study on the performance gain from incorporating pilot information have been conducted. Please refer to the extended version of this letter for details [14].

V. CONCLUSION

A graph attention method has been proposed for downlink power control in CFmMIMO systems. The proposed method not only effectively managed power control in the presence of pilot contamination but also demonstrated strong adaptability to varying communication network sizes and both fixed and dynamic numbers of UEs. Additionally, the neural network was trained in a self-supervised manner, eliminating the need for the labor-intensive process of labeling the large volume of samples. Experimental results demonstrated that the proposed method outperforms APG and transformer-based methods.

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