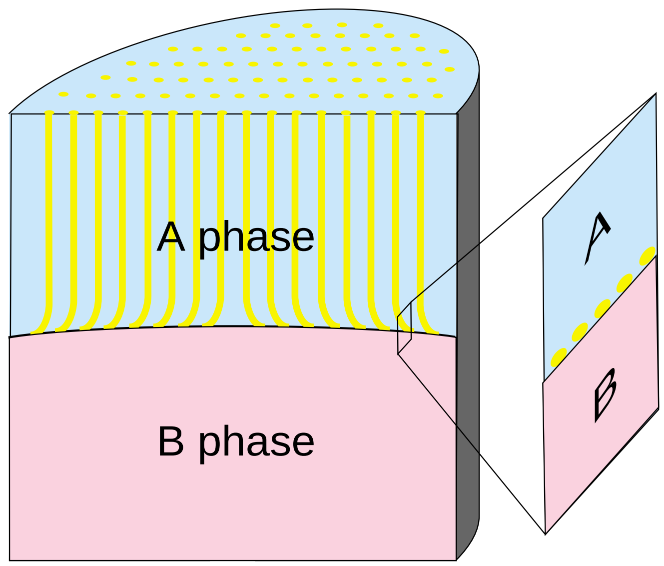
# VORTEX SHEET ON THE A-B INTERFACE OF SUPERFLUID $^3$He

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# Content

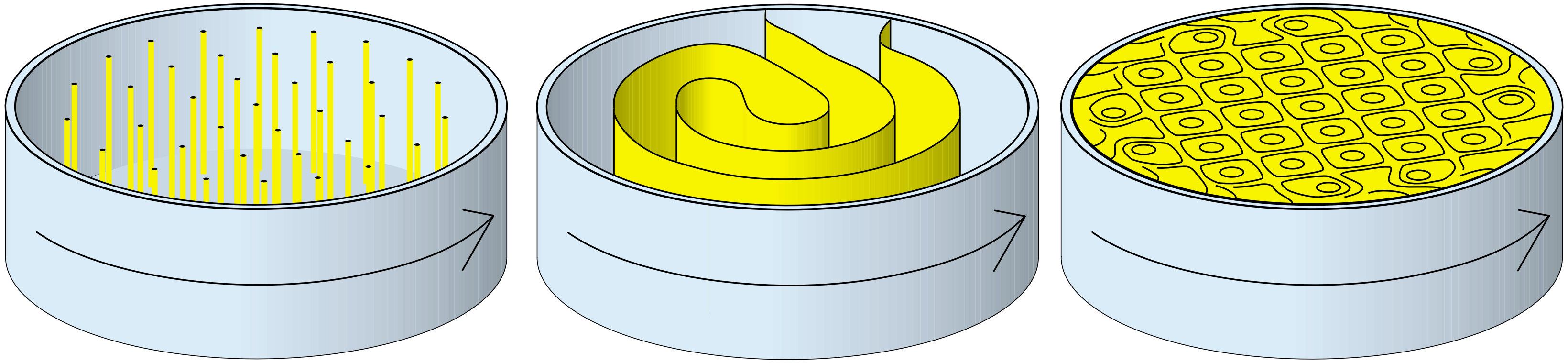
Introduction to superfluid $^3$He-A

Bending of vorticity near the interface

Structure of vorticity at the interface

Experiments

# Superfluid $^3$He-A

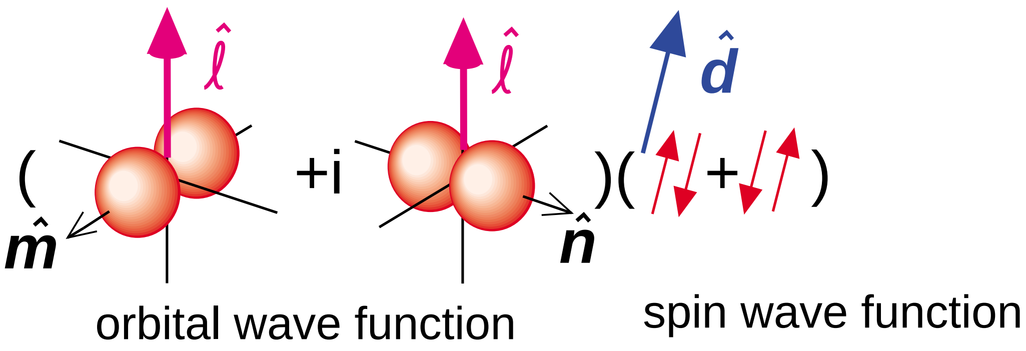


1, 2 and 3 dimensional vortex structures

large core size, $\sim 10\ \mu$m

low critical velocity, $\approx 1$ mm/s

# Cooper pairs in $^3$He-A




A phase factor $e^{i\chi}$ corresponds to rotation of $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ around $\hat{\mathbf{l}}$:

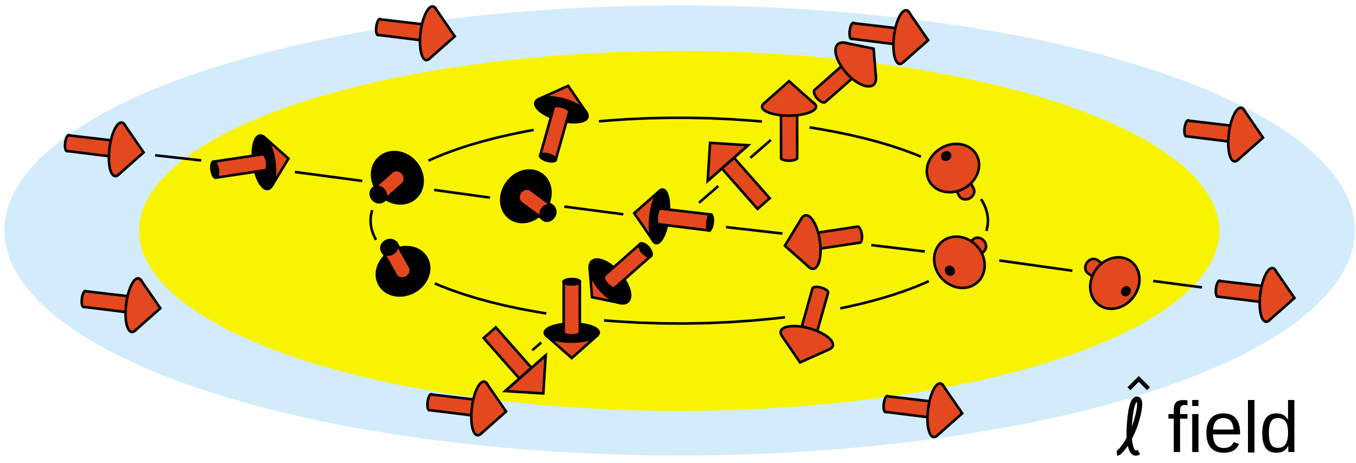
$$
\begin{aligned}
e^{i\chi}(\hat{\mathbf{m}} + i\hat{\mathbf{n}}) &= (\cos\chi + i\sin\chi)(\hat{\mathbf{m}} + i\hat{\mathbf{n}}) \\
&= (\hat{\mathbf{m}}\cos\chi - \hat{\mathbf{n}}\sin\chi) + i(\hat{\mathbf{m}}\sin\chi + \hat{\mathbf{n}}\cos\chi).
\end{aligned}
$$

Superfluid velocity

$$
\mathbf{v}_s = \frac{\hbar}{2m}\nabla\chi = \frac{\hbar}{2m}\sum_j \hat{m}_j \nabla \hat{n}_j. \tag{1}
$$

# Continuous vortices

Consider the structure




Here $\hat{\mathbf{l}}$ sweeps once trough all orientations (once a unit sphere).

$\Rightarrow$ $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ circle twice around $\hat{\mathbf{l}}$ when one goes around this object.

$\Rightarrow$ This is a two-quantum vortex. It is called *continuous*, because $\Delta$ (the amplitude of the order parameter) vanishes nowhere.

# Hydrodynamic theory of $^3$He-A

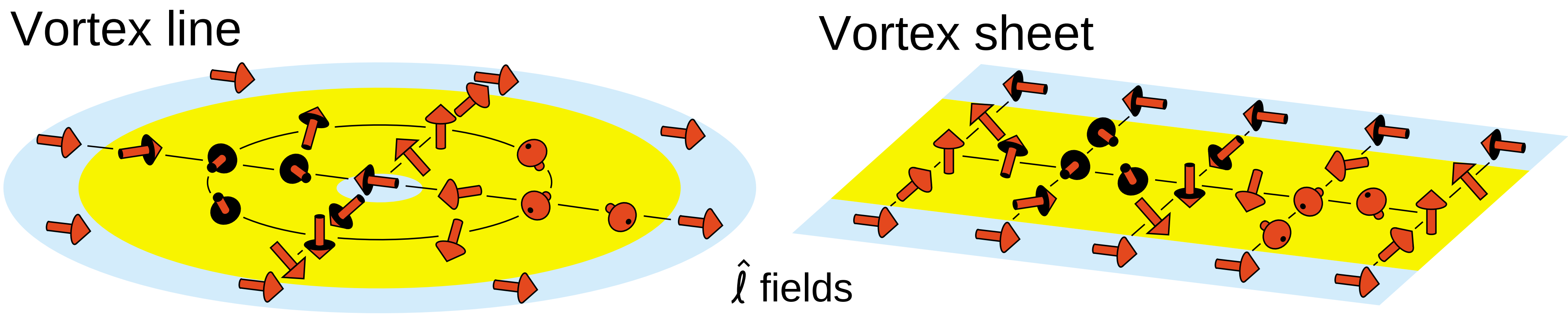
Assume the order parameter ($\hat{\mathbf{m}}$, $\hat{\mathbf{n}}$, $\hat{\mathbf{l}}$, $\hat{\mathbf{d}}$) changes slowly in space. Then one can make gradient expansion of the free energy

$$\begin{aligned} F &= \int d^3r \Big[ -\tfrac{1}{2}\lambda_{\mathsf{D}}(\hat{\mathbf{d}}\cdot\hat{\mathbf{l}})^2 + \tfrac{1}{2}\lambda_{\mathsf{H}}(\hat{\mathbf{d}}\cdot\mathbf{H})^2 \\ &+\tfrac{1}{2}\rho_\perp \mathbf{v}^2 + \tfrac{1}{2}(\rho_\parallel - \rho_\perp)(\hat{\mathbf{l}}\cdot\mathbf{v})^2 + C\mathbf{v}\cdot\nabla\times\hat{\mathbf{l}} - C_0(\hat{\mathbf{l}}\cdot\mathbf{v})(\hat{\mathbf{l}}\cdot\nabla\times\hat{\mathbf{l}}) \\ &+\tfrac{1}{2}K_{\mathsf{s}}(\nabla\cdot\hat{\mathbf{l}})^2 + \tfrac{1}{2}K_{\mathsf{t}}|\hat{\mathbf{l}}\cdot\nabla\times\hat{\mathbf{l}}|^2 + \tfrac{1}{2}K_{\mathsf{b}}|\hat{\mathbf{l}}\times(\nabla\times\hat{\mathbf{l}})|^2 \\ &+\tfrac{1}{2}K_5|(\hat{\mathbf{l}}\cdot\nabla)\hat{\mathbf{d}}|^2 + \tfrac{1}{2}K_6[(\hat{\mathbf{l}}\times\nabla)_i\hat{\mathbf{d}}_j)]^2 \Big]. \end{aligned} \qquad (2)$$

$\Rightarrow$ Theory of continuos vortex structures (including the core)

Possible to calculate structures, (intrinsic) nucleation, and dynamics of vortices

# Vortex lines and sheets



# Superfluid B phase

For present purposes the B phase is rather conventional superfluid:

- vortex lines: single quantum, small core size ($\sim$ 10 nm)

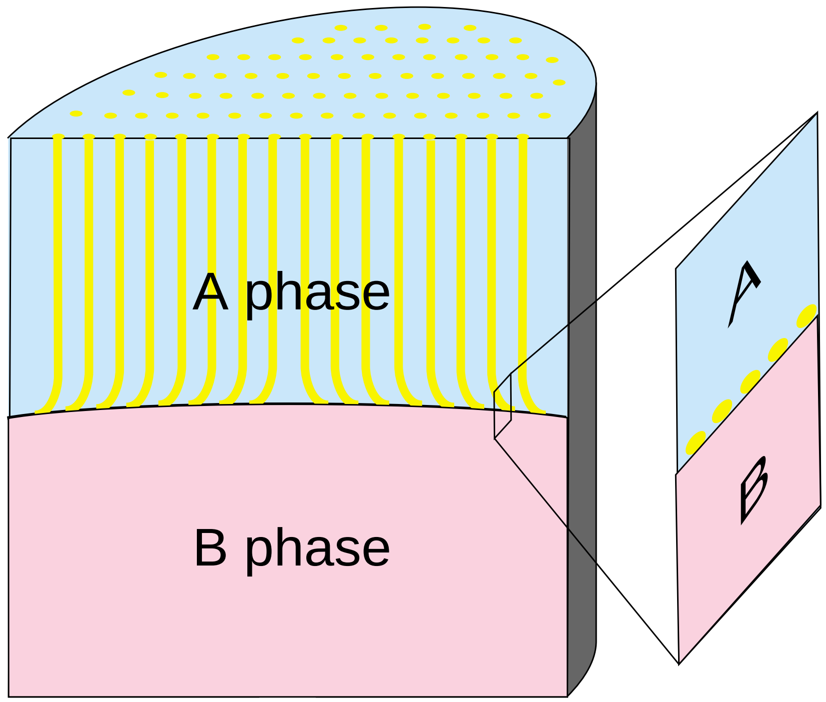
- high critical velocity for vortex nucleation (a few cm/s)

# A-B interface

A and B phases are phase coherent ⇒ Vortices cannot terminate at the interface

Difference in core size ($r_A \sim 1000 r_B$) and in quantization ($\kappa_A = 2\kappa_B$)

⇒ A-phase vortices do not easily penetrate through the interface



# Bending of vorticity at the A-B interface

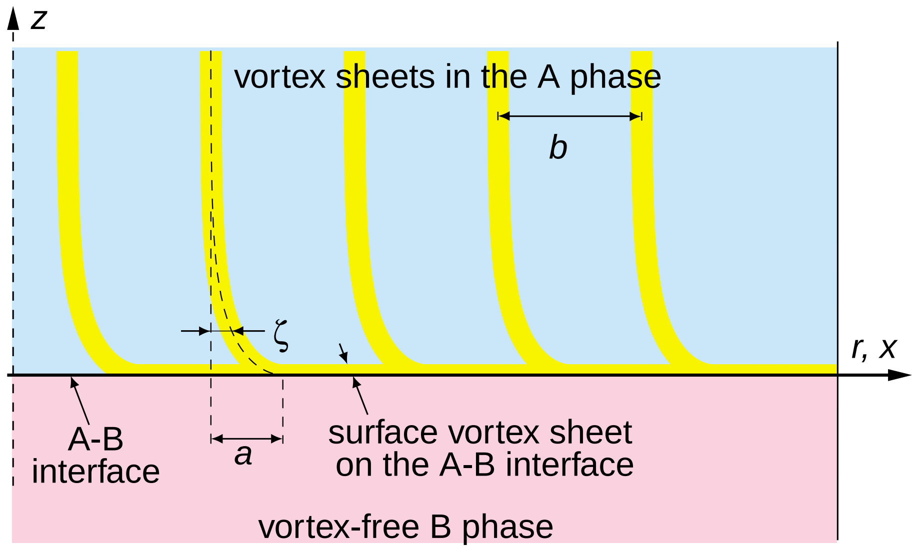
Simple model for bending vortex sheet: Minimize

$$F = \int d^3r \tfrac{1}{2}\rho_s v^2 + \sigma A. \qquad (3)$$

with constraints

$$\nabla \cdot \mathbf{v} = 0, \ \nabla \times \mathbf{v} = 2\mathbf{\Omega}. \qquad (4)$$

Here $\mathbf{v} \equiv \mathbf{v}_s - \mathbf{v}_n$. $A$ is the area of the sheet and $\sigma$ its surface tension. $\mathbf{v}$ has tangential discontinuity at the sheet.




Solution

$$\mathbf{v} = 2\Omega x \hat{\mathbf{y}} \qquad (5)$$

$$\begin{aligned}\frac{F}{L_x L_y} &= \int_0^\infty dz \Big( \frac{1}{b} \int_{-b/2+\zeta}^{b/2+\zeta} dx \frac{1}{2}\rho_s v^2 \\ &\quad + \frac{\sigma}{b}\sqrt{1+\left(\frac{d\zeta}{dz}\right)^2} \Big) \\ &= \int_0^\infty dz \Big[ \frac{1}{6}\rho_s \Omega^2 (b^2 + 12\zeta^2) \\ &\quad + \frac{\sigma}{b}\sqrt{1+\left(\frac{d\zeta}{dz}\right)^2} \Big] \end{aligned} \qquad (6)$$

$\Rightarrow$

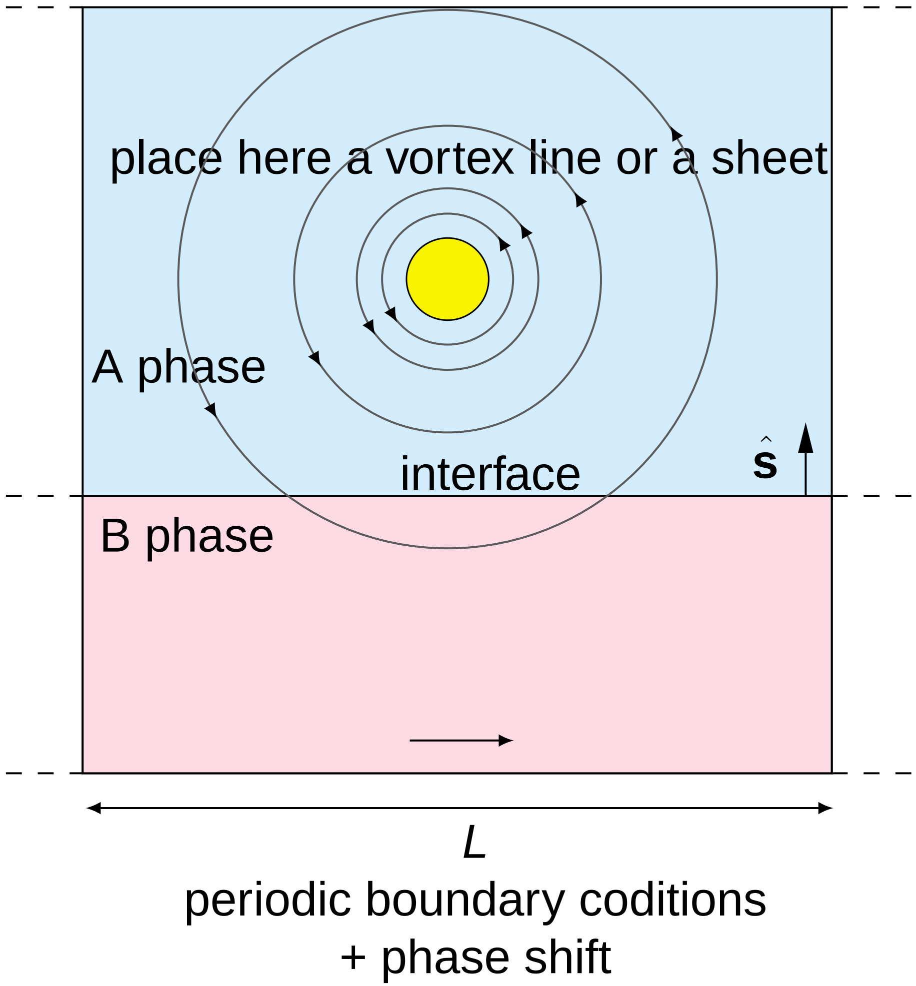
$$\frac{z}{a} = 1 - \sqrt{2 - \frac{\zeta^2}{a^2}} - \frac{1}{\sqrt{2}} \ln \frac{\sqrt{2} - \sqrt{2 - (\zeta/a)^2}}{(\sqrt{2}-1)\zeta/a}$$

where $a = b/\sqrt{6}$.

Surprisingly the Bekarevich-Khalatnikov model gives exactly the same form for vortex lines.

# Structure of the surface sheet

Numerical calculation




Boundary conditions at the A-B interface

$$(\hat{\mathbf{m}} + \mathrm{i}\hat{\mathbf{n}}) \cdot \hat{\mathbf{s}} = e^{\mathrm{i}\phi} \tag{7}$$

$$\hat{\mathbf{l}} \cdot \hat{\mathbf{s}} = 0 \tag{8}$$

$$\hat{\mathbf{d}} = R \cdot \hat{\mathbf{s}} \tag{9}$$

Penetration of superflow into B phase essential. Vortex layer on a solid wall is not stable.

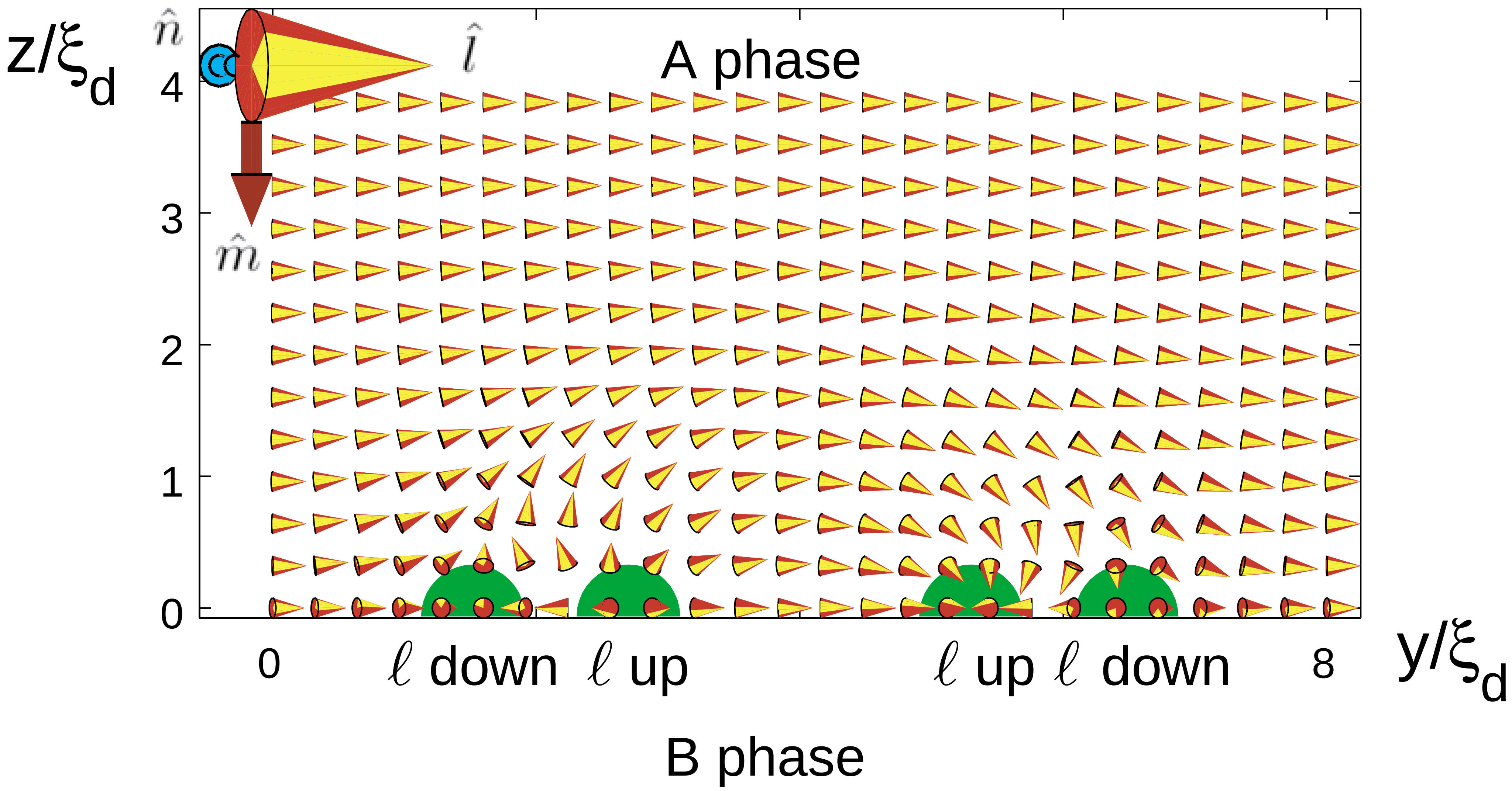
# Results

Structures independent of initial conditions (lines and sheets)

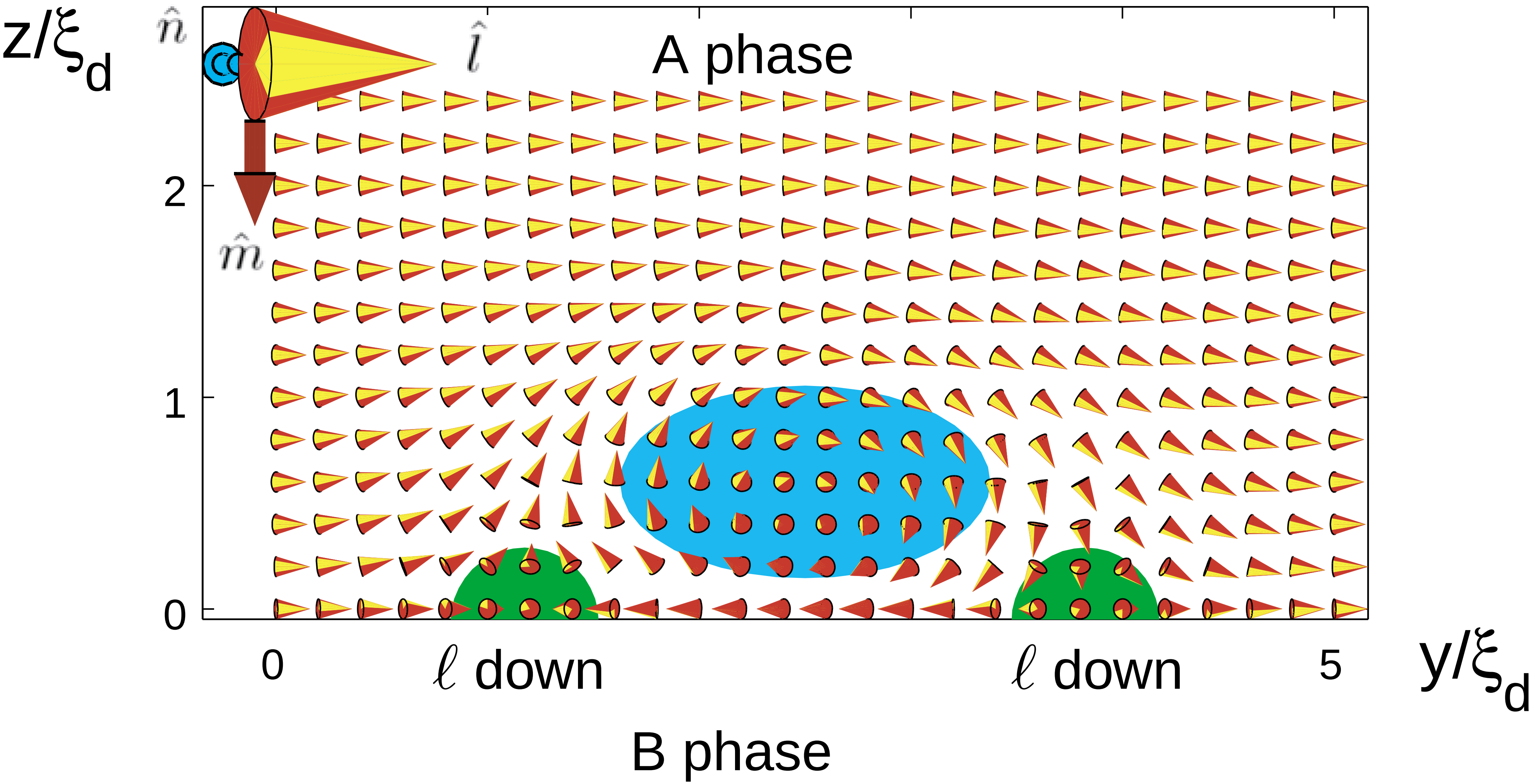
Two different structures found

They consist of units of one or one half of quantum of circulation

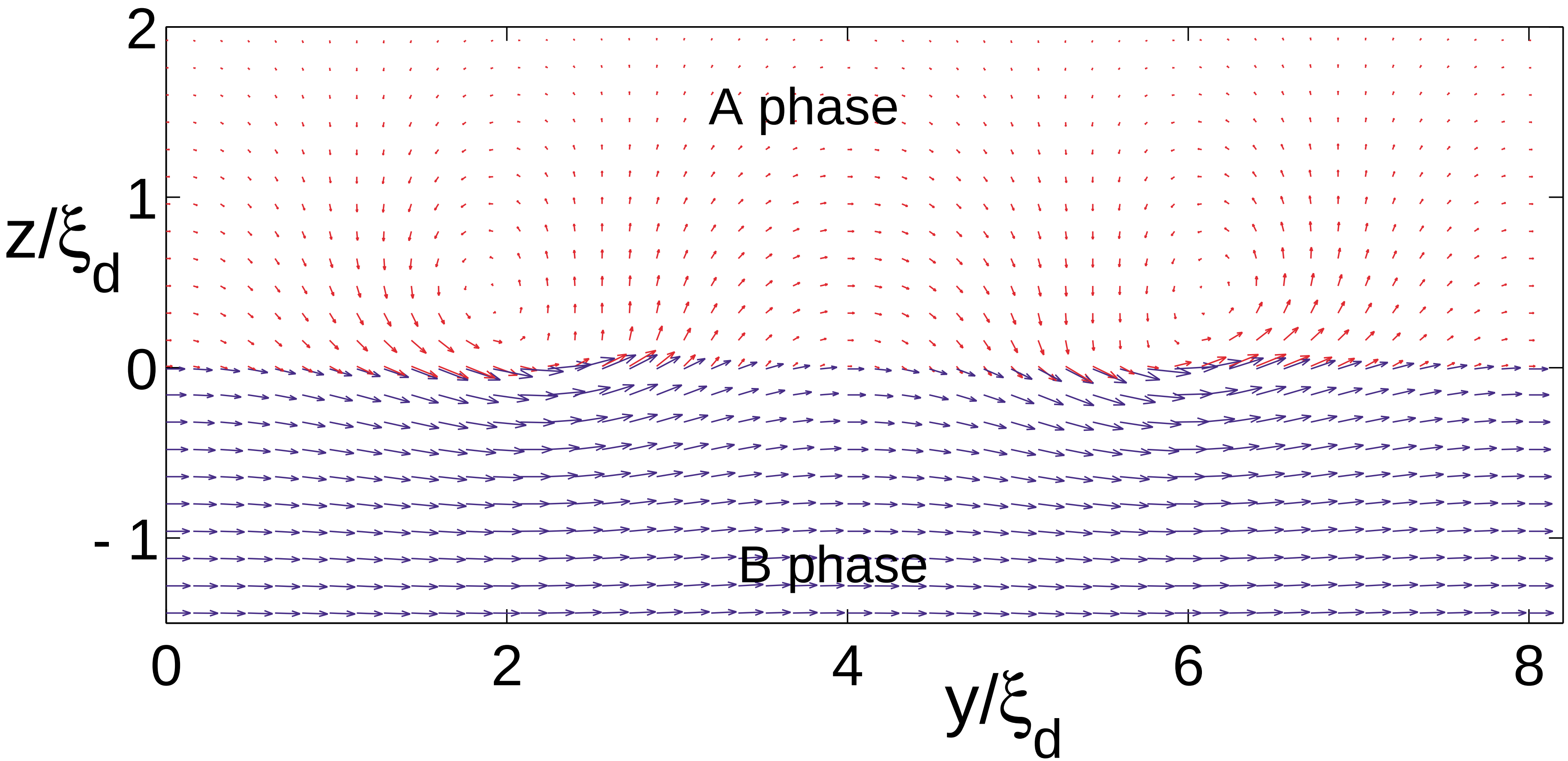
# Low density texture



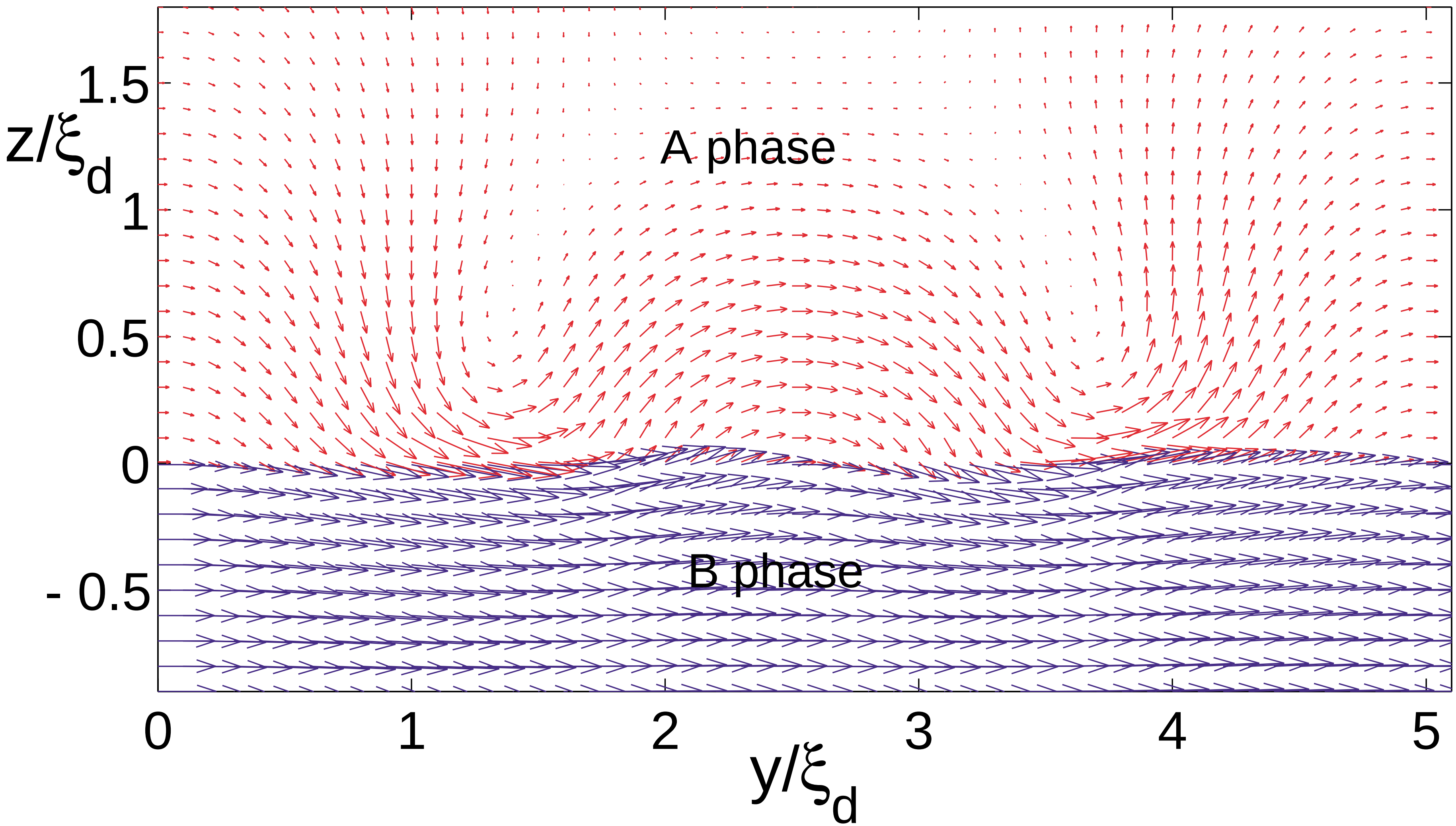
# High density texture



# Current

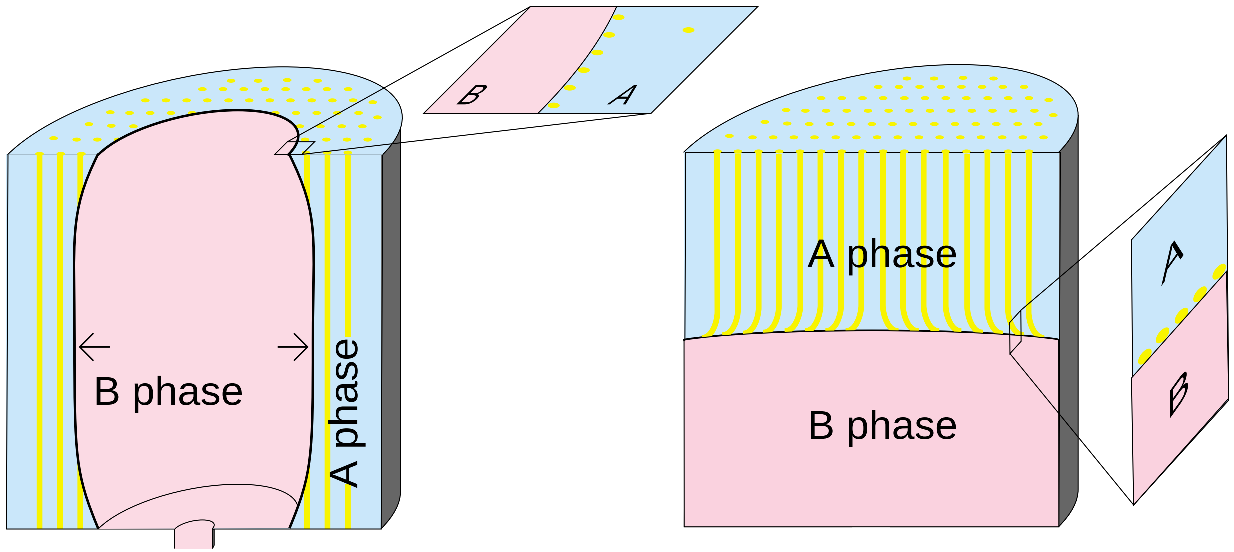



Low density texture




High density texture

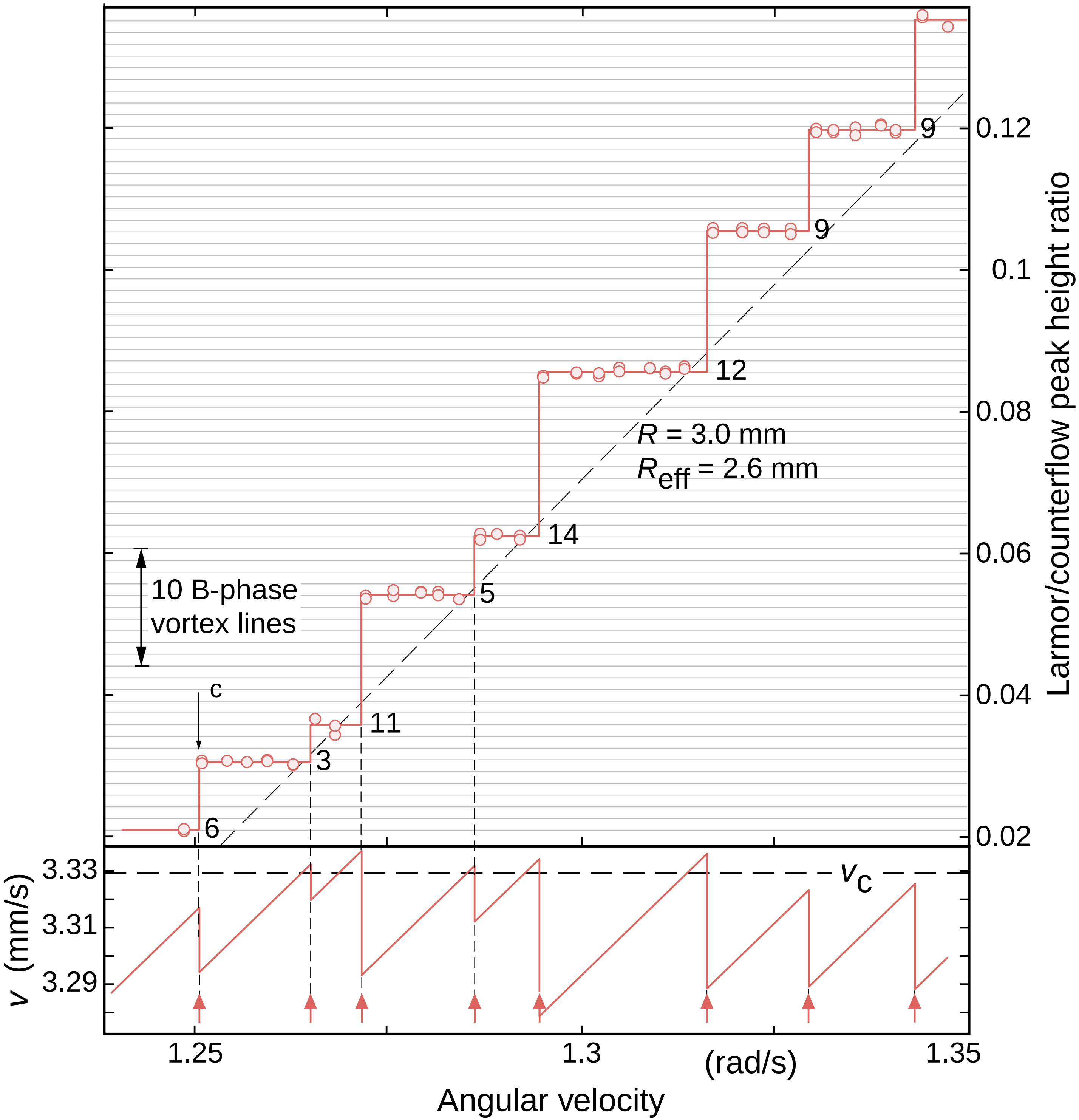
# Experiments




No NMR signal directly from the vortex layer

Can count vortices in both A and B phases
$\Rightarrow$ vortices penetrate through the interface in multiples of one circulation quantum.

This supports that the interface sheet has smaller units than vortex lines, which have circulation 2.



0.12
0.1
0.08
0.06
0.04
0.02
Larmor/counterflow peak height ratio
6
3
11
5
14
12
9
9
c
10 B-phase
vortex lines
R = 3.0 mm
R_eff = 2.6 mm
3.33
3.31
3.29
v (mm/s)
v_c
1.25
1.3
1.35
(rad/s)
Angular velocity

# Conclusions

Two types of vortex sheets in $^3$He-A:
in bulk and on the A-B interface

The interfacial sheet separates into units of one or one half circulation quantum. This is consistent with experiments.

publications and lecture transparencies available via
http://boojum.hut.fi/research/theory/