RECENT TOPICS IN THE THEORY OF SUPERFLUID ^{3}He

Lecture at ISSP Tokyo 13.5.2003

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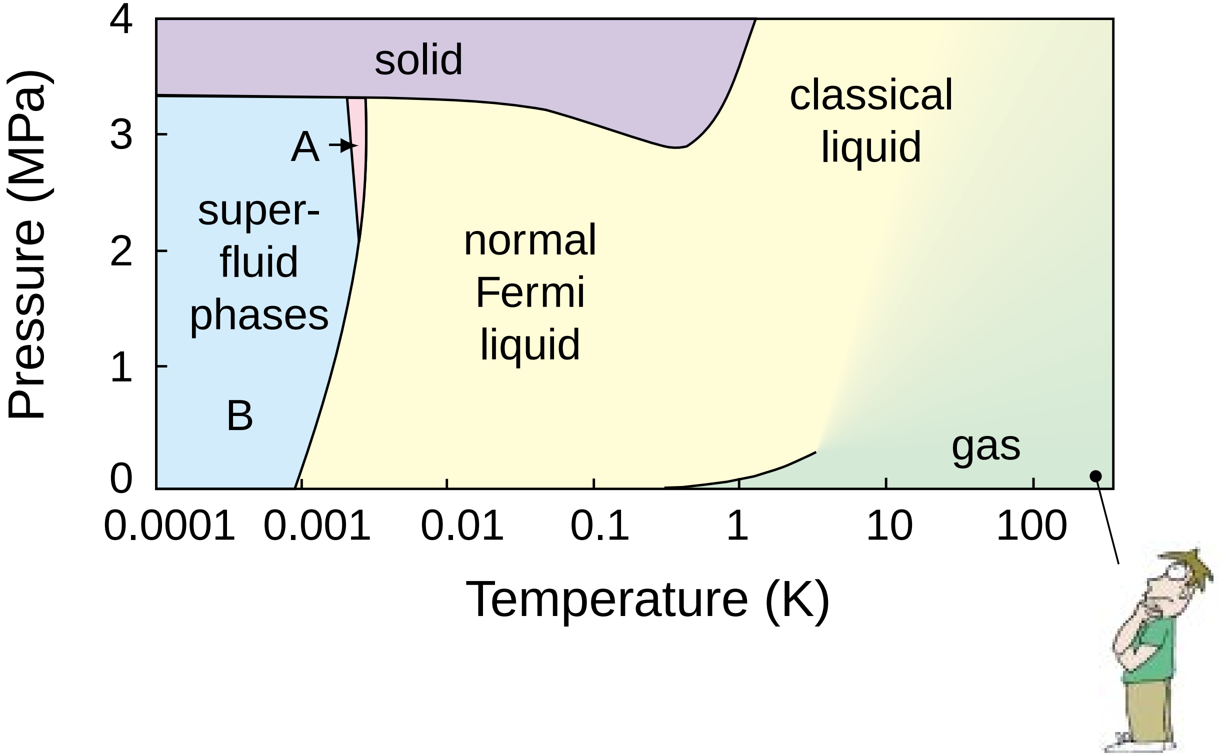
Content

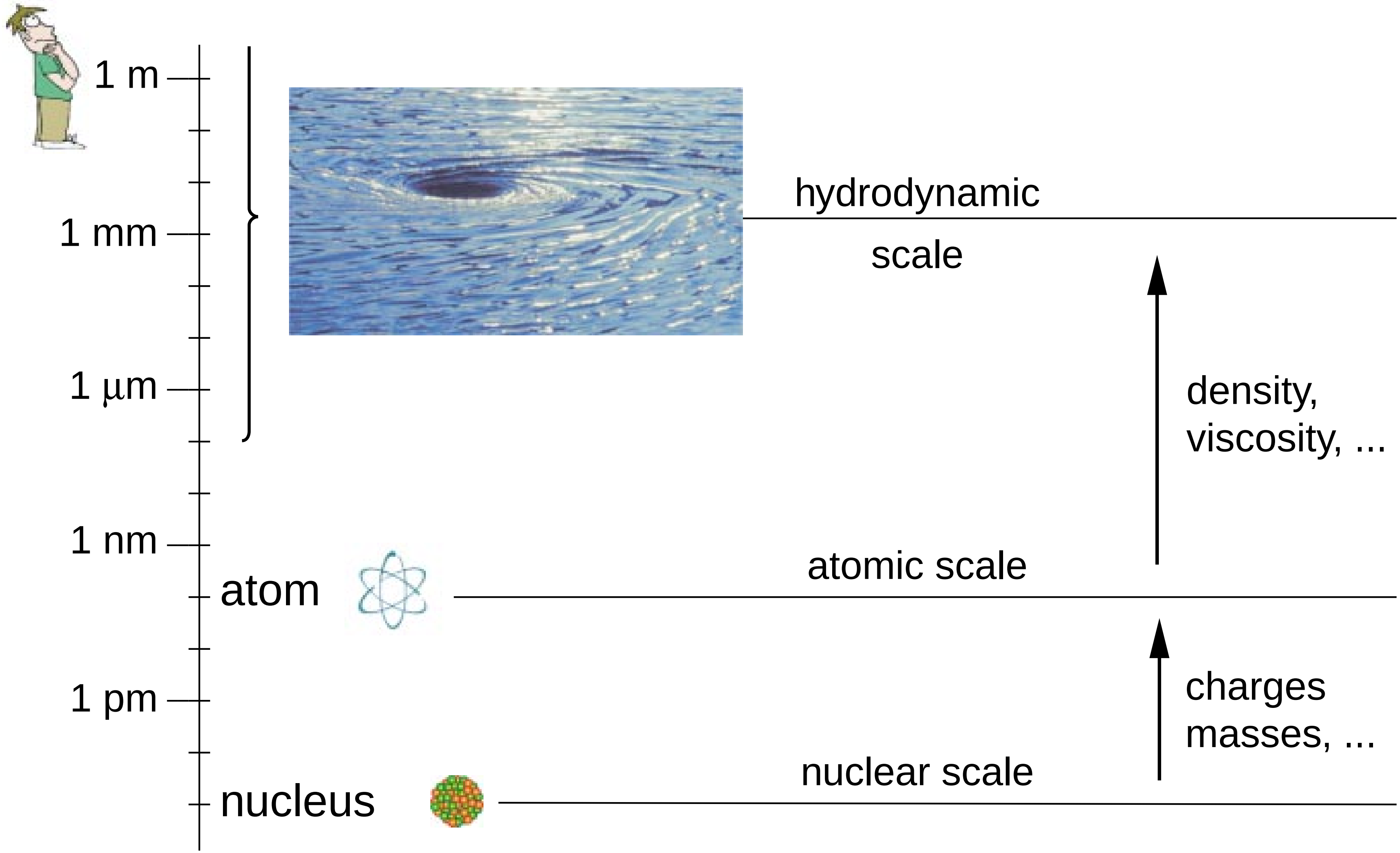
These lecture notes will be placed at
http://boojum.hut.fi/research/theory/

Phase diagram of superfluid ^{3}He

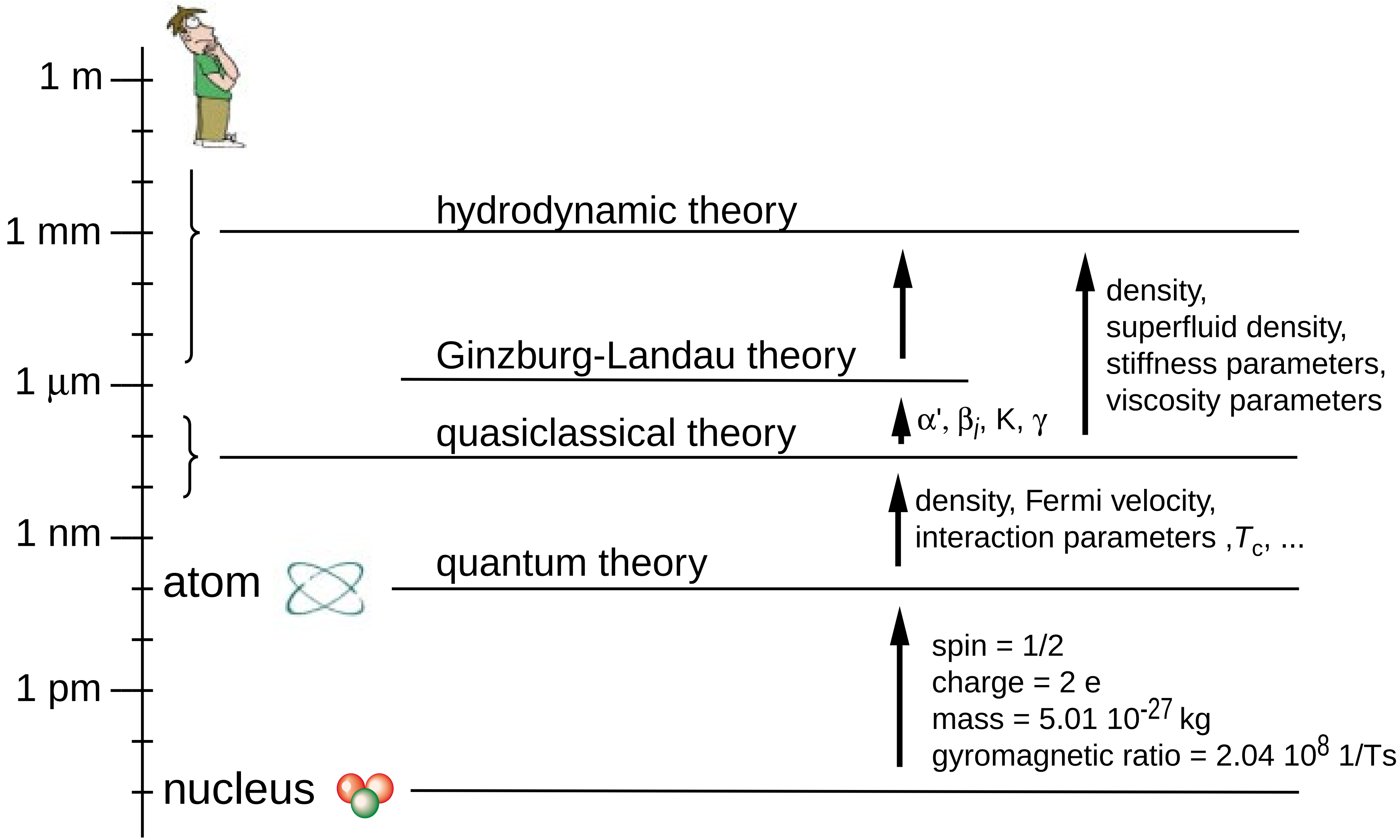
^{3}He is the less common isotope of helium. It is a fermion.



Length scales

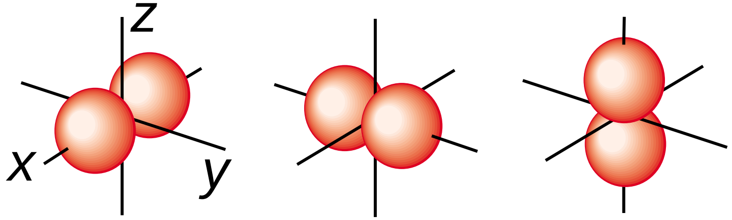


Length scales in superfluid ^{3}He



Cooper pairs in superfluid ^{3}He

Orbital wave functions (L=1)



Spin wave functions (S=1)

S_x=0: (-↑↑+↓↓)

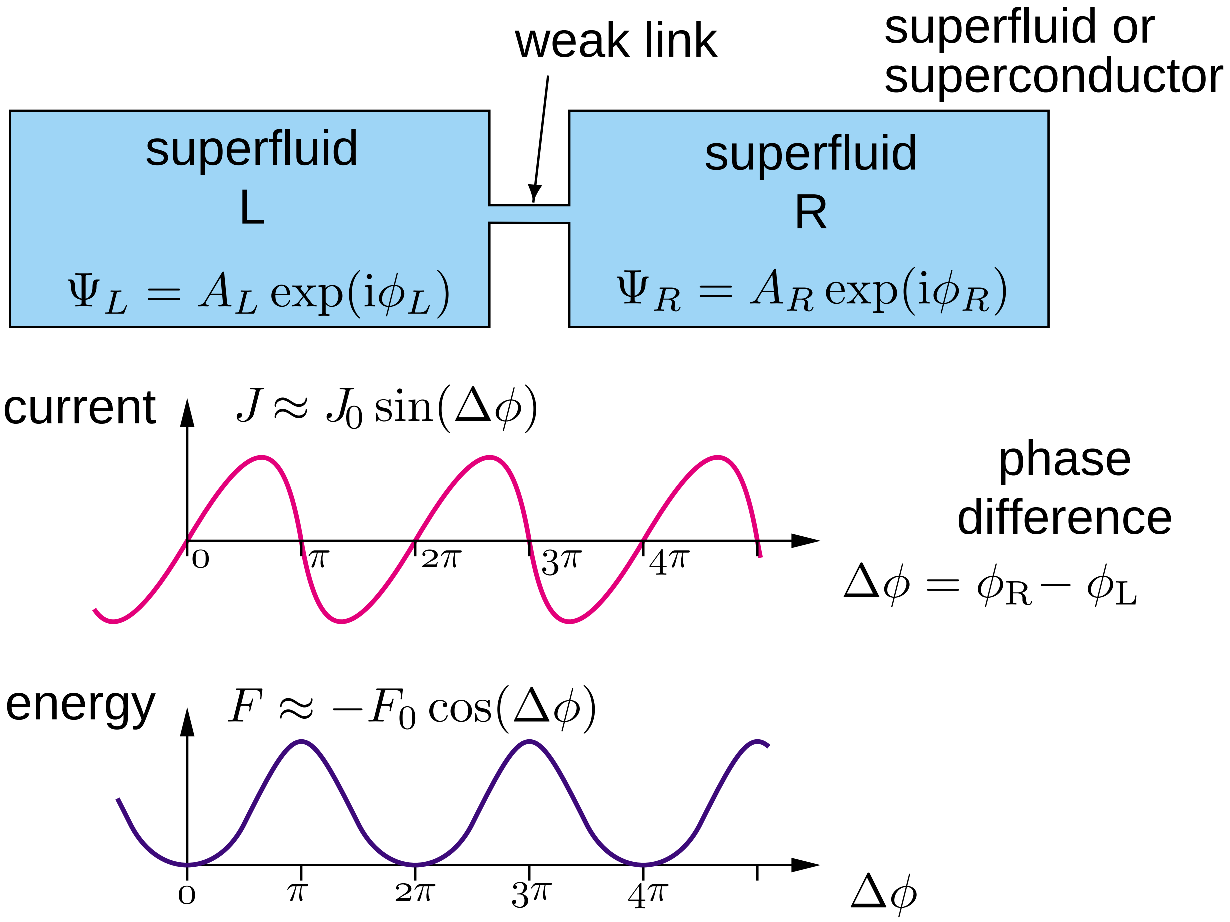
S_y=0: i(↑↑+↓↓)

S_z=0: (↑↓+↓↑)

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

order parameter

Part I. JOSEPHSON EFFECT

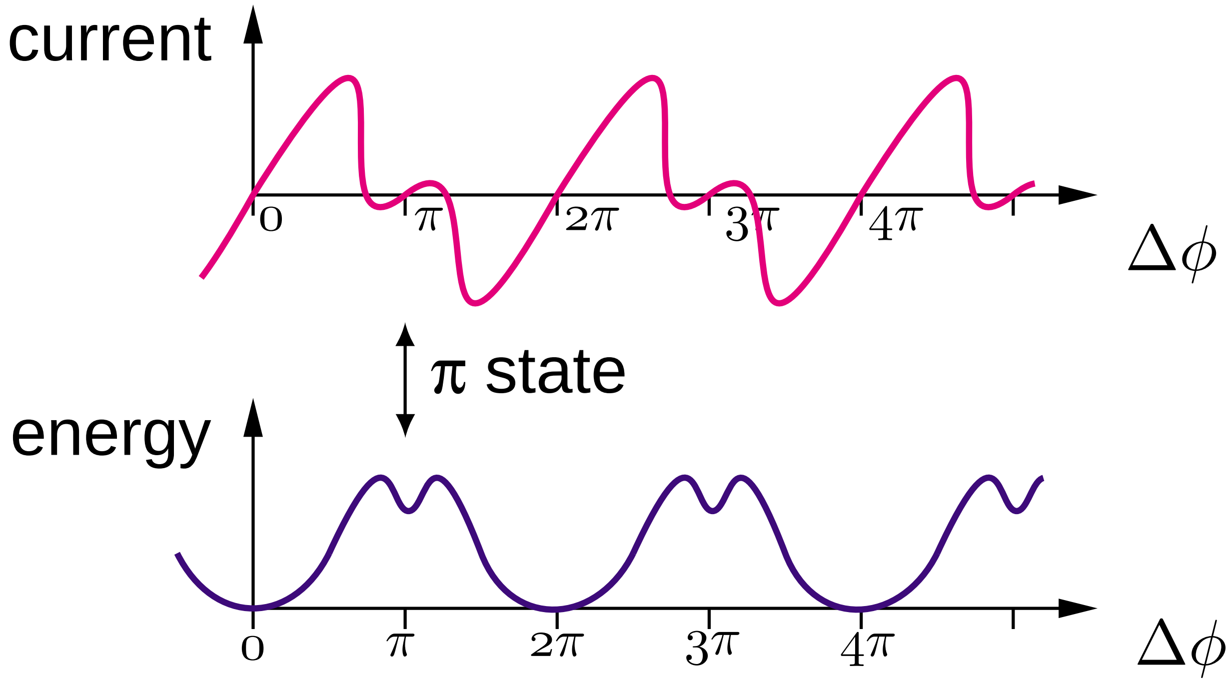


Experiments:

- superconductors: 1960's
- ^{3}He: Avenel and Varoquaux 1988

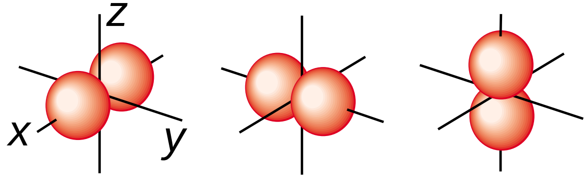
New experiments in ^{3}He-B

- S. Backhaus, S. Pereverzev, R. Simmonds, A. Loshak, J. Davis, R. Packard, A. Marchenkov (1998, 1999)
- O. Avenel, Yu. Mukharsky, E. Varoquaux (1999)



Superfluid B phase

Orbital wave functions (L=1)



Spin wave functions (S=1)

$S_x=0$: $(-\uparrow\uparrow + \downarrow\downarrow)$

$S_y=0$: $i(\uparrow\uparrow + \downarrow\downarrow)$

$S_z=0$: $(\uparrow\downarrow + \downarrow\uparrow)$

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} = A_0 e^{i\phi} \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix}$$

order parameter

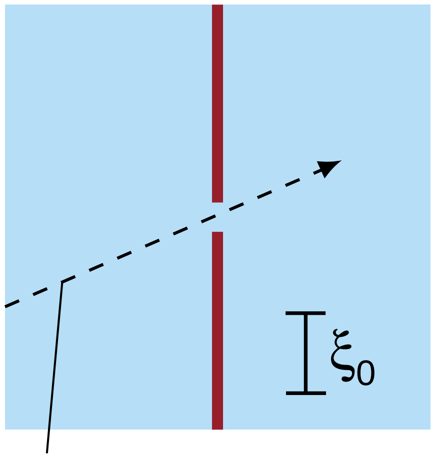
amplitude (real)

phase factor (complex)

rotation matrix (real)

$\begin{cases} \text{angle } \theta = 104^\circ \\ \text{axis } \mathbf{n} \end{cases}$

Pinhole

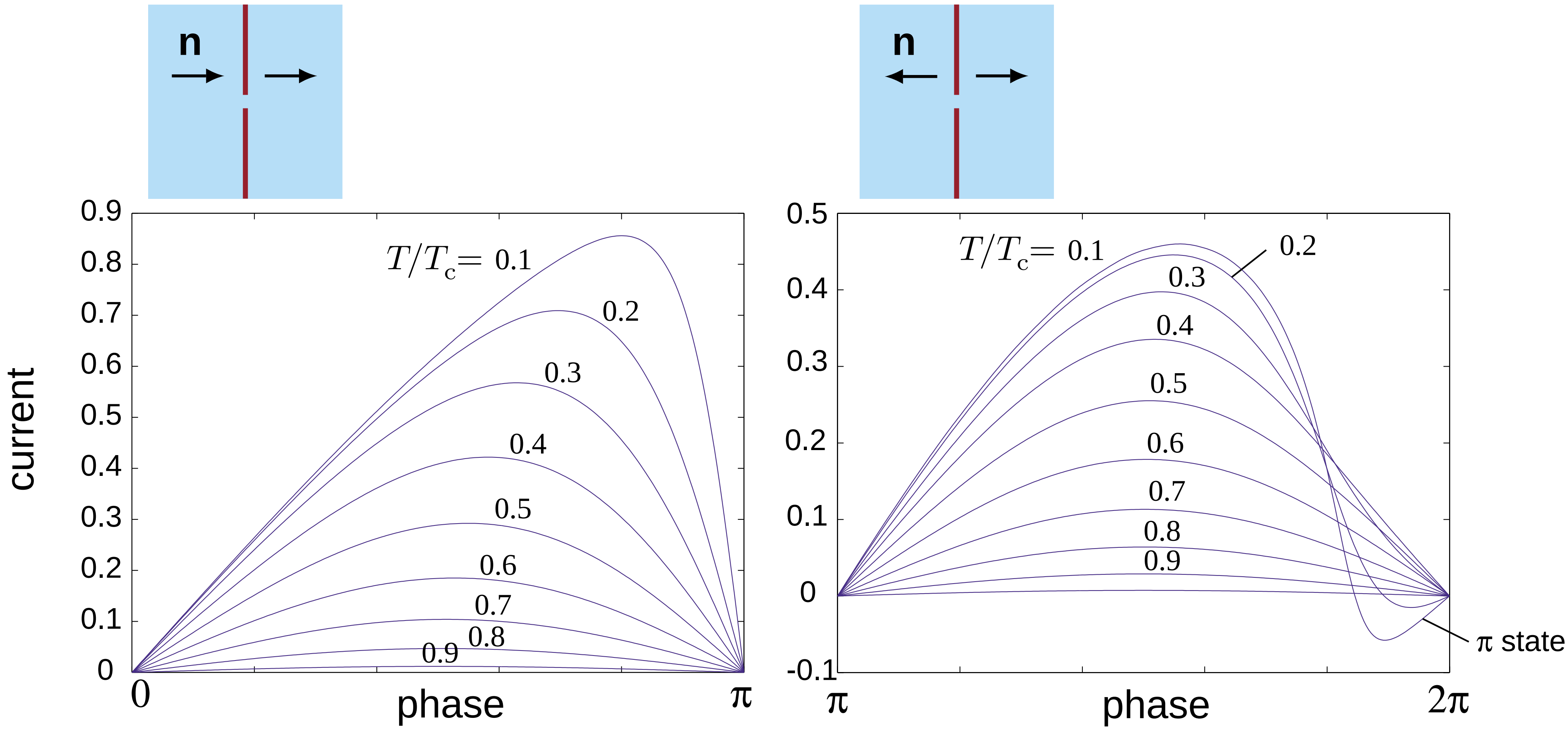


size $\ll \xi_0 \approx 10$ nm

current

$$J\left(\Delta\phi, R^L_{\mu j}, R^R_{\mu j}\right)$$
$$= 2m_3 A v_{\mathrm{F}} N(0)\pi T \sum_{\epsilon_n} \int \frac{\mathrm{d}^2\hat{k}}{4\pi} (\hat{\mathbf{k}}\cdot\hat{\mathbf{z}})\, g(\hat{\mathbf{k}}, \mathbf{R}_{\mathrm{hole}}, \epsilon_n)$$

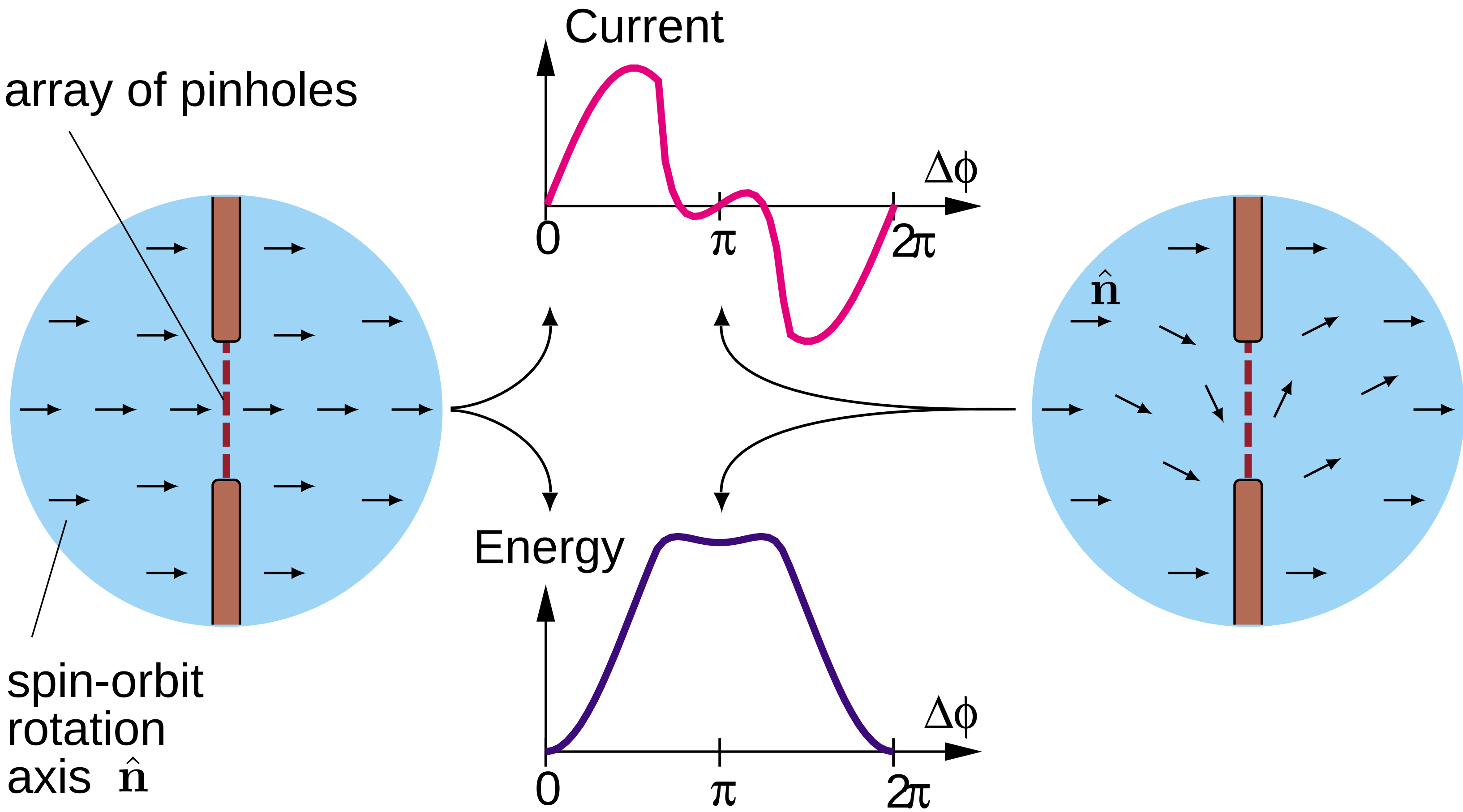
Pinhole current-phase relation



π state occurs only at very low temperatures

$\Rightarrow$ not consistent with experiments

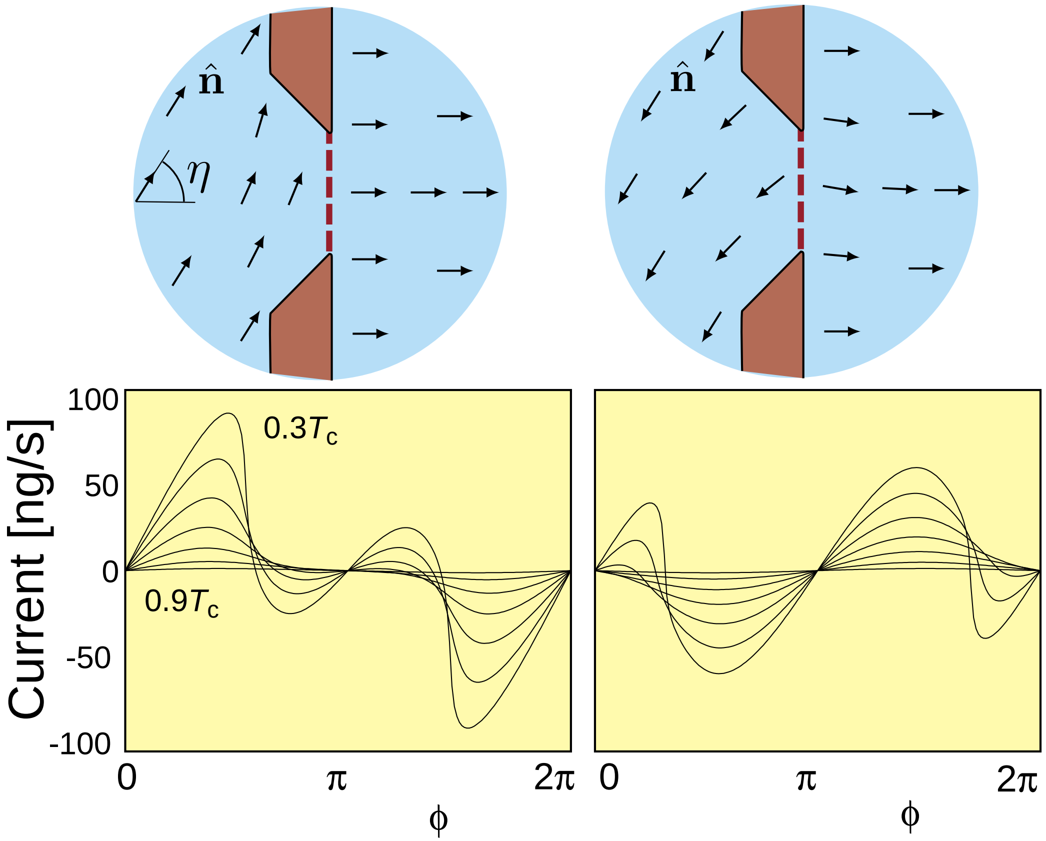
Aperture array



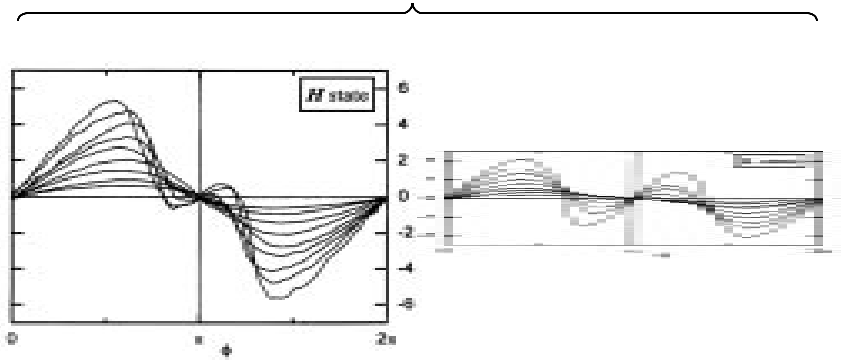
"Anisotextural" π state

Results

- asymmetry of the junction
- one fitting parameter η

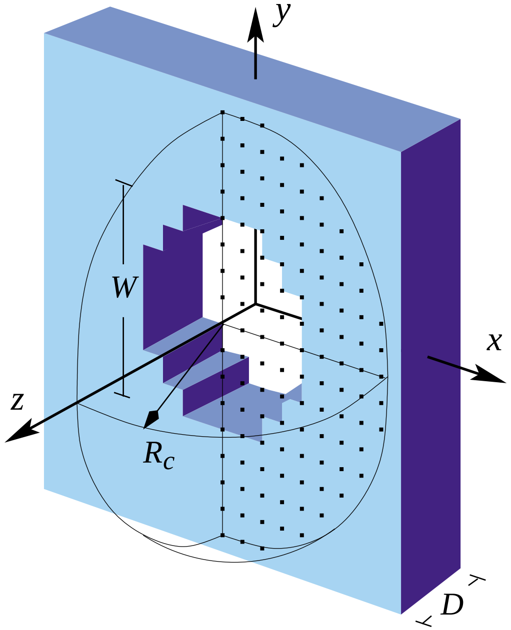


Experiment:
S. Backhaus, S. Pereverzev,
R. Simmonds, A. Loshak,
J. Davis, R. Packard,
A. Marchenkov (1998, 1999)



Theoretically strong dependence on magnetic field and surroundings
- to be tested experimentally

Large aperture



Order parameter

$$\bar{\bar{A}} = \begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}$$

Boundary condition

$$\bar{\bar{A}} = \exp(\pm \frac{\mathrm{i}}{2}\Delta\phi)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

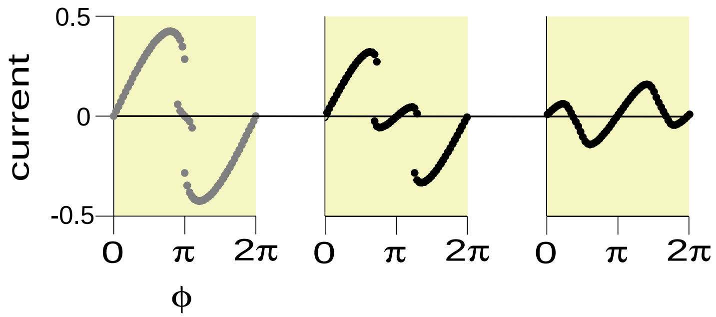
when $z \to \pm\infty$

Ginzburg-Landau theory

- valid only for T near T_c
- partial differential equation:

$$\partial_j \partial_j A_{\mu i} + (\gamma - 1)\partial_i \partial_j A_{\mu j} = [-\bar{\bar{A}} + \beta_1 \bar{\bar{A}}^* \,\mathrm{Tr}(\bar{\bar{A}}\bar{\bar{A}}^T)$$
$$+\beta_2 \bar{\bar{A}}\,\mathrm{Tr}(\bar{\bar{A}}\bar{\bar{A}}^{T*}) + \beta_3 \bar{\bar{A}}\bar{\bar{A}}^T\bar{\bar{A}}^* + \beta_4 \bar{\bar{A}}\bar{\bar{A}}^{T*}\bar{\bar{A}} + \beta_5 \bar{\bar{A}}^*\bar{\bar{A}}^T\bar{\bar{A}}]_{\mu i}$$

- at surfaces $\bar{\bar{A}} = 0$

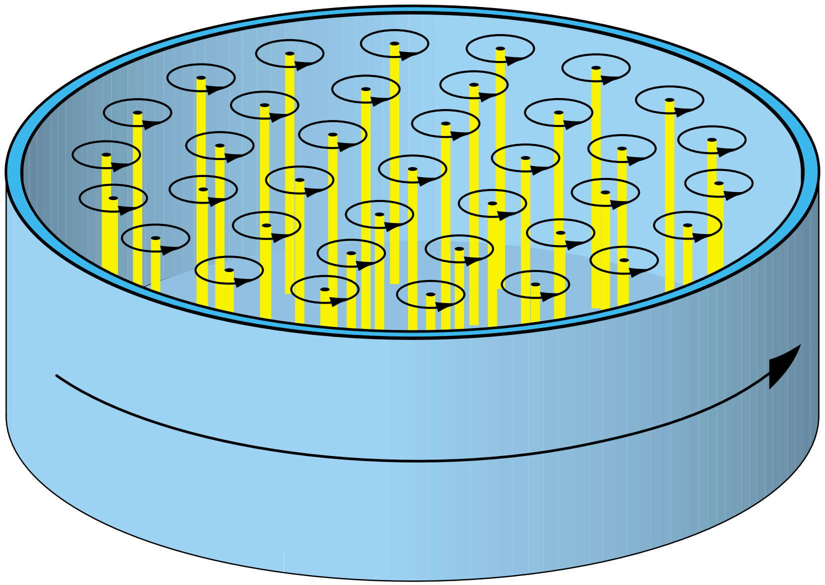


http://boojum.hut.fi/research/theory/jos.html

Part II. ROTATING SUPERFLUID

An uncharged superfluid cannot rotate homogeneously

Rotation takes place via vortex lines

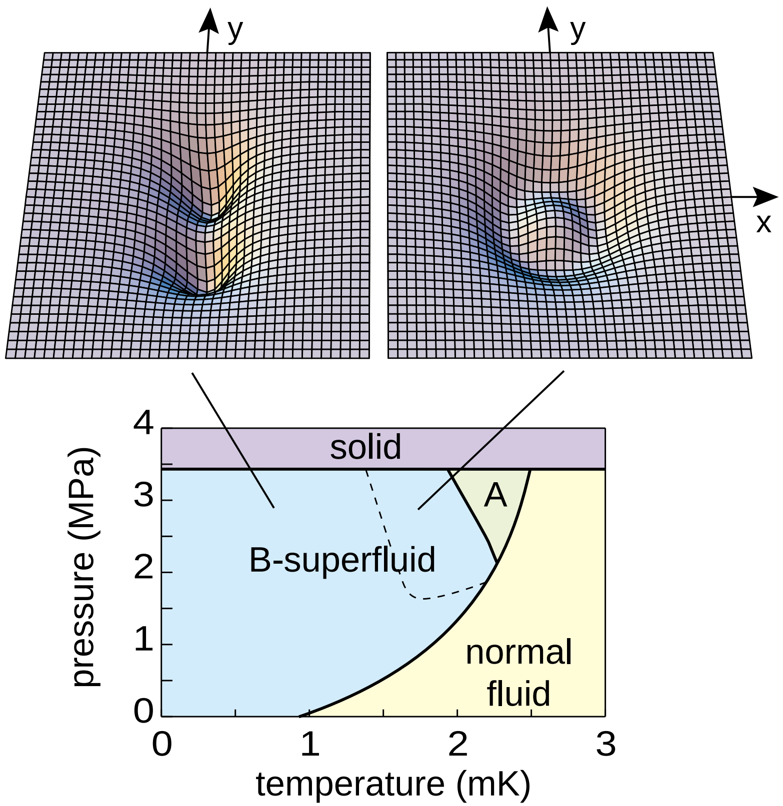


Vortex cores in ^{3}He-B

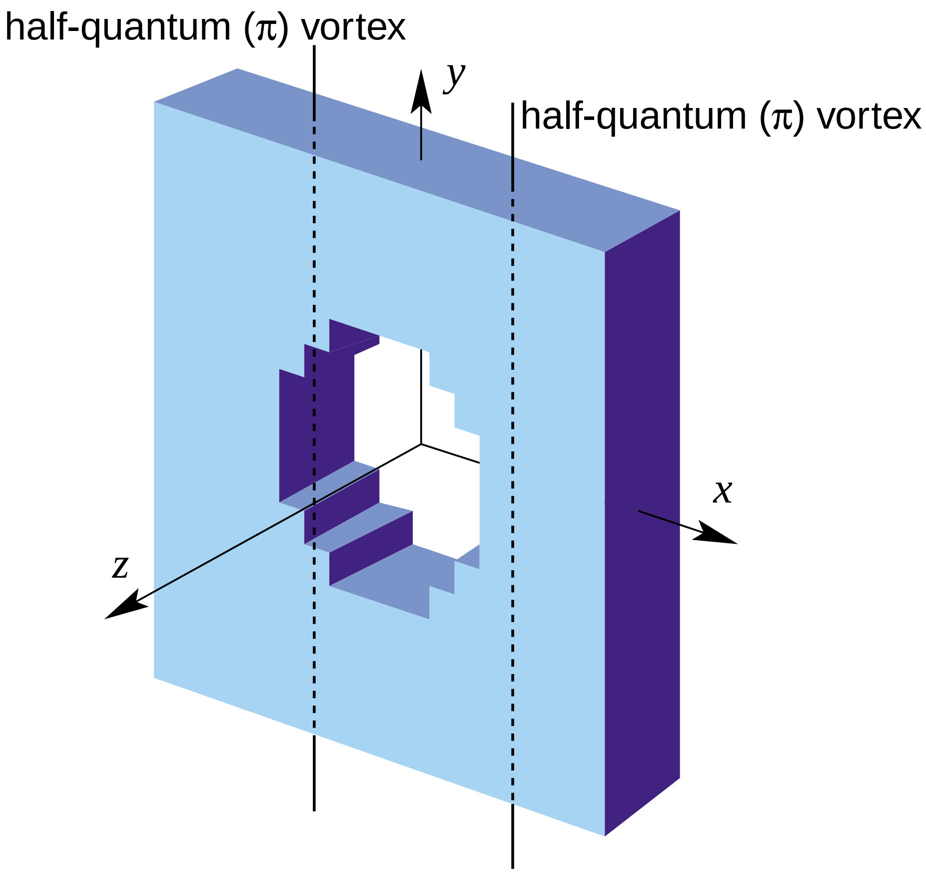
Numerical minimization of Ginzburg-Landau functional in 2 D

$$\partial_j\partial_j A_{\mu i} + (\gamma-1)\partial_i\partial_j A_{\mu j} = [-\mathsf{A} + \beta_1 \mathsf{A}^* Tr(\mathsf{A}\mathsf{A}^T) + \beta_2 \mathsf{A} Tr(\mathsf{A}\mathsf{A}^{T*}) + \beta_3 \mathsf{A}\mathsf{A}^T\mathsf{A}^* + \beta_4 \mathsf{A}\mathsf{A}^{T*}\mathsf{A} + \beta_5 \mathsf{A}^*\mathsf{A}^T\mathsf{A}]_{\mu i}. \quad (1)$$

$\Rightarrow$ broken symmetry in vortex cores

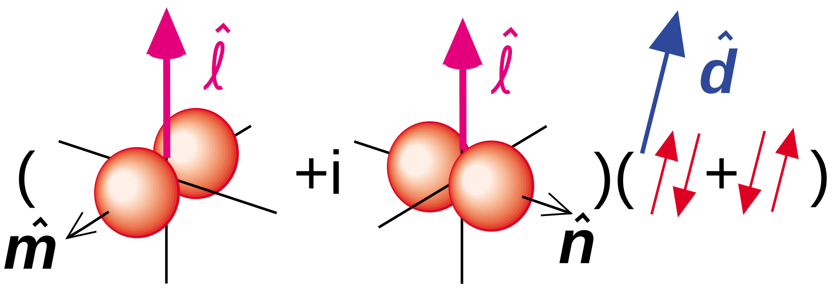


Virtual half-quantum vortices



The A phase

The order parameter $A_{\mu j} = \Delta \hat{d}_\mu (\hat{m}_j + i\hat{n}_j)$



A phase factor $e^{i\chi}$ corresponds to rotation of $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ around $\hat{\mathbf{l}}$:

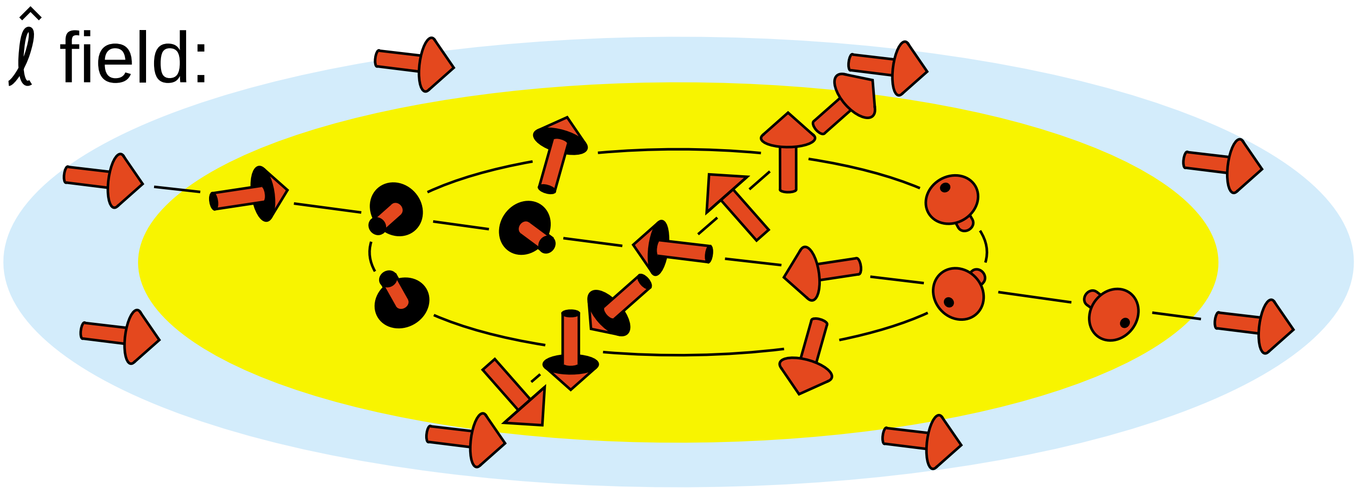
$$\begin{aligned} e^{i\chi}(\hat{\mathbf{m}} + i\hat{\mathbf{n}}) &= (\cos\chi + i\sin\chi)(\hat{\mathbf{m}} + i\hat{\mathbf{n}}) \\ &= (\hat{\mathbf{m}}\cos\chi - \hat{\mathbf{n}}\sin\chi) + i(\hat{\mathbf{m}}\sin\chi + \hat{\mathbf{n}}\cos\chi). \end{aligned}$$

Superfluid velocity

$$\mathbf{v}_\mathrm{s} = \frac{\hbar}{2m}\boldsymbol{\nabla}\chi = \frac{\hbar}{2m}\sum_j \hat{m}_j \boldsymbol{\nabla}\hat{n}_j. \qquad (2)$$

Vortices in the A phase

Consider the structure



Here $\hat{\mathbf{l}}$ sweeps once trough all orientations (once a unit sphere).

⇒ $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ circle twice around $\hat{\mathbf{l}}$ when one goes around this object.

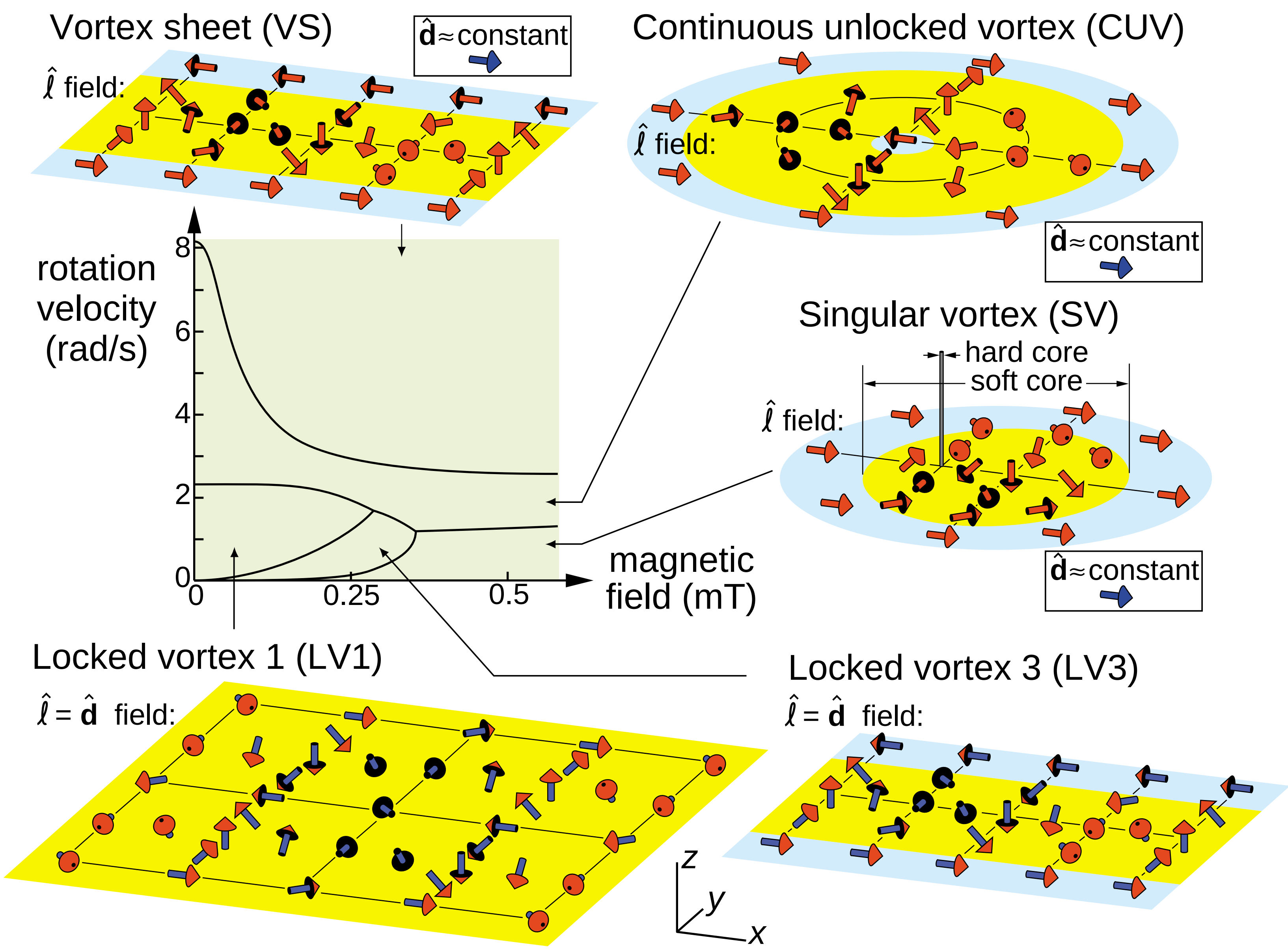
⇒ This is a two-quantum vortex. It is called *continuous*, because Δ (the amplitude of the order parameter) vanishes nowhere.

Hydrostatic theory of ^{3}He-A

Assume the order parameter ($\hat{\mathbf{m}}$, $\hat{\mathbf{n}}$, $\hat{\mathbf{l}}$, $\hat{\mathbf{d}}$) changes slowly in space. Then we can make gradient expansion of the free energy

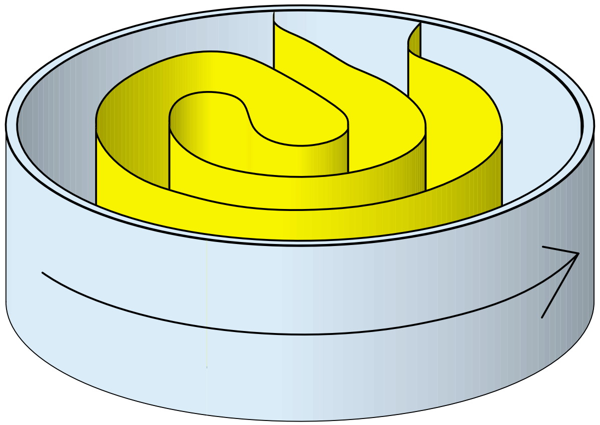
$$\begin{aligned} F &= \int d^3r \Big[-\tfrac{1}{2}\lambda_{\mathsf{D}}(\hat{\mathbf{d}}\cdot\hat{\mathbf{l}})^2 + \tfrac{1}{2}\lambda_{\mathsf{H}}(\hat{\mathbf{d}}\cdot\mathbf{H})^2 \\ &+\tfrac{1}{2}\rho_\perp \mathbf{v}^2 + \tfrac{1}{2}(\rho_{\|} - \rho_\perp)(\hat{\mathbf{l}}\cdot\mathbf{v})^2 + C\mathbf{v}\cdot\nabla\times\hat{\mathbf{l}} - C_0(\hat{\mathbf{l}}\cdot\mathbf{v})(\hat{\mathbf{l}}\cdot\nabla\times\hat{\mathbf{l}}) \\ &+\tfrac{1}{2}K_{\mathsf{s}}(\nabla\cdot\hat{\mathbf{l}})^2 + \tfrac{1}{2}K_{\mathsf{t}}|\hat{\mathbf{l}}\cdot\nabla\times\hat{\mathbf{l}}|^2 + \tfrac{1}{2}K_{\mathsf{b}}|\hat{\mathbf{l}}\times(\nabla\times\hat{\mathbf{l}})|^2 \\ &+\tfrac{1}{2}K_5|(\hat{\mathbf{l}}\cdot\nabla)\hat{\mathbf{d}}|^2 + \tfrac{1}{2}K_6[(\hat{\mathbf{l}}\times\nabla)_i\hat{\mathbf{d}}_j)]^2 \Big]. \end{aligned} \tag{3}$$

Vortex phase diagram in ^{3}He-A



Vortex sheet

Vortex sheets are possible in ^{3}He-A



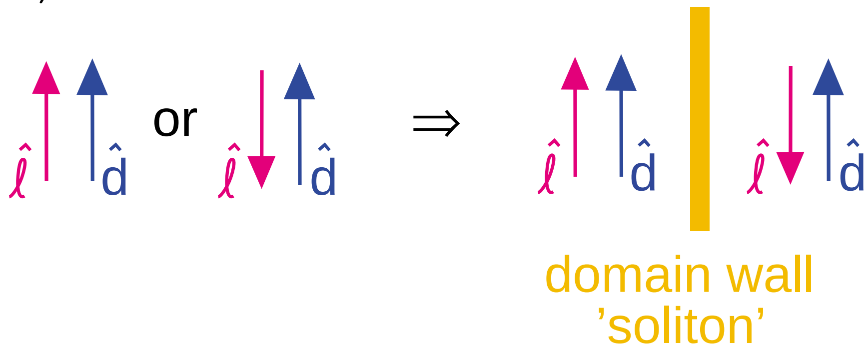
Sheets were first suggested to exist in ^{4}He, but they were found to be unstable.

Why stable in ^{3}He-A?

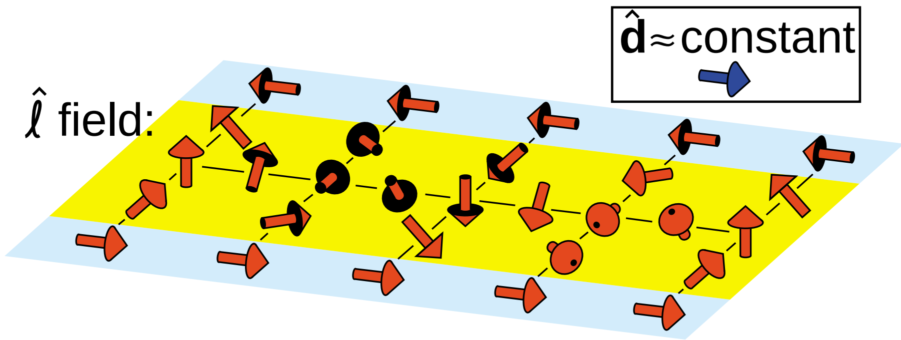
Dipole-dipole interaction (3)

$$f_D = -\frac{1}{2}\lambda_D(\hat{\mathbf{d}}\cdot\hat{\mathbf{l}})^2$$

$\Rightarrow$



Vortex sheet = soliton wall to which the vortices are bound.



Shape of the vortex sheet

The equilibrium configuration of the sheet is determined by the minimum of

$$F = \int d^3r \tfrac{1}{2}\rho_s(\mathbf{v}_n - \mathbf{v}_s)^2 + \sigma A.$$

Here A is the area of the sheet and σ its surface tension.

The equilibrium distance b between two sheets is

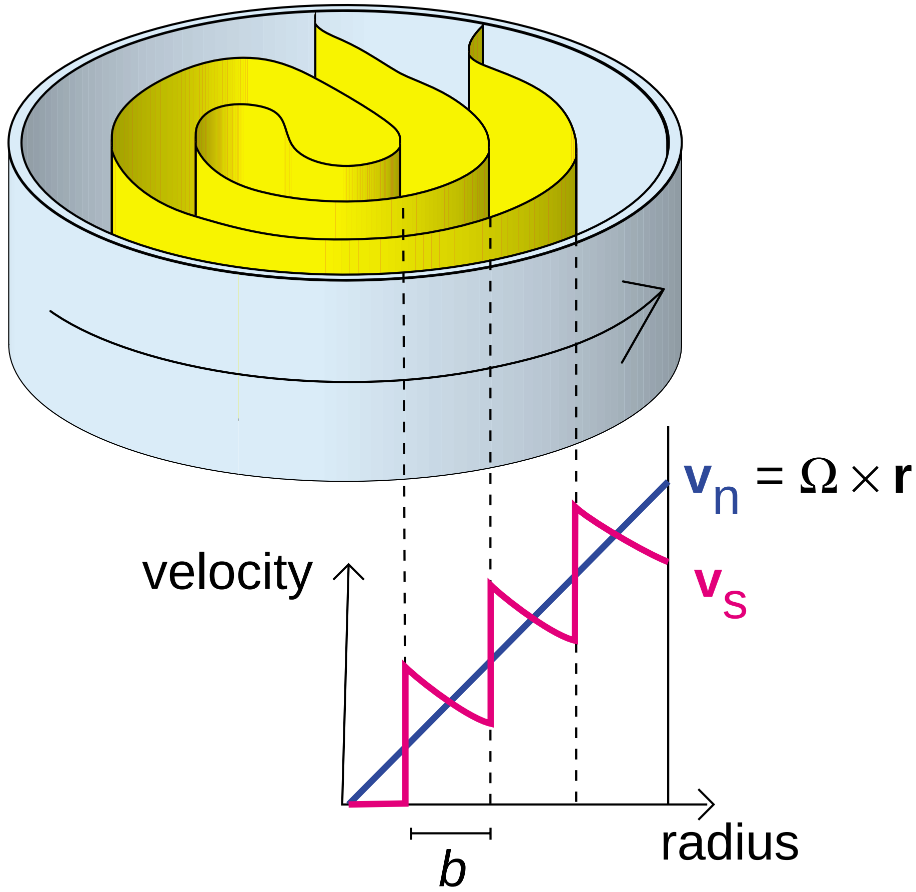
$$b = \left(\frac{3\sigma}{\rho_\mathrm{s}\Omega^2}\right)^{1/3}. \qquad (4)$$

This gives 0.36 mm at $\Omega = 1$ rad/s.

The area of the sheet

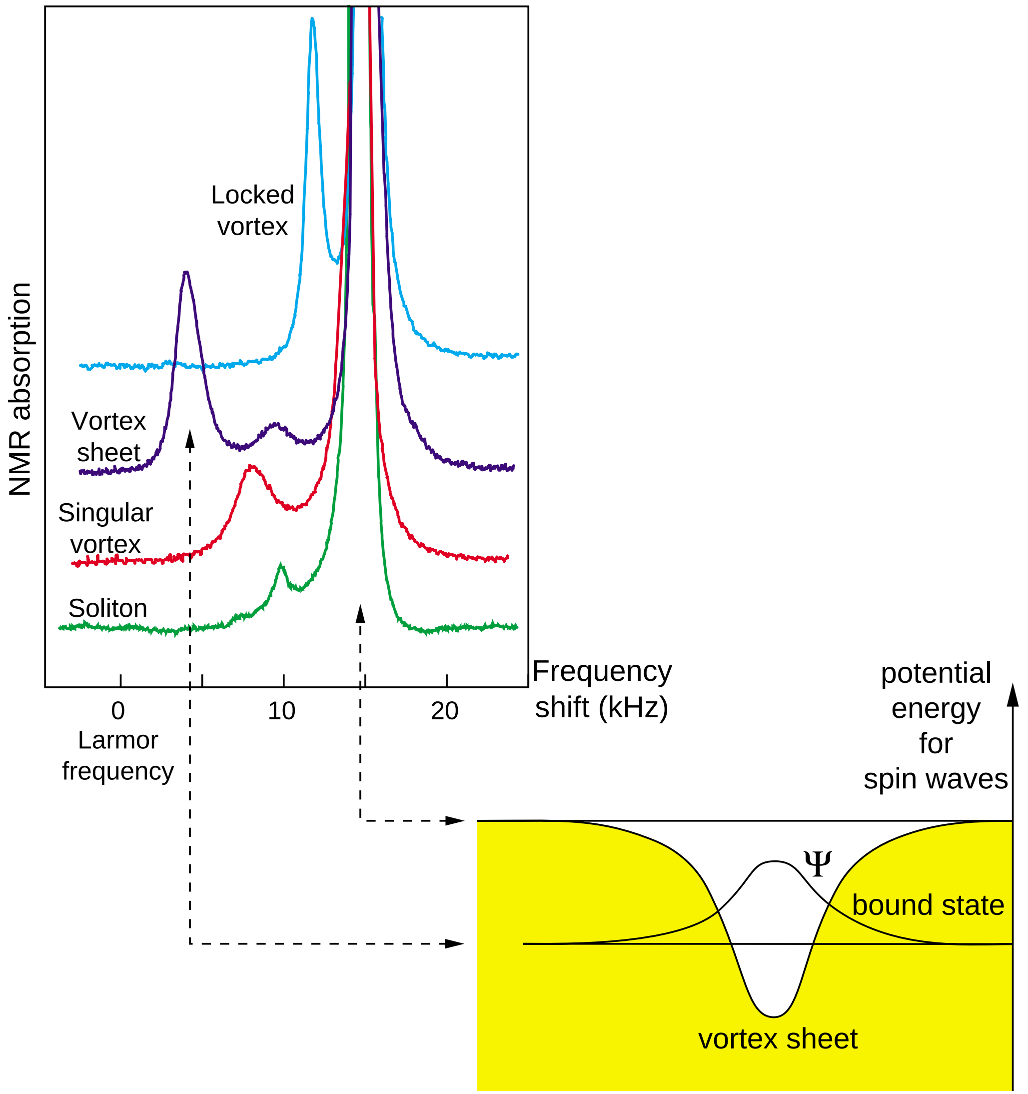
$$A \propto \frac{1}{b} \propto \Omega^{2/3}, \qquad (5)$$

as compared to $N_{\text{vortex line}} \propto \Omega$.



Connection lines with the side wall allow the vortex sheet to grow and shrink when angular velocity Ω changes.

NMR spectra of vortices

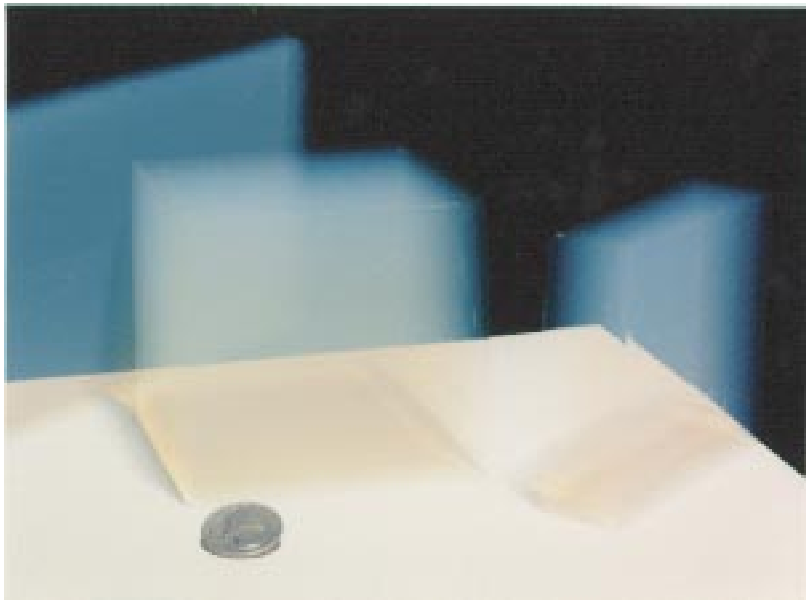


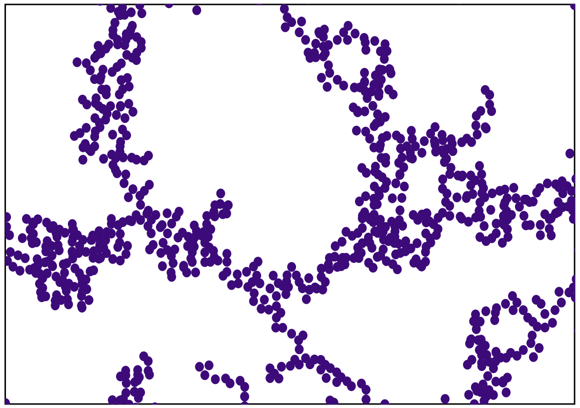
http://boojum.hut.fi/research/theory/sheet.html

Part III. IMPURE SUPERFLUID ^{3}He

^{3}He is a naturally pure substance

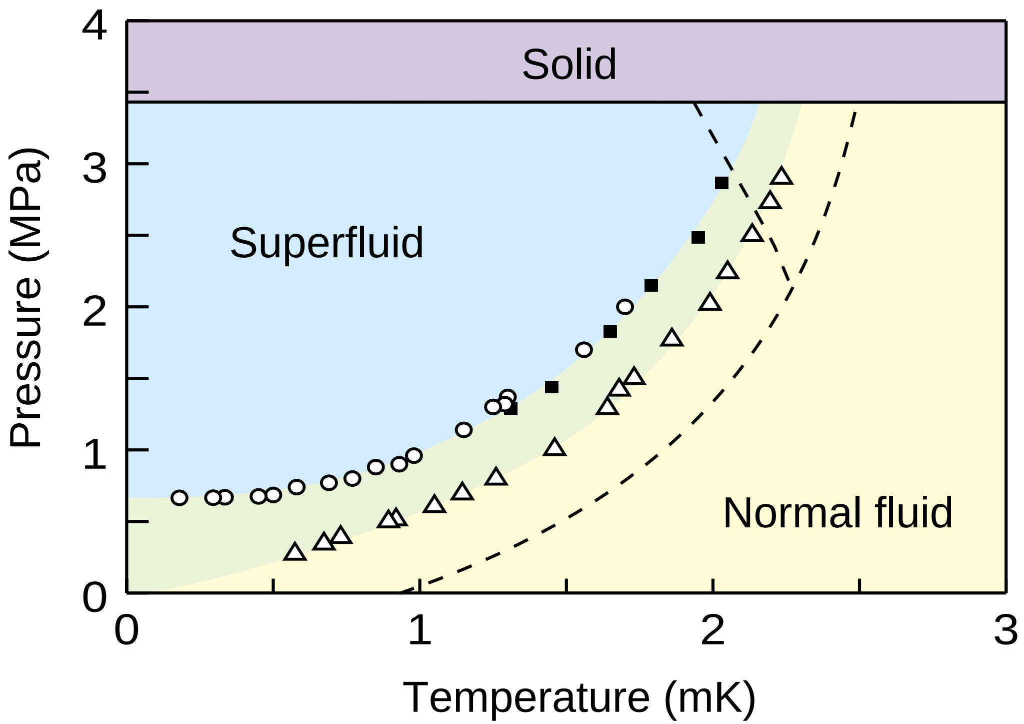
Impurity can be introduced by porous aerogel





- strands of SiO_2
- typically 98% empty
- small angle x-ray scattering $\Rightarrow$ homogeneous on scale above ≈ 100 nm

Experimental phase diagram

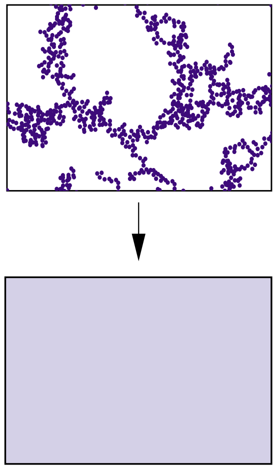


Porto et al (1995), Sprague et al (1995), Matsumoto et al (1997)

Models

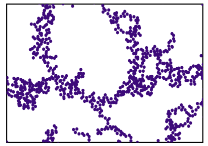
Main effect of aerogel is to scatter quasiparticles

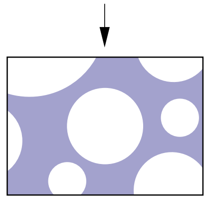
Simplest model: homogeneous scattering model



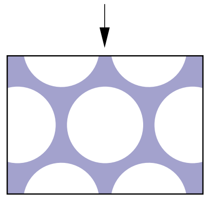
⇒ only qualitative agreement with experiments

Better models:

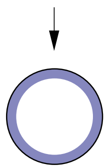




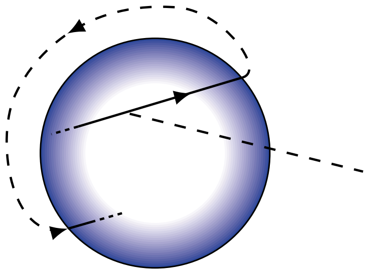
Random voids
- not simple enough to calculate



Periodic voids
- not simple enough to calculate



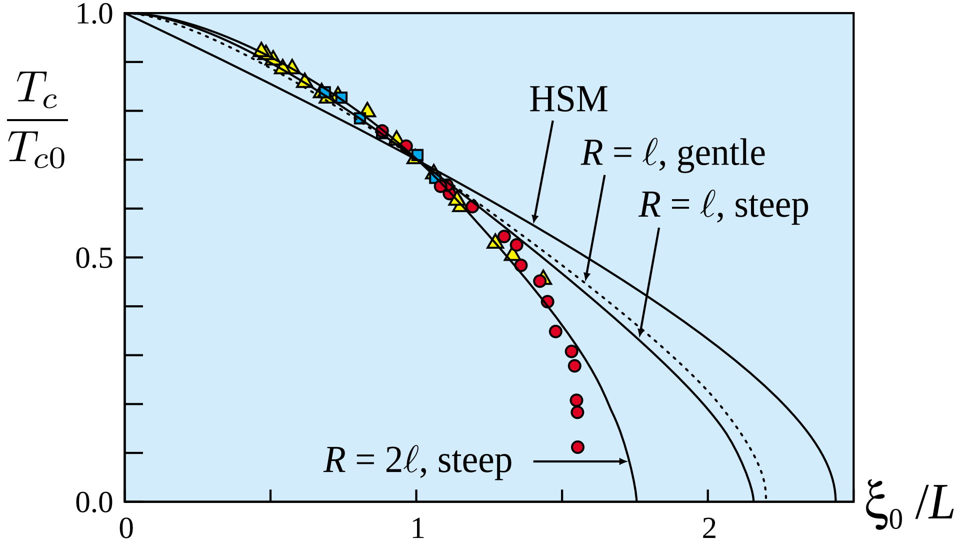
Isotropic inhomogeneous scattering
- spherical unit-cell approximation
- (quasi)periodic boundary condition



Isotropic inhomogeneous scattering model (IISM)

Isotropic on large scale
$\Rightarrow$ Hydrodynamics the same as in bulk but renormalized parameter values

Better fit with experiment than in HSM
$\Rightarrow$ realistic parameter values



http://boojum.hut.fi/research/theory/aerogel.html

Conclusion

Superfluid ^{3}He forms a very rich system because of the several different length scales.