Workshop on Vortices in Superfluids and Superconductors
Oulu 4.-8.1.2003

INHOMOGENEOUS IMPURITY DISTRIBUTION IN FERMI SUPERFLUIDS

http://boojum.hut.fi/research/theory/aerogel.html

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Content

Impurities in Fermi superfluids

- quasiclassical theory
- impurity averaging

Homogeneous distribution of impurities

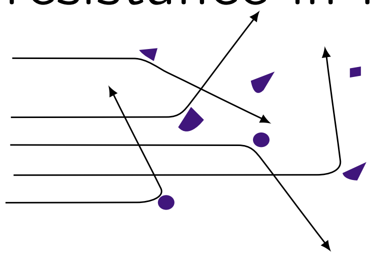
A model of inhomogeneous impurity distribution

Application to superfluid ^{3}He in aerogel

Impurities in metals

Parameters:
- scattering cross section σ
- density of impurities n
- mean free path for quasiparticles $\ell = 1/n\sigma$

Scattering causes electrical resistance in normal metals



Superconductors (usual case)

- T_c not changed (Anderson theorem)

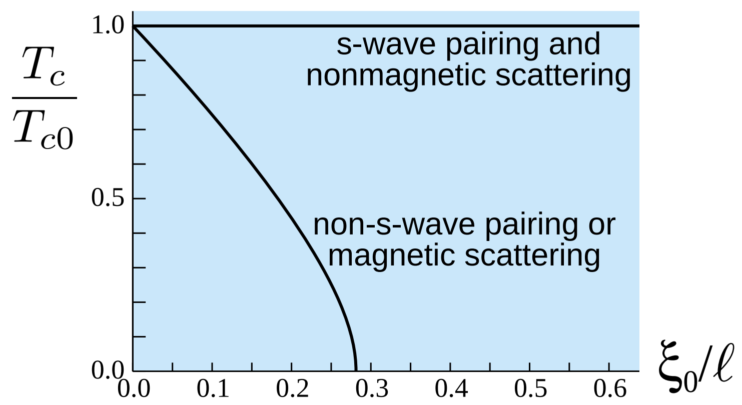
- Ginzburg-Landau parameter increases

but

- magnetic impurities in usual superconductors, or

- usual impurities in unconventional superconductors (p, d, ... wave pairing)

$\Rightarrow$



(Abrikosov-Gorkov 1961)

Quasiclassical scattering theory

parameters:

- Fermi wave length λ_F
- coherence length $\xi_0 = \frac{\hbar v_F}{2\pi k_B T_c}$
- scattering cross section σ
- density of impurities n
- mean free path $\ell = 1/n\sigma$

Assumption:

$$\lambda_F, \sqrt{\sigma} \ll \xi_0, \ell \qquad (1)$$

Take leading terms, ignore terms that are smaller by factors λ_F/ξ_0, etc.

Technical tool: take an average over the locations of the impurities.

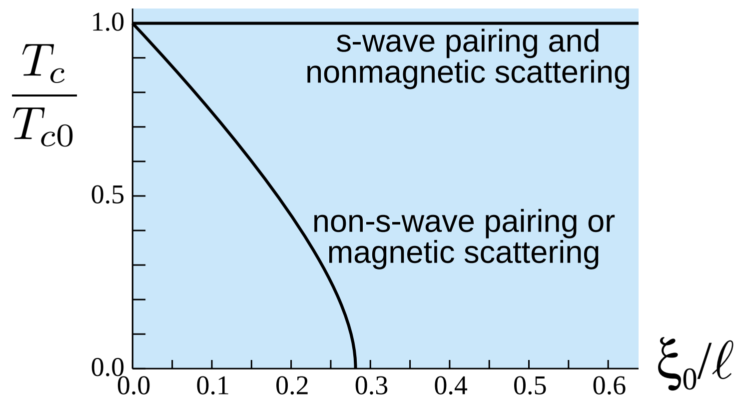
However, due to assumptions (1), this implies that all fluctuations in the impurity density are lost.

$\Rightarrow$

Instead of true discrete impurities there is a continuous scattering medium.

$\Rightarrow$

"Homogeneous scattering model" (HSM)



Quasiclassical equations

4x4 matrix Green's function $\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m)$:

$$[i\epsilon_m\widehat{\tau}_3 - \widehat{\nu} - \widehat{\Delta} - \widehat{\rho}, \widehat{g}] + i\hbar\mathbf{v}_\mathsf{F} \cdot \nabla_\mathbf{r}\widehat{g} = 0$$

$$\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m)\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m) = -\pi^2$$

$$\widehat{\rho}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m) = n(\mathbf{r})\widehat{t}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m).$$

Equation for the scattering matrix $\widehat{t}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}', \epsilon_m)$

$$\widehat{t}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}', \mathbf{r}, \epsilon_m) = \widehat{v}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}') + N(0)\langle\widehat{v}(\widehat{\mathbf{k}}, \widehat{\mathbf{k}}'')\widehat{g}(\widehat{\mathbf{k}}'', \mathbf{r}, \epsilon_m)\widehat{t}(\widehat{\mathbf{k}}'', \widehat{\mathbf{k}}', \mathbf{r}, \epsilon_m)\rangle_{\widehat{\mathbf{k}}''}.$$

Energy functional

$$\Omega = \int d^3r[\frac{1}{V_{BCS}}\langle|\Delta(\widehat{\mathbf{k}}, \mathbf{r})|^2\rangle_{\widehat{\mathbf{k}}} + \frac{1}{2}N(0)T\sum_{\epsilon_m}\int_0^\Delta \frac{d\Delta}{\Delta}\langle\mathrm{Tr}_4[\widehat{g}(\widehat{\mathbf{k}}, \mathbf{r}, \epsilon_m)\widehat{\Delta}(\widehat{\mathbf{k}}, \mathbf{r})]\rangle_{\widehat{\mathbf{k}}}]$$

$+$ terms arising from Fermi-liquid corrections $\widehat{\nu}$.

Need a better theory?

- in high T_c superconductors $\sqrt{\sigma}/\xi_0$ is not negligible.

$\Rightarrow$

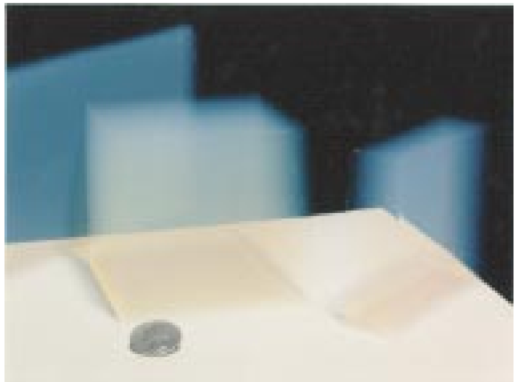
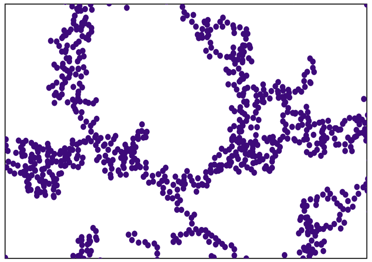
Fluctuations of the impurity density are important.
(Franz, Kallin, Berlinsky, and Salkola 1997)

- superfluid ^{3}He in aerogel

Impure superfluid ^{3}He

^{3}He is a naturally pure substance

Impurity can be introduced by porous aerogel

- strands of SiO_2
- typically 98% empty
- small angle x-ray scattering $\Rightarrow$ homogeneous on scale above ≈ 100 nm

Compare that to

$$\xi_0 = 16 \ldots 74 \text{ nm},$$

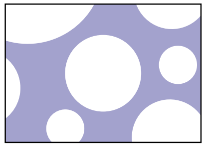
depending on pressure.

Experiments:
HSM has qualitative success, but insufficient quantitatively.

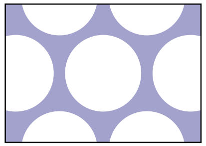
Models

Use quasiclassical theory with a location dependent impurity density $n(\mathbf{r})$.

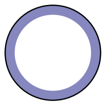
Homogeneous scattering model



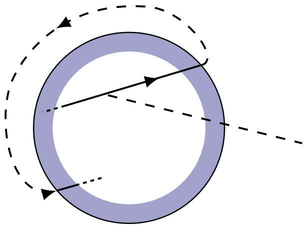
Random voids
- not simple enough to calculate



Periodic voids
- not simple enough to calculate



Isotropic inhomogeneous scattering
- spherical unit-cell approximation
- (quasi)periodic boundary condition

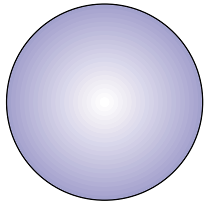


trajectory of a quasiparticle

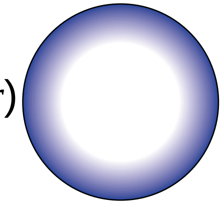
Isotropic inhomogeneous scattering model (IISM)

parameters:
- R, radius of the unit cell
- $n(r)$, scattering profile
- ℓ, average mean free path

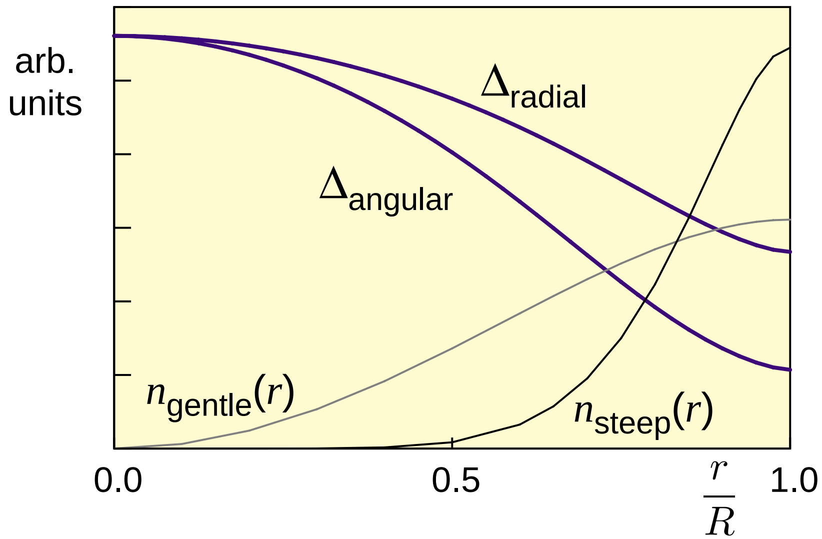
$n_{\text{gentle}}(r)$



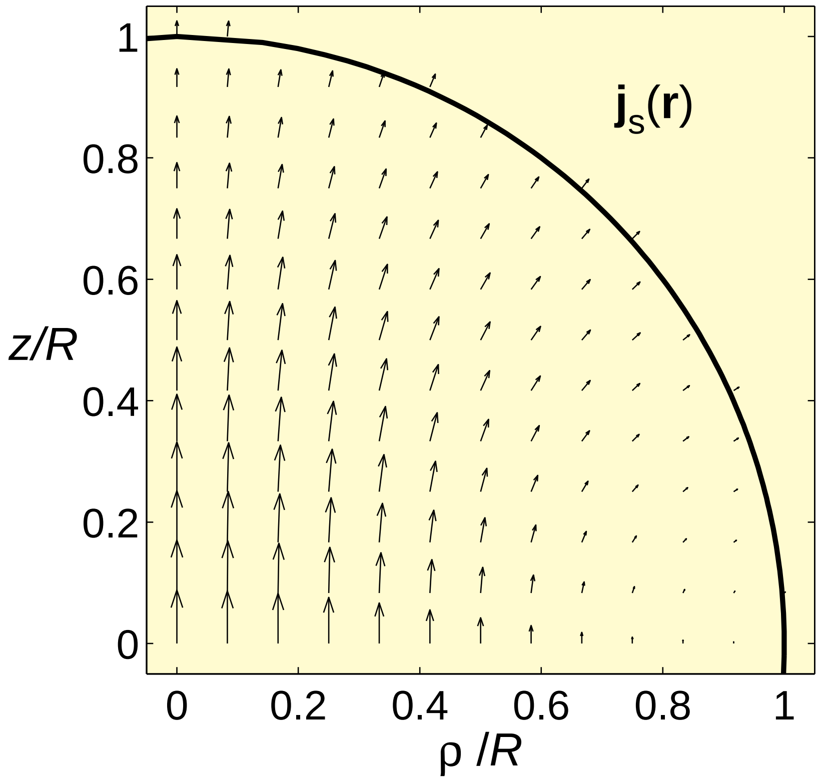
$n_{\text{steep}}(r)$



Order parameter in distorted B phase

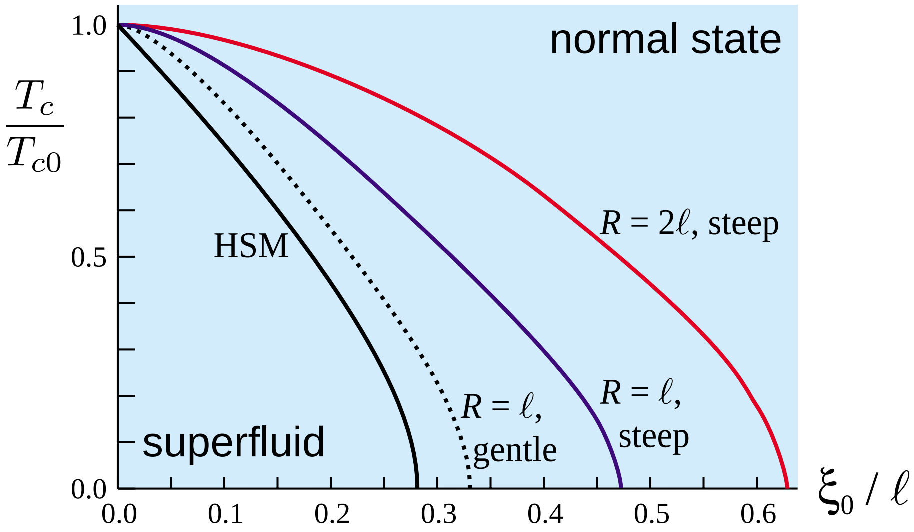


Supercurrent



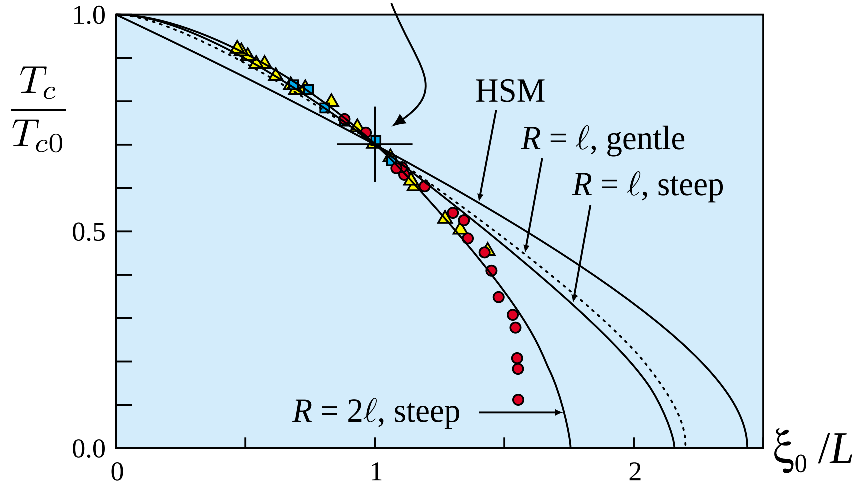
$\mathbf{j}_S$ in cylindrical coordinates ρ, ϕ, z.

Transition temperature



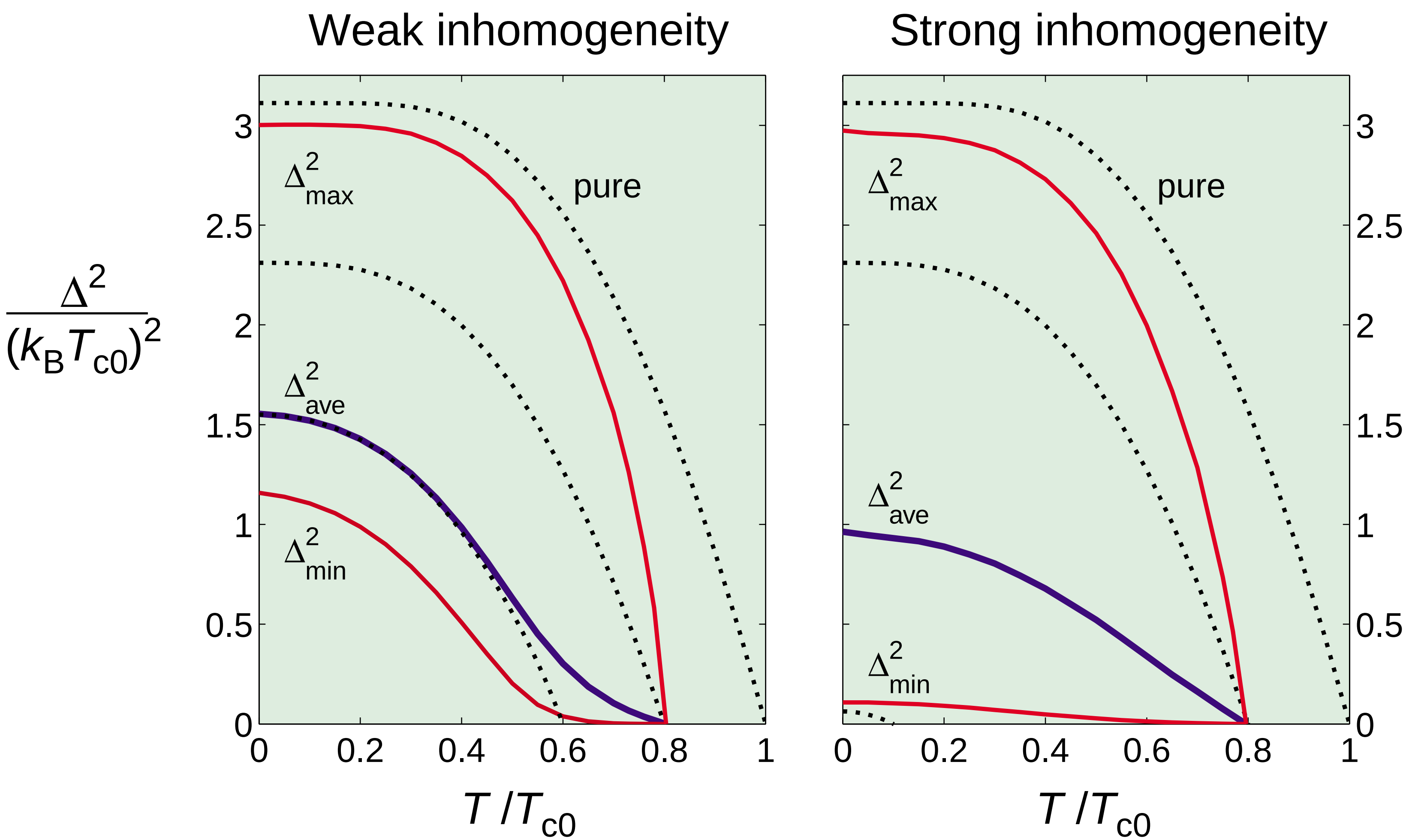
Experiments (vary pressure)

data made to coincide here by scaling the *x*-axis

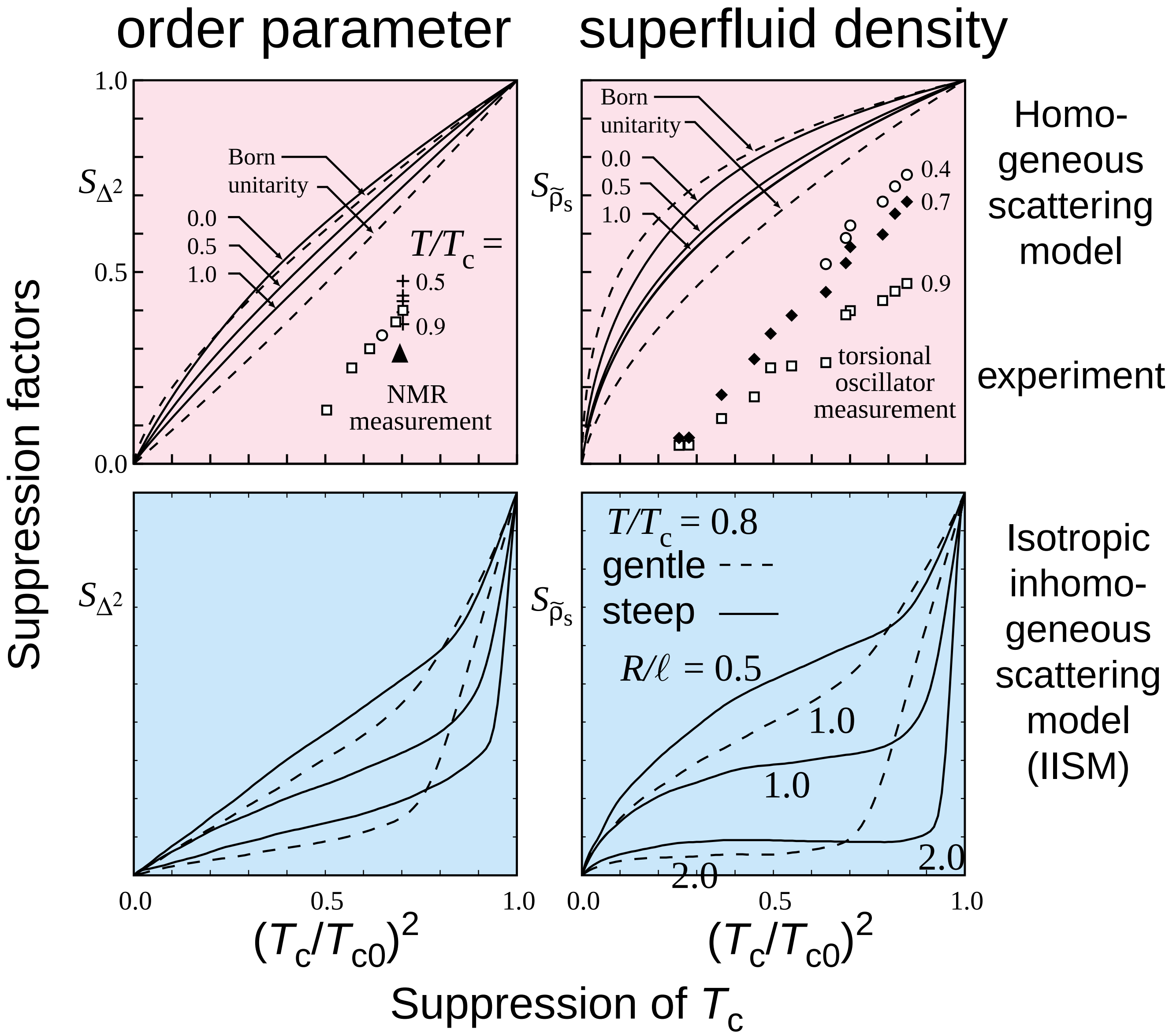


Porto et al (1995), Sprague et al (1995), Matsumoto et al (1997)

Temperature dependence



Suppression of



Discussion

The IISM gives better fit to experiment than HSM.

The optimal radius of the unit cell $R \approx 140$ nm is on the same order of magnitude than the aerogel correlation length ≈ 84 nm.

No perfect fit to experiments
- possible reason: IISM has only one length scale R.

Conclusions

Quasiclassical theory: inhomogeneous scattering modelled by $n(\mathbf{r})$.

Isotropic inhomogeneous scattering model (IISM):

- the simplest model of inhomogeneous scattering that reduces to homogeneous medium on a large scale

- computationally much heavier than HSM

- ^{3}He in aerogel: IISM clearly better than HSM, but still not perfect

- how to calculate vortex states?

- application to other superfluids?

Links: http://boojum.hut.fi/research/theory/aerogel.html