Dissipative Currents in Superfluid ^{3}He-B Weak Links

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Recent measurements of dissipative currents in pressure-biased weak links of superfluid ^{3}He-B are discussed. It is pointed out that the theoretical understanding of their results is unsatisfactory. As one candidate model to explain them, we consider the process of multiple Andreev reflections (MAR). Connection of MAR to bound quasiparticle states inside ballistic contacts is discussed. As an explicit example we analyze the current in a short pressure-biased ballistic ^{3}He-B constriction. It is shown that the dissipative part of the current does not depend on the spin-orbit rotation matrices.

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1. PRESSURE-BIASED ^{3}HE-B WEAK LINKS

Flow of superfluid ^{3}He through a weak link under a pressure bias was recently studied by Simmonds *et al.*[1] The experimentally observed current-pressure ($I - P$) characteristics have thus far been explained only by phenomenological models.[1] The problem is that these models lack proper justification. For example, one model assumes the existence of A phase inside the apertures, which would cause dissipation through precession of the $\hat{\mathbf{l}}$ vector and the associated orbital viscosity. However, the apertures are so small (with dimensions on the order of the zero-temperature coherence length ξ_0) that the order parameter is strongly suppressed inside them. Thus no identifiable phase whatsoever is likely to exist there, the A phase included. Possibly this assumption has its roots in the calculations which have found that sometimes an A-phase-like state is favored close to surfaces, where it may, for example, act as a seed for nucleating the A phase from the B phase.[2]

There also exist earlier measurements of pressure-biased weak links of ^{3}He-B. The authors of Ref. 3 analyzed their results with a model based on an assumed analogy between their weak link and a tunneling junction with a resistor in series. The comparison proved to be difficult. Thus for several years the behavior of superfluid ^{3}He weak links subjected to pressure biases has remained somewhat of a mystery. This is rather disturbing, given that superfluid ^{3}He is generally well explained by theoretical models. While it is possible that the experimental results involve new, undiscovered physics, it appears that not all of the pre-existing ideas have been properly tested either. In what follows we discuss the most obvious one, which by now is well accepted in the theory of superconductors. Quite surprisingly, to our knowledge, it has previously not been discussed in the ^{3}He literature.

2. MULTIPLE ANDREEV REFLECTIONS

The equilibrium properties of a weak link in superfluid ^{3}He-B may successfully be described in terms of the free energy of the contact, which yields the so-called current-phase relation (CPR).[2] In the case of small "pinhole" contacts close to the superfluid transition temperature T_c one may even restrict to a simple "tunneling model",[4,5] where the CPR is purely sinusoidal, as though in a tunneling junction. However, for the description of dc currents in a junction biased by a pressure head P, or chemical potential difference $U = (m_3/\rho)P$, models based on tunneling junctions are insufficient. This is because there is a fundamental difference between ballistic contacts and those with tunneling barriers. In a tunneling contact between two superconductors, direct currents are mostly due to thermally excited quasiparticles. Consequently, at low temperatures the currents are very small for biases below the "gap voltage" $U = 2\Delta$. On the other hand, the dc currents in a ballistic superconducting weak link are determined by the process of multiple Andreev reflections (MAR),[6,7] and this may be expected to be the case in weak links of superfluid ^{3}He-B as well.

Even in an infinitely short constriction there are some localized quasiparticle states available below the gap (see below). In the MAR process quasiparticles trapped in these states inside the junction get accelerated by the local gradient in chemical potential and undergo several reflections where the quasiparticle is transformed from particle-like to hole-like and vice versa, gaining an energy $2U$ on each round. This process continues until the quasiparticle escapes above the gap or scatters inelastically, having traveled a length comparable with the inelastic mean free path $l_{\rm in}$. The average number of reflections is then roughly proportional to the smaller of Δ/U or $l_{\rm in}/(2\overline{\lambda} + d)$, where $\overline{\lambda} \sim \xi_0$ is an average penetration depth for sub-gap quasiparticles and d is the length of the junction. Since each back-and-forth

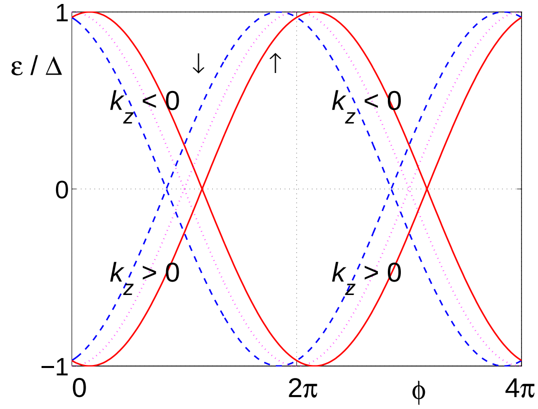

Fig. 1. The bound states $\epsilon(\hat{\mathbf{k}}, \sigma, \phi)$ for $\uparrow$ and $\downarrow$ ($\sigma = \pm 1$) quasiparticles of each $\hat{\mathbf{k}}$-channel are shifted by $\mp\chi_{\hat{\mathbf{k}}}$ with respect to the average phase difference ϕ. When $\epsilon \approx \pm\Delta$, the states "thermalize", *i.e.*, their occupation returns to the equilibrium value determined by the bulk superfluids.

round effectively transports two particle-like excitations in the same direction, this leads to strong dissipative currents even for $U \ll 2\Delta$. In fact, were it not for the cutoff due to inelastic scattering, the differential conductance (the slope of the $I - P$ curve) would *diverge* for small U. In the case of ^{3}He, the inelastic scattering is due to inter-quasiparticle collisions.

In the "adiabatic limit" $U \ll 2\Delta$, where the phase difference changes slowly, one may describe the above process in terms of the occupation of slowly moving bound states as follows.[6] Assume a short ($d = 0$) point contact between two (l and r) volumes of ^{3}He-B. Counting energies from the chemical potential of the l side, there is a shift in the r-side chemical potential by $\delta\mu^r = -U$, where $U = (m_3/\rho)P$. This corresponds to a Josephson frequency $\omega_J = 2U/\hbar$. Neglecting the gap suppression close to surfaces, gap vectors for momentum direction $\hat{\mathbf{k}}$ are given by $\mathbf{\Delta}^{l,r}(\hat{\mathbf{k}}) = \Delta e^{i\varphi^{l,r}}\hat{\mathbf{d}}^{l,r}(\hat{\mathbf{k}})$, where $\hat{\mathbf{d}}^{l,r}(\hat{\mathbf{k}}) = R^{l,r}\hat{\mathbf{k}}$ and $R^{l,r}$ are rotation matrices. If, for each $\hat{\mathbf{k}}$, we choose the spin quantization axis parallel to $\hat{\mathbf{d}}^l \times \hat{\mathbf{d}}^r$, the condensates may be divided into $\uparrow\uparrow$ and $\downarrow\downarrow$ parts, which behave much like two independent s-wave systems.[9] Their phase differences over the contact are given by $\phi_{\hat{\mathbf{k}},\sigma} = \phi - \sigma\chi_{\hat{\mathbf{k}}}$, where $\sigma = \pm 1$, $\phi = \varphi^r - \varphi^l = \omega_J t$, and $\chi_{\hat{\mathbf{k}}} = \arccos(\hat{\mathbf{d}}^l \cdot \hat{\mathbf{d}}^r)$. The energies of the bound states are then given by (cf. Fig. 1)

$$\epsilon(\hat{\mathbf{k}}, \sigma, \phi) = -\,\mathrm{Sign}(\hat{k}_z \sin(\phi_{\hat{\mathbf{k}},\sigma}/2))\Delta\cos(\phi_{\hat{\mathbf{k}},\sigma}/2), \quad \sigma = \pm 1 \tag{1}$$

where the z axis points from l to r. Since $\phi(t)$ is growing ($\dot{\phi} > 0$), these levels move up or down between the two gap edges and "pump" quasiparticles from

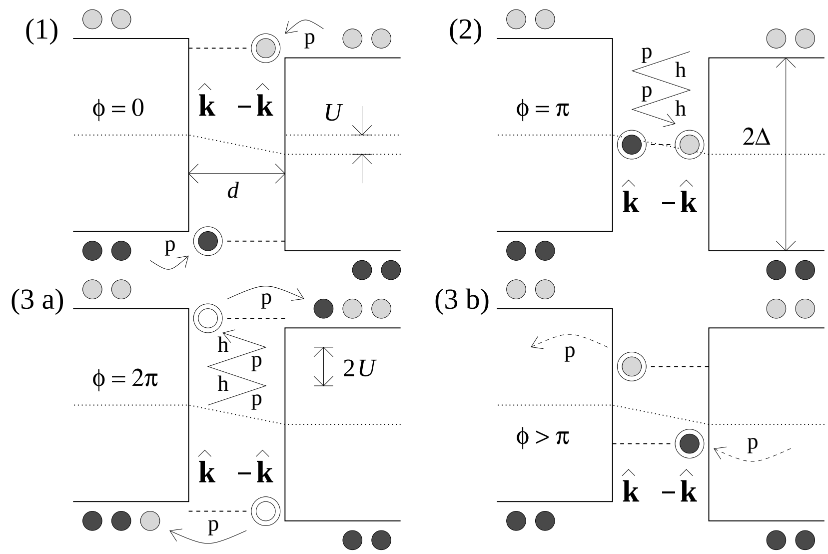

Fig. 2. Schematic illustration of the formation of a dissipative current of the $\uparrow$-quasiparticles in one $\pm\hat{\mathbf{k}}$ channel, with $\hat{k}_z > 0$ and $\chi_{\hat{\mathbf{k}}} = 0$. We assume $d = 0$, although a finite d is drawn for clarity. Panels 1 through 3a describe the transport between two thermalizations of the bound states of Eq. (1). Panel 3b shows one alternative outcome, where particles are returned to thermal equilibrium by intermediate inelastic processes involving other $\hat{\mathbf{k}}$ directions. (This is important at low biases.) The level of gray shading of the "particles" describes the thermal weight of the process (cf. the particles "in queue" outside the gap). A difference in average occupation of the $\pm\hat{\mathbf{k}}$ states is associated with a net current. At higher temperatures, opposite processes begin to cancel each other and the current is reduced. The zigzag trajectory refers to the MAR interpretation of Eq. (6) below.

one side to the other – see Figs. 1 and 2. The signs in Eq. (1) and the figures are such that (with our conventions) a state moving upward is transporting a particle with $\hat{k}_z > 0$, since this is the one being accelerated by the gradient of chemical potential. A particle with $\hat{k}_z < 0$ is being decelerated and hence moving downward in energy.

3. CALCULATION OF CURRENT

Our mathematical treatment of the MAR current is based on the parametrization of the quasiclassical Green functions with the so-called coherence functions $\underline{\gamma}$.[8] As shown in Ref. 6, these may be interpreted as Andreev reflection amplitudes which have been generalized to account for inelastic scattering. For homogeneous unitary states the retarded (R) and advanced (A) amplitudes are given by

$$\underline{\gamma}^{R,A} = -\underline{\Delta}^{R,A} / \left[\epsilon^{R,A} \pm \mathrm{i} \sqrt{-\underline{\Delta}^{R,A} \underline{\tilde{\Delta}}^{R,A} - (\epsilon^{R,A})^2} \right]. \tag{2}$$

Here $\epsilon^{R,A} = \epsilon \pm \mathrm{i}\Gamma_1$, where the imaginary part $\Gamma_1(\epsilon) = \Gamma_1(-\epsilon)$ is due to inelastic scattering. Using this we may define a lifetime $\tau_{\mathrm{in}} = \hbar/\Gamma_1$ and the scattering length $l_{\mathrm{in}} = v_{\mathrm{F}}\tau_{\mathrm{in}}$. These are often assumed to be energy-independent,[6,7] but that is not necessary. However, for simplicity we shall neglect strong-coupling effects on the gap matrix with the assumption $\underline{\Delta}^{R,A}(\hat{\mathbf{k}}, \epsilon) = \underline{\Delta}(\hat{\mathbf{k}}) = \mathbf{\Delta}(\hat{\mathbf{k}}) \cdot \underline{\boldsymbol{\sigma}} \mathrm{i} \underline{\sigma}_2$ and continue to ignore its suppression close to surfaces, which makes (2) valid everywhere. Choosing the spin quantization axes as explained in the previous section, the $\underline{\Delta}_{l,r}$ and $\underline{\gamma}_{l,r}$ matrices are diagonal in spin space ($\varphi^{l,r} = 0$),

$$\underline{\Delta}_{l,r} = \Delta \begin{bmatrix} -e^{-\mathrm{i}\phi^{l,r}_{\hat{\mathbf{k}}}} & 0 \\ 0 & e^{\mathrm{i}\phi^{l,r}_{\hat{\mathbf{k}}}} \end{bmatrix}, \quad \underline{\gamma}^{R,A}_{l,r} = \gamma^{R,A} \begin{bmatrix} -e^{-\mathrm{i}\phi^{l,r}_{\hat{\mathbf{k}}}} & 0 \\ 0 & e^{\mathrm{i}\phi^{l,r}_{\hat{\mathbf{k}}}} \end{bmatrix}. \tag{3}$$

Here $\gamma^{R,A} = -\Delta/(\epsilon^{R,A} \pm \mathrm{i}\sqrt{\Delta^2 - (\epsilon^{R,A})^2})$ and $\phi^{l,r}_{\hat{\mathbf{k}}}$ are the azimuthal angles of $\hat{\mathbf{d}}^{l,r}$ in the plane perpendicular to $\hat{\mathbf{d}}^l \times \hat{\mathbf{d}}^r$, which satisfy $\phi^i_{-\hat{\mathbf{k}}} = \phi^i_{\hat{\mathbf{k}}} + \pi$. Using these simplifications we calculate inside the contact the current

$$I_C(t) = \pi\hbar q_C v_{\mathrm{F}} N(0) S \langle \hat{k}_z I_C(\hat{\mathbf{k}}, t) \rangle_{\hat{k}_z > 0} \tag{4}$$

where $\langle \cdots \rangle_{\hat{k}_z>0} = \int_{\hat{k}_z>0} \mathrm{d}\Omega_{\hat{\mathbf{k}}}/4\pi$, S is the area of the contact, and $q_C = m_3$ (mass current) or $q_C = \hbar/2$ (spin current). Assuming $I_C(t)$ to be periodic with the Josephson period $T_J = 2\pi/\omega_J$, we expand

$$I_C(\hat{\mathbf{k}}, t) = \sum_{k=-\infty}^{\infty} I_k(\hat{\mathbf{k}}) e^{\mathrm{i}k\omega_J t} \tag{5}$$

such that $I_k(\hat{\mathbf{k}}) = I^*_{-k}(\hat{\mathbf{k}})$. For $k \geqslant 0$ we then find

$$\begin{aligned} I_k(\hat{\mathbf{k}}) = \mathrm{Tr}\, \underline{C} \Bigg\{ & \frac{U}{\pi\hbar}\delta_{k0} + 2 \begin{bmatrix} e^{\mathrm{i}k\chi_{\hat{\mathbf{k}}}} & 0 \\ 0 & e^{-\mathrm{i}k\chi_{\hat{\mathbf{k}}}} \end{bmatrix} \mathcal{P} \int \frac{\mathrm{d}\epsilon}{2\pi\hbar} \tanh(\beta\epsilon/2) \\ & \times (1 - |\gamma^R(\epsilon)|^2) \sum_{l=0}^{\infty} \prod_{q=1}^{l} |\gamma^R(\epsilon - qU)|^2 \prod_{p=l+1}^{l+2k} \gamma^R(\epsilon - pU) \Bigg\}, \end{aligned} \tag{6}$$

where $\underline{C} = \underline{1}$ (mass current) or $\underline{C} = \underline{\boldsymbol{\sigma}}$ (spin current), and we noticed that $\chi_{\hat{\mathbf{k}}} = \phi^r_{\hat{\mathbf{k}}} - \phi^l_{\hat{\mathbf{k}}}$. The summation index l runs over the number of Andreev reflections. We note that while the interpretation related to Eq. (1) was restricted to the adiabatic limit, Eq. (6) is in principle valid for arbitrary P.

4. CONCLUSION

For the case of mass current, Eq. (6) is the p-wave equivalent of the s-wave result presented in Ref. 6. For $\chi_{\hat{\mathbf{k}}} = 0$ the results coincide exactly. Thus we again reproduce the result that the B phase junction with equal spin-orbit rotations is essentially equivalent with the s-wave system. In addition we note that the $k = 0$ term which is responsible for the dc current component does not depend on $\chi_{\hat{\mathbf{k}}}$, *i.e.*, the rotation matrices at all. Therefore, at least within the approximation where gap suppression at surfaces is neglected, the dc current should be independent of textural configurations, and all the s-wave results of Refs. 6 and 7 should be applicable. In particular, when τ_{in} is energy-independent, the $I - P$ curve is linear for $U \ll (\bar{\lambda}/l_{in})\Delta$.

Although the above details are not exactly supported by experiments, the observed magnitudes and temperature-dependences of the currents seem to be well accounted for. However, space does not permit us a more detailed analysis here, and a fuller comparison to experiments is postponed until a more complete calculation has been carried out. Most importantly, one should take into account the gap suppression, which is always present in a p-wave superfluid close to surfaces.[5] In addition, apertures of finite length d could be considered. One should also notice that the lifetime of quasiparticles is in general energy-dependent. Finally, there could be corrections due to the anisotextural effect,[4,5] which is essential for explaining the equilibrium properties of the aperture arrays used in measurements of Ref. 1. All these considered, it seems premature to invoke any new dissipation mechanisms for ^{3}He in a weak link before the MAR process is properly investigated.

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