The Difference Between Semiconductor and Superconductor

Erkki Thuneberg

QTF Centre of Excellence, Department of Applied Physics, Aalto University, FI-00076 Aalto, Finland

Abstract

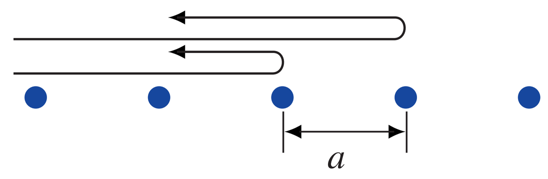
Both semiconductors and superconductors have an energy gap for electronic excitations. Why is there dissipationless current only in the latter?

Background for semiconductors and superconductors

Consider a free electron. Its wave function and energy are

$$\psi_k(r) = A\exp(ik\cdot r), \quad E(k) = \frac{\hbar^2 k^2}{2m}. \qquad (1)$$

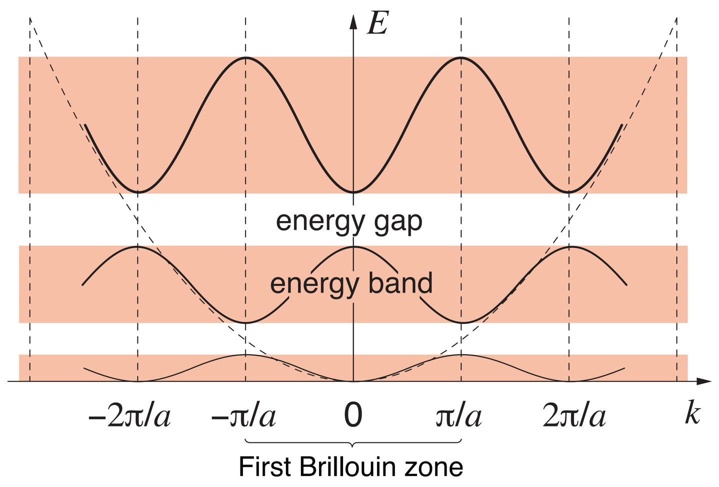
In a crystal lattice this is modified by the periodic potential formed by the ions (nuclei and core electrons).



The electron levels become superpositions. In the simplest case (integer n)

$$\psi(x) = u_k\exp(ikx) + v_k\exp\left(i\left(k-\frac{2\pi n}{a}\right)x\right). \qquad (2)$$

This makes energy gaps to open when the unperturbed levels are degenerate.



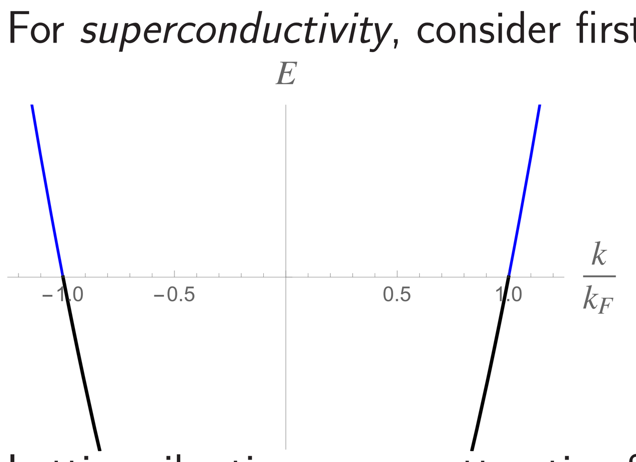
The crystal wave vector k can be limited to the first Brillouin zone. Alternatively, the discontinuity at the zone boundary can be avoided using the extended-zone scheme. Then the energy levels shifted by $k \to k + \frac{2\pi n}{a}$ are not independent, but represent the same level. Thus, the occupation of the levels $f(E,k)$ has to satisfy

$$f(E,k) \equiv f(E, k+\tfrac{2\pi n}{a}). \qquad (3)$$

The energy levels are filled with electrons following the Pauli principle. Solids can be divided in three groups.

- Metal: there is one partially filled band.
- Insulator: The highest filled band is separated from the lowest empty band by an energy gap of several electron volts.
- *Semiconductor*: like insulator, but the band gap is smaller.

For *superconductivity*, consider first a normal metal.

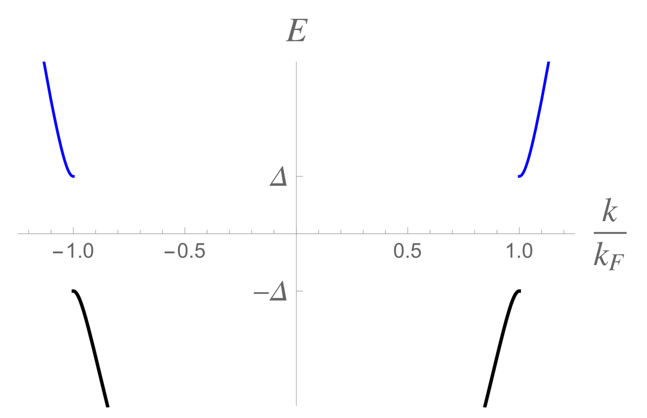


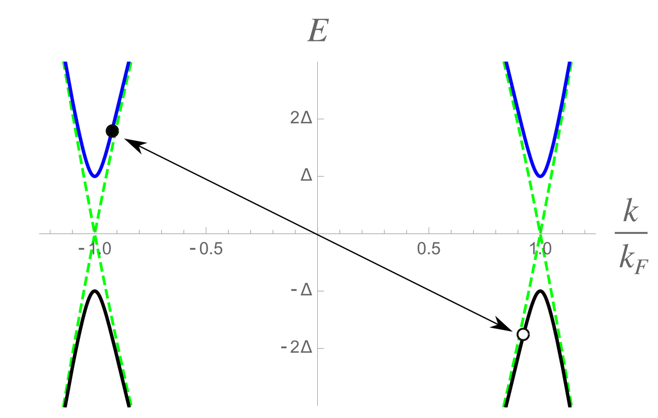
The electron $E(k)$ near the Fermi surface. Black lines denote occupied levels and the blue lines unoccupied levels at zero temperature ($T = 0$). It is also convenient to choose the Fermi level as the zero of energy, $E = 0$.

Lattice vibrations cause attractive force between the electrons. This makes electron pairs, where the occupations of states of $\boldsymbol{k}$ and $-\boldsymbol{k}$ are correlated:

$$\psi = u_k|0,0\rangle + v_k|1,1\rangle. \qquad (4)$$

Here $|0,0\rangle$, $|1,0\rangle$, $|0,1\rangle$ and $|1,1\rangle$ form the basis states of the occupations at levels $\boldsymbol{k}$ and $-\boldsymbol{k}$. The correlation leads to the opening of an energy gap when the unperturbed states are degenerate, that is, at the Fermi level.

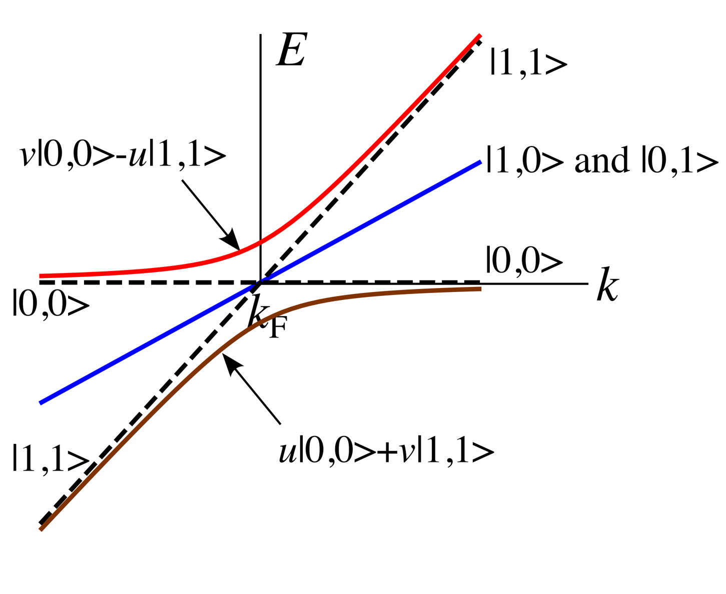




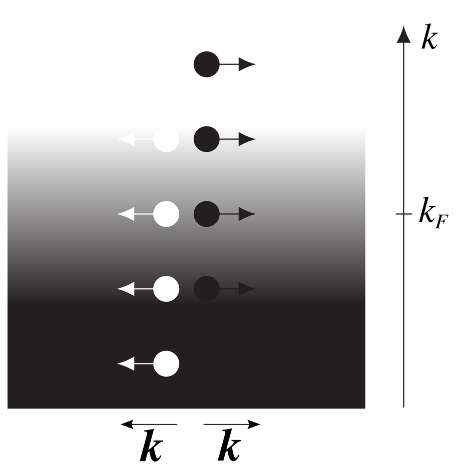
The left picture is discontinuous at $k = \pm k_F$. The right picture uses an extended scheme (and also shows the normal state by dashed lines). The extended scheme has double representation of each physical level so that

$$f(E,\boldsymbol{k}) \equiv 1 - f(-E,-\boldsymbol{k}). \qquad (5)$$

That is, a hole (missing electron) at a negative energy level is equivalent to an electron at the corresponding positive energy level of the opposite wave vector (shown by arrow). The extended scheme is useful because depending on the need, one can select different "reduced schemes". Selecting only positive energies gives the standard BCS excitation spectrum. Allowing both positive and negative energies is called *semiconductor model*.



The left figure illustrates how the excitation spectrum of a superconductor can be obtained from the avoided crossing of the levels in Eq. (4). The excitation energy is the energy difference between the blue (elementary excited state) and brown (ground state) lines. The nature of the quasiparticle $|1,0\rangle$ is illustrated on the right.



References

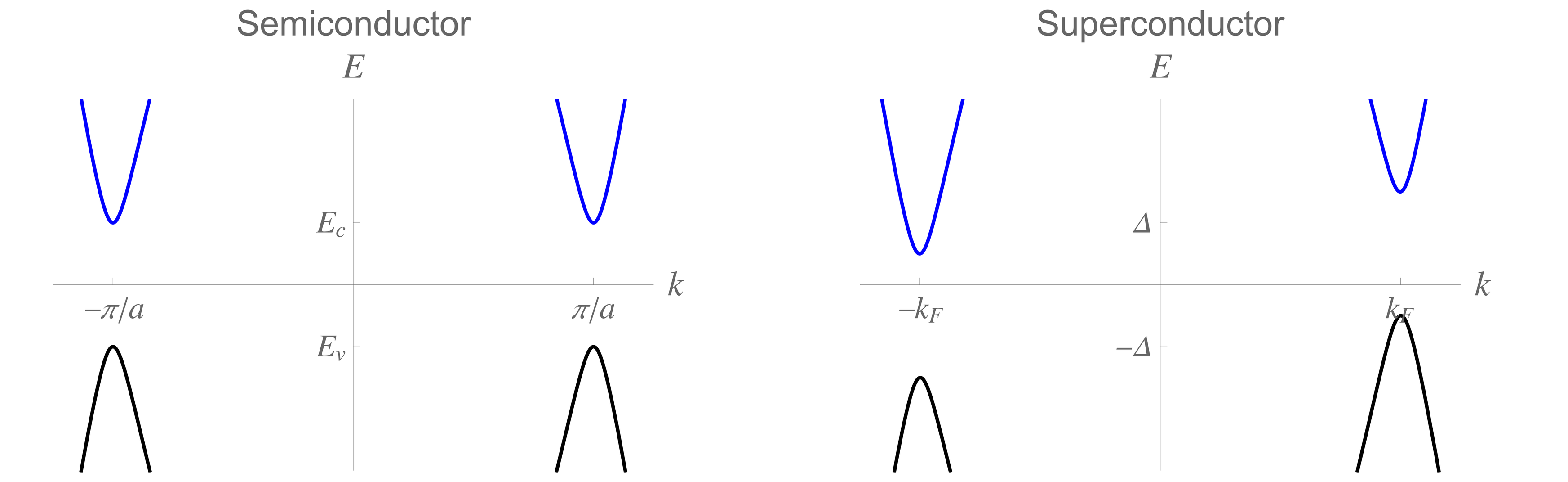
[1] J.W. Serene and D. Rainer, Phys. Rep. **101**, 221 (1983).

[2] W. Belzig, F.K. Wilhelm, C. Bruder, G. Schön and A.D. Zaikin, Superlattices and microstructures **25**, 1251 (1999).

[3] J.A. Kuorelahti, S.M. Laine, and E.V. Thuneberg, Phys. Rev. B **98**,144512 (2018).

The essential difference

In semiconductors the gap is horizontal in the crystal frame (the rest frame of the crystal), but in superconductors it can be inclined.

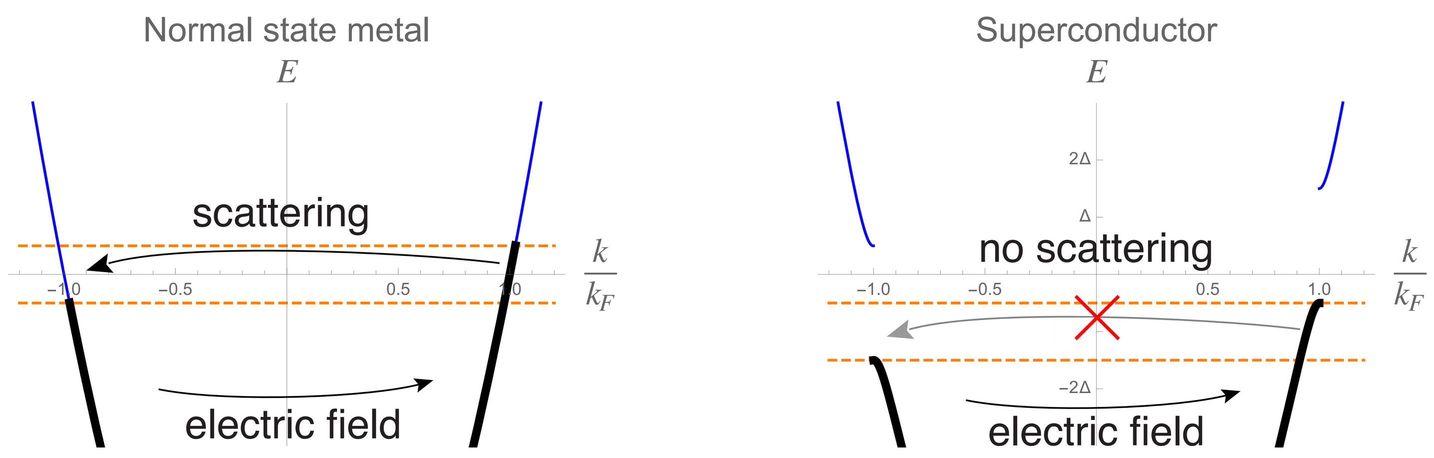


In semiconductors the gap arises from the crystal lattice and thus is balanced in the crystal frame.

In superconductors, the gap arises from interaction of electrons. Although the interaction is mediated by the lattice, the gap need not be horizontal in the crystal frame. The inclined gap is equivalent that the rest frame of the conduction electrons is moving relative to the crystal frame.

Dissipationless current

The electric field drives the electron distribution to one side. In the normal state the electrons are scattered by impurities and lattice vibrations to the empty states of the same or lower energy.



The electric field drives electrons also in a superconductor. In addition, it also tilts the gap. There is net electron flow to the right because the levels with $k > 0$ are filled to higher energy than the levels with $k < 0$. This current is dissipationless because of the tilted energy gap: there are no available states for electrons at the same (or lower) energy.

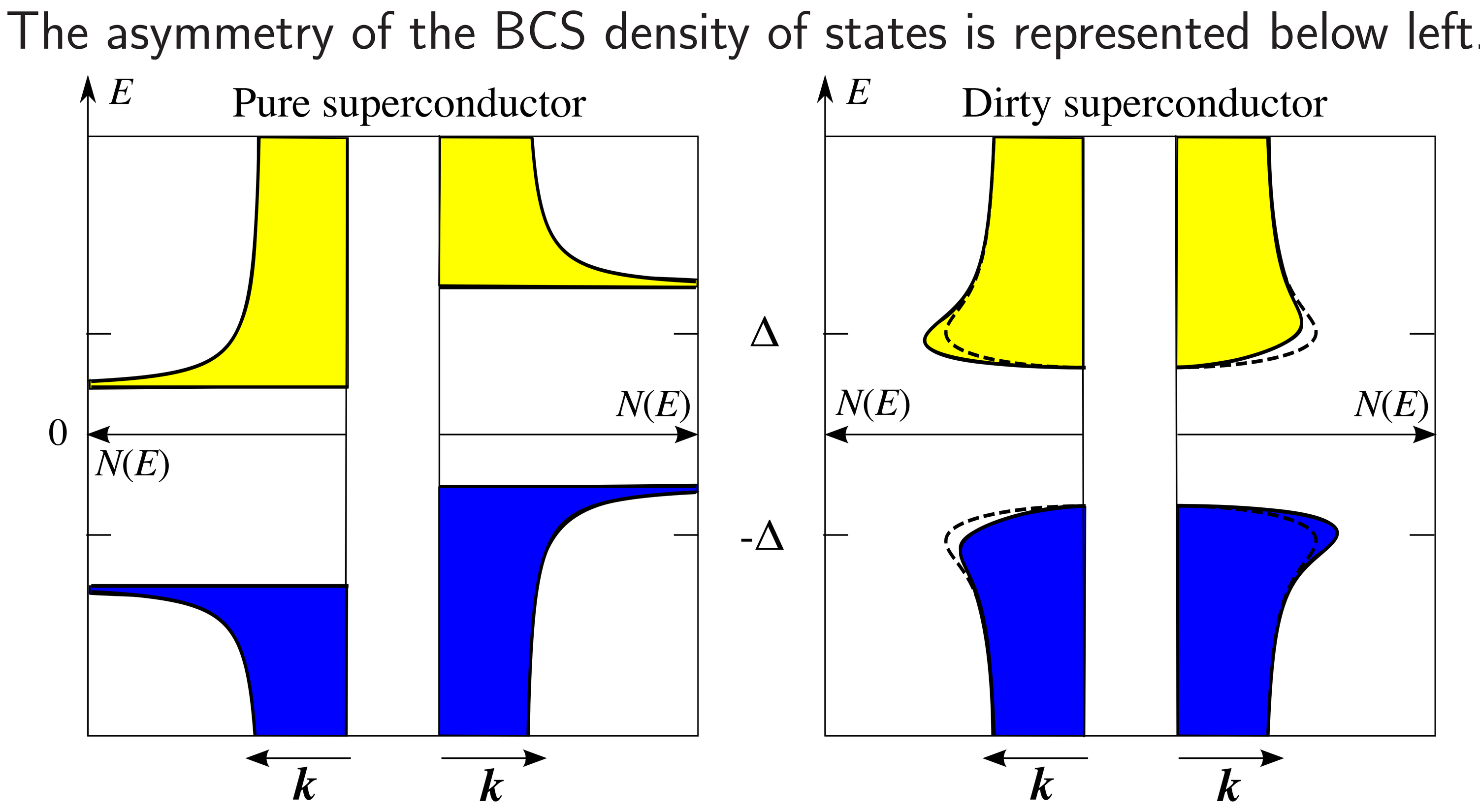
- The common explanation for supercurrent is the flow of Cooper pairs. Here we advocate the alternative view of the supercurrent as *the asymmetry of the electron levels under the gap*.

Microscopic justification

The connection of the inclined gap with the supercurrent is justified by [1]

$$j = ev_F N(0)\int\frac{d\Omega_k}{4\pi}\hat{k}\int_{-E_c}^{E_c} d\epsilon\frac{|\tilde{\epsilon}|}{\sqrt{\tilde{\epsilon}^2-|\Delta|^2}}\theta(\tilde{\epsilon}^2-|\Delta|^2)(\phi_{B1}+\phi_{B2}). \qquad (6)$$

Most importantly, $\tilde{\epsilon} = \epsilon - a$, where $a(\boldsymbol{k}) \approx \boldsymbol{\alpha}\cdot\hat{\boldsymbol{k}}$ gives the inclination of the gap. The asymmetry of the BCS density of states is represented below left.



The right panel shows the density of states modified by strong impurity scattering, obtained by solving Usadel equations [2]. The scattering makes the gap isotropic, but the asymmetry of the electron levels still gives the supercurrent.

Applications: pair breaking critical current, critical velocity for moving objects in superfluids [3].