Exercise Round 1

The deadline of this exercise round is **November 3, 2016**. The solutions will be gone through during the exercise session at 14:15-16:00. The problems should be *solved before the exercise session*, and during the session those who have completed the exercises may be asked to present their solutions on the board/screen.

The exercise markings should be posted to the "Exercise markings for session" forum on the MyCourses web page by 14:15 on the exercise day. Alternatively you can return PDF solutions to the "Mailbox for pre-exercise reports" on the MyCourses page before 14:15 on the exercise day.

Exercise 1 (Mean and covariance equations)

(a) Complete the missing steps in the derivation of the covariance (2.37).

(b) Derive the mean and covariance differential equations (2.38) by differentiating the equations (2.36) and (2.37).

Exercise 2 (Solution of an Ornstein–Uhlenbeck process)

(a) Find the complete solution $x(t)$ as well as the mean $m(t)$ and variance $P(t)$ of the following scalar stochastic differential equation:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -\lambda\, x(t) + w(t), \quad x(0) = x_0, \tag{1}$$

where x_0 and $\lambda > 0$ are given constants and the white noise $w(t)$ has spectral density q.

(b) Compute the limit of the mean and variance when $t \to \infty$ (i) directly via $\lim_{t\to\infty} P(t)$, and (ii) by solving the stationary state of the variance differential equation $\mathrm{d}P/\mathrm{d}t = 0$.

Exercise 3 (Euler–Maruyama solution of an O–U process)

Simulate 1000 trajectories on the time interval $t \in [0, 1]$ from the Ornstein–Uhlenbeck process in the previous exercise using the Euler–Maruyama method with $\lambda = 1/2$, $q = 1$, $\Delta t = 1/100$, $x_0 = 1$ and check that the mean and covariance trajectories approximately agree with the theoretical values.