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# Rao-Blackwellized Monte Carlo Data Association for Multiple Target Tracking

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# Outline

- Bayesian Tracking
- Approaches to Multiple Target Tracking
- Particle Filtering
- Rao-Blackwellization
- RBMCDA in Practice
- MHT and RBMCDA
- Applications

## Bayesian Tracking [1/2]

- Uncertainties in dynamics and measurements are modeled as **probability distributions**
- In multiple target tracking the **state**  $\mathbf{x}_k$  is stacked vector of targets states, data association indicators and possibly a set of unknown model parameters
- The ultimate goal is to compute **posterior distribution** of the state:

$$p(\mathbf{x}_k \mid \mathbf{y}_1, \dots, \mathbf{y}_k)$$

## Bayesian Tracking [2/2]

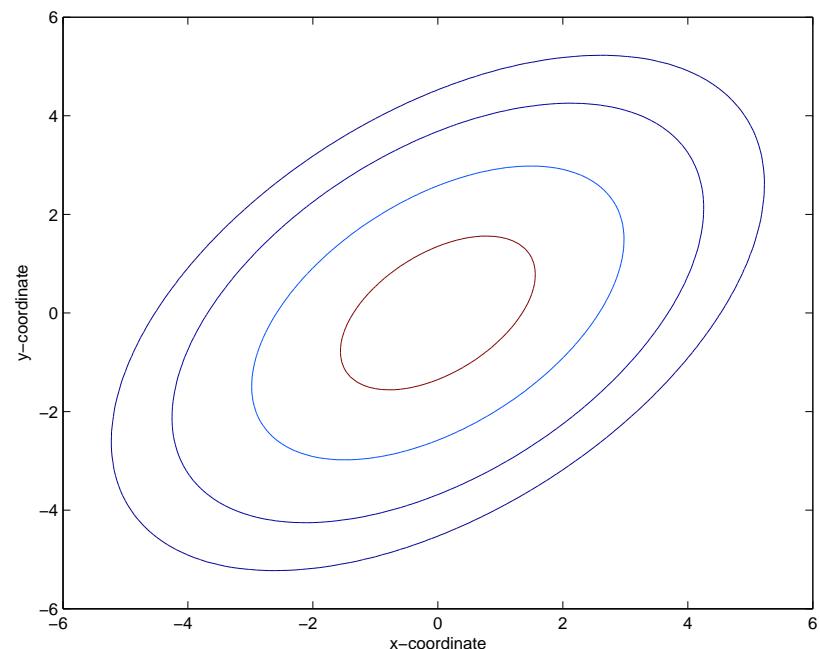
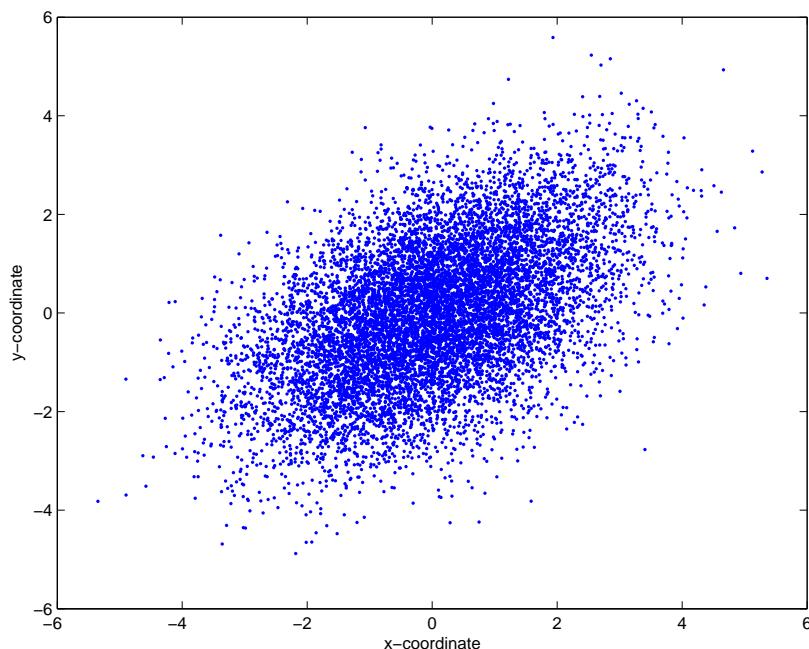
- The posterior can be computed from the **optimal filtering equations**, which can be derived from the Bayesian Theory
- If the model is completely linear and Gaussian, the equations reduce to **Kalman filter**, otherwise approximations required
- **Filtering algorithms** are different ways to approximate the optimal filtering equations and the posterior state distribution

# Approaches to Multiple Target Tracking

- **Joint Probabilistic Data Association (JPDA):** Integrate over data associations, form separate Gaussian approximations for target states
- **Multiple Hypothesis Tracking (MHT):** Find the most probable data association history and compute the state estimates conditionally to that
- **Particle Filtering:** Integrate optimal filtering equations with Monte Carlo to form sample representation of the posterior state distribution

# Particle Filtering [1/3]

- **Sequential Importance Resampling (SIR)** is a sequential version of Importance Sampling with additional resampling stage
- The posterior distribution representation is a **weighted set of particles**, expectations can be computed as sample averages



## Particle Filtering [2/3]

- Sample representation has **no limitations in shape or analytic form** of the distribution
- **Multimodal distributions** can be represented - they may arise when data associations are very uncertain
- When number of particles  $n \rightarrow \infty$ , Monte Carlo **approaches the exact solution**

## Particle Filtering [3/3]

- If distribution can be handled analytically, Monte Carlo **should not be used** - it should be used as the last effort
- Efficiency of sampling depends heavily on the **quality of the importance distribution**
- There exists **optimal importance distribution**, which minimizes variance of the importance weights
- When **dimensionality of the state** grows, more particles are needed, especially if importance distribution is not very good

## Rao-Blackwellization [1/3]

- Rao-Blackwellization of Monte Carlo sampling: Use **closed form computations** always when possible and sample only part of the state
- Analytic calculations are always more accurate - they correspond to the case of **infinite number of particles**
- Rao-Blackwellized particle filter combines the benefits of **Kalman filters** and **particle filters**

## Rao-Blackwellization [2/3]

- If the dynamic and measurement models of single targets are linear Gaussian, **given the data associations** the estimation could be performed by Kalman filter
- When the data associations are unknown the **joint distribution of data associations and states is non-Gaussian**
- RBMCDA: Compute Gaussian parts of the model in closed form by Kalman filter and **sample only the data association indicators**

## Rao-Blackwellization [3/3]

- The space of data association indicators is finite and thus the **optimal importance distribution** can be used
- The marginal distributions needed by sampling procedure are **“by-products” of Kalman filter equations**
- Because the data association indicators are a priori independent and the importance distribution is good, **sampling is very efficient**

## RBM CDA in Practice [1/5]

- An **association event**  $c_k$  is represented with an integer variable with  $T + 1$  values

$c_k = 0 \Rightarrow$  clutter association at time step  $k$

$c_k = 1 \Rightarrow$  target 1 association at time step  $k$

$c_k = 2 \Rightarrow$  target 2 association at time step  $k$

⋮

$c_k = T \Rightarrow$  target  $T$  association at time step  $k$

## RBMCDA in Practice [2/5]

- Indicators may have **prior distribution**:

$$p(c = 0) = \text{false alarm prior}$$
$$p(c = 1) = \text{association to target 1}$$
$$\vdots$$
$$p(c = T) = \text{association to target } T$$

- Uniform prior** can be used for a representing lack of prior information

## RBMCDA in Practice [3/5]

- The **clutter** originated measurements

$$p(\mathbf{y}_k \mid \mathbf{X}_k, \mathbf{y}_k \text{ is clutter}) = 1/V$$

- The **target** originated measurements

$$\begin{aligned} p(\mathbf{y}_k \mid \mathbf{X}_k, \mathbf{y}_k \text{ is from target } j) \\ = N(\mathbf{y}_k \mid \mathbf{H}_{j,k} \mathbf{x}_{j,k}, \mathbf{R}_{j,k}) \end{aligned}$$

- Target dynamics

$$p(\mathbf{x}_{j,k} \mid \mathbf{x}_{j,k-1}) = N(\mathbf{x}_{j,k} \mid \mathbf{A}_{j,k-1} \mathbf{x}_{j,k-1}, \mathbf{Q}_{j,k-1})$$

## RBMCDA in Practice [4/5]

- Particles contain the state **means and covariances** for each target on time step  $k$ , and **importance weights**:

particle 1 :  $\{\mathbf{m}_{1,k}^{(1)}, \mathbf{P}_{1,k}^{(1)}, \dots, \mathbf{m}_{T,k}^{(1)}, \mathbf{P}_{T,k}^{(1)}, w_k^{(1)}\}$

⋮

particle N :  $\{\mathbf{m}_{1,k}^{(N)}, \mathbf{P}_{1,k}^{(N)}, \dots, \mathbf{m}_{T,k}^{(N)}, \mathbf{P}_{T,k}^{(N)}, w_k^{(N)}\}$

- Particles are conditional to different data association histories.

## RBMCDA in Practice [5/5]

1. Predict means and covariances of each particle using the **Kalman filter prediction** equations
2. Compute **association likelihoods** for each target association hypothesis
3. **Draw association hypothesis** for each particle from the optimal importance distribution
4. Update the particle weights and perform **Kalman filter update step** for each particle with the given data association
5. If the estimated effective number of weights is too low, perform **resampling**

## MHT and RBMCDA

- Rao-Blackwellized Monte Carlo Data Association (RBMCDA) and Multiple Hypothesis Tracking (MHT) are very similar, but **theoretical backgrounds** are different
- In both methods the system state is a set of Gaussian hypotheses. The practical **difference is in hypothesis processing**
- In theory, RBMCDA is Minimum Mean Square Error estimator and MHT is Minimum Probability of Error estimator

## Application: Outlier Detection [1/3]

- Simulated process is:

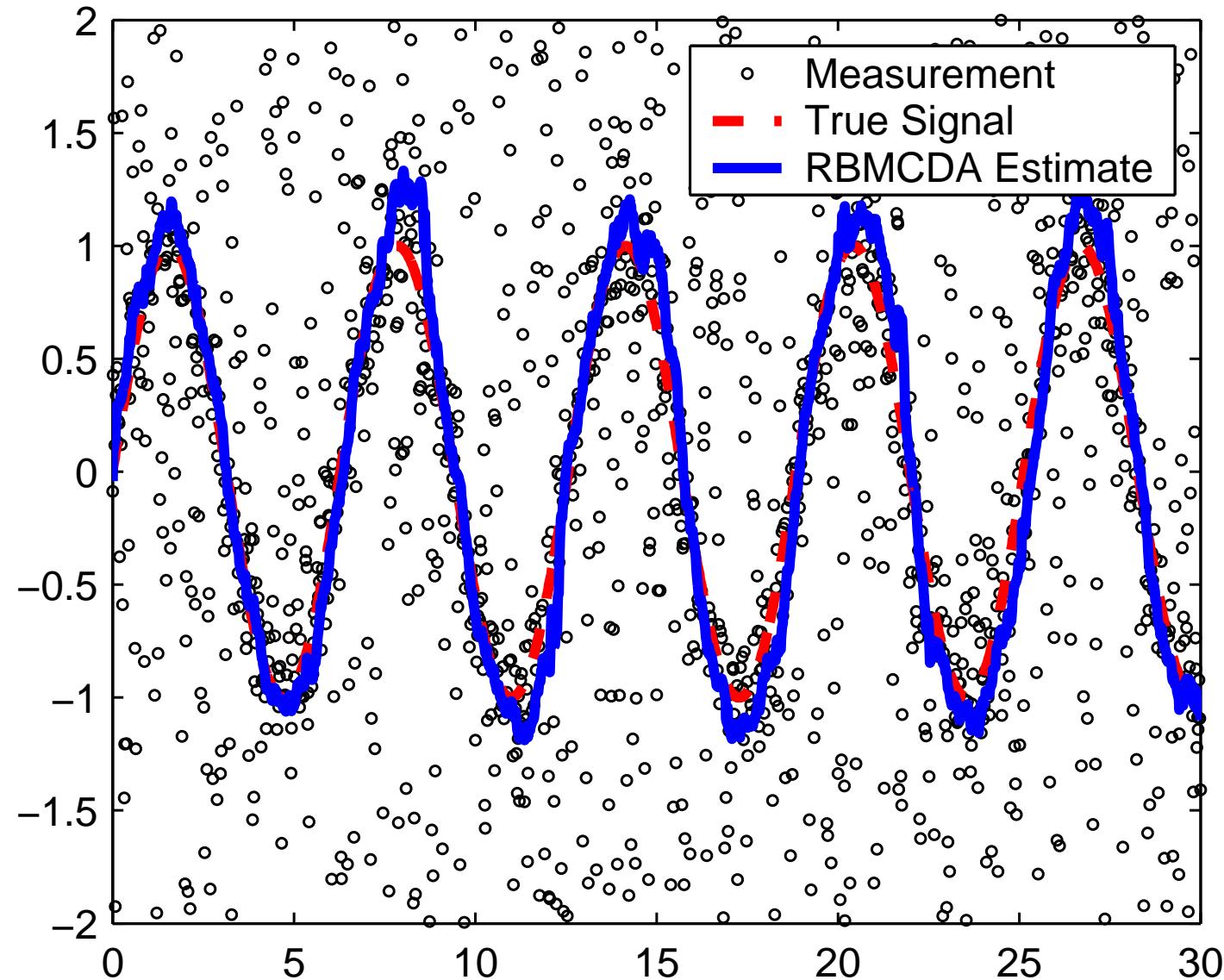
$$x(t) = \sin(\omega t)$$

- Gaussian measurements noise and 50% of clutter measurements, uniformly distributed on range  $[-2, 2]$ :

$$\begin{pmatrix} x_k \\ \dot{x}_k \end{pmatrix} = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_{k-1} \\ \dot{x}_{k-1} \end{pmatrix} + \mathbf{q}_{k-1}$$

$$p(y_k \mid \mathbf{x}_k, c_k) = \begin{cases} 1/4 & , \text{ if } c_k = 0 \\ N(y_k \mid (1 \ 0)\mathbf{x}_k, R) & , \text{ if } c_k = 1 \end{cases}$$

## Application: Outlier Detection [2/3]



## Application: Outlier Detection [3/3]

<i>Method</i>	<i>RMSE</i>	<i>STD</i>
RBMFDA, 10 particles	0.16	0.02
RBMFDA, 100 particles	0.15	0.01
Bootstrap filter, 1000 particles	2.07	2.31
Bootstrap filter, 10000 particles	0.16	0.02
Kalman filter, assuming no clutter	0.39	0.02
Kalman filter, clutter modeled	0.32	0.03
Kalman filter, perfect associations	0.11	0.01

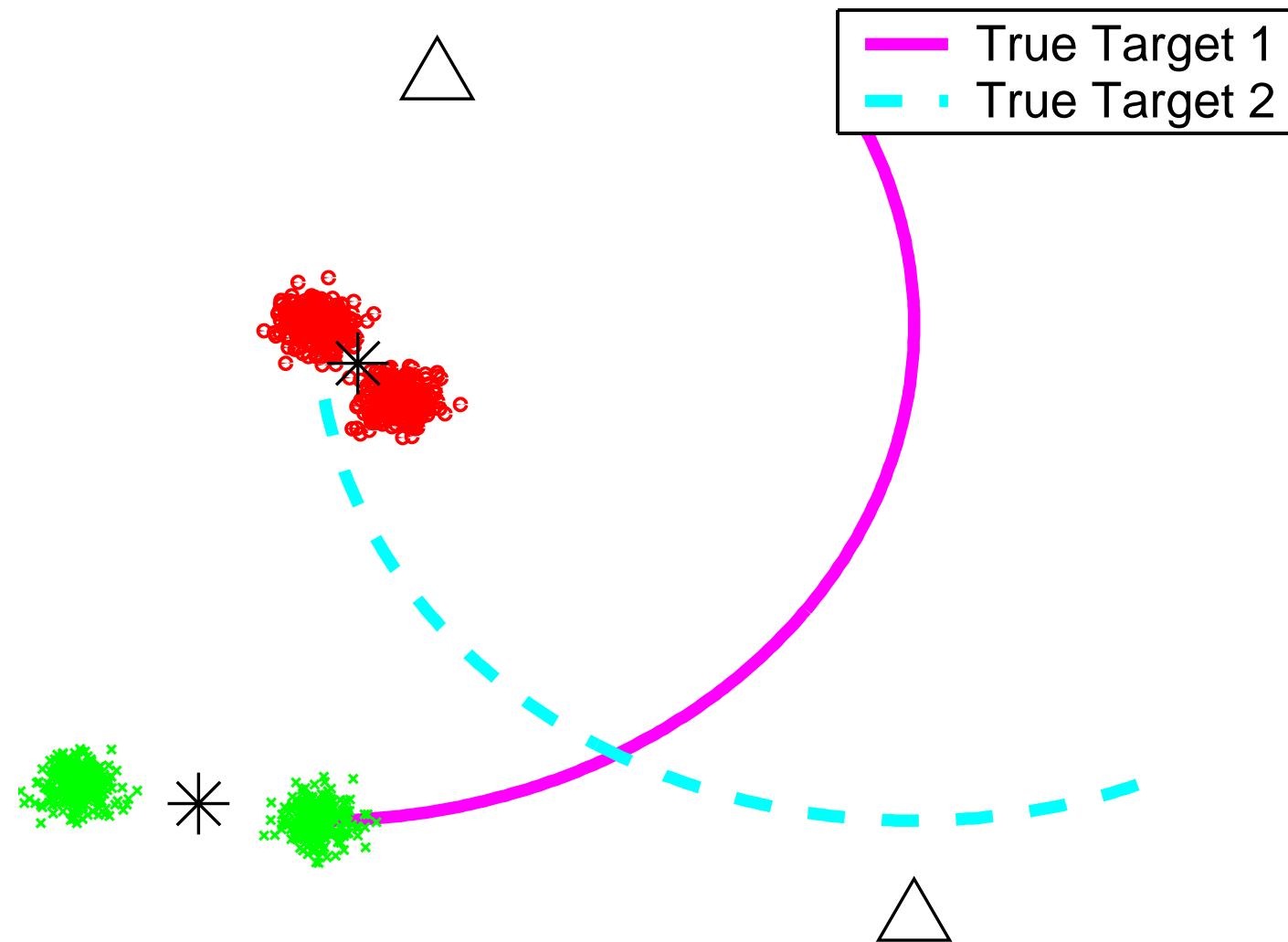
## Application: Multiple Target Tracking [Model]

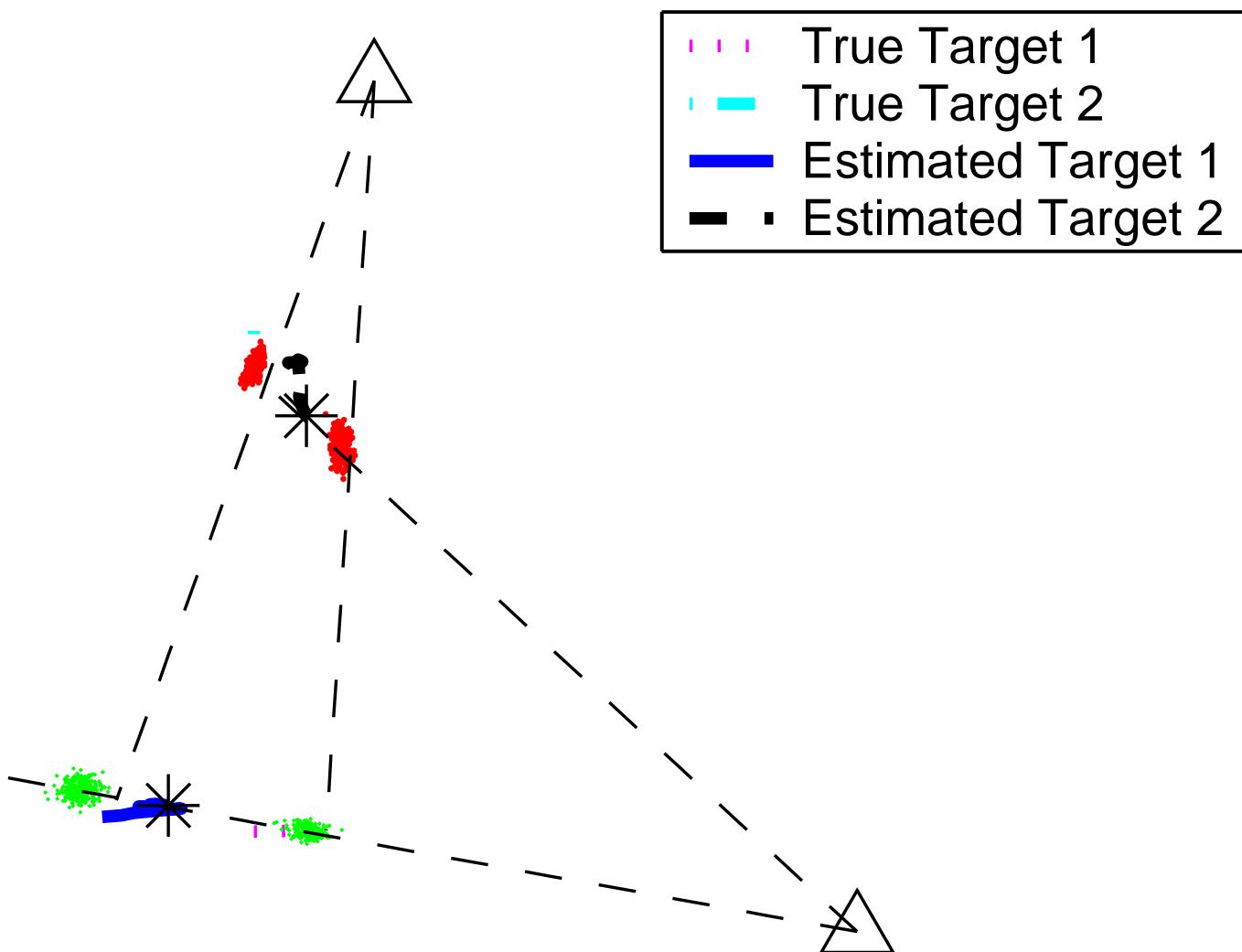
- Model for each target  $j$ :

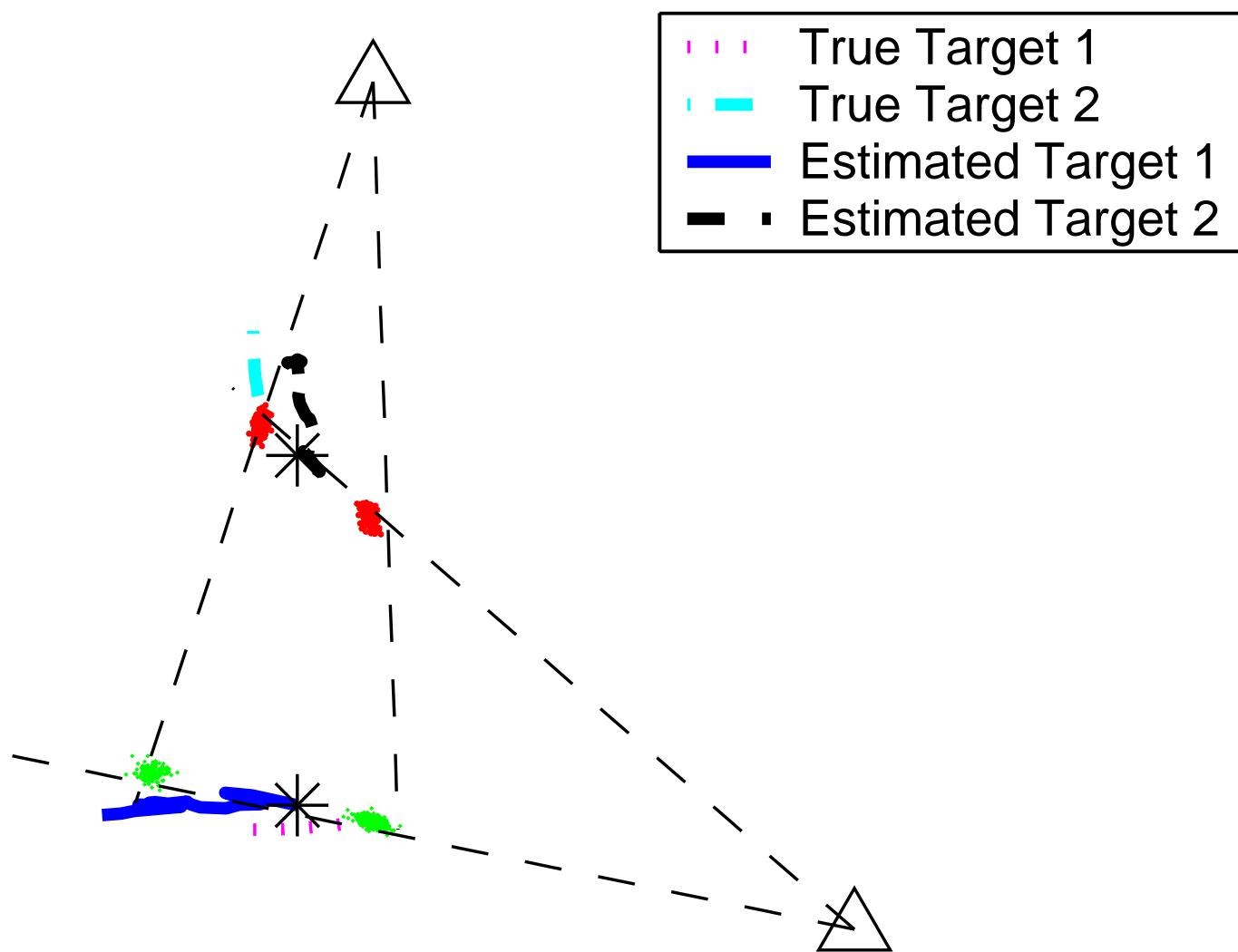
$$\begin{pmatrix} x_{j,k} \\ y_{j,k} \\ \dot{x}_{j,k} \\ \dot{y}_{j,k} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{j,k-1} \\ y_{j,k-1} \\ \dot{x}_{j,k-1} \\ \dot{y}_{j,k-1} \end{pmatrix} + \mathbf{q}_{k-1}$$

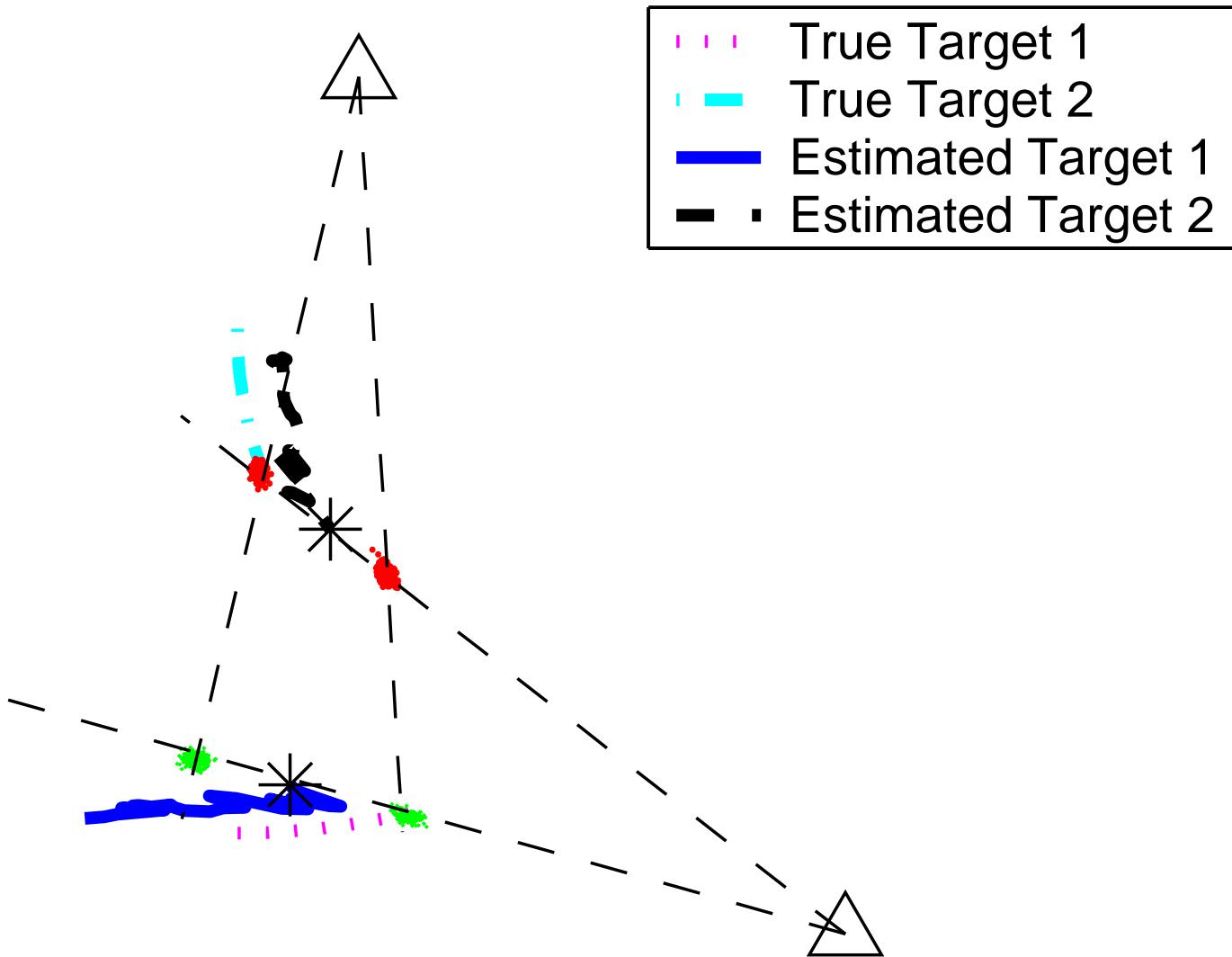
$$\hat{\theta}_k = \arctan \left( \frac{y_{j,k} - s_y^i}{x_{j,k} - s_x^i} \right) + r_k$$

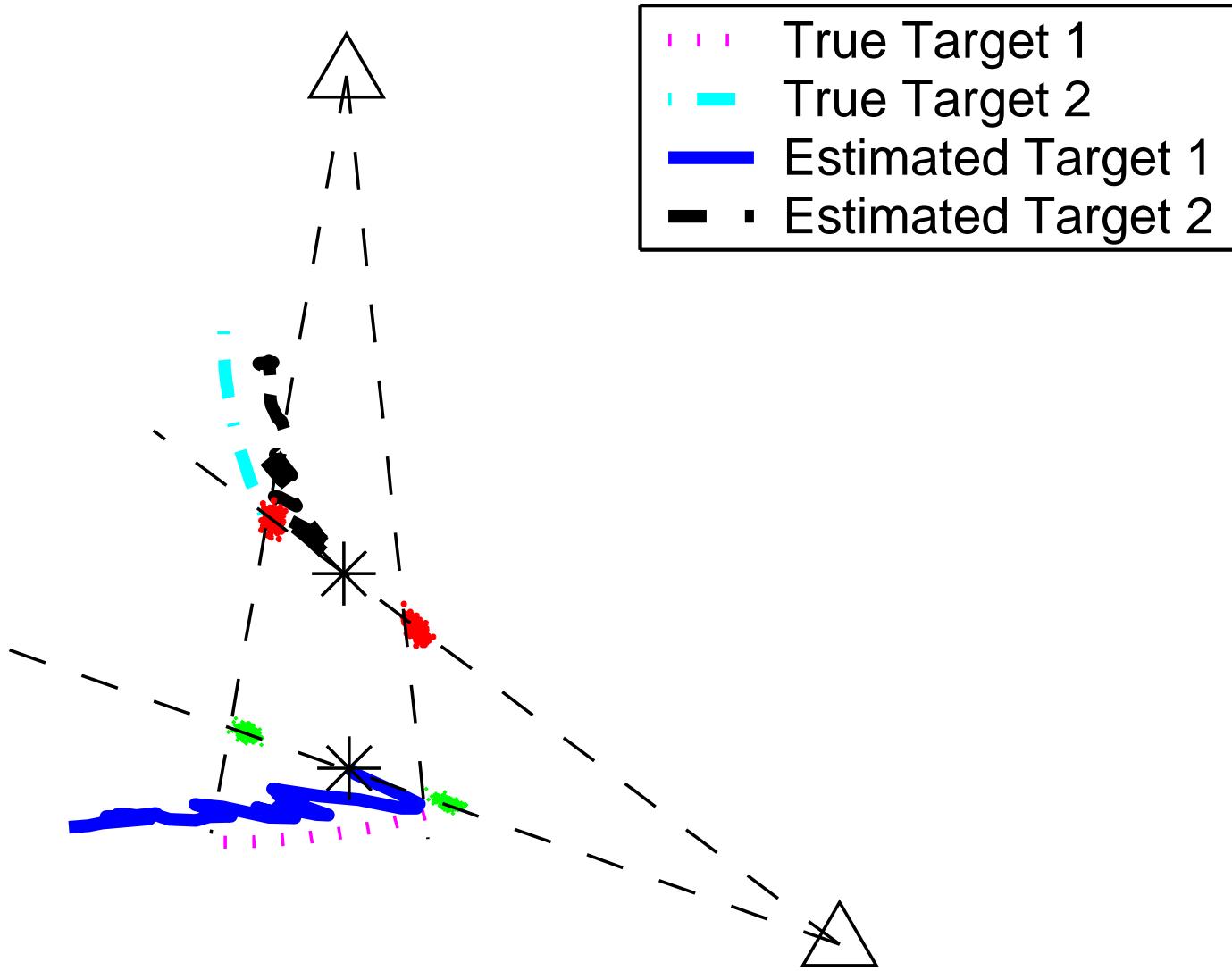
# Application: Multiple Target Tracking [Prior]

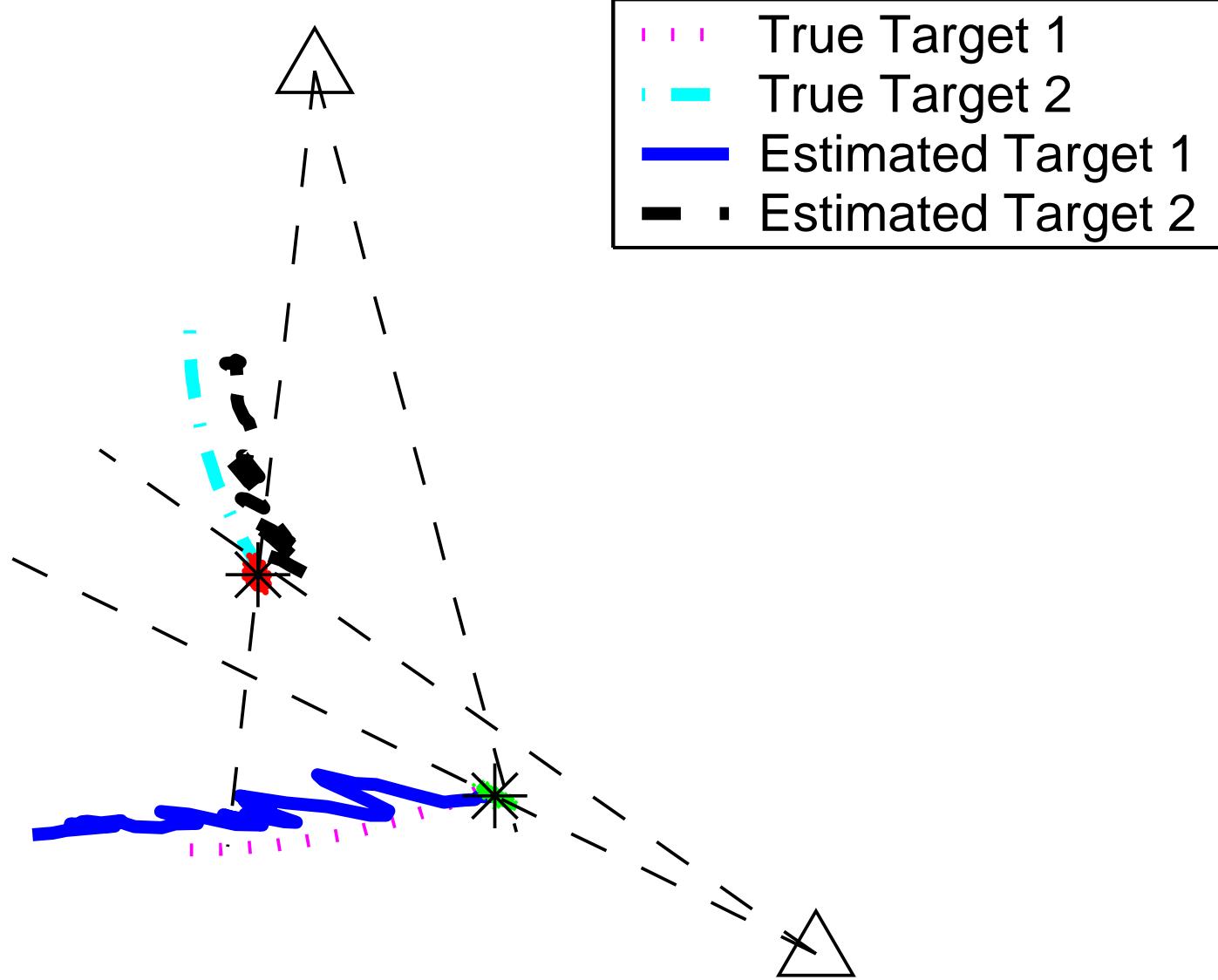


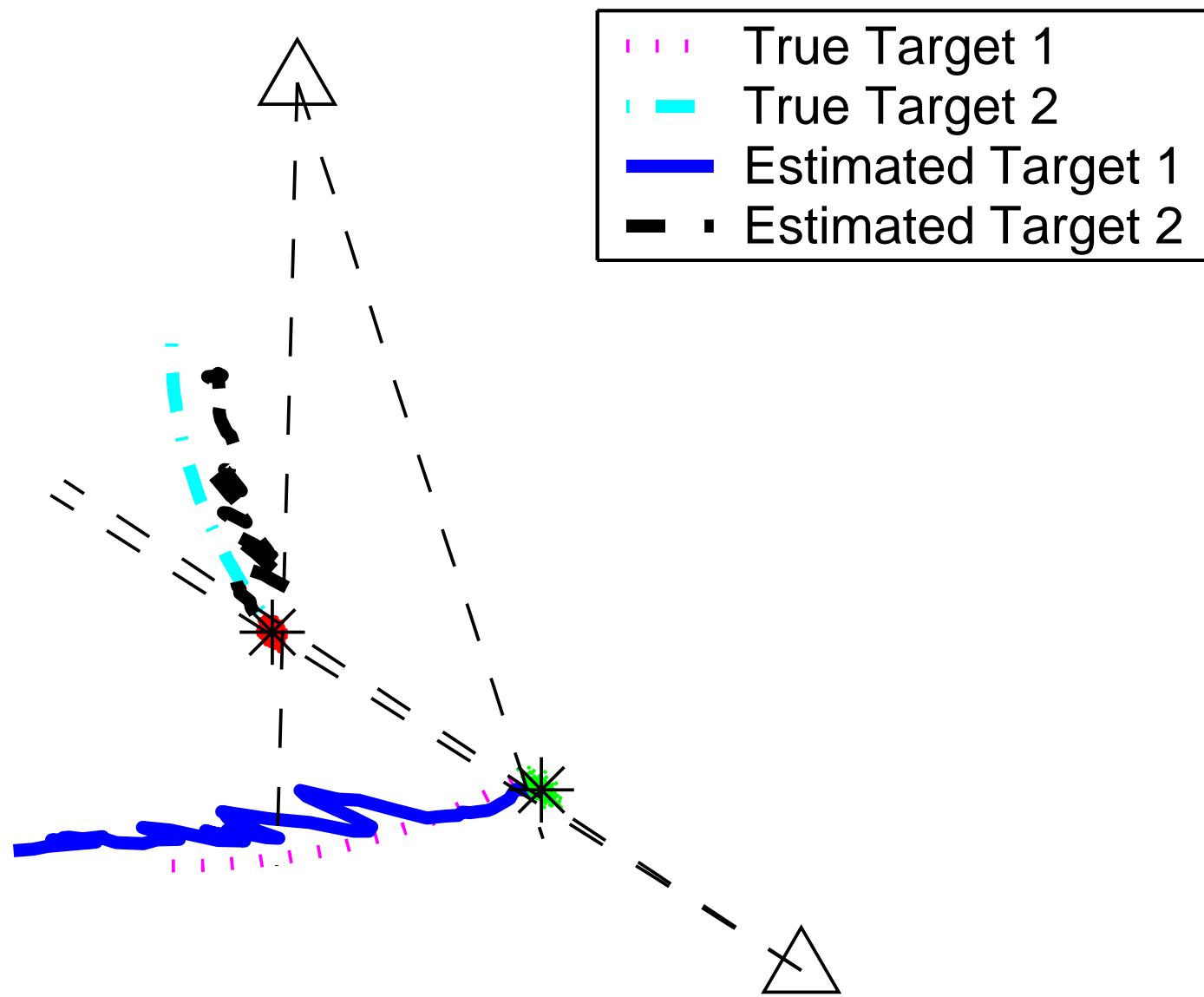


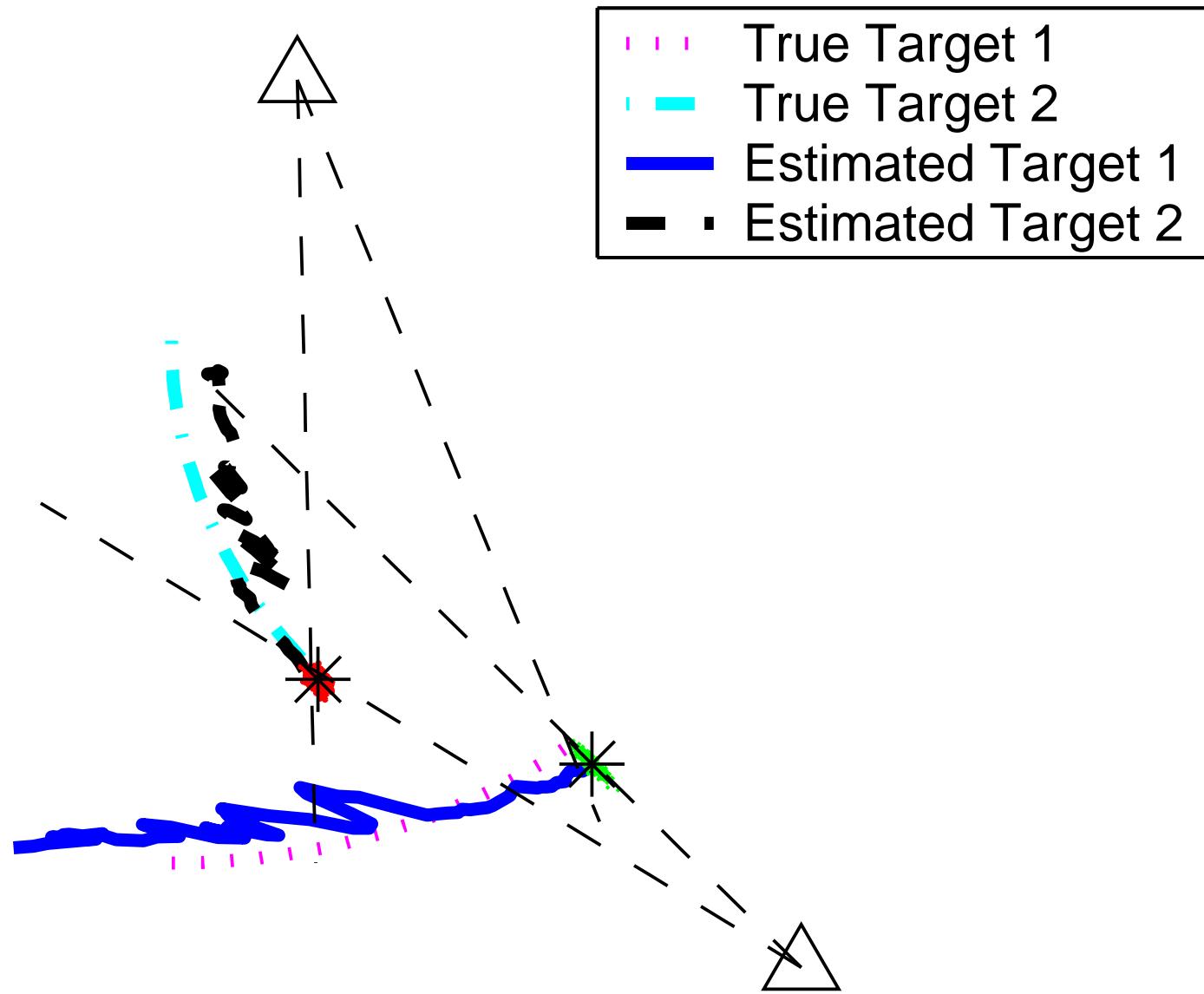


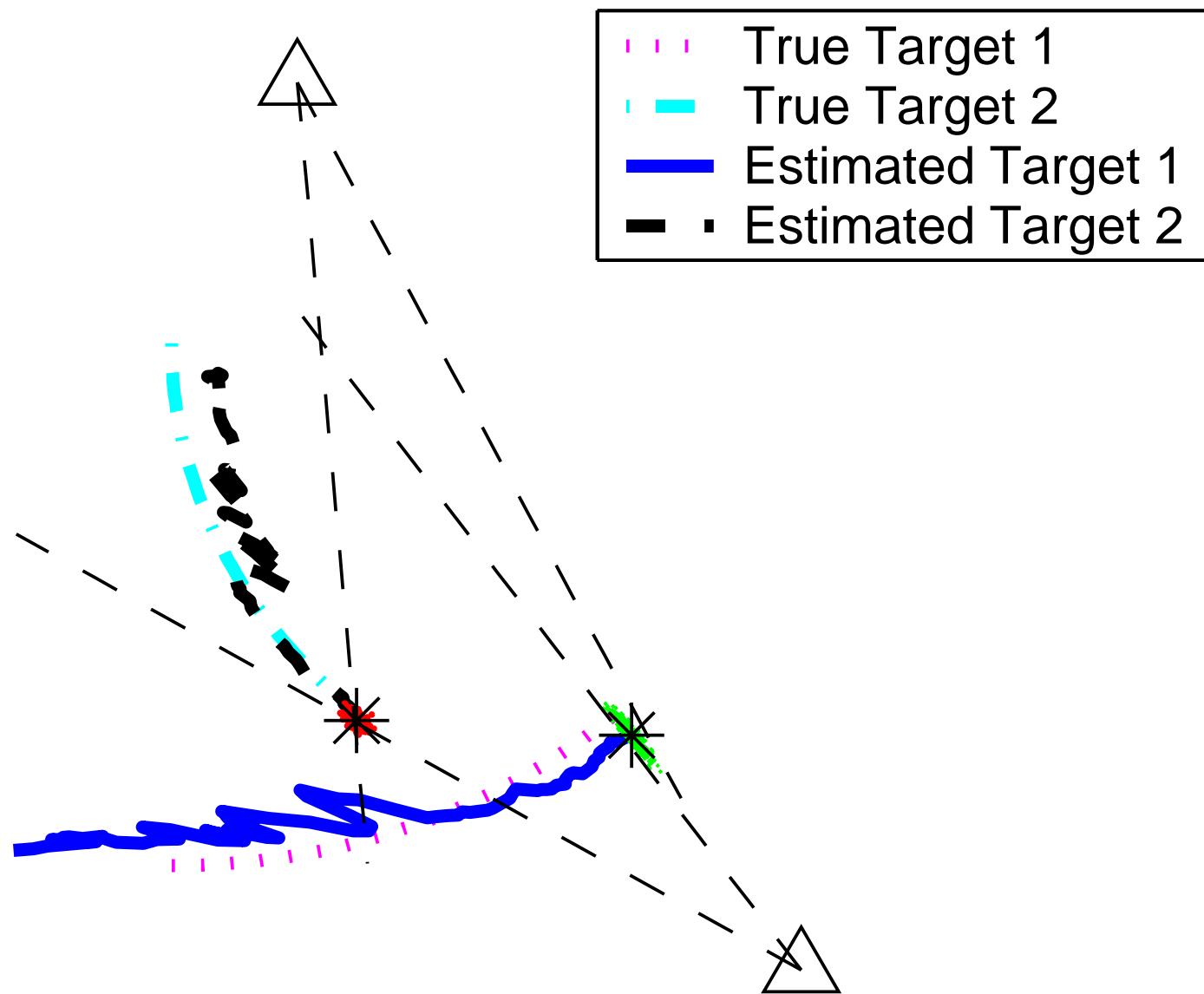


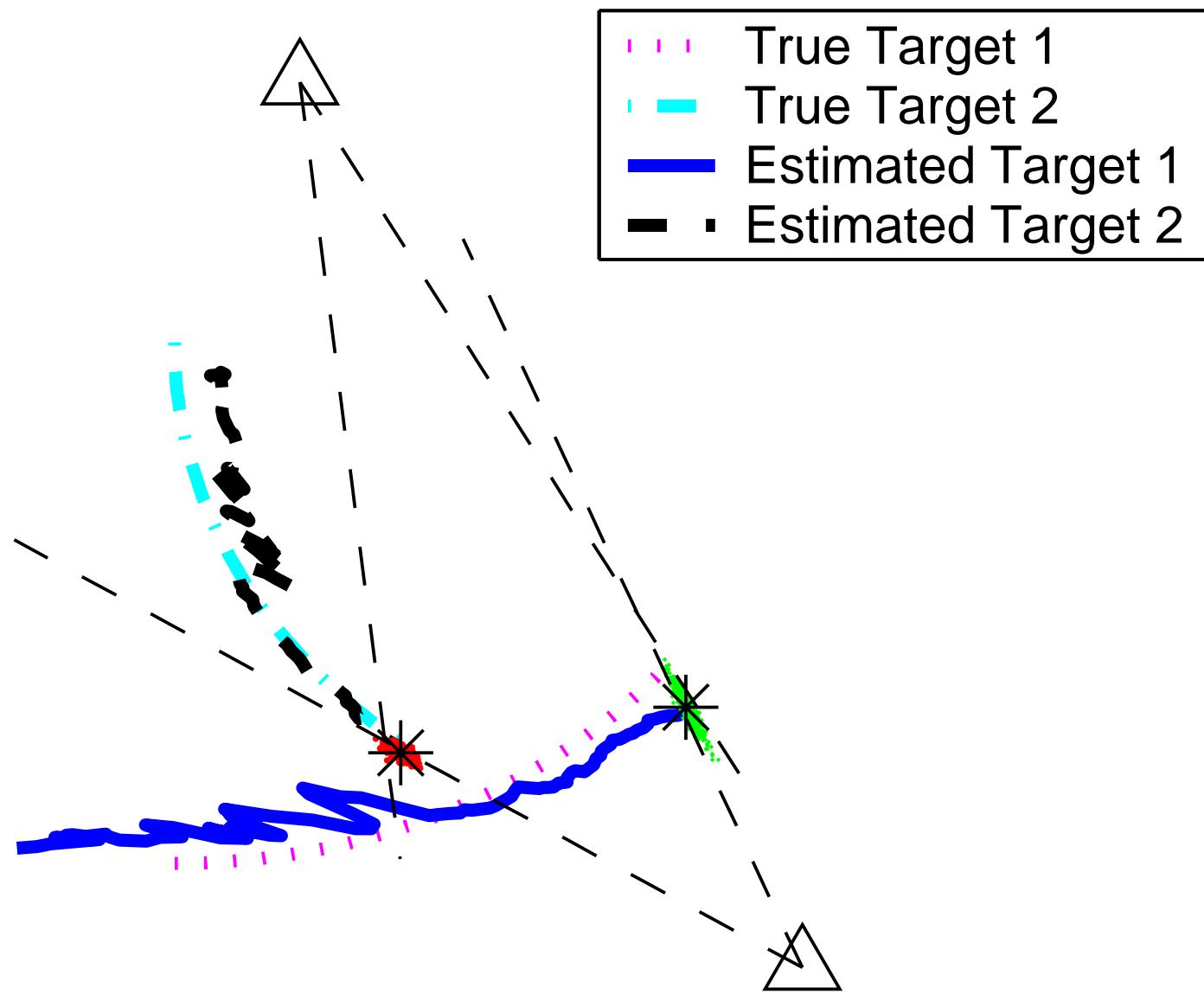


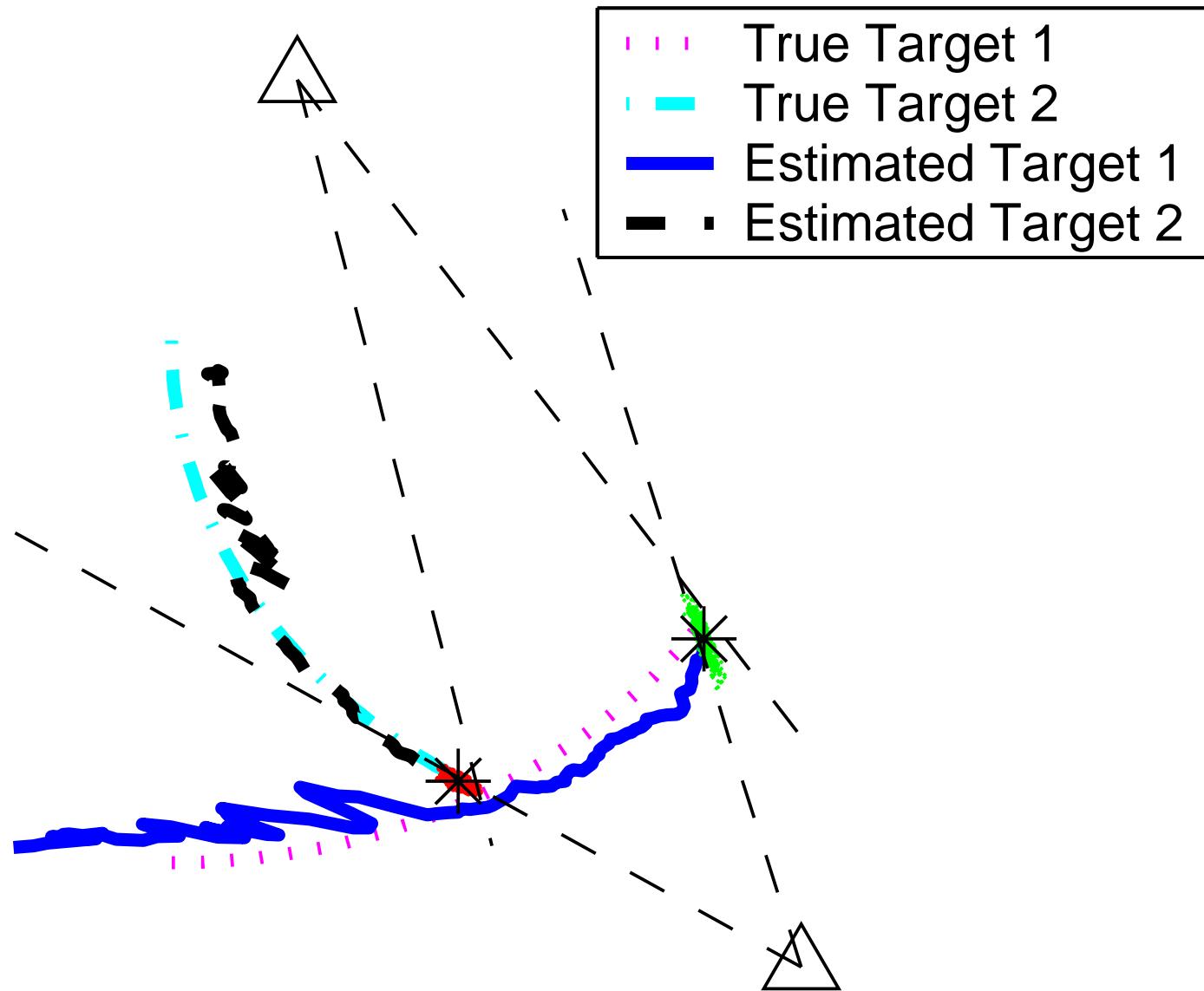


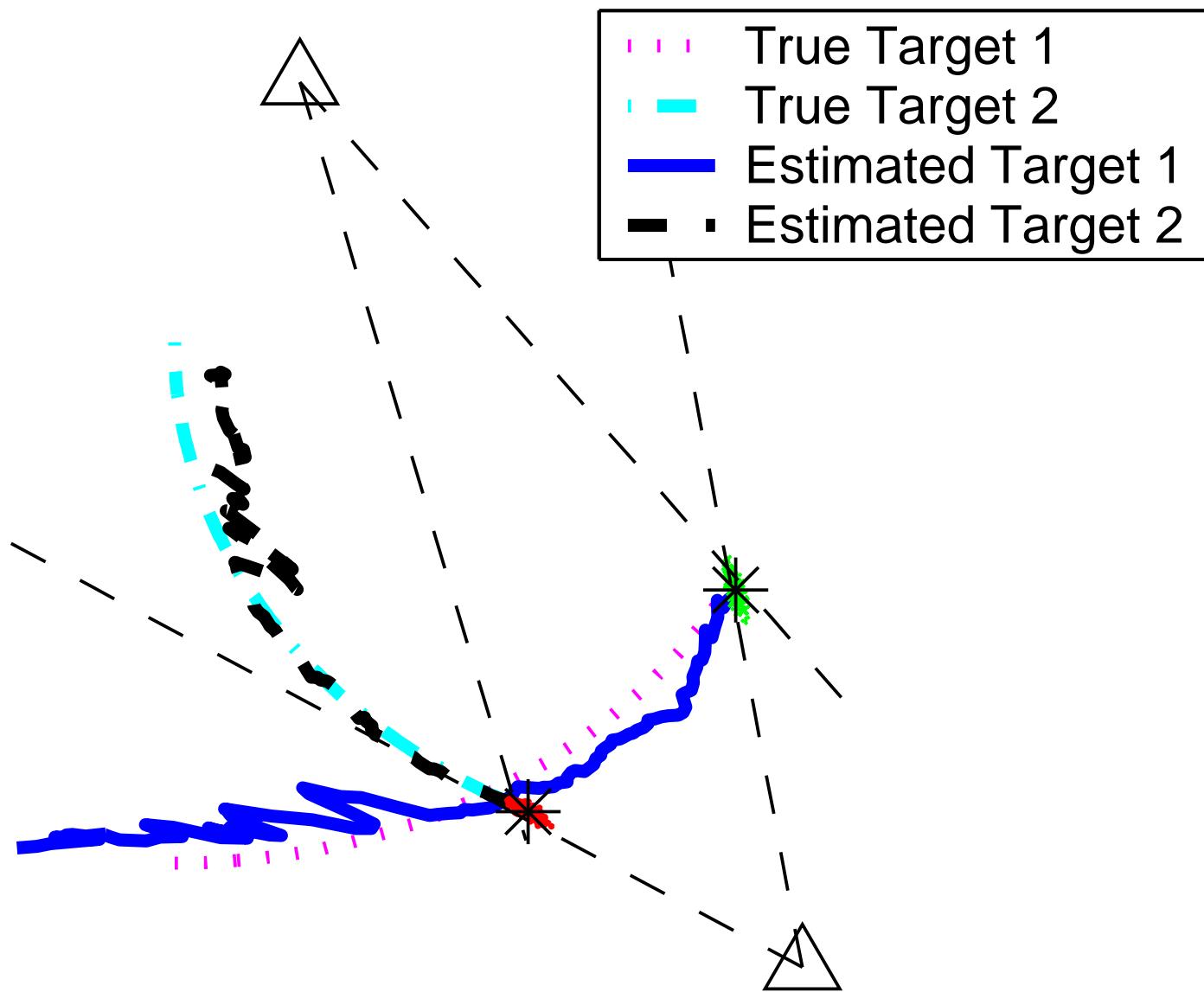


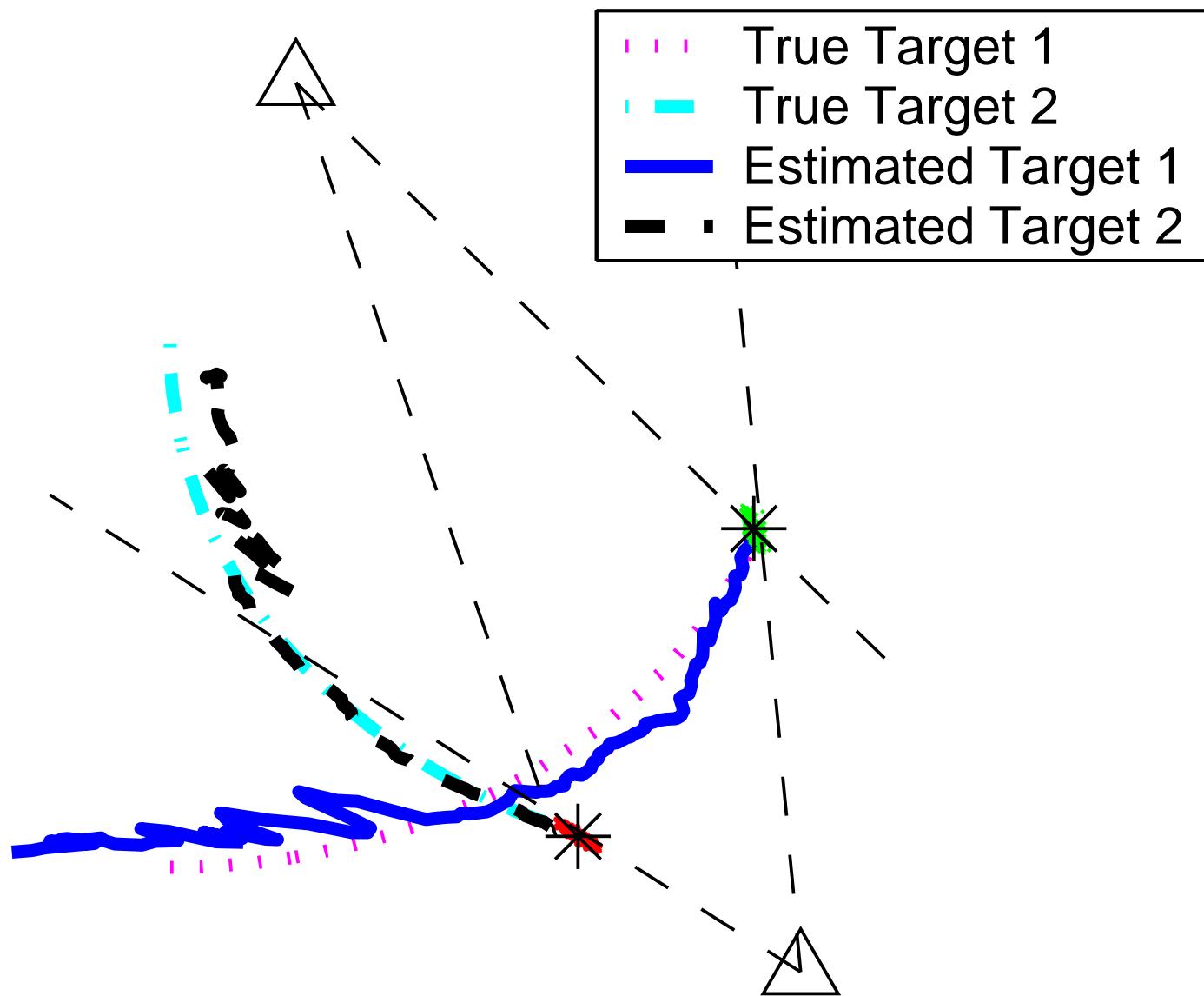


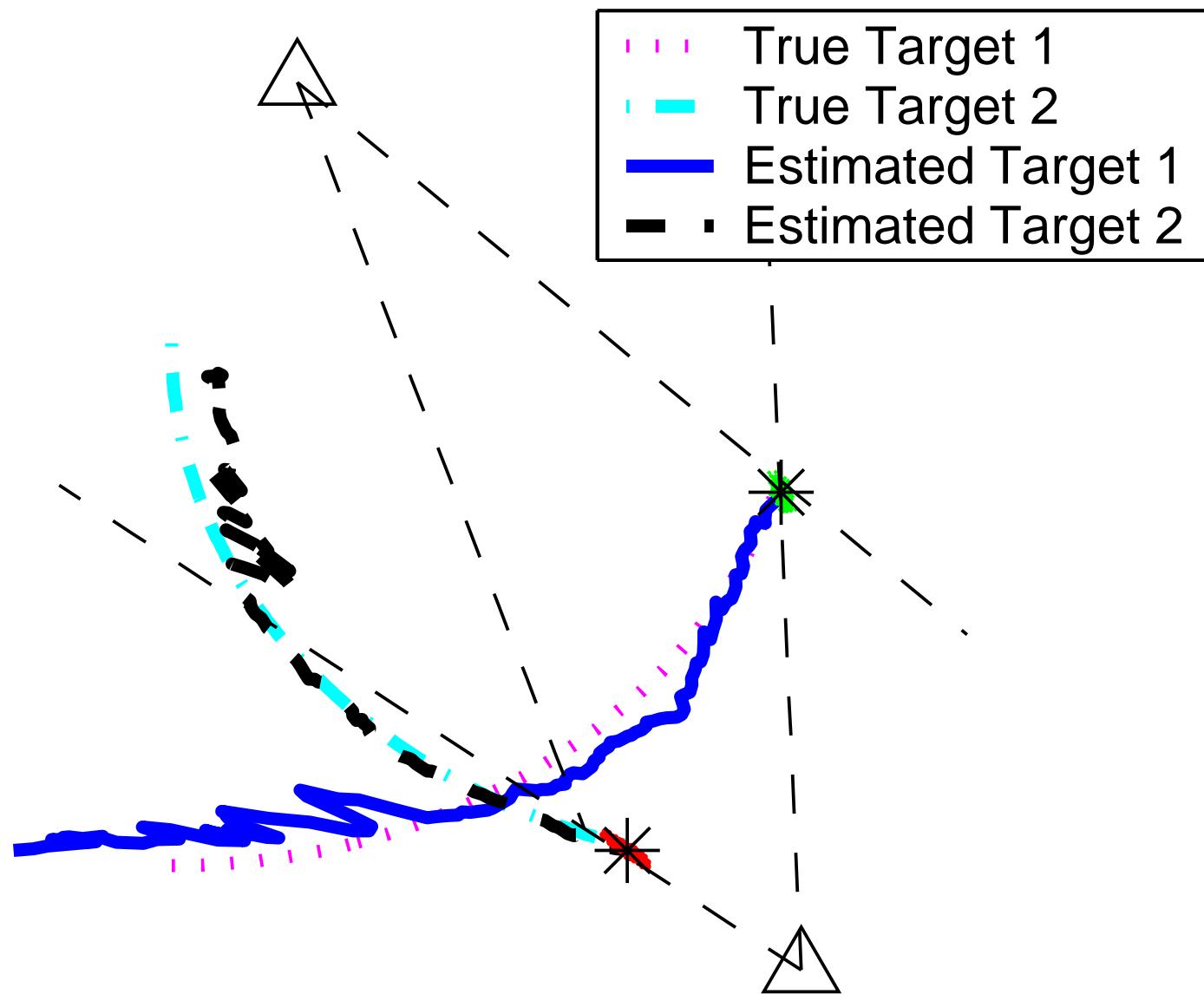


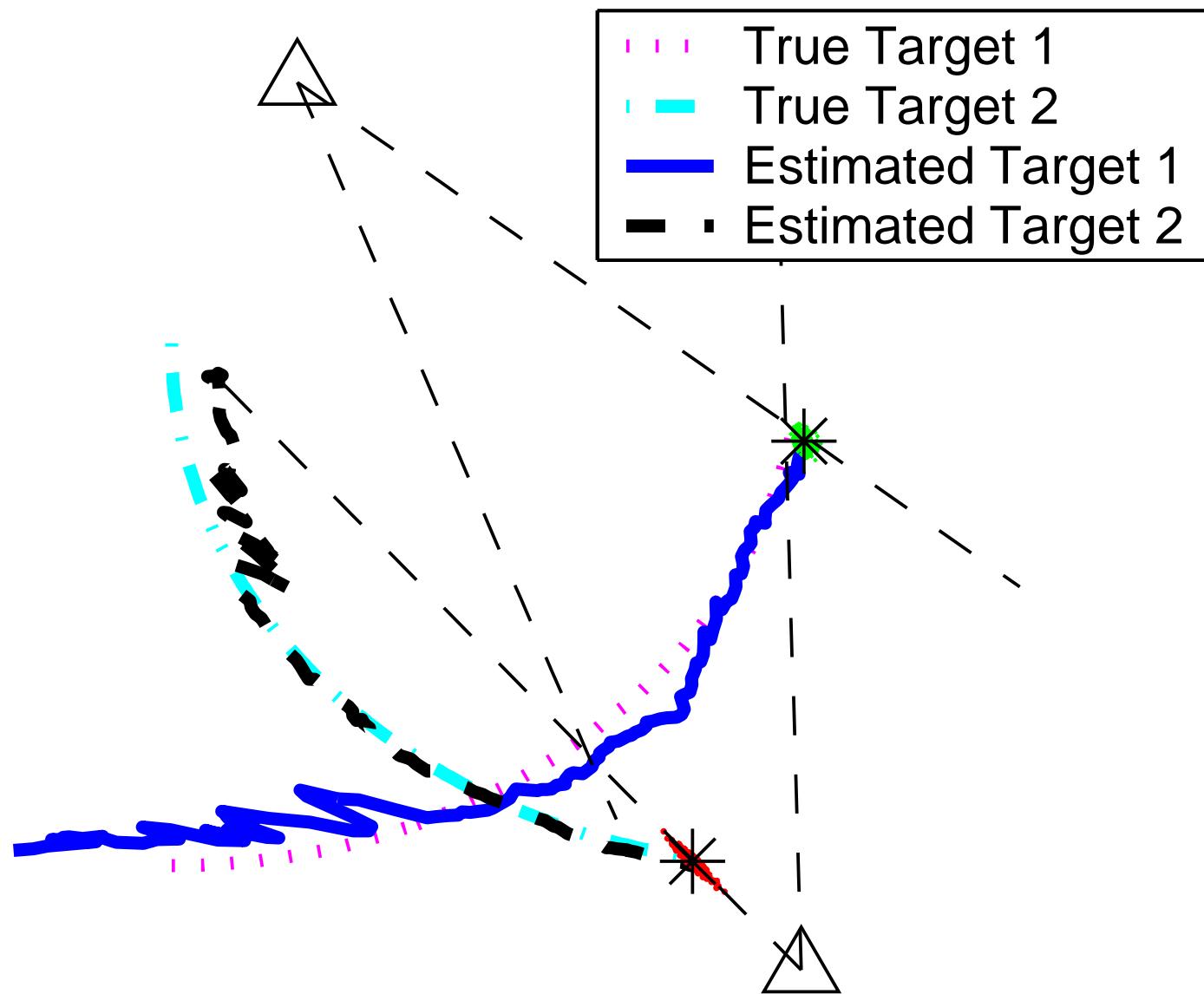


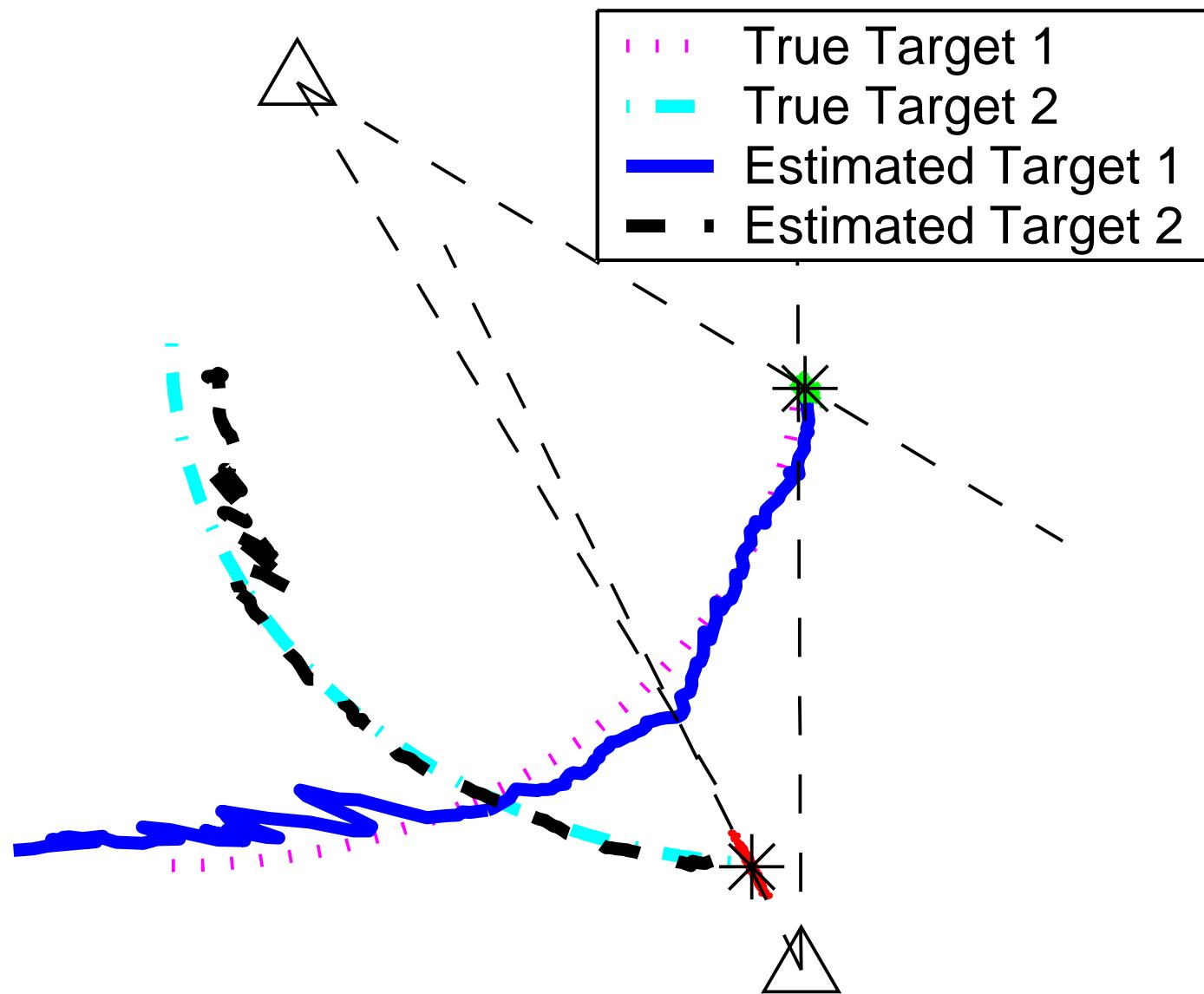


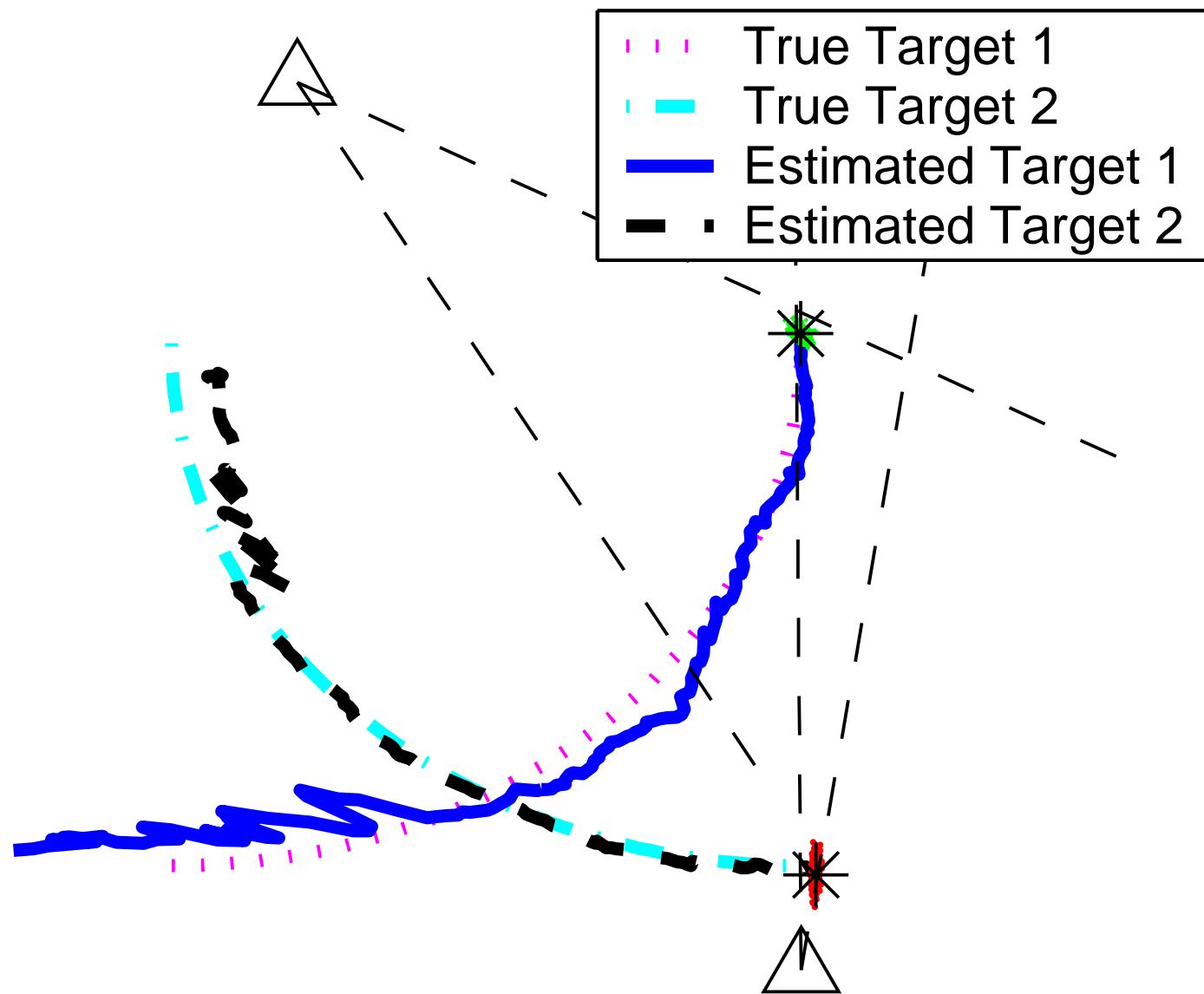


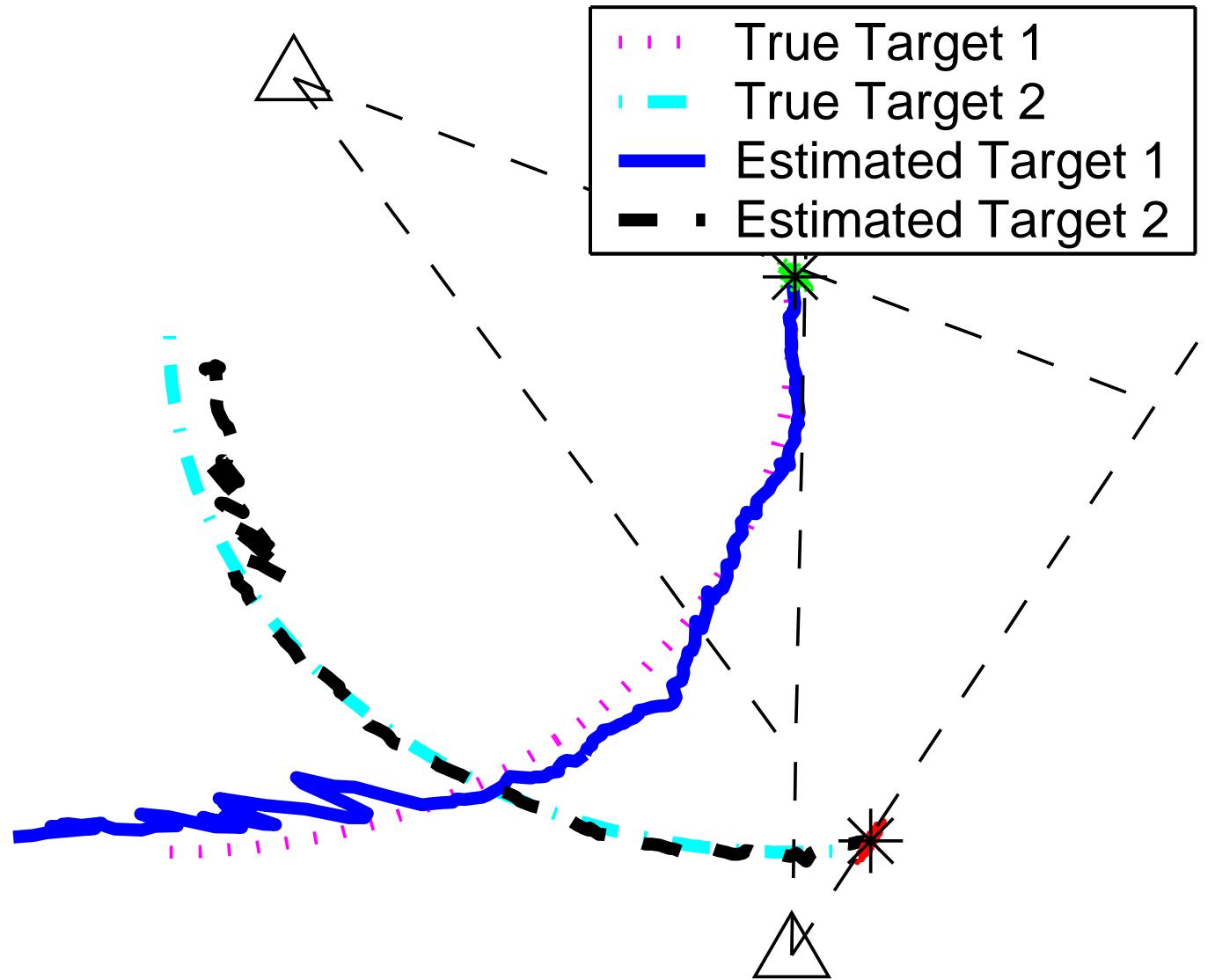


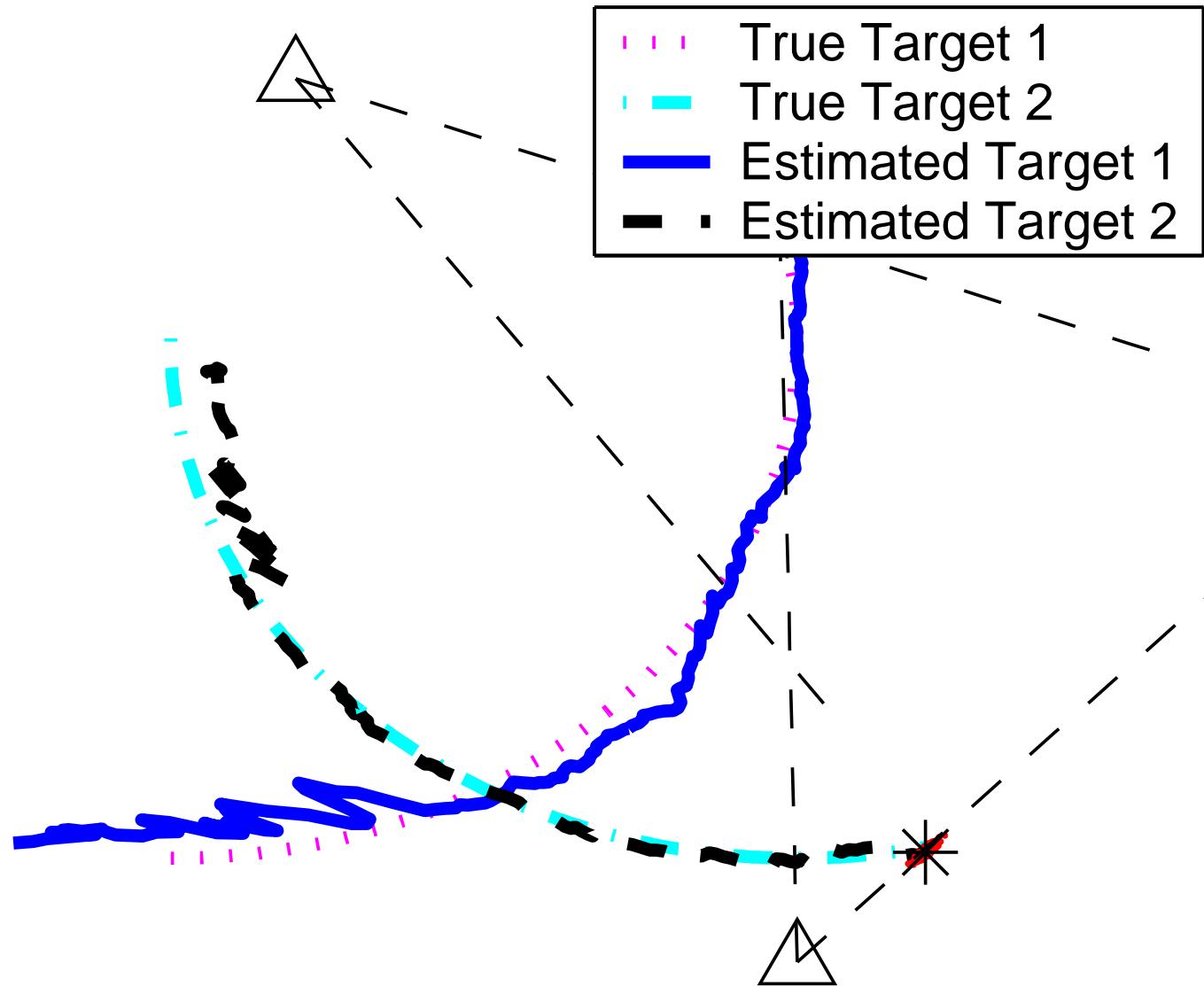


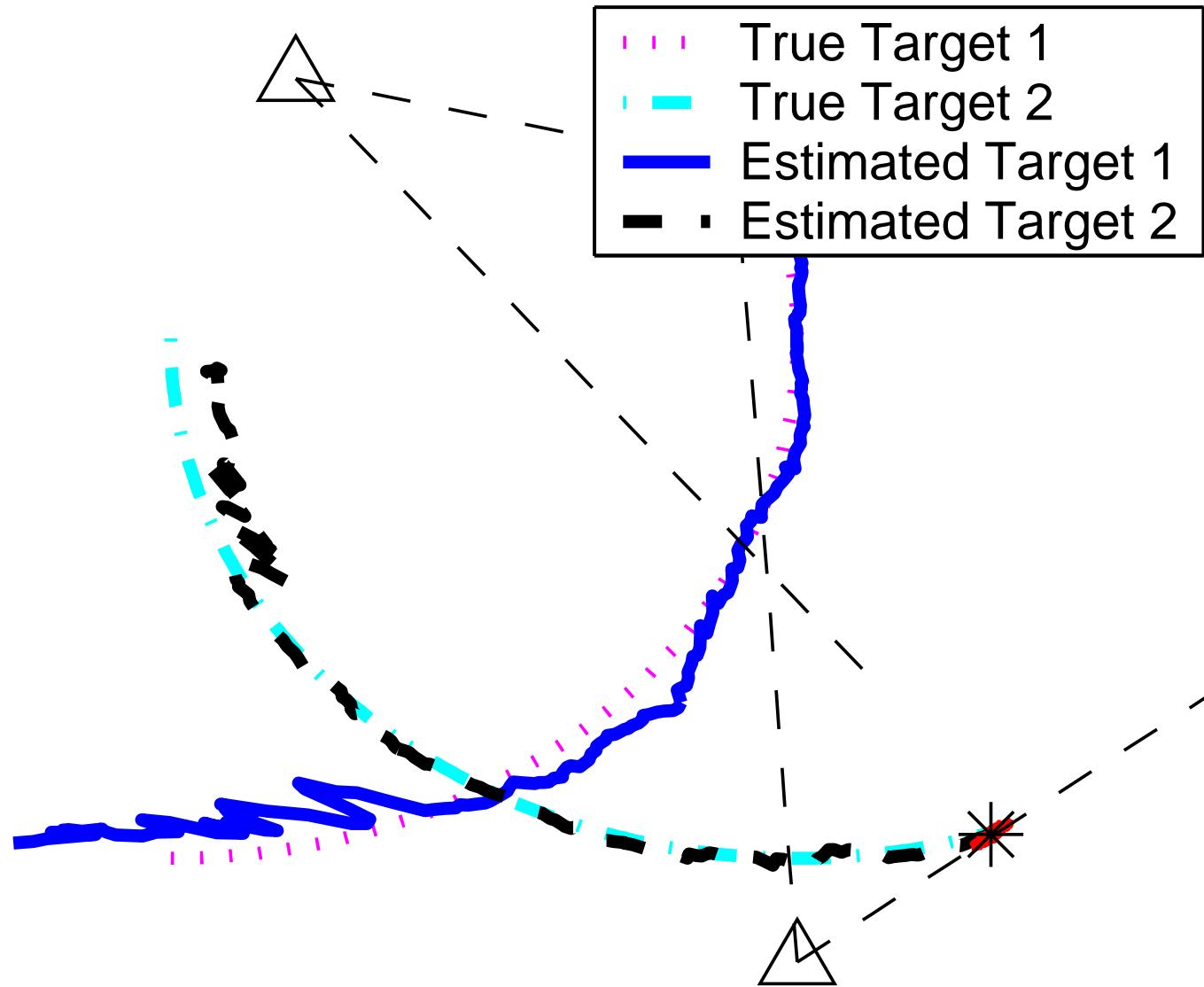


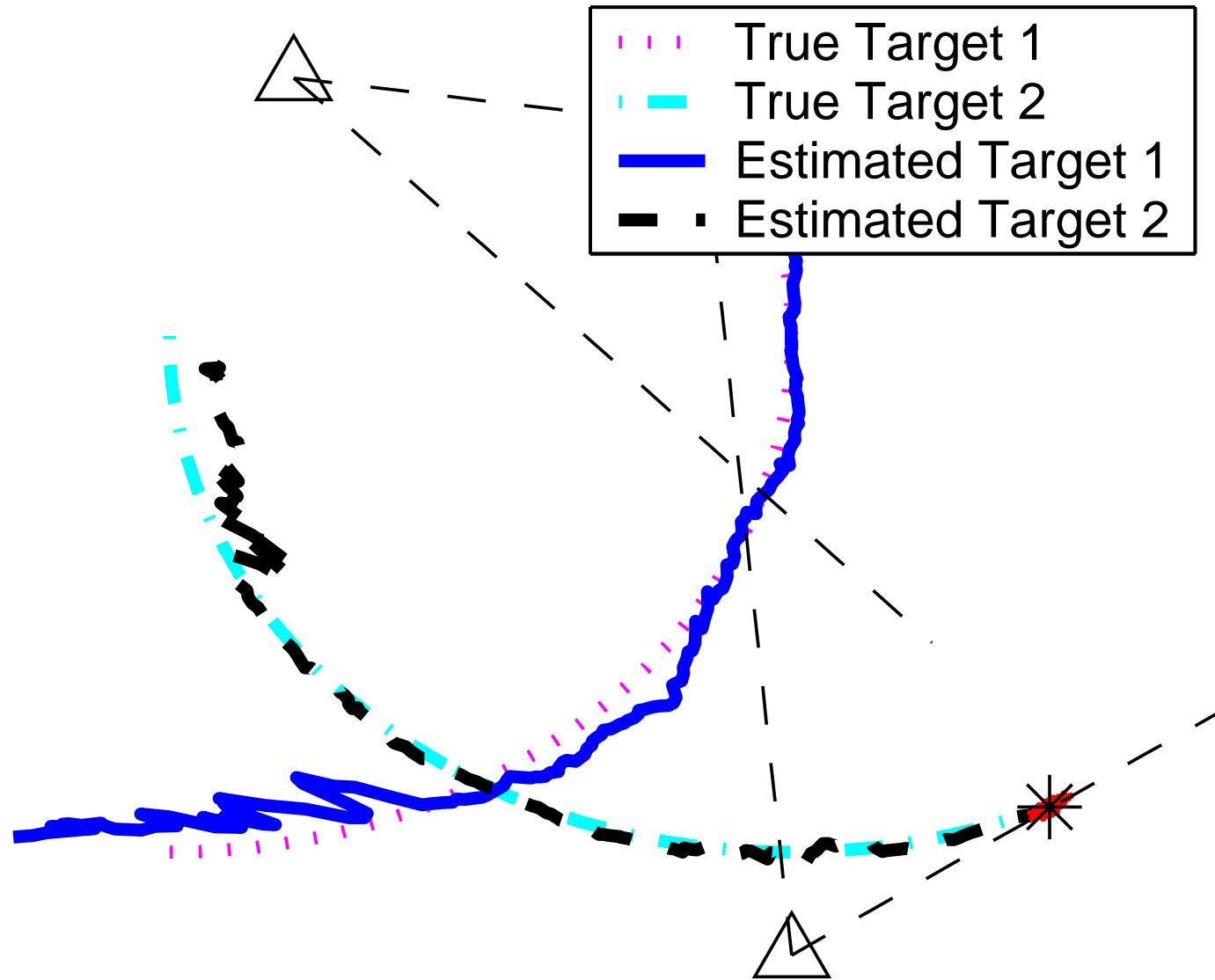




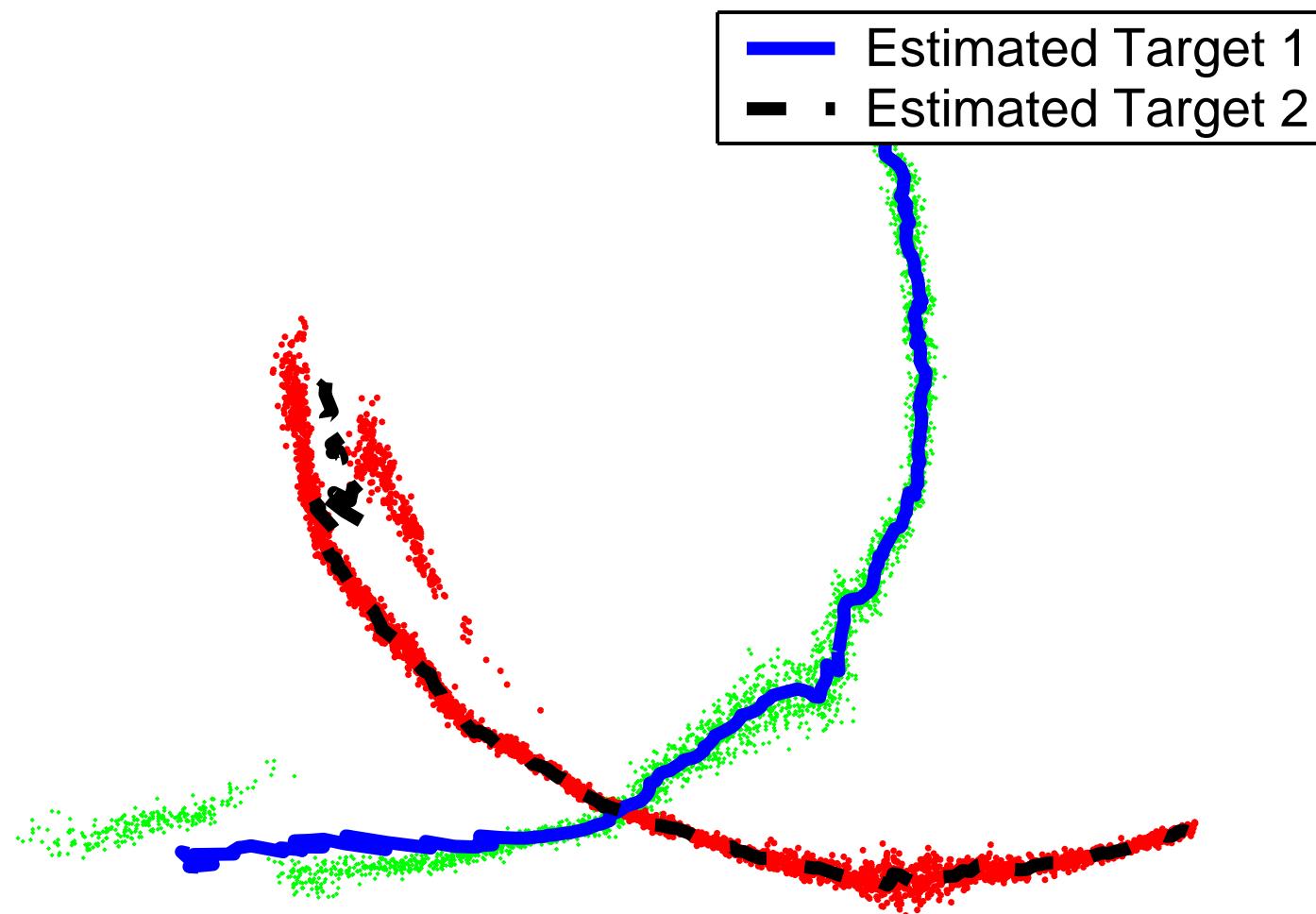




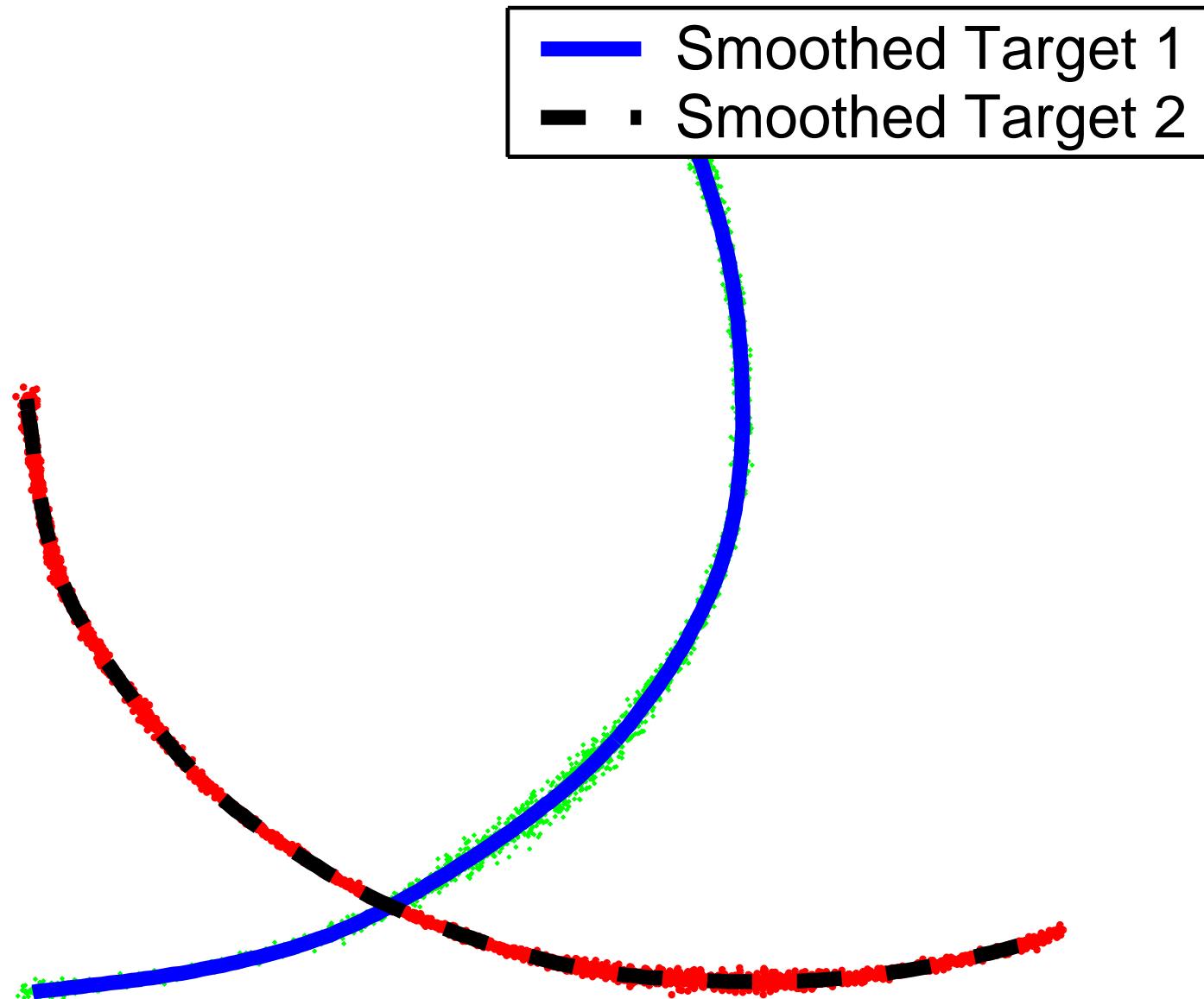




## Application: Multiple Target Tracking [Filtered]



## Application: Multiple Target Tracking [Smoothed]



# Summary

- The proposed RBMCDA is based on computing approximate solution to the joint data association and state estimation problem by a particle filter
- Efficiency is ensured by using Rao-Blackwellization of the particle filter, such that only the data associations need to be sampled
- The resulting algorithm is a combination of Kalman filtering of target states and particle filtering of data associations
- Kalman filter can be replaced by Extended Kalman Filter (EKF) or Unscented Kalman Filter (UKF) if the model is non-linear