Rao-Blackwellized Particle Filter for Tracking Unknown Number of Targets in Clutter

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Introduction

- Rao-Blackwellized particle filtering based algorithm for tracking an unknown number of targets.
- The algorithm is based on formulating probabilistic stochastic process models for target states, data associations, and birth and death processes.
- The tracking of these stochastic processes is implemented using sequential Monte Carlo sampling, and the efficiency of the sampling is improved by using Rao-Blackwellization.

Multiple target tracking considers problems where the purpose is to estimate the dynamic states (positions, velocities, accelerations) of multiple targets based on indirect sensor measurements. The problem can be roughly divided into the following sub-problems, which should be solved jointly:

- Single target tracking: The classical problem of tracking single target with multiple sensors.
- Data association: Given a measurement, which target produced it, if any?
- Unknown number of targets: How many targets are there at the scene?

Rao-Blackwellization

The estimation problem is solved using a *Rao-Blackwellized particle filter*:

- We approximate the posterior distribution of the measurement conditional state process by a *particle filter*, where the distribution is approximated by a weighted set of particles.
- We use *Rao-Blackwellization* or *marginalization*, where we integrate out the single target tracking sub-problem, which improves the particle filter efficiency because less Monte Carlo samples (i.e., particles) are needed.
- In practice, the marginalization is performed such that we solve the single target tracking subproblems in each particle conditional to the data association history in the particle.
- In the single target tracking subproblem we can use any *classical single target tracking algorithms* such as Kalman filter, extended Kalman filter (EKF), unscented Kalman filter (UKF) or interacting multiple models (IMM).
- The classical methods tend work very well (even better than particle filters) in the single target tracking problems, and in our approach we use the *particle filter* only to the *non-linear part* (data associations / number of targets) and *classical Gaussian approximation based filters* to the *almost linear part* (single target tracking).

Target Dynamics

Because the sensor measurements arrive at *irregular intervals* it is necessary to use *continuous-discrete filtering*, where the target dynamics are modeled as *stochastic differential equations* (i.e., Itô processes)

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{f}(\mathbf{x}, t) + \mathbf{L}(t)\,\mathbf{w}(t). \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state. We approximate the non-linear *Itô process* by a *Gaussian process* using the *extended Kalman filter (EKF)* or the *unscented Kalman filter (UKF)*. The *interacting multiple models (IMM)* algorithm could be also applied for generating a mixture of Gaussians approximation.

Sensor Measurements

The sensor measurements $\mathbf{z}_k \in \mathbb{R}^s$, *given the data associations*, are modeled as

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}(t_k), t) + \mathbf{r}_k, \quad (2)$$

where $\mathbf{r}_k$ is a Gaussian random variable. In our approach non-linear models are approximated by *EKF or UKF*, which produce *Gaussian approximation* to the single target (data association conditional) posterior.

Note that due to the *continuous-discrete approach* several *time steps* k may occur at the *same time instance* $t_k = t_{k+1} = \ldots = t_{k+m}$ and thus we may always assume at most one measurement at a time step.

Data Associations and Clutter

Data associations are modeled with *contact variable* c_k, which holds the index of the target, which is associated with the measurement $\mathbf{z}_k$. The contacts are modeled as *m'th order Markov chain*

$$p(c_k \mid c_{k-1}, \ldots, c_{k-m}). \quad (3)$$

With this, we are able to model the following:

- The targets may have a certain known *detection probability* p_d.
- Some of the measurements are *false alarms* and the number of these *clutter measurements* per time step is Poisson distributed.
- Maximum of *one measurement per target per time instance* is possible due to physical (sensor) constraints.

These can be modeled by first constructing the joint model $p(c_k, c_{k-1}, \ldots, c_{k-m})$ and then computing analytically the conditional distributions.

Unknown Number of Targets

The unknown number of targets is handled by the following model:

- At time of a measurement, target *birth may happen with probability* p_b.
- Target birth may only happen *jointly with a data association* with the newborn target.
- After associating a measurement with a target, the *life time t_d (or time to death) of the target* has the probability density

$$t_d \sim p(t_d), \quad (4)$$

  which can be, for example, an exponential or *gamma distribution*.

Example: Bearings Only Tracking in Presence of Clutter

- Dynamic model is a Wiener velocity model.
- Measurement model for target j is:

$$\hat{\theta}_k = \arctan\left(\frac{y_{j,k} - s_y}{x_{j,k} - s_x}\right) + r_k, \quad (5)$$

  where r_k is a Gaussian random variable and (s_x, s_y) is the position of the sensor.
- Target detection probability $p_d = 80\%$.
- Number of clutter measurements per time step is Poisson with mean 5.

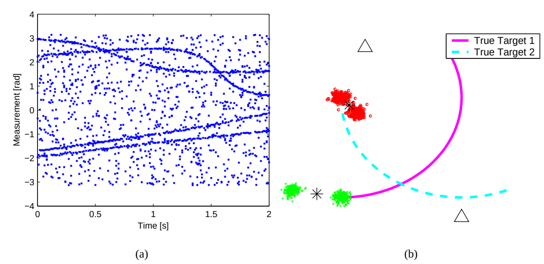

Figure 1: (a) Measurement data. (b) The sensor locations and prior distributions.

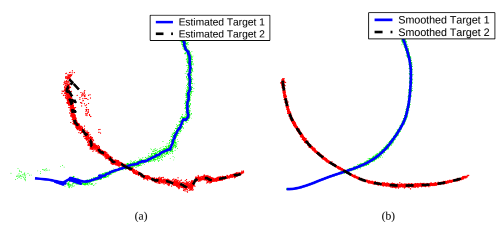

Figure 2: (a) Filter estimates for each time step. (b) Smoother estimates for each time step.

Example: Unknown Number of 1D Signals

- Dynamic model is a Wiener velocity model.
- Measurement model for target j is:

$$y_{k,j} = x^{(j)}(t_k) + r_k. \quad (6)$$

- Every measurement has 1% change of being a corrupted measurement uniformly distributed on the area $[-5, 5]$.
- The prior probability of birth $p_b = 1/100$.
- A priori time to death t_d has the gamma distribution $t_d \sim \mathrm{Gamma}(t_d \mid 2, 1)$.

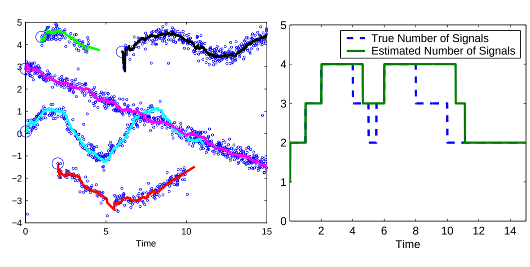

Figure 3: (a) Filtering result. (b) Estimated number of signals.