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A LINEAR STOCHASTIC STATE SPACE MODEL FOR ELECTROCARDIOGRAMS

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ABSTRACT

This paper proposes a linear stochastic state space model for electrocardiogram signal processing and analysis. The model is obtained as a discretized version of Wiener process acceleration model. The model is combined with a fixed-lag Rauch-Tung-Striebel smoother to perform on-line signal denoising, feature extraction, and beat classification. The results indicate that the proposed approach outperforms a conventional FIR filter in terms of improved signal-to-noise ratio, and that the approach can be used for highly accurate on-line classification of normal beats and premature ventricular contractions. The benefits of the model include the possibility to use closed-form solutions to the optimal filtering and smoothing problems, quick adaptation to sudden changes in beat morphology and heart rate, simple and fast initialization, preprocessing-free operation, intuitive interpretation of the system state, and more.

Index Terms— Electrocardiography, state space model, Bayesian smoothing, signal denoising, feature extraction, beat classification.

1. INTRODUCTION

Stochastic state space models (SSMs) and Bayesian filtering have been used successfully in many applications to electrocardiogram (ECG) signal processing and analysis. A particularly popular group of SSMs is based on the dynamical model introduced in [1], originally designed for synthetic ECG generation. This dynamical model has found use in noise removal [2–7], lossy compression [8], ECG contaminant filtration [9], premature ventricular contraction (PVC) detection [10, 11], atrial fibrillation modeling [12], beat segmentation [13, 14], and more. The dynamical model uses five Gaussian functions to represent the P, Q, R, S, and T waves that can usually be seen in the ECG during a single heartbeat.

Despite the many advantages of the dynamical model [1], there are a number of drawbacks in using it for ECG signal processing and analysis. Firstly, the model is highly non-linear, necessitating the use of approximative solutions to the optimal filtering equations. Secondly, the model cannot adapt itself to sudden changes in beat morphology that may occur due to PVCs, bundle branch block (BBB) beats, or paced beats. Thirdly, the model requires careful initialization of the model parameters, typically by combining a priori information with non-linear optimization.

In [15], an adaptive Kalman filter is used with a linear SSM to allow changes in beat morphology. This approach relies on beat averaging and assumes the beats to be normally distributed around their mean. A particular issue with this approach is that it requires extensive preprocessing, involving low-pass filtering, high-pass filtering, notch filtering, QRS detection, beat separation, and principal component analysis.

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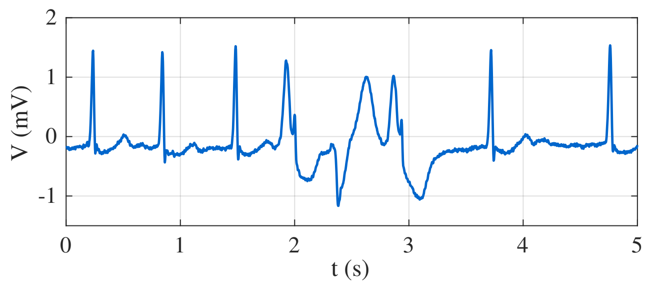

Fig. 1. An example ECG segment, extracted from record 221 in the MITDB. This segment features atrial fibrillation and a short episode of ventricular tachycardia. Note the large variations in beat morphology and the heart rate.

The shortcomings mentioned in the previous paragraphs have been addressed in [16, 17]. In these papers, the authors propose to use interacting multiple models (IMMs), each of which is a polynomial of order 1, 2, or 3. However, the system state in [16, 17] is defined as a set of polynomial coefficients whose interpretation with respect to the signal is rather complicated. Also, the initial mode probabilities and transition probabilities for the IMMs need to be specified. Despite the complex structure of the IMMs, only one polynomial coefficient is used for beat classification.

In this paper we propose to use a linear stochastic state space model, based on Wiener process acceleration model, to avoid the challenges related to the previous approaches. The use of the proposed model is demonstrated in ECG denoising, feature extraction, and beat classification, all of which are performed on-line. The advantages of the proposed model include: (1) the possibility to use closed-form solutions to the optimal filtering and smoothing problems; (2) quick adaptation to sudden changes in beat morphology and heart rate; (3) simple and fast initialization; (4) preprocessing-free operation; (5) intuitive interpretation of the system state; (6) avoidance of defining mode transition probabilities; (7) ease of online implementation; and (8) versatility.

The structure of this paper is as follows. The remainder of this section describes the proposed state space model and its advantages. The second section presents the materials and methods, the third section illustrates the results, and the fourth and fifth section comprise a discussion and a conclusion, respectively.

1.1. The proposed state space model

We propose to use a disretized version of the continuous Wiener process acceleration (CWPA) model [18] to ECG signal processing and analysis. The CWPA model provides a natural representation of a moving object's trajectory. However, it can also be used to represent the dynamics of a continuous ECG signal, as will be described within this subsection.

In the CWPA model, the system state is defined as $\mathbf{x}(t) = [s(t) \;\; \dot{s}(t) \;\; \ddot{s}(t)]^\top$ where $s(t) \in \mathbb{R}$ is the displacement of the object or signal at time $t \in \mathbb{R}$. The derivatives $\dot{s}$ and $\ddot{s}$ represent the velocity and the acceleration, respectively. The dynamics of the model are determined by the differential equation

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{F}\mathbf{x}(t) + \mathbf{L}w(t) \tag{1}$$

where

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{2}$$

and $w(t)$ is zero-mean white noise with power spectral density q. Eqns. (1) and (2) imply that the derivative of the acceleration, $\dddot{s}$, is driven by white noise. Hence, the acceleration, which is the integral of $\dddot{s}$, is a Wiener process.

The CWPA model defines a continous-time linear time-invariant system. Thus, given a time step Δt, the discretized versions of $\mathbf{F}$ and $\mathbf{Q}$ are obtained [18, 19] as

$$\mathbf{F}_k = \exp(\mathbf{F}\Delta t)$$

and

$$\mathbf{Q}_k = \int_0^{\Delta t} \exp\left(\mathbf{F}(\Delta t - \tau)\right) \mathbf{L}\mathbf{Q}\mathbf{L}^\top \exp\left(\mathbf{F}(\Delta t - \tau)\right)^\top d\tau,$$

respectively. The definitions of $\mathbf{F}$ and $\mathbf{L}$ in (2) allow $\mathbf{F}_k$ and $\mathbf{Q}_k$ to be evaluated in closed form, yielding

$$\mathbf{F}_k = \begin{bmatrix} 1 & \Delta t & \frac{(\Delta t)^2}{2} \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

and

$$\mathbf{Q}_k = \begin{bmatrix} \frac{1}{20}(\Delta t)^5 & \frac{1}{8}(\Delta t)^4 & \frac{1}{6}(\Delta t)^3 \\ \frac{1}{8}(\Delta t)^4 & \frac{1}{3}(\Delta t)^3 & \frac{1}{2}(\Delta t)^2 \\ \frac{1}{6}(\Delta t)^3 & \frac{1}{2}(\Delta t)^2 & \Delta t \end{bmatrix} q. \tag{4}$$

The discretized version of Eqn. (1) then reads as

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \mathbf{q}_{k-1}, \tag{5}$$

where $\mathbf{x}_k = [s_k \;\; \dot{s}_k \;\; \ddot{s}_k]^\top$ and $\mathbf{q}_{k-1}$ is a process noise term with covariance $\mathbf{Q}_k$.

Interestingly, one can also arrive at the matrix $\mathbf{F}_k$ by considering the Taylor series representation of the continous-time signal. If s is an analytic function, then

$$s(t) = \sum_{n=0}^{\infty} \frac{s^{(n)}(t_0)}{n!}(t - t_0)^n.$$

By taking only a finite number of terms from the series, one can obtain arbitrarily accurate approximations to $s(t)$; in that case only a finite number of derivatives need to exist. In particular, using only two terms yields the linear approximation

$$s(t) \approx s(t_0) + \dot{s}(t_0)(t - t_0),$$

and using three terms yields the quadratic approximation

$$s(t) \approx s(t_0) + \dot{s}(t_0)(t - t_0) + \frac{\ddot{s}(t_0)}{2}(t - t_0)^2.$$

These approximations can be found in the product $\mathbf{F}_{k-1}\mathbf{x}_{k-1}$ by noting that $\Delta t = t - t_0$.

The state space model (3)–(5) allows optimal filtering and smoothing [20]. In order to enable closed-form solution, we assume that $\mathbf{q}_{k-1}$ is normally distributed, and define the measurement model as

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{r}_k,$$

where $\mathbf{y}_k$ is the measurement, $\mathbf{H} = [1 \;\; 0 \;\; 0]$, and $\mathbf{r}_k \sim \mathrm{N}(\mathbf{0}, \mathbf{R}_k)$ with some covariance $\mathbf{R}_k$. Note that although the model (3)–(5) is non-linear with respect to the time, it is linear with respect to the system state. We refer to (3)–(5) as the discretized Wiener process acceleration (DWPA) model.

The DWPA model has been considered, for example, in [18] and [19], but to our best knowledge there are no publications so far on its use in ECG signal processing and analysis. Although the SSM in [15] is also linear, it differs from (3)–(5) by two important factors: firstly, the time step k in [15] corresponds to an entire heartbeat, as opposed to an individual sample; secondly, the signal dynamics in [15] follow a random walk, whereas the DWPA model uses a double-integrated Wiener process.

1.2. Advantages of the proposed model

The formulation of the DWPA model provides many advantages over the non-linear models (NLMs) in [2]– [14], the linear random walk (LRW) model in [15], and the IMMs in [16, 17].

In comparison to the NLMs, the DWPA model allows the use of closed-form solutions to the optimal filtering and smoothing problems. More specifically, the structure of the dynamic and measurement equations combined with the assumption about the process and measurement noises enable the use of the ordinary Kalman filter (KF), which is the optimal solution in the linear Gaussian case. Similarly, the proposed model enables the use of the Rauch-Tung-Striebel smoother (RTSS), which is the optimal solution to the linear-Gaussian smoothing problem. Both the KF and the RTSS algorithm can be expressed in closed form without the need for linear approximations as when using the extended Kalman filter (EKF), or for computationally expensive sampling as when using particle filters (PFs). Also, the proposed model eliminates the need to compute any derivatives as with the EKF, which may be cumbersome and error-prone, or to specify any sampling distributions as with the PFs.

Another advantage of the DWPA model over the NLMs, as well as the LRW, is its ability to adapt itself quickly to sudden changes in beat morphology. Since the NLMs and the LRW model are based on normal beat templates, they are unlikely to perform well in the presence of single PVCs, BBB beats, paced beats, or false QRS detections [5]. In contrast, the proposed model makes no assumptions about the expected beat morphology, making the model much more flexible and robust. The model is also insensitive to fluctuations in the heart rate due to atrial fibrillation or other arrhythmias. An example of highly varying beat morphology and heart rate is shown in Fig. 1.

The third benefit of the DWPA model is simple and fast initialization: the only parameters to be tuned are the noise variances q and $\mathbf{R}_k$. By contrast, the NLMs also require the estimation of the angular rate ω as well as the Gaussian functions' locations θ_i, amplitudes α_i, and widths β_i. Typically, $i \in \{P, Q, R, S, T\}$, and the estimation involves non-linear optimization where the initial values are again to be specified.

The fourth advantage of the proposed model is preprocessing-free operation. Whereas the LRW model requires low-pass filtering, high-pass filtering, notch filtering, QRS detection, beat separation, and principal component analysis, the DWPA model can be used directly on raw ECG data.

The fifth and sixth benefit of the DWPA model are the ease of interpretation of the system state and the fact that there is no need for manual definition of probabilities, respectively. In [16, 17], the system state consists of polynomial coefficients and the user has to define the mode transition probabilities a priori. However, in the proposed approach the system state simply consists of the filtered signal and its derivatives, providing a much more intuitive interpretation; also, since there is no IMM structure, no transition probabilities need to be defined either.

The seventh asset of the proposed model is its ease of on-line implementation. As opposed to the NLMs and the LRW model, the DWPA model can be used for signal denoising without any knowledge of the QRS complex locations or the beat boundaries, whose estimation is typically done in off-line mode. In fact, the DWPA model can be used to detect the QRS complexes and the beat boundaries as a side-product of denoising the raw ECG signal in an on-line manner. This versatility is the eighth benefit; the model also lends itself to beat classification and clustering, since the signal derivatives can be used as features to distinguish between different beat types. Section 2 provides examples of using the model for signal denoising, feature extraction, and beat classification.

2. MATERIALS AND METHODS

In order to assess the usability of the DWPA model, a number of experiments was conducted on a publicly available dataset. In each experiment, artificial Gaussian white noise was added to the signal, yielding a signal-to-noise ratio (SNR) of 20, 15, 10, or 5 dB. The original signal, although not completely noise-free, was used as the golden truth.

The noisy signal was fed into a Rauch-Tung-Striebel (RTS) smoother that utilized the DWPA model. The smoother was implemented with a fixed lag to enable on-line processing. The output of the smoother was used to estimate the original signal and the derivatives, thereby enabling signal denoising, feature extraction, and on-line beat classification. In addition, two conventional denoising methods were implemented for comparison. The MATLAB software was used in all experiments.

2.1. The dataset

The experiments were made on the MIT-BIH Arrhythmia Database (MITDB) [21, 22]. The MITDB is a benchmark dataset for designing arrhythmia detection algorithms. However, it also lends itself to other purposes due to the many examples of normal and abnormal beats and rhythms. The dataset contains a total of 48 records, each of which has two channels and is about 30 minutes in duration. The records have been annotated for beats and rhythm by two cardiologists. The records contain examples of normal beats, BBB beats, PVCs, paced beats, and more.

2.2. The Rauch-Tung-Striebel smoother

The RTS smoother [23] is the closed-form solution of the optimal smoothing problem [20] when the state-space and measurement models are linear and the process and measurement noises are normally distributed: in probabilistic notation,

$$\begin{aligned} p(\mathbf{x}_k \mid \mathbf{x}_{k-1}) &= \mathrm{N}(\mathbf{x}_k \mid \mathbf{F}_{k-1}\mathbf{x}_{k-1}, \mathbf{Q}_{k-1}), \\ p(\mathbf{y}_k \mid \mathbf{x}_k) &= \mathrm{N}(\mathbf{y}_k \mid \mathbf{H}_k\mathbf{x}_k, \mathbf{R}_k), \end{aligned}$$

where $\mathbf{x}_k, \mathbf{F}_{k-1}, \mathbf{Q}_{k-1}, \mathbf{y}_k, \mathbf{H}_k$ and $\mathbf{R}_k$ are defined as in subsection 1.1. The smoother computes the distibution

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:T}) = \mathrm{N}(\mathbf{x}_k \mid \mathbf{m}_k^{\mathrm{s}}, \mathbf{P}_k^{\mathrm{s}}),$$

thereby giving an estimate of the system state (the mean $\mathbf{m}_k^{\mathrm{s}}$) and its uncertainty (the covariance $\mathbf{P}_k^{\mathrm{s}}$). Above, $T > k$, implying that future observations are used to estimate the current state. In practice, a number of observations $1 : T$ is first obtained, followed by the estimation of a past state $\mathbf{x}_k$.

The RTSS algorithm can be summarized as follows [20]:

$$\begin{aligned} \mathbf{m}_{k+1}^- &= \mathbf{F}_k\mathbf{m}_k, \\ \mathbf{P}_{k+1}^- &= \mathbf{F}_k\mathbf{P}_k\mathbf{F}_k^\top + \mathbf{Q}_k, \\ \mathbf{G}_k &= \mathbf{P}_k\mathbf{F}_k^\top \left(\mathbf{P}_{k+1}^-\right)^{-1}, \\ \mathbf{m}_k^{\mathrm{s}} &= \mathbf{m}_k + \mathbf{G}_k\left(\mathbf{m}_{k+1}^{\mathrm{s}} - \mathbf{m}_{k+1}^-\right), \\ \mathbf{P}_k^{\mathrm{s}} &= \mathbf{P}_k + \mathbf{G}_k\left(\mathbf{P}_{k+1}^{\mathrm{s}} - \mathbf{P}_{k+1}^-\right)\mathbf{G}_k^\top. \end{aligned}$$

Above, $\mathbf{m}_{k+1}^-$ and $\mathbf{P}_{k+1}^-$ are the predicted mean and covariance, respectively; they can be obtained by ordinary Kalman filtering [20]. Once the predictions are available, the RTSS algorithm is run backwards in time, from $k = T$ to $k = 1$.

2.3. Fixed-lag smoothing

In order to facilitate on-line processing, the RTSS can be implemented with a fixed lag. That is, the predicted mean and covariance are computed at time step k using a Kalman filter, after which the RTSS is run from k to $k - L + 1$ where L is the lag length of the smoother. This way the number of computations remains constant even though the number of observations grows without a bound. At each k, the fixed-lag smoother produces a sequence of state estimates, but only the one related to $k - L + 1$ is used.

2.4. Signal denoising

The denoising capability of the proposed approach was investigated in a two-phase experiment. The first phase was qualitative and aimed at assessing the denoising results visually. The second phase was quantitative with the results measured as improvements in signal-to-noise ratio (ISNR), defined as

$$\mathrm{ISNR} = \mathrm{SNR}_{\mathrm{out}} - \mathrm{SNR}_{\mathrm{in}}.$$

In both phases, the input signals were chosen from the MITDB so that they contained only a small amount of noise.

In the quantitative phase, the signal segments were chosen to represent different rhythms and beat types, such as normal sinus rhythm, single PVCs, ventricular bigeminy, atrial fibrillation, and ventricular tachycardia. The length of each segment was 60 seconds. The DWPA approach was compared to two conventional denoising methods: a band-pass finite impulse response (FIR) filter

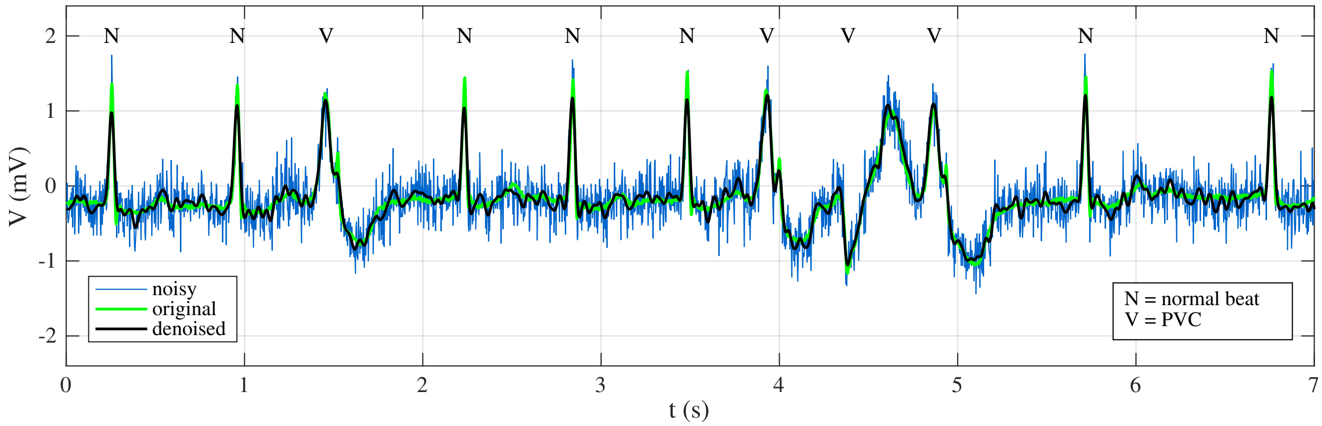

Fig. 2. An example of ECG denoising with the proposed state space model and an RTS smoother. The noisy signal was obtained by adding artificial Gaussian white noise to record 221 from the MITDB. The input SNR is 5 dB. Note the ability of the model to adapt itself to the abnormal morphology of the PVCs.

Table 1. Denoising results on six records from the MITDB

Record	Time segment (s)	20 dB			15 dB			10 dB			5 dB		
		FIR	WD	DWPA	FIR	WD	DWPA	FIR	WD	DWPA	FIR	WD	DWPA
115	0-60	-8.97	**6.57**	5.76	-3.98	**7.91**	6.42	0.30	**8.79**	7.02	3.73	**9.55**	7.76
116	240-300	-1.61	**6.29**	5.79	2.44	**7.70**	6.68	5.21	**8.87**	7.64	6.44	**9.78**	8.63
119	0-60	-1.77	**6.84**	6.48	2.18	**8.44**	7.32	5.09	**9.69**	8.17	6.54	**10.61**	9.01
201	480-540	-3.55	**4.71**	4.50	0.72	**7.11**	5.43	3.96	**8.78**	6.39	6.09	**9.80**	7.28
203	180-240	-10.66	4.58	**5.08**	-5.78	**7.19**	6.18	-1.17	**9.13**	7.27	2.73	**10.34**	8.19
205	270-330	-3.89	3.43	**5.42**	0.39	5.86	**6.23**	3.89	**7.59**	6.99	5.76	**8.87**	7.77
Average		-5.08	5.40	**5.51**	-0.67	**7.37**	6.38	2.88	**8.81**	7.25	5.22	**9.83**	8.11

The dB columns (20 dB to 5 dB) refer to input SNRs; the results are given in dBs of SNR improvement.

and wavelet denoising (WD). The FIR filter was of first-order Butterworth type, and had a passband of 0.4–40 Hz as in [5]. The WD method utilized the Symlet 4 wavelet, decomposition level 4, soft thresholding, and threshold rescaling.

2.5. Feature extraction and classification

The feature extraction capabilities of the proposed approach were investigated in a beat classification framework. Motivated by the findings in [16, 17], we hypothesized that the second state component of the DWPA model (that is, the signal derivative), is a feasible feature for distinguishing between different beat types. In [16, 17] the investigated beat types were the normal sinus beat, the left BBB beat, the right BBB beat, the ventricular escape beat, and the junctional escape beat. In our experiment, we decided to address the problem of distinguishing between normal beats and PVCs.

The feature extraction was performed in an on-line manner together with the signal denoising. Five records from the MITDB, selected because they only contain normal beats and PVCs, were used in this experiment. Artificial Gaussian white noise was added to the signals, yielding an initial SNR of 20 dB. Only the first channel (modified lead II) was used.

To perform beat classification, we again chose an on-line approach. The idea in the approach is to use the derivative of the first beat as a template and to compare the derivatives of the subsequent beats to this template. If the mean squared error (MSE) between the two exceeds a given threshold, the subsequent beat is classified as a PVC; otherwise, it is classified as a normal. The beat annotations in the MITDB were used for defining the locations of the QRS complexes, but this step could be replaced by extending the DWPA model as described in Section 4. The time windows for feature extraction were set from QRS - 75 ms to QRS + 425 ms after some experimenting.

The chosen classification approach required the use of an additional beat template for records 106 and 230. The reason for this is that the normal beats in these records undergo substantial changes in their morphology. It would, of course, be possible to utilize a more sophisticated approach with a proper classifier and a training set of hundreds of beats, but at this point the aim was just to demonstrate the usability of the DWPA model for on-line feature extraction and classification with a minimal amount of training.

3. RESULTS

The results of the signal denoising experiment are presented in Table 1. The results indicate that the DWPA approach outperforms the FIR filter in all test cases. The best overall performance is given by WD, but the DWPA still yields the highest average ISNR for an input SNR

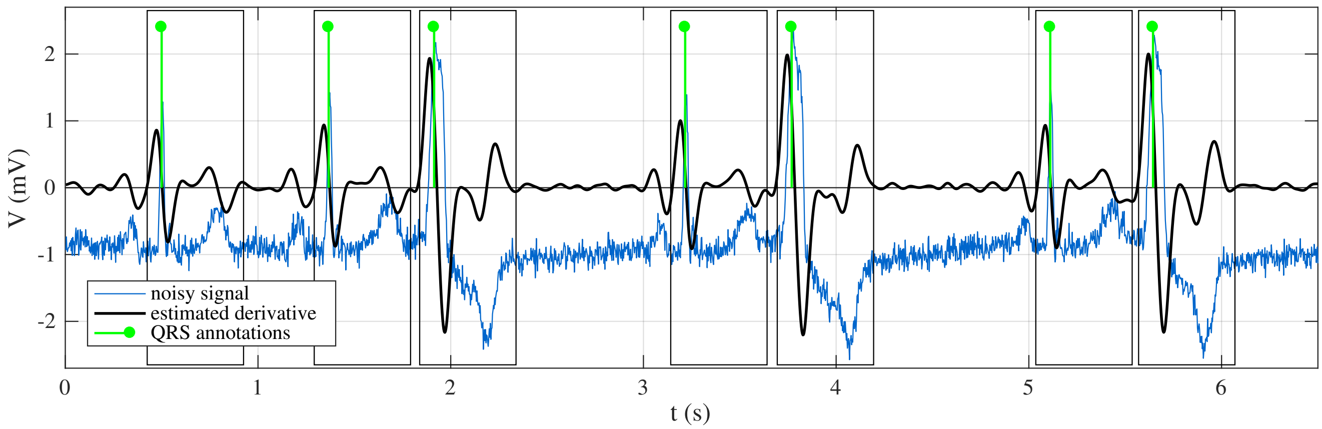

Fig. 3. An example of feature extraction with the proposed state space model. The noisy signal was obtained by adding artificial Gaussian white noise to record 119 from the MITDB, giving an input SNR of 20 dB. The signal exhibits ventricular bigeminy starting at about 1.8 s: the 3rd, 5th, and 7th beat are PVCs. The rectangles represent time windows from QRS - 75 ms to QRS + 425 ms. Note the difference in the derivatives between the normal beats and the PVCs, the smoothness of the estimates, and the fact that the features are unaffected by the signal baseline being non-zero. The estimates have been scaled for clarity.

of 20 dB.

Fig. 2 shows an example of signal denoising with the DWPA model. As can be seen, the model is able to adapt itself to sudden changes is signal morphology, in this case due to PVCs.

The results of the feature extraction and classification experiment are listed in Table 2. For comparison, the table also shows the results reported in [10] on the same five records. Although the NLM based method [10] is slightly better for two records, both approaches perform excellently.

Fig. 3 shows an example of feature extraction with the DWPA model. It can be seen that, with a suitable set of parameters, the proposed approach yields smooth features that need not be corrected for a non-zero signal baseline.

Table 2. Classification results on five records from the MITDB

Record	N	V	Sayadi et al. [10]		DWPA	
			Se	PPV	Se	PPV
106	1507	520	**1.00**	**1.00**	0.99	0.99
119	1543	444	1.00	1.00	1.00	1.00
123	1515	3	1.00	1.00	1.00	1.00
221	2031	396	1.00	**1.00**	1.00	0.99
230	2255	1	1.00	1.00	1.00	1.00
Average	1770	273	1.00	1.00	1.00	1.00

N = number of normal beats, V = number of PVCs,
Se = sensitivity, PPV = positive predictive value

4. DISCUSSION

Modeling an ECG signal with Wiener process acceleration is a very different approach as compared to the use of the NLMs [2]– [14]. While the NLMs utilize a priori information about the typical ECG beat morphology, the DWPA model may be applied to an arbitrary signal. Although the use of a priori information is likely to provide superior results if the information is valid, in this paper we have specifically considered situations in which the typical assumptions do not necessarily hold.

It would, however, be possible to extend the present model by replacing the Wiener acceleration model with a more complicated linear stochastic state-space model as a state-space representation of a suitable Gaussian process regressor [24]. When combined with parameter estimation methods for state-space models [20] this would lead to a model that learns the shape of the signal automatically and hence provides an automatic learning of the prior information currently lacking in the model. The fixed-lag smoother inference could still be applied exactly analogously to the present approach. The price is, though, the decreased flexibility of the model and the requirement for an additional parameter estimation phase. For that reason, we prefer to use the DWPA as a generic and flexible model for ECG signals.

The classification results indicate that the DWPA model lends itself to classifying beats with excellent accuracy in records that only contain normal beats and PVCs. Even though the requirement for only two beat types may seem restricting, one should note that these results were obtained by on-line classification with a training set of just one or two beats and extra noise added to the already noisy signals.

As mentioned in Subsection 2.5, the beat annotations in the MITDB were used for defining the locations of the QRS complexes in the classification experiment. If desired, this step could be replaced by using a standard QRS detector such as the Pan-Tompkins (PT) algorithm [25]. In fact, it would be possible to implement the PT algorithm to use a modified version of the DWPA model: after all, the PT algorithm involves filtering, differentation, squaring, and computing a moving average (MA). The state in the DWPA model already contains the denoised signal and its derivative, the squaring operation is trivial, and the short-time memory required by the MA operation can be incorporated as extra states in the model. The estimates of $\dot{s}_k$ and $\ddot{s}_k$ could also be used for detecting beat boundaries, since a wave apex is associated with a zero-crossing of $\dot{s}_k$, and the type of the apex (positive or negative) is determined by the sign of $\ddot{s}_k$.

5. CONCLUSION

In this paper we have proposed to use a linear stochastic state space model for ECG signal processing and analysis. The proposed DWPA model is based on Wiener process acceleration model and provides numerous advantages over the previous approaches. The results show that the model can be used effectively for on-line denoising, feature extraction, and beat classification.

In the future we intend to extend the model to enable on-line QRS detection and wave delineation. The feature extraction capabilities of the model could be investigated further by using a larger training set together with proper classification algorithms such as support vector machines or Gaussian process classifiers. It would also be of interest to extend the classification framework to multiple beat types, such as BBBs, ventricular escape beats, and junctional escape beats.

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