Exercise Round 4 (22.11.2012).

Exercise 1. (1.5 order Itô–Taylor)

Simulate trajectories from the following SDE with 1.5 order strong Itô–Taylor series based method

$$dx = \tanh(x) \, dt + dB, \quad x(0) = 0,$$

where $B(t)$ is a standard Brownian motion, and compare the resulting histogram to the exact solution

$$p(x,t) = \frac{1}{\sqrt{2\pi t}} \cosh(x) \exp \left( -\frac{1}{2} t \right) \exp \left( -\frac{1}{2t} x^2 \right).$$

Exercise 2. (Milstein’s method)

Consider the following scalar SDE:

$$dx = -cx \, dt + gx \, dB$$

$$x(0) = x_0$$

where $a$, $g$ and $x_0$ are positive constants and $B(t)$ is a standard Brownian motion.

A) Check using the Itô formula that the solution to this equation is

$$x(t) = x_0 \exp \left( (-c - g^2/2) t + g B(t) \right)$$

Hint: $\phi(B(t), t) = x_0 \exp \left( (-c - g^2/2) t + g B(t) \right)$.

B) Simulate the equation using Milstein’s method with parameters $x_0 = 1$, $c = 1/10$, $g = 1/10$, and check that the histogram at $t = 1$ looks the same as obtained by simulating the above exact solution.

Exercise 3. (Gaussian approximation of SDE)

A) Form a Gaussian assumed density approximation to the SDE in Equation (1) on times $t \in [0, 5]$ and compare it to the exact solution. Compute the Gaussian integrals numerically on a uniform grid.

B) Form Gaussian assumed density approximation to the Equation (2) and numerically compare it to the histogram obtained in 2 B).