Exercise Round 6.

Exercise 1. (Optimal Importance Distribution)

Recall the following state space model from the Execise 3 of Round 1:

$$\begin{aligned} \mathbf{x}_k &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{x}_{k-1} + \mathbf{w}_{k-1} \\ y_k &= \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}_k + v_k \end{aligned} \tag{1}$$

where $\mathbf{x}_k = (x_k \;\; \dot{x}_k)^T$ is the state, y_k is the measurement, and $\mathbf{w}_k \sim \mathrm{N}(\mathbf{0}, \mathrm{diag}(1/10^2, 1^2))$ and $v_k \sim \mathrm{N}(0, 10^2)$ are white Gaussian noise processes.

A) Write down the Kalman filter equations for this model.

B) Derive expression for the optimal importance distribution for the model:

$$\pi(\mathbf{x}_k) = p(\mathbf{x}_k \,|\, \mathbf{x}_{k-1}, \mathbf{y}_{1:k}). \tag{2}$$

C) Write pseudo code for the corresponding particle filter algorithm (Sequential Importance Resampling algorithm). Also write down the equations for the weight update.

D) Compare the number of CPU steps (multiplications / additions) needed by the particle filter and Kalman filter. Which implementation would you choose for a real implementation?

Exercise 2. (Kalman Filter Based Importance Distribution)

Implement bootstrap filter to the model in Exercise 1 of Round 4. Also implement a sequential importance resampling filter, where you use one of the Kalman filters (EKF, UKF, GHKF or CKF) for forming the importance distribution. Note that you might want to use a small non-zero covariance as the prior of previous step instead of plain zero to get the filters work better.

Exercise 3. (Bearings Only Tracking with SIR)

Implement bootstrap filter and SIR with CKF importance distribution to the bearing only target tracking model in Exercises 4.3 and 5.3. Plot the results and compare RMSE values of different methods.