Exercise Round 5.

The answers to the exercises should be returned as follows:

- The deadline for exercise rounds 4–6 (there are 3 exercises on each round) is **May 21, 2010**.

The answers should be sent as email to the teacher (ssarkka@lce.hut.fi) in PDF form. When sending the email, please add "S-114.4202" or "1144202" to subject. The answers can also be returned on paper to the teacher.

Exercise 1. (Optimal Importance Distribution)

Recall the following state space model from the Execise 3 of Round 1:

$$\begin{aligned} \mathbf{x}_k &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{x}_{k-1} + \mathbf{w}_{k-1} \\ y_k &= \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}_k + v_k \end{aligned} \tag{1}$$

where $\mathbf{x}_k = (x_k \;\; \dot{x}_k)^T$ is the state, y_k is the measurement, and $\mathbf{w}_k \sim \mathrm{N}(\mathbf{0}, \mathrm{diag}(1/10^2, 1^2))$ and $v_k \sim \mathrm{N}(0, 10^2)$ are white Gaussian noise processes.

A) Write down the Kalman filter equations for this model.

B) Derive expression for the optimal importance distribution for the model:

$$\pi(\mathbf{x}_k) = p(\mathbf{x}_k \,|\, \mathbf{x}_{k-1}, \mathbf{y}_{1:k}). \tag{2}$$

C) Write pseudo code for the corresponding particle filter algorithm (Sequential Importance Resampling algorithm). Also write down the equations for the weight update.

D) Compare the number of CPU steps (multiplications / additions) needed by the particle filter and Kalman filter. Which implementation would you choose for a real implementation?

Exercise 2. (Unscented Kalman filter)

Derive and implement UKF to the model in Exercise 2 of Round 4, that is, to the same problem where you implemented EKF and SLF. Implement the UKF equations yourself, i,e., do not use the EKF/UKF toolbox or similar. Plot the results and compare the RMSE values of the methods. Also include the Matlab codes to the returned document.

Exercise 3. (Bearings Only Target Tracking with UKF and BF)

Implement UKF and BF (bootstrap filter) to the bearings only target tracking model in Exercise 3 of Round 4. You can use the EKF/UKF toolbox, if it helps. Plot the results and compare RMSE values of different methods. Also include the Matlab codes to the returned document.