Exercise Round 1.

The answers to the exercises should be returned as follows:

- The deadline for exercise rounds 1–3 (there are 3 exercises on each round) is **April 16, 2010**.

The answers should be sent as email to the teacher (ssarkka@lce.hut.fi) in PDF form. When sending the email, please add "S-114.4202" or "1144202" to subject. The answers can also be returned on paper to the teacher.

Exercise 1. (Linear Least Squares Estimation)

Assume that we have obtained n measurement pairs (y_k, x_k) from the linear regression model

$$y_k = a_1\, x_k + a_2, \qquad k = 1, \ldots, n. \tag{1}$$

The purpose is now to derive an estimate to the parameters a_1 and a_2 such that the following error is minimized (LS-estimate):

$$E(a_1, a_2) = \sum_{k=1}^{n} (y_k - a_1\, x_k - a_2)^2. \tag{2}$$

A) Define $\mathbf{y} = (y_1 \ \ldots \ y_n)^T$ and $\mathbf{a} = (a_1 \ a_2)^T$. Show that the set of Equations (1) can be written in matrix form

$$\mathbf{y} = \mathbf{X}\,\mathbf{a},$$

with a suitably defined matrix $\mathbf{X}$.

B) Write the error function in Equation (2) in matrix form in terms of $\mathbf{y}$, $\mathbf{X}$ and $\mathbf{a}$.

C) Compute the gradient of the matrix form error function and solve the LS-estimate of the parameter $\mathbf{a}$ by finding the point where the gradient is zero.

Exercise 2. (Installation of EKF/UKF Toolbox)

A) Download and install the EKF/UKF toolbox to some Matlab computer from the web page:

```
http://www.lce.hut.fi/research/mm/ekfukf/
```

B) Run the following demonstrations:

```
demos/kf_sine_demo/kf_sine_demo.m
demos/kf_cwpa_demo/kf_cwpa_demo.m
```

After running them read the contents of these files and try to understand how they have been implemented. Also read the documentations of functions kf_predict and kf_update (type e.g. “doc kf_predict” in Matlab).

Exercise 3. (Kalman filtering with EKF/UKF Toolbox)

A) Consider the following state space model:

$$\begin{aligned} \mathbf{x}_k &= \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{x}_{k-1} + \mathbf{w}_{k-1} \\ y_k &= \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}_k + v_k \end{aligned} \tag{3}$$

where $\mathbf{x}_k = (x_k \;\; \dot{x}_k)^T$ is the state, y_k is the measurement, and $\mathbf{w}_k \sim \mathrm{N}(\mathbf{0}, \mathrm{diag}(1/10^2, 1^2))$ and $v_k \sim \mathrm{N}(0, 10^2)$ are white Gaussian noise processes.

B) Simulate 100 step state sequence from the model and plot the signal x_k, signal derivative $\dot{x}_k$ and the simulated measurements y_k. Start from initial state drawn from zero-mean 2d-Gaussian distribution with identity covariance.

C) Use Kalman filter for computing the state estimates $\mathbf{m}_k$ using the following kind of Matlab-code:

```
m = [0;0];  % Initial mean
P = eye(2); % Initial covariance
for k = 1:100
   [m,P] = kf_predict(m,P,A,Q);
   [m,P] = kf_update(m,P,y(k),H,R);
   % Store the estimate m of state x_k here
end
```

Include the Matlab code to the PDF document that you return.

D) Plot the state estimates $\mathbf{m}_k$, the true states $\mathbf{x}_k$ and measurements y_k. Compute the RMSE error of using the first components of vectors $\mathbf{m}_k$ as the estimates of first components of states $\mathbf{x}_k$. Also compute the RMSE error that we would have if we used the measurements as the estimates.