

## Research on negative refraction and backward-wave media: A historical perspective

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In this presentation I will give a historical overview of the developments of understanding of the phenomenon of *negative refraction*. This text is a result of my findings in the literature (made by chance) and of discussions with many colleagues, and not a result of a systematic historical study. Here, I present some interesting observations about the history of this research area, not attempting to make any statements about priority of any discovery.

$$k \sin \varphi = k_1 \sin \varphi_1.$$

Однако последнее равенство удовлетворяется как при угле  $\varphi_1$ , так и при угле  $\pi - \varphi_1$ .

Требую по-прежнему, чтобы энергия во второй среде *оттекала* от границы раздела, мы приходим тогда к тому, что фаза должна *набегать* на эту границу и, следовательно, направление распространения преломленной волны будет составлять с нормалью угол

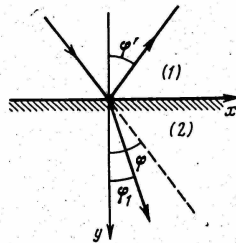


Рис. 12

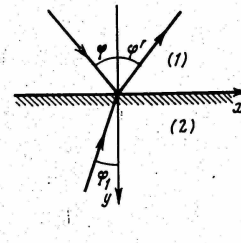


Рис. 13

$\pi - \varphi_1$ . Как ни непривычно такое построение, но, конечно, ничего удивительного в нем нет, ибо фазовая скорость еще ничего не говорит о направлении потока энергии.

ЧЕТВЕРТАЯ ЛЕКЦИЯ

(5.V 1944 г.)

Figure 1: Prof. L.I. Mandelshtam, photo 1940; An extract from his book [1]. The text reads: “...However, the last equation is satisfied not only at  $\varphi_1$ , but also at  $\pi - \varphi_1$ . Demanding as before that the energy in the second medium propagates *from* the boundary, we arrive to the conclusion that the phase must propagate *towards* the boundary and, consequently, the propagation direction of the refracted wave will make with the normal the angle  $\pi - \varphi_1$ . Although this derivation appears to be unusual, but of course there is no wonder, because the phase velocity still tells us nothing about the direction of the energy transfer.”

The earliest publication on negative refraction that came to my attention was not a journal paper, but lecture notes of Prof. L.I. Mandelshtam (1879-1944) from Moscow University, see Figure 1. He noticed that since the phase velocity does not have to have the same direction as the

power flow vector, “negative refraction” is possible. Mandelshtam [with a reference to Lamb (1904) who “gave examples of *fictional* one-dimensional media with negative group velocity] presented physical examples of structures supporting waves with negative group velocity [2], such as materials with periodically varying in space effective permittivity.

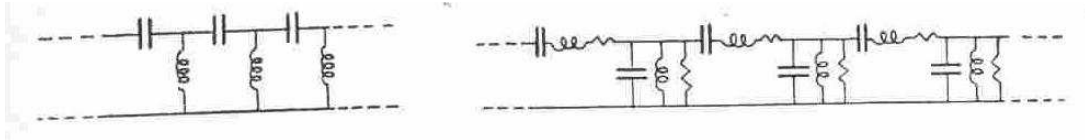


Figure 2: Backward-wave transmission lines from a paper by Malyuzhinets (1951) [3].

G.D. Malyuzhinets from the Institute of Radiotechnics and Electronics (Moscow) published a paper on the Sommerfeld radiation condition in backward-wave media in 1951 [3]. He noted that in such media the phase velocity of waves at infinity should point from infinity to the source. What was perhaps even more interesting, he considered as an example a one-dimensional artificial transmission line shown in Figure 2. Such structures are very actively studied at present [4, 5].

Materials with negative parameters as backward-wave materials were mentioned by D.V. Sivukhin in 1957 [6]: He noticed that media with negative parameters are backward-wave media, but had to state that “media with  $\epsilon < 0$  and  $\mu < 0$  are not known. The question on the possibility of their existence has not been clarified.”

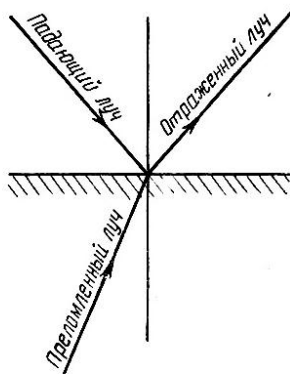


Рис. 14. Иллюстрация к закону преломления луча в среде с отрицательной дисперсией.

Figure 3: Negative refraction in periodical media, from a paper by Silin (1959) [8]. The text reads: “An illustration to the refraction law in a medium with negative dispersion.”

During the 60s, backward-wave structures were very much studied in connection with the design of microwave tubes, see e.g. [7]. Let me refer to an interesting paper by R.A. Silin (1959) [8], where the negative refraction phenomenon in periodical media was discussed (see Figure 3).

An important step was made by V.G. Veselago (also from Moscow) in 1967<sup>1</sup>, see Figure 4. Prof. Veselago made a systematic study of electromagnetic properties of materials with negative parameters and reported on his unsuccessful search for such media [9].

<sup>1</sup>Note that there is a misprint about the date of the original publication in the English translation of his paper. It was published in 1967, and not in 1964.

II. РАСПРОСТРАНЕНИЕ ВОЛН В ВЕЩЕСТВЕ С  $\epsilon < 0$  И  $\mu < 0$ .  
«ПРАВЫЕ» И «ЛЕВЫЕ» ВЕЩЕСТВА

Для того чтобы выявить электродинамические закономерности, существенно связанные со знаком  $\epsilon$  и  $\mu$ , следует обратиться к тем соотношениям, в которых  $\epsilon$  и  $\mu$  выступают раздельно, а не в виде произведения, как это имеет место в (1) — (3). Такими соотношениями являются прежде всего уравнения Максвелла и материальные уравнения

$$\left. \begin{aligned} \text{rot } \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \\ \text{rot } \mathbf{H} &= \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}, \\ \mathbf{B} &= \mu \mathbf{H}, \\ \mathbf{D} &= \epsilon \mathbf{E}. \end{aligned} \right\} (4)$$

Для плоской монохроматической волны, у которой все величины пропорциональны  $e^{i(kz - \omega t)}$ , выражения (4) и (4') сводятся к

$$\left. \begin{aligned} [\mathbf{kE}] &= \frac{\omega}{c} \mu \mathbf{H}, \\ [\mathbf{kH}] &= -\frac{\omega}{c} \epsilon \mathbf{E}. \end{aligned} \right\} (5)$$

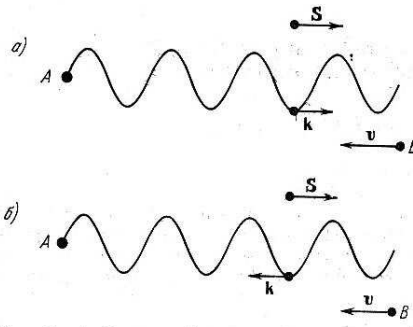
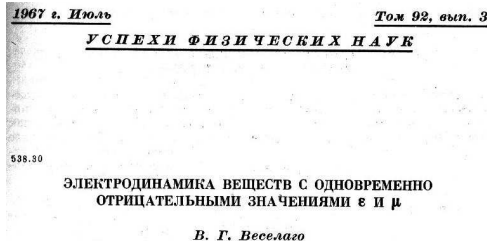
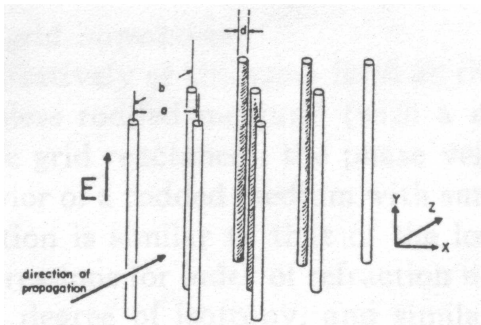


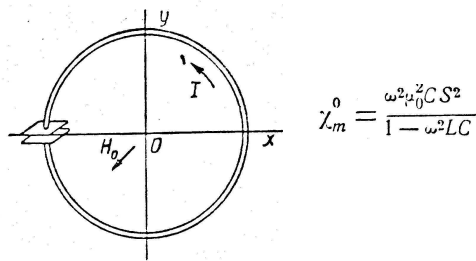
Рис. 1. а) Допплер-эффект в правом веществе. б) Допплер-эффект в левом веществе. Буквой А обозначен источник излучения, буквой В —

Figure 4: From the famous review paper by Prof. V.G. Veselago (1967) [9].



$$\epsilon_p = \epsilon_0 \left( 1 - \frac{\omega_p^2}{\nu^2 + \omega^2} + j \frac{\omega_p^2 \nu / \omega}{\nu^2 + \omega^2} \right)$$

Figure 5: Wire media in the 1960s [12].



$$\chi_m^0 = \frac{\omega^2 \nu_0^2 C S^2}{1 - \omega^2 LC}$$

Figure 6: Split rings in the 1950s [13].

It is interesting that both components of the first actual realization of a material with negative parameters [10] have been known for a long time, but no attempts to combine them have been made. The wire medium as an artificial dielectric was invented probably in the 1950s as a material for microwave lenses (J. Brown, 1953; W. Rotman, 1961, and many others) [11, 12], see Figure 5.

A split ring as a magnetic particle was shown in the well-known antenna text book by Schelkunoff and Friis [13], see Figure 6. Particles with loops of various shapes and in combination with other shapes were studied quite a lot in the 80s and 90s in connection with developments of artificial chiral materials for microwave applications. Polarizabilities of these bianisotropic particles in

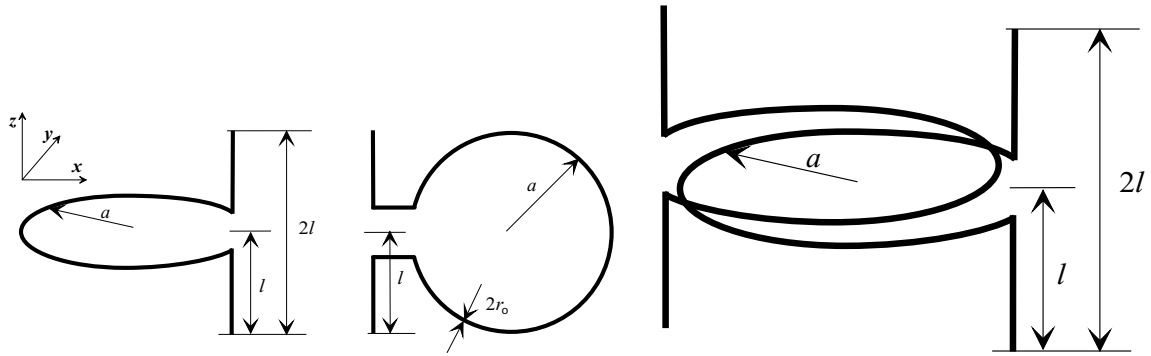


Figure 7: Chiral, omega, and double-helix inclusions of artificial media with magnetic- electric- and magnetoelectric response (1994–1997) [14, 15, 16]. Note that removing the straight-wire portions from the double helix shown on the right one arrives to the double split ring [17, 18].

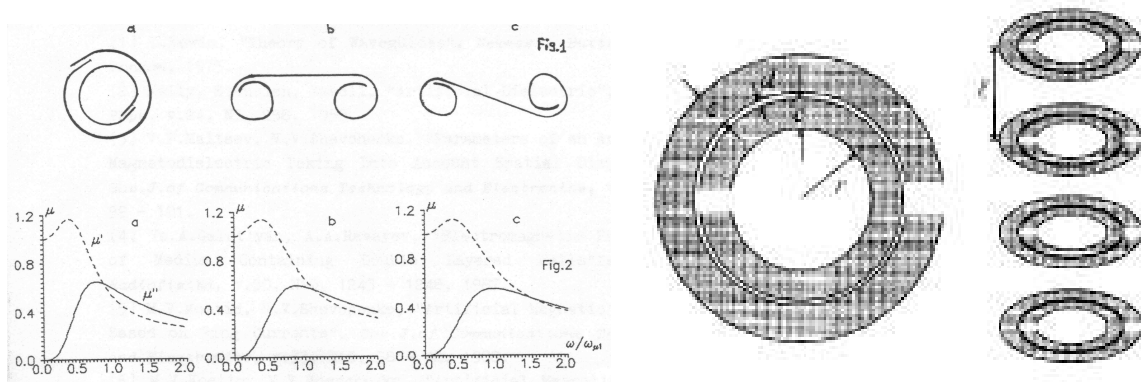


Figure 8: First double split rings [17, 18].

magnetic and electric fields were studied analytically, numerically, and experimentally. Figure 7 shows some inclusions of artificial bianisotropic materials [14, 15, 16]. The double split ring was used in the design of microwave absorbers (Figure 8, left [17]) and the same geometry with a very strong coupling between loops (Figure 8, right [18]) was used in the first realization of materials with negative parameters [10].

At radio frequencies, very strong magnetic response can be realized using variants of so called *Swiss rolls*. Some of such tubes and rolls are shown in Figure 9. The earliest known to me variant is shown on the left (suggested for NMR applications) [19, 20]. The Swiss roll introduced by J. Pendry [18] is shown in the middle, and the *metasolenoid* [21] is on the right. The last particle is not conducting along the axis, and its resonant frequency is determined primarily by the geometry of one period of the array.

Recent studies have shown that backward waves can propagate in materials with positive parameters provided that one of the materials is *chiral*. This result was established in papers [22, 23], see an illustration in Figure 10. So, not only the old studies of complex-shaped particles used in chiral composites, but also the knowledge about chiral media as such can be applied in the science of negative refraction.

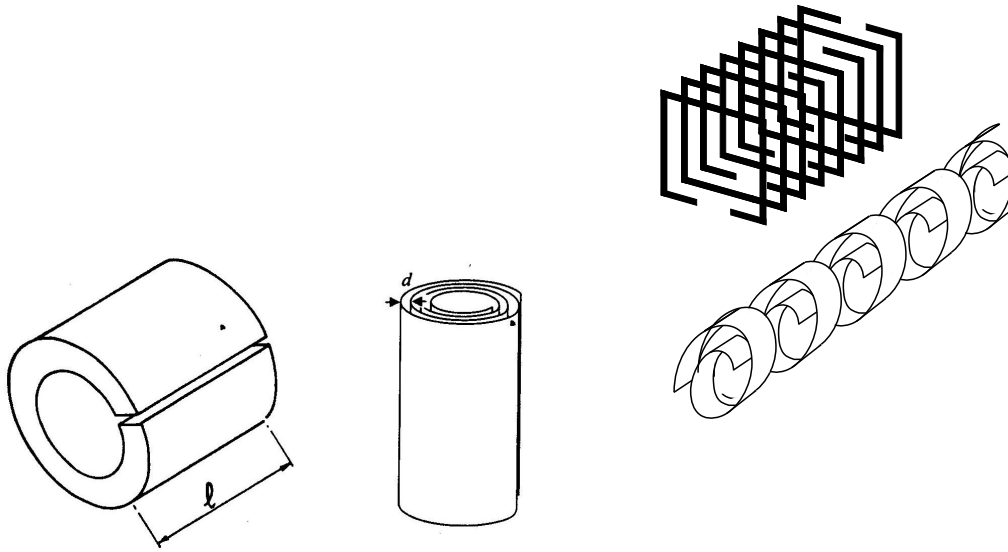


Figure 9: The evolution of Swiss rolls (1977-1999-2003, from left to right) [18, 19, 20, 21].

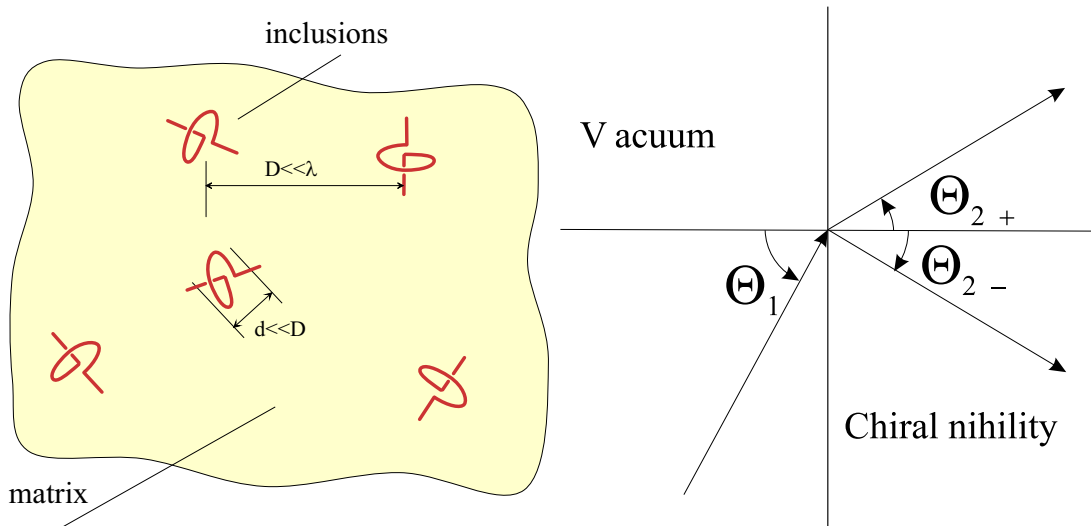


Figure 10: Typical geometry of an artificial chiral material (left) and refracted waves for a special case when the refractive index  $n = 0$  ("chiral nihility", right) [22].

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