Metasurfaces

Sergei A. Tretyakov

Department of Electronics and Nanoengineering
School of Electrical Engineering
Aalto University (Finland)

URSI General Assembly, August 2017
Outline

▶ Metasurfaces
▶ Physical optics approach
  ▶ General design methodology
  ▶ Engineering transmission: matched single-layer transmitarrays and non-reflecting absorbers
  ▶ Engineering reflection: metamirrors
▶ Exact synthesis: towards perfect performance
  ▶ Anomalous reflection
  ▶ Multi-port reflectors
  ▶ Managing transmission
Metamaterial: an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties

All the field vectors are averaged over electrically (optically) small volumes each containing many meta-atoms.
**Metasurface:**
electrically thin composite material layer, designed and optimized to function as a tool to control and transform electromagnetic waves

Two-dimensional versions of metamaterials...

\[ J_e = \frac{j\omega p}{S}, \quad J_m = \frac{j\omega m}{S} \]

(S is the unit-cell area). Electric and magnetic current sheets
Perfect lens

Propagating modes — negative refraction

$$\epsilon_r = -1, \quad \mu_r = -1$$

$$n = \sqrt{\epsilon_r \mu_r} = -1$$

Evanescent modes — plasmon resonance

$$R_{\text{half}} \to \infty$$

Two resonant grids

Two resonant grids (metasurfaces!) instead of a backward-wave medium slab


A resonant particle

- 5 mm
- 2.5 mm

Aalto University
School of Electrical Engineering
Can we replace all metamaterial devices with metasurface devices?

Huygens’ principle

Metasurfaces: Many functionalities

Probably the first metasurface: 1898

The intensities of the reflected and transmitted waves, in terms of that of the primary wave, are therefore

\[ I' = \frac{k^2c^3}{1+k^2c^2}. \]  

(95)

If the wave-length is at all large compared with \( c \), \( kc \) is small, and the reflection is almost total. But, for any given wave-length (large compared with \( a \)), \( kc \) may be made as great as we please by sufficiently diminishing the radius \( b \) of the wires. In this way we can pass to the case of free transmission.
Current work: Software-defined metasurfaces

Metasurface: Electric and magnetic current sheets

\[ J_e = \frac{j\omega p}{S}, \quad J_m = \frac{j\omega m}{S} \]

\( S \) is the unit-cell area.
Engineering reflection and transmission
Locally periodical arrays (physical optics)
Only electric current
(only electric polarization)

\[ \mathbf{p} = \overline{\alpha}_{ee} \cdot \mathbf{E}_{\text{inc}} = \overline{\alpha}_{ee} \cdot \mathbf{E}_{\text{loc}} \]

\[ \overline{I}_t + \overline{R} = \overline{T} \]

Possible functionalities: FSS and some polarizers

Impossible functionalities: Absorbers (50% max absorption), twist polarizers, mirrors with controlled reflection phase, . . .

Both electric and magnetic currents: (electric and magnetic polarizations)

\[
\begin{bmatrix}
    p \\
    m
\end{bmatrix} = \begin{bmatrix}
    \bar{\alpha}_{ee} & 0 \\
    0 & \bar{\alpha}_{mm}
\end{bmatrix} \cdot \begin{bmatrix}
    E_{inc} \\
    H_{inc}
\end{bmatrix} = \begin{bmatrix}
    \bar{\alpha}_{ee} & 0 \\
    0 & \bar{\alpha}_{mm}
\end{bmatrix} \cdot \begin{bmatrix}
    E_{loc} \\
    H_{loc}
\end{bmatrix}
\]
Reflected and transmitted waves

\[ E_{\text{ref}} = -\frac{\eta_0}{2} \mathbf{J}_e \pm \frac{1}{2} \mathbf{z}_0 \times \mathbf{J}_m = -\frac{j \omega}{2S} [\eta_0 \mathbf{p} \mp \mathbf{z}_0 \times \mathbf{m}] \]

\[ E_{\text{tr}} = E_{\text{inc}} - \frac{\eta_0}{2} \mathbf{J}_e \mp \frac{1}{2} \mathbf{z}_0 \times \mathbf{J}_m = E_{\text{inc}} - \frac{j \omega}{2S} [\eta_0 \mathbf{p} \pm \mathbf{z}_0 \times \mathbf{m}] \]

(S is the unit-cell area, \( \eta_0 \) is the free-space impedance)

The sheet generates different secondary fields at its two sides, and we can control reflection independently of transmission (\( T \neq 1 + R \)).
Non-reflecting thin layers: Huygens’ sheets

\[ E_{\text{ref}} = 0 \quad \Rightarrow \quad \eta_0 J_e = \pm z_0 \times J_m, \quad \eta_0 p = \pm z_0 \times m \]

The same relation as between the fields in the incident plane wave.

All (properly designed) absorbers, polarizers, non-reflecting FSS, phase-shifting surfaces, . . . are Huygens’ sheets.
Special case: Uniaxial symmetry

\[
\tilde{\alpha}_{ee} = \tilde{\alpha}_{ee}^{co} \tilde{I}_t + \tilde{\alpha}_{ee}^{cr} \tilde{J}_t, \quad \tilde{\alpha}_{mm} = \tilde{\alpha}_{mm}^{co} \tilde{I}_t + \tilde{\alpha}_{mm}^{cr} \tilde{J}_t
\]

\(\tilde{I}_t = \tilde{I} - z_0 z_0\) is the two-dimensional unit dyadic, and \(\tilde{J}_t = z_0 \times \tilde{I}_t\) is the vector-product operator.

Reflected and transmitted fields (normally incident plane waves):

\[
E_{\text{ref}} = -\frac{j \omega}{2S} \left[ \left( \eta_0 \tilde{\alpha}_{ee}^{co} - \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{co} \right) \tilde{I}_t + \left( \eta_0 \tilde{\alpha}_{ee}^{cr} - \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{cr} \right) \tilde{J}_t \right] \cdot E_{\text{inc}}
\]

\[
E_{\text{tr}} = \left\{ \left[ 1 - \frac{j \omega}{2S} \left( \eta_0 \tilde{\alpha}_{ee}^{co} + \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{co} \right) \right] \tilde{I}_t - \frac{j \omega}{2S} \left( \eta_0 \tilde{\alpha}_{ee}^{cr} + \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{cr} \right) \tilde{J}_t \right\} \cdot E_{\text{inc}}
\]

Red = non-reciprocal effect
Magneto-dielectric sheets

Additional possible functionalities: zero-reflection devices (absorbers, FSS, phase-shifting sheets, some polarizers); zero-transmission devices (absorbers, mirrors with controlled reflection phase)

Still impossible: twist-polarizers (except using nonreciprocity), and all devices which require different response when illuminated from different sides
Example: Reciprocal perfect absorbers

Desired performance: Reflected field is zero (both co- and cross-polarized components); Transmitted field is zero (both co- and cross-polarized components)

To ensure zero transmission:

\[ 1 - \frac{j\omega}{2S} \left( \eta_0 \alpha_{ee}^c + \frac{1}{\eta_0} \alpha_{mm}^c \right) = 0 \]

To ensure zero reflection:

\[ \eta_0 \alpha_{ee}^c - \frac{1}{\eta_0} \alpha_{mm}^c = 0 \]

Solution:

\[ \eta_0 \alpha_{ee}^c = \frac{1}{\eta_0} \alpha_{mm}^c = \frac{S}{j\omega} \]

Need unit cells with balanced electric and magnetic moments, both at resonance.
From collective polarizabilities to the properties of individual unit cells

Local fields and interaction constants...

\[ p = \overline{\alpha}_{ee} \cdot E_{\text{inc}}, \quad p = \overline{\alpha}_{ee} \cdot E_{\text{loc}} \]

\[ m = \overline{\alpha}_{mm} \cdot H_{\text{inc}}, \quad m = \overline{\alpha}_{mm} \cdot H_{\text{loc}} \]

\[ E_{\text{loc}} = E_{\text{inc}} + \overline{\beta}_{e} \cdot p \]

\[ H_{\text{loc}} = H_{\text{inc}} + \overline{\beta}_{m} \cdot m \]

For our simple case

\[ \frac{1}{\alpha_{ee}} = \frac{1}{\alpha_{ee}^{\text{co}}} + \beta_{e}, \quad \frac{1}{\alpha_{mm}} = \frac{1}{\alpha_{mm}^{\text{co}}} + \frac{\beta_{e}}{\eta_{0}^{2}} \]
We need particles with the polarizabilities equal to

$$\frac{1}{\eta_0 \alpha_{ee}} = \frac{1}{\alpha_{mm} / \eta_0} = \text{Re}\left(\frac{\beta_e}{\eta_0}\right) + j \frac{\omega^3}{6\pi c^2} + j \frac{\omega}{2S}$$

Balanced (optimal) lossy particles. Resonance frequency of particles in free space is different from that in the array.

For dense arrays,

$$\frac{1}{\eta_0} \text{Re}(\beta_e) \approx \frac{0.36}{\sqrt{\epsilon_0 \mu_0} a^3}$$

($a$ is the array period, $S = a^2$ is the cell area)
Symmetric all-frequency-matched single-layer absorber: topology

Polarizabilities are balanced
Frequency response

![Graph showing frequency response with R, T, and A labels.](https://meta.aalto.fi)
Reciprocal symmetric perfect absorbers

Experiment
Engineering transmission: Matched transmitarrays (Huygens’ metasurfaces)
Locally periodical arrays (physical optics)
Matched (Huygens’) transmitarrays: Design target

Desired performance: Reflected field is zero (both co- and cross-polarized components); Transmitted field is co-polarized and its phase is shifted by the angle $\phi$.

There is no reflection if

$$\eta_0 \hat{\alpha}_{ee}^{co} - \frac{1}{\eta_0} \hat{\alpha}_{mm}^{co} = 0, \quad \eta_0 \hat{\alpha}_{ee}^{cr} - \frac{1}{\eta_0} \hat{\alpha}_{mm}^{cr} = 0$$

There is no cross-polarized transmission if

$$\eta_0 \hat{\alpha}_{ee}^{cr} + \frac{1}{\eta_0} \hat{\alpha}_{mm}^{cr} = 0$$

The transmitted field has the desired phase shift (and the same amplitude as the incident field) if

$$1 - \frac{j \omega}{2S} \left( \eta_0 \hat{\alpha}_{ee}^{co} + \frac{1}{\eta_0} \hat{\alpha}_{mm}^{co} \right) = e^{j \phi}$$
The collective polarizabilities should satisfy

\[
\eta_0 \hat{\alpha}_{ee}^{\text{co}} - \frac{1}{\eta_0} \hat{\alpha}_{mm}^{\text{co}} = 0
\]

\[
\eta_0 \hat{\alpha}_{ee}^{\text{cr}} - \frac{1}{\eta_0} \hat{\alpha}_{mm}^{\text{cr}} = 0, \quad \eta_0 \hat{\alpha}_{ee}^{\text{cr}} + \frac{1}{\eta_0} \hat{\alpha}_{mm}^{\text{cr}} = 0
\]

\[
1 - \frac{j\omega}{2S} \left( \eta_0 \hat{\alpha}_{ee}^{\text{co}} + \frac{1}{\eta_0} \hat{\alpha}_{mm}^{\text{co}} \right) = e^{j\phi}
\]

Solution:

\[
\hat{\alpha}_{ee}^{\text{cr}} = \hat{\alpha}_{mm}^{\text{cr}} = 0, \quad \eta_0 \hat{\alpha}_{ee}^{\text{co}} = \frac{1}{\eta_0} \hat{\alpha}_{mm}^{\text{co}} = \frac{S}{j\omega} \left(1 - e^{j\phi}\right)
\]
What should be the individual polarizabilities of array particles?

We again use the connection between the polarizabilities of particles in infinite arrays and the same particles in free space:

\[
\frac{1}{\eta_0 \alpha_{ee}} = \frac{1}{\eta_0 \tilde{\alpha}_{ee}^{co}} + \frac{\beta_e}{\eta_0}, \quad \frac{1}{\alpha_{mm}/\eta_0} = \frac{1}{\tilde{\alpha}_{mm}^{co}/\eta_0} + \frac{\beta_e}{\eta_0}
\]

From here,

\[
\frac{1}{\eta_0 \alpha_{ee}} = \frac{1}{\alpha_{mm}/\eta_0} = \frac{1}{\eta_0} \text{Re}(\beta_e) - \frac{\omega}{2S} \frac{\sin \phi}{1 - \cos \phi} + j \frac{k^3}{6\pi \sqrt{\varepsilon_0 \mu_0}}
\]
Designing particles and the transmitarray

Balanced spirals, racemic arrangement

Experimental set-up
Measured performance

General bianisotropic sheets
(electric, magnetic, and magnetoelectric properties)

\[
\begin{bmatrix}
J_e \\
J_m
\end{bmatrix} = \begin{bmatrix}
\Xi & \Xi \\
\Xi & \Xi
\end{bmatrix} \begin{bmatrix}
\hat{Y}_{ee} & \hat{Y}_{em} \\
\hat{Y}_{me} & \hat{Y}_{mm}
\end{bmatrix} \cdot \begin{bmatrix}
E_{\text{inc}} \\
H_{\text{inc}}
\end{bmatrix}
\]

The same in terms of the dipole moments of unit cells \( p \) and \( m \):

\[
\begin{bmatrix}
J_e \\
J_m
\end{bmatrix} = \begin{bmatrix}
\Xi \\
\Xi
\end{bmatrix} \begin{bmatrix}
\alpha_{ee} & \alpha_{em} \\
\alpha_{me} & \alpha_{mm}
\end{bmatrix} \cdot \begin{bmatrix}
E_{\text{inc}} \\
H_{\text{inc}}
\end{bmatrix}
\]

\[
J_e = j\omega p / S, \quad J_m = j\omega m / S
\]

\[
\begin{bmatrix}
p \\
m
\end{bmatrix} = \begin{bmatrix}
\Xi \\
\Xi
\end{bmatrix} \begin{bmatrix}
\alpha_{ee} & \alpha_{em} \\
\alpha_{me} & \alpha_{mm}
\end{bmatrix} \cdot \begin{bmatrix}
E_{\text{inc}} \\
H_{\text{inc}}
\end{bmatrix}
\]
Uniaxial symmetry

Electric and magnetic polarization:

\[ \vec{\alpha}_{ee} = \vec{\alpha}_{ee}^{co} \vec{I}_t + \vec{\alpha}_{ee}^{cr} \vec{J}_t, \quad \vec{\alpha}_{mm} = \vec{\alpha}_{mm}^{co} \vec{I}_t + \vec{\alpha}_{mm}^{cr} \vec{J}_t \]

Magnetoelectric coupling:

\[ \vec{\alpha}_{em} = \vec{\alpha}_{em}^{co} \vec{I}_t + \vec{\alpha}_{em}^{cr} \vec{J}_t, \quad \vec{\alpha}_{me} = \vec{\alpha}_{me}^{co} \vec{I}_t + \vec{\alpha}_{me}^{cr} \vec{J}_t \]

\( \vec{I}_t = \vec{I} - z_0 z_0 \) is the two-dimensional unit dyadic, and \( \vec{J}_t = z_0 \times \vec{I}_t \) is the vector-product operator.

Reciprocal and nonreciprocal coupling:

\[ \vec{\alpha}_{em} = (\vec{\chi} + j\vec{\kappa}) \vec{I}_t + (\vec{\nabla} + j\vec{\Omega}) \vec{J}_t, \quad \vec{\alpha}_{me} = (\vec{\chi} - j\vec{\kappa}) \vec{I}_t + (-\vec{\nabla} + j\vec{\Omega}) \vec{J}_t. \]

Red = non-reciprocal effect
Reflected and transmitted fields
Most general uniaxial sheets

Normally incident plane wave:

\[
E_{\text{ref}} = - \frac{j \omega}{2S} \left[ \left( \eta_0 \tilde{\alpha}_{ee}^\text{co} \pm \tilde{\alpha}_{em}^\text{cr} \pm \tilde{\alpha}_{me}^\text{cr} - \frac{1}{\eta_0} \tilde{\alpha}_{mm}^\text{co} \right) \mathbb{I}_t \right. \\
+ \left( \eta_0 \tilde{\alpha}_{ee}^\text{cr} \mp \tilde{\alpha}_{em}^\text{co} \mp \tilde{\alpha}_{me}^\text{co} - \frac{1}{\eta_0} \tilde{\alpha}_{mm}^\text{cr} \right) \mathbb{J}_t \left. \right] \cdot E_{\text{inc}}
\]

\[
E_{\text{tr}} = \left\{ \left[ 1 - \frac{j \omega}{2S} \left( \eta_0 \tilde{\alpha}_{ee}^\text{co} \pm \tilde{\alpha}_{em}^\text{cr} \mp \tilde{\alpha}_{me}^\text{cr} + \frac{1}{\eta_0} \tilde{\alpha}_{mm}^\text{co} \right) \mathbb{I}_t \\
- \frac{j \omega}{2S} \left( \eta_0 \tilde{\alpha}_{ee}^\text{cr} \mp \tilde{\alpha}_{em}^\text{co} \pm \tilde{\alpha}_{me}^\text{co} + \frac{1}{\eta_0} \tilde{\alpha}_{mm}^\text{cr} \right) \mathbb{J}_t \right\} \cdot E_{\text{inc}}
\]

This allows the most general device synthesis, within the physical optics approximation.
Reflected and transmitted fields

Reciprocal sheets

\[ \hat{\alpha}^{\text{cr}}_{\text{em}} + \hat{\alpha}^{\text{cr}}_{\text{me}} = 2j\Omega, \quad \hat{\alpha}^{\text{cr}}_{\text{me}} - \hat{\alpha}^{\text{cr}}_{\text{em}} = 2\kappa \]

\[ E_{\text{ref}} = -\frac{j\omega}{2S} \left( \eta_0 \hat{\alpha}^\text{co}_{\text{ee}} \pm 2j\Omega - \frac{1}{\eta_0} \hat{\alpha}^\text{co}_{\text{mm}} \right) E_{\text{inc}} \]

\[ E_{\text{tr}} = \left[ 1 - \frac{j\omega}{2S} \left( \eta_0 \hat{\alpha}^\text{co}_{\text{ee}} + \frac{1}{\eta_0} \hat{\alpha}^\text{co}_{\text{mm}} \right) \right] E_{\text{inc}} \mp \frac{\omega}{S} \kappa z_0 \times E_{\text{inc}} \]

Magnetic response \((\alpha_{\text{mm}})\) controls matching; omega coupling \((\Omega)\) controls asymmetry in reflections from two sides; chirality \((\kappa)\) controls polarization transformation in transmission.
General bianisotropic sheets

Additional possible functionalities: All what was still impossible with magneto-dielectric sheets

Still impossible: Nothing (if not forbidden by basic physics)
Engineering reflections: Metamirrors

Required collective polarizabilities

Desired performance: Transmitted field is zero; Lossless reflection; Reflection phase $\phi$ for one side and $\theta$ for the other.

No transmission:

$$1 - \frac{j\omega}{2S} \left( \eta_0 \tilde{\alpha}_{ee}^{co} \pm \tilde{\alpha}_{em}^{cr} \mp \tilde{\alpha}_{me}^{cr} + \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{co} \right) = 0, \quad \eta_0 \tilde{\alpha}_{ee}^{cr} + \tilde{\alpha}_{em}^{co} \pm \tilde{\alpha}_{me}^{co} + \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{cr} = 0$$

Desired reflection coefficients:

$$- \frac{j\omega}{2S} \left( \eta_0 \tilde{\alpha}_{ee}^{co} + \tilde{\alpha}_{em}^{cr} + \tilde{\alpha}_{me}^{cr} - \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{co} \right) = e^{j\phi}$$

$$- \frac{j\omega}{2S} \left( \eta_0 \tilde{\alpha}_{ee}^{co} - \tilde{\alpha}_{em}^{cr} - \tilde{\alpha}_{me}^{cr} - \frac{1}{\eta_0} \tilde{\alpha}_{mm}^{co} \right) = e^{j\theta}$$
Several possible realizations. Assuming that we will use only reciprocal unit cells:

\[
\begin{align*}
\eta_0 \hat{\alpha}_{ee}^{co} &= \frac{S}{j\omega} \left(1 - \frac{e^{j\phi} + e^{j\theta}}{2}\right) \\
\hat{\alpha}_{em}^{cr} &= -\frac{S}{j\omega} \left(1 - \frac{e^{j\phi} - e^{j\theta}}{2}\right) \\
\frac{1}{\eta_0} \hat{\alpha}_{mm}^{co} &= \frac{S}{j\omega} \left(1 + \frac{e^{j\phi} + e^{j\theta}}{2}\right)
\end{align*}
\]

Need lossless bianisotropic unit cells (omega coupling, no chirality).
Shapes of wire “omega” particles
Example: Deflecting metamirror

Example: Focusing metamirror

Focal length $\approx 0.65\lambda_0$
Measured results: Focusing metamirror

Operating frequency 5 GHz, bandwidth $\approx$ 5\%, reflectivity $\approx$ 86\%, focal distance $0.65\lambda$, f-number $f/D = 0.23$, focal spot size $2.8\lambda \times 0.9\lambda$, metasurface thickness $\lambda/7.6$, diameter $2.8\lambda$, the focusing reflector is transparent outside of the resonant band.
Oblique incidence: Focal spot shift

Simulation results

0°  5°  10°  15°
Three primary feeds at different positions create three beams in different directions.

Multifunctional metasurfaces

Example three-layer surface

\[ H < \lambda \]

\[ \vec{E}_{\text{inc}} \]

\[ \vec{k} \]

\[ X \]

\[ Y \]

\[ Z \]

\[ \frac{E}{E_{\text{inc}}} \]

\[ \left| \frac{E}{E_{\text{inc}}} \right|^2 \]
Physical optics is an approximation!

How one can create metasurfaces for *perfect* control of reflection and transmission?
Conventional design approach: Physical optics (the phased-array principle)

The incident wave is $e^{-jkx \sin \theta_i}$ and the reflected wave is $e^{-jkx \sin \theta_r}$.

The reflection coefficient is set to

$$R = \exp[jkx(\sin \theta_i - \sin \theta_r)] = \exp(j\Phi_r(x))$$

At every point we want to have full power reflection (or transmission):

$$|R| = 1 \quad \text{or} \quad |T| = 1$$

Gradient phase ("the generalized reflection law"): 

$$\sin \theta_i - \sin \theta_r = \frac{1}{k} \frac{d\Phi_r(x)}{dx}$$
But such reflectors do not produce the desired reflected fields!

\[ |R| \neq 1 \quad \text{or} \quad |T| \neq 1 \]

Power efficiency:

\[ \zeta_r = 1 - \left( \frac{Z_r - Z_i}{Z_r + Z_i} \right)^2 = \frac{4 \cos \theta_i \cos \theta_r}{(\cos \theta_i + \cos \theta_r)^2} \]

Efficiency results

Conventional design ("generalized reflection law")

\[
\sin \theta_i - \sin \theta_r = \frac{1}{k_1} \frac{d\Phi_r(x)}{dx}, \quad Z_s(x) = j \frac{\eta_1}{\cos \theta_i} \cot[\Phi_r(x)/2]
\]

\(\theta_i = 0^\circ, \quad \theta_r = 70^\circ.\) Efficiency 75.7\%.
“Active-lossy design”

\[ E_r = E_i \frac{\sqrt{\cos \theta_i}}{\sqrt{\cos \theta_r}}, \quad Z_s(x) = \frac{\eta_1}{\sqrt{\cos \theta_i \cos \theta_r}} \frac{\sqrt{\cos \theta_r} + \sqrt{\cos \theta_i} e^{j\Phi_r(x)}}{\sqrt{\cos \theta_i} - \sqrt{\cos \theta_r} e^{j\Phi_r(x)}} \]

Efficiency 100%
But how can we realize it?

We need either active elements (gain) or strong non-locality (receiving in “lossy regions” and radiating in “active regions”)

Design concept: Inhomogeneous leaky-wave antenna

Select reactive surface impedance $Z_{s0}$ such that a surface wave is supported:

$$\beta_s = k_1 \sqrt{1 - \frac{Z_1^2}{Z_{s0}^2}}$$

Periodically modulating the surface, couple to plane waves with

$$\sin \theta_n = \frac{\beta_s}{k_1} = \sqrt{1 - \frac{Z_1^2}{Z_{s0}^2}} + n \sin \theta_r$$

Next we LINEARLY modulate the reflection phase, to receive and radiate from/to the desired directions:
Numerical results

\[ \Phi_r(x) = \begin{cases} 
(sin \theta_r - sin \theta_n) k_1 x - \Phi_0 & 0 \leq x < x_1 \\
(sin \theta_i - sin \theta_n) k_1 (x - D) - \Phi_0 & x_1 \leq x < D. 
\end{cases} \]

Efficiency 100% with a lossless reflector: an inhomogeneous reactive boundary.
Design for experimental realization

Array of rectangular patches on a grounded dielectric substrate.

Numerically calculated efficiency 94%
(< 100% due to losses in copper and dielectric)

Target operational frequency 8 GHz.

1.575 mm thick Rogers 5880 substrate ($\varepsilon_r = 2.2$, $\tan \delta = 0.0009$), copper patches.

The sample size $11.7\lambda \times 7\lambda$ (440 mm $\times$ 262.5 mm).
Experimental results

Fixed bi-static antenna positions, at $0^\circ$ and $70^\circ$.
Rotating sample ($\phi = 0$ corresponds to the normal incidence).

-90 -60 -30 0 30 60 90
$\phi$ (deg)

|$S_{21}$| (dB)

-90 -80 -70 -60 -50 -40 -30

Metamirror sample

Experimentally measured power efficiency 93.8% at 8.08 GHz
(numerically simulated: 94% at 8 GHz).

Reference metal plate

-90 -60 -30 0 30 60 90
$\phi$ (deg)

|$S_{21}$| (dB)
Simple special case $\theta_r = \pm \theta_i$

General case: “active-lossy” surface

$$E_r = E_i \frac{\sqrt{\cos \theta_i}}{\sqrt{\cos \theta_r}}, \quad Z_s(x) = \frac{\eta_1}{\sqrt{\cos \theta_i \cos \theta_r}} \frac{\sqrt{\cos \theta_r} + \sqrt{\cos \theta_i} e^{j\Phi_r(x)}}{\sqrt{\cos \theta_i} - \sqrt{\cos \theta_r} e^{j\Phi_r(x)}}$$

If $\theta_r = \pm \theta_i$, the surface is purely reactive at every point:

$$E_r = E_i,$$
$$Z_s(x) = j \frac{\eta_1}{\sqrt{\cos \theta_i}} \cot \frac{\Phi_r(x)}{2}$$

Specular reflection or retroreflection
Three-channel mirror

At $\theta_i = 0^\circ$ and $\theta_i = \pm 70^\circ$ angles, the observer sees himself as in a mirror normally oriented in respect to him.
Results

Exact synthesis of transmitting metasurfaces

\[ \zeta_{r,TE} = 1 - \left( \frac{Z_t - Z_i}{Z_t + Z_i} \right)^2 = \frac{4\eta_1\eta_2 \cos \theta_i \cos \theta_t}{(\eta_2 \cos \theta_i + \eta_1 \cos \theta_t)^2}, \quad \zeta_{r,TM} = \frac{4\eta_1\eta_2 \cos \theta_i \cos \theta_t}{(\eta_2 \cos \theta_i + \eta_1 \cos \theta_t)^2} \]

Homogenization model

Involving only *tangential* fields:

\[ E_{t1} = \overline{Z}_{11} \cdot \mathbf{n} \times \mathbf{H}_{t1} + \overline{Z}_{12} \cdot (-\mathbf{n} \times \mathbf{H}_{t2}) \]

\[ E_{t2} = \overline{Z}_{21} \cdot \mathbf{n} \times \mathbf{H}_{t1} + \overline{Z}_{22} \cdot (-\mathbf{n} \times \mathbf{H}_{t2}) \]
Equations for $Z$-parameters

Equating the normal components of the Poynting vector:

$$E_t = E_i \sqrt{\frac{\cos \theta_i}{\cos \theta_t}} \sqrt{\frac{\eta_2}{\eta_1}} e^{j\phi_t}$$

Substituting the desired field values:

$$e^{-jk_1 \sin \theta_i z} = Z_{11} \frac{1}{\eta_1} \cos \theta_i e^{-jk_1 \sin \theta_i z} - Z_{12} \frac{1}{\sqrt{\eta_1 \eta_2}} \sqrt{\cos \theta_i \cos \theta_t} e^{-jk_2 \sin \theta_t z + j\phi_t}$$

$$e^{-jk_2 \sin \theta_t z + j\phi_t} = Z_{21} \frac{1}{\sqrt{\eta_1 \eta_2}} \sqrt{\cos \theta_i \cos \theta_t} e^{-jk_1 \sin \theta_i z} - Z_{22} \frac{\cos \theta_t}{\eta_2} e^{-jk_2 \sin \theta_t z + j\phi_t}$$

Two equations, four design parameters.

Perfect lossless design

\[ e^{-jk_1 \sin \theta_i z} = jX_{11} \frac{1}{\eta_1} \cos \theta_i e^{-jk_1 \sin \theta_i z} - jX_{12} \frac{1}{\sqrt{\eta_1 \eta_2}} \sqrt{\cos \theta_i \cos \theta_t} e^{-jk_2 \sin \theta_t z + j\phi_t} \]

\[ e^{-jk_2 \sin \theta_t z + j\phi_t} = jX_{21} \frac{1}{\sqrt{\eta_1 \eta_2}} \sqrt{\cos \theta_i \cos \theta_t} e^{-jk_1 \sin \theta_i z} - jX_{22} \frac{\cos \theta_t}{\eta_2} e^{-jk_2 \sin \theta_t z + j\phi_t} \]

Unique solution [where \( \Phi_t(z) = -k_2 \sin \theta_t z + \phi_t + k_1 \sin \theta_i z \)]:

\[ X_{11} = -\frac{\eta_1}{\cos \theta_i} \cot \Phi_t \]

\[ X_{22} = -\frac{\eta_2}{\cos \theta_t} \cot \Phi_t \]

\[ X_{12} = X_{21} = -\frac{\sqrt{\eta_1 \eta_2}}{\sqrt{\cos \theta_i \cos \theta_t}} \frac{1}{\sin \Phi_t} \]
Anomalous refraction requires bianisotropy (omega coupling)

Unit-cell polarizabilities

\[
\tilde{\alpha}_{\text{ee}}^{yy} = \frac{S}{j\omega \eta_1 \cos \theta_t + \eta_2 \cos \theta_i} \cos \theta_i \cos \theta_t \left[ 2 - \left( \sqrt{\frac{\eta_1 \cos \theta_t}{\eta_2 \cos \theta_i}} + \sqrt{\frac{\eta_2 \cos \theta_i}{\eta_1 \cos \theta_t}} \right) e^{-j\Phi_t(z)} \right]
\]

\[
\tilde{\alpha}_{\text{mm}}^{zz} = \frac{S}{j\omega \eta_1 \cos \theta_t + \eta_2 \cos \theta_i} \frac{\eta_1 \eta_2}{\eta_1 \cos \theta_t + \eta_2 \cos \theta_i} \left[ 2 - \left( \sqrt{\frac{\eta_1 \cos \theta_t}{\eta_2 \cos \theta_i}} + \sqrt{\frac{\eta_2 \cos \theta_i}{\eta_1 \cos \theta_t}} \right) e^{-j\Phi_t(z)} \right]
\]

\[
\tilde{\alpha}_{\text{em}}^{yz} = -\tilde{\alpha}_{\text{me}}^{yz} = \frac{S}{j\omega \eta_1 \cos \theta_t + \eta_2 \cos \theta_i} \eta_2 \cos \theta_i - \eta_1 \cos \theta_t
\]

where \( S \) is the unit-cell area.

Bianisotropic omega layers, e.g. arrays of \( \Omega \)-shaped particles, arrays of split rings, double arrays of patches (different patches on the two sides of a thin dielectric substrate), asymmetric three-layer structures, …
Experimental realization

Ultimate efficiency for conventional Huygens’ metasurfaces: 75.7%

Measured efficiency: 81%


Conclusions

▶ Metasurfaces are electrically thin composite material layers, designed and optimized to function as tools to control and transform electromagnetic waves

▶ Bianisotropic metasurfaces allow (nearly) full control of reflection and transmission properties, but advanced functionalities require strongly non-local structures

This work has been supported in part by the Academy of Finland (project METAMIRROR) and EU H2020 program (FET OPEN project VISORSURF).