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Metasurfaces

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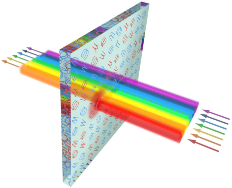
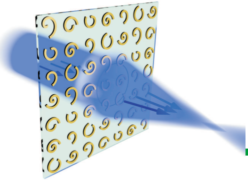
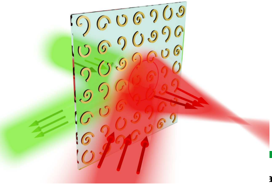
Outline

- Metasurfaces
- Physical optics approach
  - General design methodology
  - Engineering transmission: matched single-layer transmitarrays and non-reflecting absorbers
  - Engineering reflection: metamirrors
- Exact synthesis: towards perfect performance
  - Anomalous reflection
  - Multi-port reflectors
  - Managing transmission

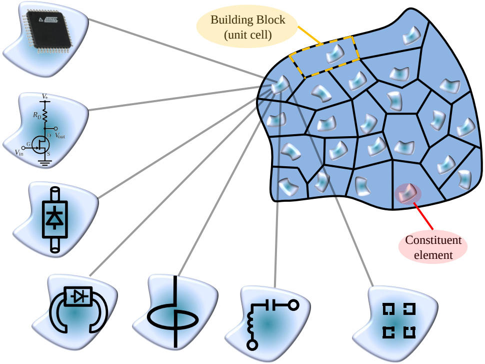






Metamaterial:

an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties

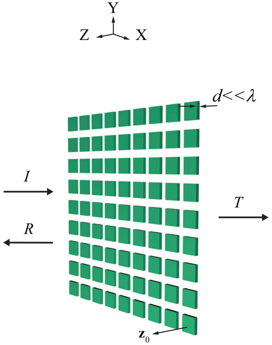


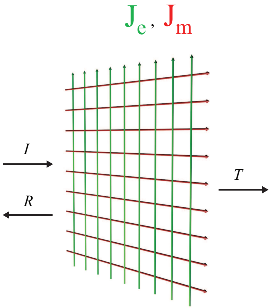
All the field vectors are averaged over electrically (optically) small volumes each containing many meta-atoms.

Metasurface:

electrically thin composite material layer, designed and optimized to function as a tool to control and transform electromagnetic waves

Two-dimensional versions of metamaterials. . .

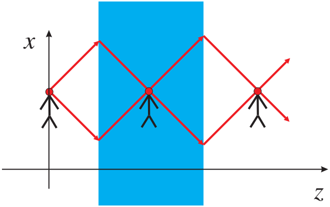




$$\mathbf{J}_\mathrm{e} = \frac{j\omega \mathbf{p}}{S}, \qquad \mathbf{J}_\mathrm{m} = \frac{j\omega \mathbf{m}}{S}$$

(S is the unit-cell area). Electric and magnetic current sheets

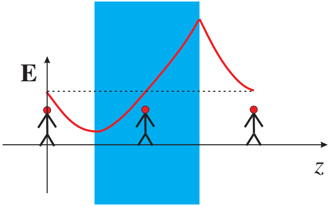
Perfect lens



Propagating modes
— negative refraction

$$\epsilon_r = -1, \qquad \mu_r = -1$$

$$n = \sqrt{\epsilon_r \mu_r} = -1$$



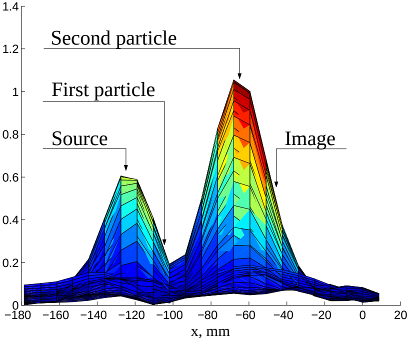
Evanescent modes
— plasmon resonance

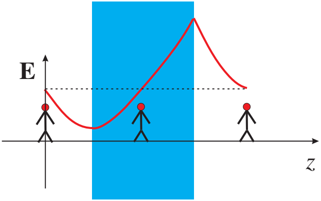
$$R_{\mathrm{half}} \to \infty$$

V. Veselago (1967), J. Pendry (2000)

Two resonant grids

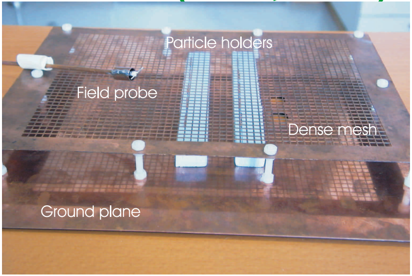
Two resonant grids (metasurfaces!) instead of a backward-wave medium slab



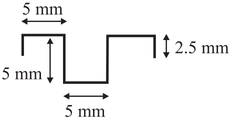


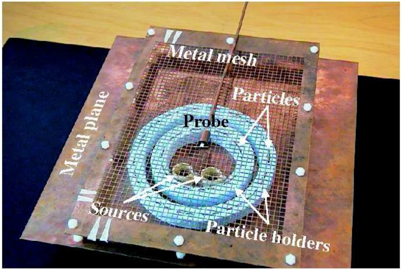
S. Maslovski, S. Tretyakov, P. Alitalo, Near-field enhancement and imaging in double planar polariton-resonant structures, J. Applied Physics, vol. 96, no. 3, pp. 1293-1300, 2004.

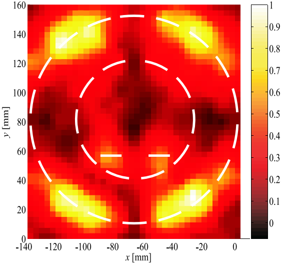
Experiments (2004, 2005)



A resonant particle

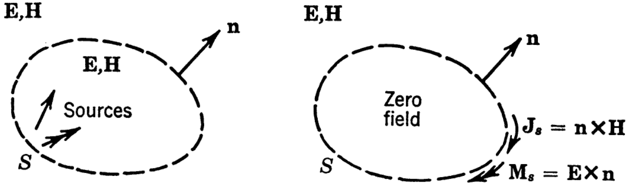






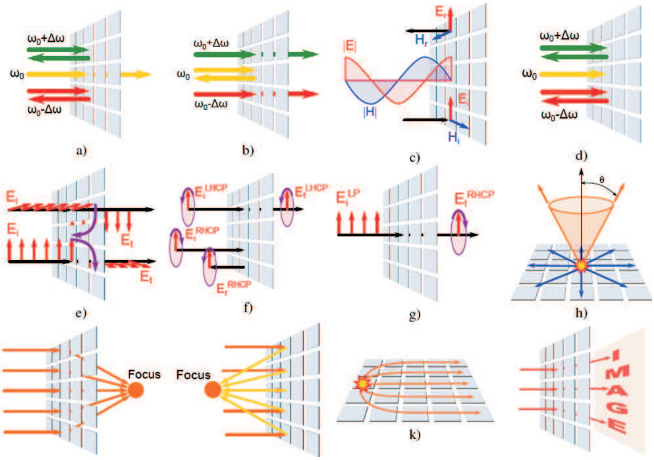
Can we replace all metamaterial devices with metasurface devices?

Huygens' principle



Picture from R.F. Harringthon, Time-Harmonic Electromagnetic Fields, IEEE Press, 2001.

Metasurfaces: Many functionalities



S.B. Glybovski, S.A. Tretyakov, P.A. Belov, Y.S. Kivshar, C.R. Simovski, Metasurfaces: From microwaves to visible, Physics Reports, vol. 634, pp. 1-72, 2016.

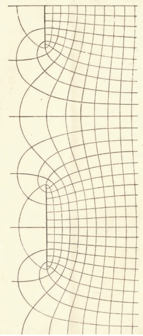
Probably the first metasurface: 1898

On the Reflection and Transmission of Electric Waves by a Metallic Grating.

By Prof. HORACE LAMB, F.R.S.

[*Extracted from the Proceedings of the London Mathematical Society, Vol. XXIX., Nos.* 644, 645.]

The main problem of this paper consists in the calculation of the disturbance produced in a train of electric waves by a plane grating composed of parallel, equal, and equidistant metallic strips. The treatment is approximate, and involves the assumption that the wave-length is large compared with the distance between the centres of consecutive strips; the application is, therefore, rather to Hertzian waves than to phenomena of ordinary Optics. The previous mathe-



In the above investigation, the coefficient of the primary wave has been taken to be unity. On the same scale, the coefficients of the reflected and transmitted waves are $-1+B_0$ and B_0, or

$$-\frac{1}{1+ikc} \quad \text{and} \quad \frac{ikc}{1+ikc},$$

respectively. Hence the *intensities* I, I' (say) of these waves, in

$$c = \frac{a}{\pi} \log \frac{a}{2\pi b}$$

$$c = \frac{a+b}{\pi} \log \sec \frac{\pi a}{2(a+b)}.$$

H. Lamb, On the reflection and transmission of electric waves by a metallic grating, Proc. London Math. Soc., vol. 29, ser. 1, 523-544, 1898.

Engineering properties of a metasurface: 1898

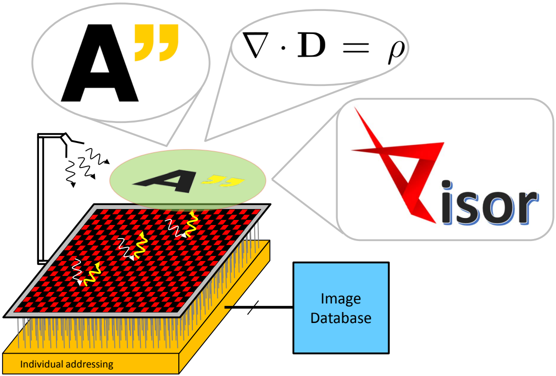
The intensities of the reflected and transmitted waves, in terms of that of the primary wave, are therefore

$$= \frac{1}{1+k^2c^2}, \quad I' = \frac{k^2c^2}{1+k^2c^2}. \qquad (95)$$

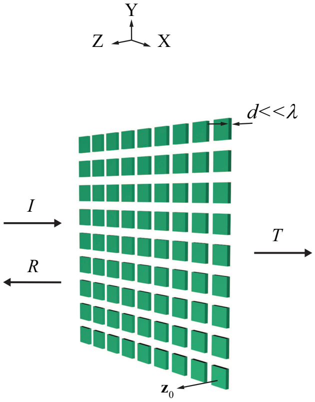
If the wave-length is at all large compared with c, kc is small, and the reflection is almost total. But, for any given wave-length (large compared with a), kc may be made as great as we please by sufficiently diminishing the radius b of the wires. In this way we can pass to the case of free transmission.

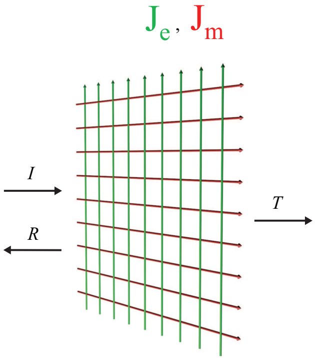
Current work: Software-defined metasurfaces

VISORSURF project: A Hardware Platform for Software-driven Functional Metasurfaces, www.visorsurf.eu/



Metasurface: Electric and magnetic current sheets

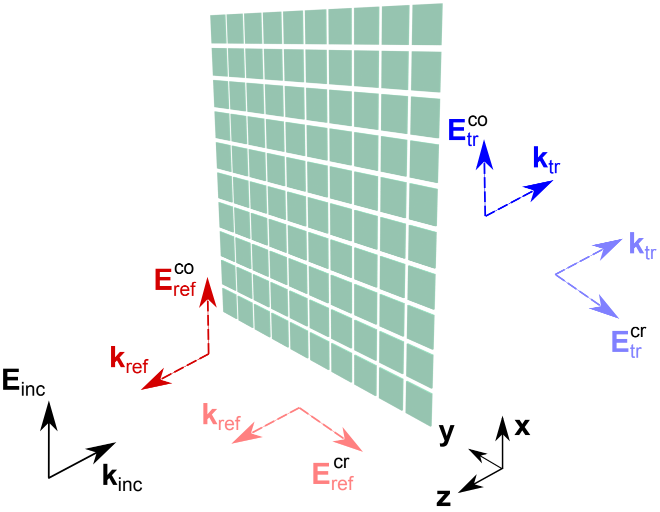




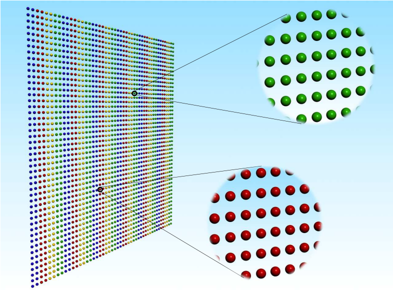
$$\mathrm{J_e} = \frac{j\omega \mathbf{p}}{S}, \qquad \mathrm{J_m} = \frac{j\omega \mathbf{m}}{S}$$

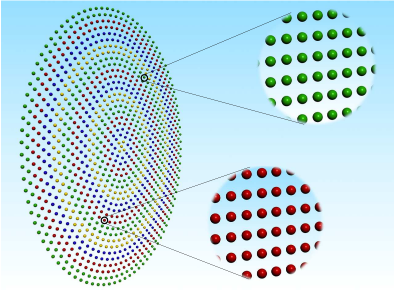
S is the unit-cell area.

Engineering reflection and transmission



Locally periodical arrays (physical optics)





Only electric current (only electric polarization)

$$\mathbf{p} = \overline{\overline{\widehat{\alpha}}}_{\rm ee} \cdot \mathbf{E}_{\rm inc} = \overline{\overline{\alpha}}_{\rm ee} \cdot \mathbf{E}_{\rm loc}$$

$$\overline{\overline{\rm I}}_{\rm t} + \overline{\overline{R}} = \overline{\overline{T}}$$

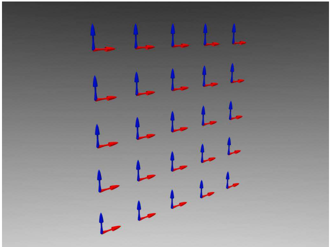
Possible functionalities: FSS and some polarizers

Impossible functionalities: Absorbers (50% max absorption), twist polarizers, mirrors with controlled reflection phase,...

Y. Vardaxoglou, Frequency Selective Surfaces: Analysis and Design, John Wiley & Sons, 1997

B.A. Munk, Frequency Selective Surfaces: Theory and Design, John Wiley & Sons, 2000

Both electric and magnetic currents: (electric and magnetic polarizations)



$$\begin{bmatrix} \mathbf{p} \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} \widehat{\overline{\overline{\alpha}}}_{\rm ee} & 0 \\ 0 & \widehat{\overline{\overline{\alpha}}}_{\rm mm} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{E}_{\rm inc} \\ \mathbf{H}_{\rm inc} \end{bmatrix} = \begin{bmatrix} \overline{\overline{\alpha}}_{\rm ee} & 0 \\ 0 & \overline{\overline{\alpha}}_{\rm mm} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{E}_{\rm loc} \\ \mathbf{H}_{\rm loc} \end{bmatrix}$$

Reflected and transmitted waves

$$\mathbf{E}_{\rm ref} = -\frac{\eta_0}{2}\mathbf{J}_{\rm e} \pm \frac{1}{2}\mathbf{z}_0 \times \mathbf{J}_{\rm m} \quad = -\frac{j\omega}{2S}[\eta_0 \mathbf{p} \mp \mathbf{z}_0 \times \mathbf{m}]$$

$$\mathbf{E}_{\rm tr} = \mathbf{E}_{\rm inc} - \frac{\eta_0}{2}\mathbf{J}_{\rm e} \mp \frac{1}{2}\mathbf{z}_0 \times \mathbf{J}_{\rm m} \quad = \mathbf{E}_{\rm inc} - \frac{j\omega}{2S}[\eta_0 \mathbf{p} \pm \mathbf{z}_0 \times \mathbf{m}]$$

(S is the unit-cell area, η_0 is the free-space impedance)

The sheet generates different secondary fields at its two sides, and we can control reflection independently of transmission ($T \neq 1 + R$).

Non-reflecting thin layers: Huygens' sheets

$$\mathbf{E}_{\rm ref} = 0 \qquad \Rightarrow \eta_0 \mathbf{J}_{\rm e} = \pm \mathbf{z}_0 \times \mathbf{J}_{\rm m}, \quad \eta_0 \mathbf{p} = \pm \mathbf{z}_0 \times \mathbf{m}$$

The same relation as between the fields in the incident plane wave.

All (properly designed) absorbers, polarizers, non-reflecting FSS, phase-shifting surfaces,... are Huygens' sheets.

Special case: Uniaxial symmetry

$$\overline{\overline{\widehat{\alpha}}}_{\rm ee} = \widehat{\alpha}_{\rm ee}^{\rm co}\overline{\overline{\rm I}}_{\rm t} + \widehat{\alpha}_{\rm ee}^{\rm cr}\overline{\overline{\rm J}}_{\rm t}, \qquad \overline{\overline{\widehat{\alpha}}}_{\rm mm} = \widehat{\alpha}_{\rm mm}^{\rm co}\overline{\overline{\rm I}}_{\rm t} + \widehat{\alpha}_{\rm mm}^{\rm cr}\overline{\overline{\rm J}}_{\rm t}$$

$\overline{\overline{\rm I}}_{\rm t} = \overline{\overline{\rm I}} - \mathbf{z}_0\mathbf{z}_0$ is the two-dimensional unit dyadic, and $\overline{\overline{\rm J}}_{\rm t} = \mathbf{z}_0 \times \overline{\overline{\rm I}}_{\rm t}$ is the vector-product operator.

Reflected and transmitted fields (normally incident plane waves):

$$\mathbf{E}_{\rm ref} = -\frac{j\omega}{2S}\left[\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co} - \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right)\overline{\overline{\rm I}}_{\rm t} + \left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm cr} - \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm cr}\right)\overline{\overline{\rm J}}_{\rm t}\right]\cdot\mathbf{E}_{\rm inc}$$

$$\mathbf{E}_{\rm tr} = \left\{\left[1 - \frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co} + \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right)\right]\overline{\overline{\rm I}}_{\rm t} - \frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm cr} + \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm cr}\right)\overline{\overline{\rm J}}_{\rm t}\right\}\cdot\mathbf{E}_{\rm inc}$$

Red=non-reciprocal effect

Magneto-dielectric sheets

Additional possible functionalities: zero-reflection devices (absorbers, FSS, phase-shifting sheets, some polarizers); zero-transmission devices (absorbers, mirrors with controlled reflection phase)

Still impossible: twist-polarizers (except using nonreciprocity), and all devices which require different response when illuminated from different sides

Example: Reciprocal perfect absorbers

Desired performance: Reflected field is zero (both co- and cross-polarized components); Transmitted field is zero (both co- and cross-polarized components)

To ensure zero transmission:

$$1-\frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co}+\frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right)=0$$

To ensure zero reflection:

$$\eta_0\widehat{\alpha}_{\rm ee}^{\rm co}-\frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}=0$$

Solution:

$$\eta_0\widehat{\alpha}_{\rm ee}^{\rm co}=\frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}=\frac{S}{j\omega}$$

Need unit cells with balanced electric and magnetic moments, both at resonance.

From collective polarizabilities to the properties of individual unit cells

Local fields and interaction constants. . .

$$\mathbf{p} = \overline{\overline{\widehat{\alpha}}}_{\rm ee} \cdot \mathbf{E}_{\rm inc}, \qquad \mathbf{p} = \overline{\overline{\alpha}}_{\rm ee} \cdot \mathbf{E}_{\rm loc}$$

$$\mathbf{m} = \overline{\overline{\widehat{\alpha}}}_{\rm mm} \cdot \mathbf{H}_{\rm inc}, \qquad \mathbf{m} = \overline{\overline{\alpha}}_{\rm mm} \cdot \mathbf{H}_{\rm loc}$$

$$\mathbf{E}_{\rm loc} = \mathbf{E}_{\rm inc} + \overline{\overline{\beta}}_{\rm e} \cdot \mathbf{p}$$

$$\mathbf{H}_{\rm loc} = \mathbf{H}_{\rm inc} + \overline{\overline{\beta}}_{\rm m} \cdot \mathbf{m}$$

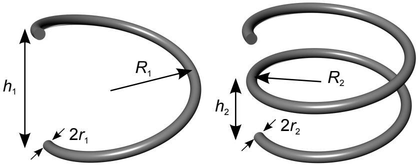
For our simple case

$$\frac{1}{\alpha_{\rm ee}} = \frac{1}{\widehat{\alpha}_{\rm ee}^{\rm co}} + \beta_{\rm e}, \qquad \frac{1}{\alpha_{\rm mm}} = \frac{1}{\widehat{\alpha}_{\rm mm}^{\rm co}} + \frac{\beta_{\rm e}}{\eta_0^2}$$

We need particles with the polarizabilities equal to

$$\frac{1}{\eta_0 \alpha_{\rm ee}} = \frac{1}{\alpha_{\rm mm}/\eta_0} = {\rm Re}\left(\frac{\beta_{\rm e}}{\eta_0}\right) + j\frac{\omega^3}{6\pi c^2} + j\frac{\omega}{2S}$$

Balanced (optimal) lossy particles. Resonance frequency of particles in free space is different from that in the array.

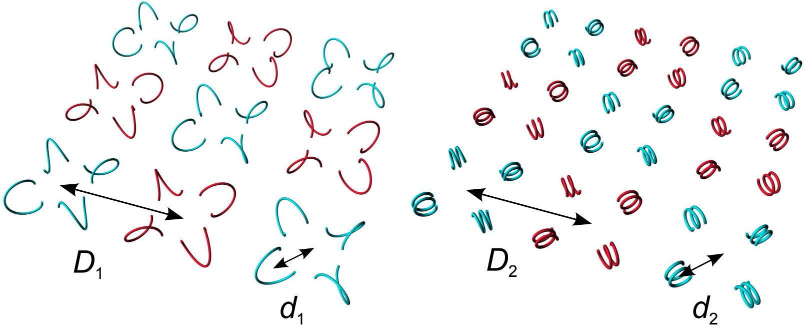


For dense arrays,

$$\frac{1}{\eta_0}{\rm Re}(\beta_{\rm e}) \approx \frac{0.36}{\sqrt{\epsilon_0\mu_0}\,a^3}$$

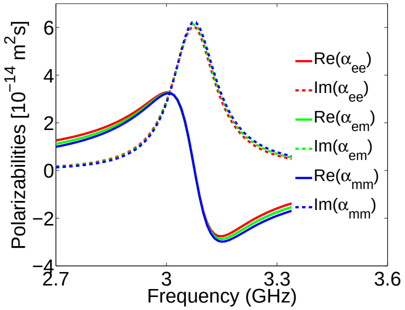
(a is the array period, $S = a^2$ is the cell area)

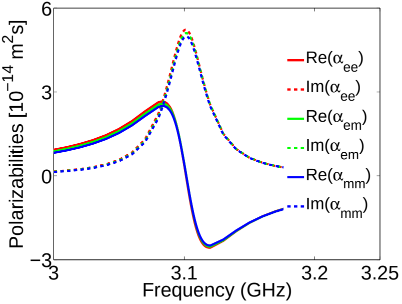
Symmetric *all-frequency-matched* single-layer absorber: topology



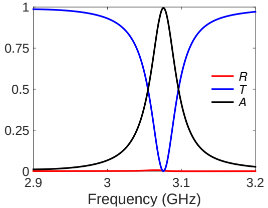
V.S. Asadchy, I.A. Faniayeu, Y. Ra'di, S.A. Khakhomov, I.V. Semchenko, S.A. Tretyakov, Broadband reflectionless metasheets: Frequency-selective transmission and perfect absorption, Phys. Rev. X, vol. 5, p. 031005, 2015.

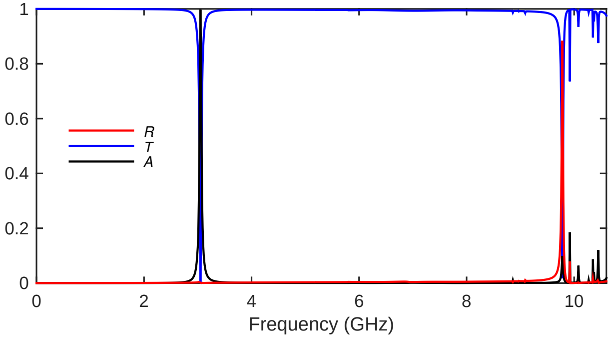
Polarizabilities are balanced





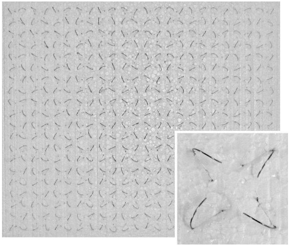
Frequency response

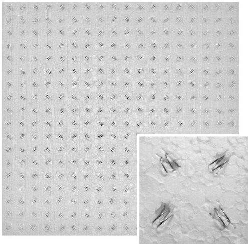


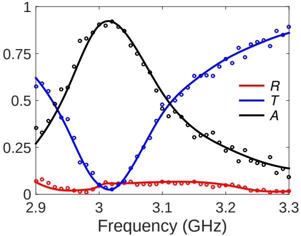


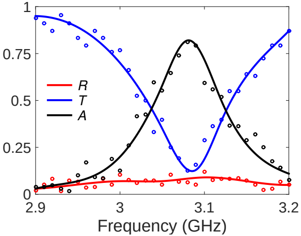
Reciprocal symmetric perfect absorbers

Experiment

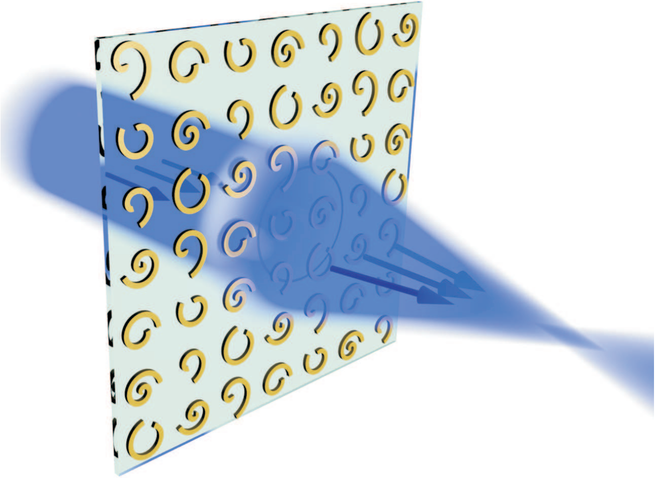




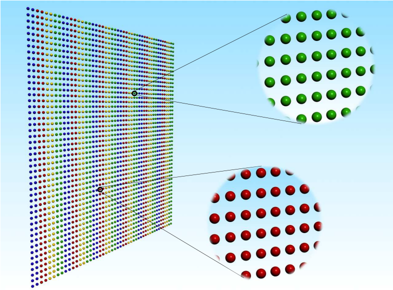


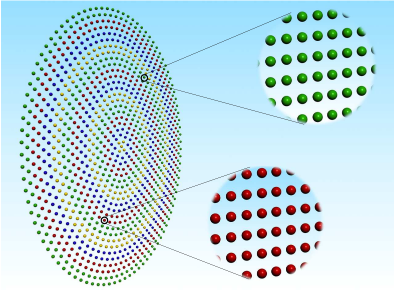


Engineering transmission: Matched transmitarrays (Huygens' metasurfaces)



Locally periodical arrays (physical optics)





Matched (Huygens') transmitarrays: Design target

Desired performance: Reflected field is zero (both co- and cross-polarized components); Transmitted field is co-polarized and its phase is shifted by the angle ϕ.

There is no reflection if

$$\eta_0 \widehat{\alpha}_{\rm ee}^{\rm co} - \frac{1}{\eta_0} \widehat{\alpha}_{\rm mm}^{\rm co} = 0, \qquad \eta_0 \widehat{\alpha}_{\rm ee}^{\rm cr} - \frac{1}{\eta_0} \widehat{\alpha}_{\rm mm}^{\rm cr} = 0$$

There is no cross-polarized transmission if

$$\eta_0 \widehat{\alpha}_{\rm ee}^{\rm cr} + \frac{1}{\eta_0} \widehat{\alpha}_{\rm mm}^{\rm cr} = 0$$

The transmitted field has the desired phase shift (and the same amplitude as the incident field) if

$$1 - \frac{j\omega}{2S} \left(\eta_0 \widehat{\alpha}_{\rm ee}^{\rm co} + \frac{1}{\eta_0} \widehat{\alpha}_{\rm mm}^{\rm co} \right) = e^{j\phi}$$

The collective polarizabilities should satisfy

$$\eta_0 \widehat{\alpha}_{\rm ee}^{\rm co} - \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co} = 0$$

$$\eta_0 \widehat{\alpha}_{\rm ee}^{\rm cr} - \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm cr} = 0, \qquad \eta_0 \widehat{\alpha}_{\rm ee}^{\rm cr} + \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm cr} = 0$$

$$1 - \frac{j\omega}{2S}\left(\eta_0 \widehat{\alpha}_{\rm ee}^{\rm co} + \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right) = e^{j\phi}$$

Solution:

$$\widehat{\alpha}_{\rm ee}^{\rm cr} = \widehat{\alpha}_{\rm mm}^{\rm cr} = 0, \qquad \eta_0 \widehat{\alpha}_{\rm ee}^{\rm co} = \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co} = \frac{S}{j\omega}\left(1 - e^{j\phi}\right)$$

What should be the individual polarizabilities of array particles?

We again use the connection between the polarizabilities of particles in infinite arrays and the same particles in free space:

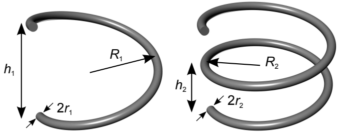
$$\frac{1}{\eta_0 \alpha_{\rm ee}} = \frac{1}{\eta_0 \widehat{\alpha}_{\rm ee}^{\rm co}} + \frac{\beta_{\rm e}}{\eta_0}, \qquad \frac{1}{\alpha_{\rm mm}/\eta_0} = \frac{1}{\widehat{\alpha}_{\rm mm}^{\rm co}/\eta_0} + \frac{\beta_{\rm e}}{\eta_0}$$

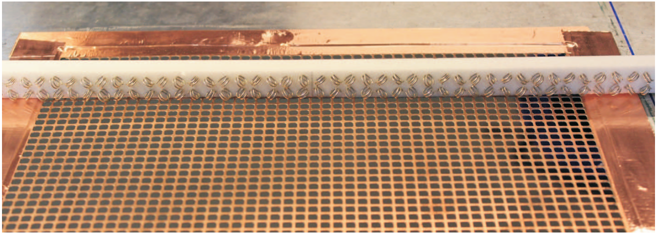
From here,

$$\frac{1}{\eta_0 \alpha_{\rm ee}} = \frac{1}{\alpha_{\rm mm}/\eta_0} = \frac{1}{\eta_0}{\rm Re}(\beta_{\rm e}) - \frac{\omega}{2S}\frac{\sin\phi}{1-\cos\phi} + j\frac{k^3}{6\pi\sqrt{\epsilon_0\mu_0}}$$

Designing particles and the transmitarray

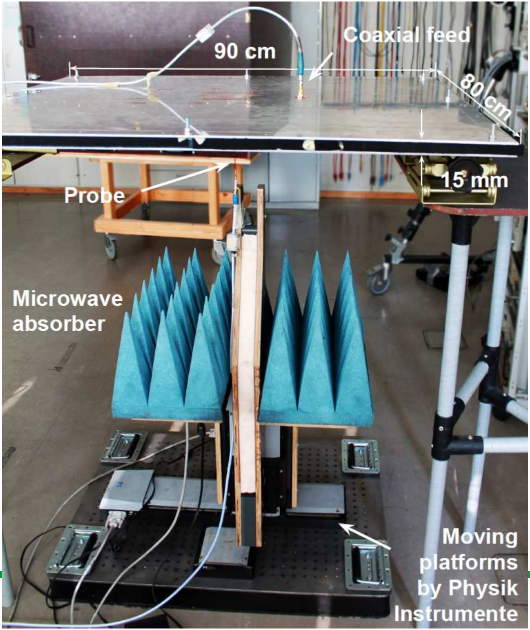
Balanced spirals, racemic arrangement



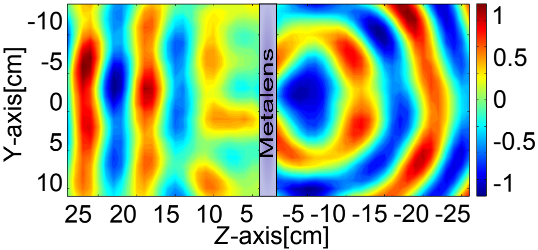


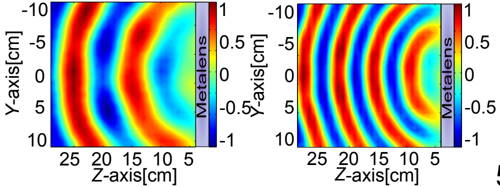
V.S. Asadchy, I.A. Faniayeu, Y. Ra'di, S.A. Khakhomov, I.V. Semchenko, S.A. Tretyakov, Broadband reflectionless metasheets: Frequency-selective transmission and perfect absorption, Phys. Rev. X, vol. 5, p. 031005, 2015.

Experimental set-up



Measured performance





3.5 GHz

5 GHz

A.A. Elsakka, V.S. Asadchy, I.A. Faniayeu, S.N. Tcvetkova, and S.A. Tretyakov, Multifunctional cascaded metamaterials: Integrated transmitarrays, IEEE Trans. Antennas Propag., vol. 64, no. 10, pp. 4266-4276, 2016.

General bianisotropic sheets (electric, magnetic, and magnetoelectric properties)

$$\begin{bmatrix} \mathbf{J}_{\rm e} \\ \mathbf{J}_{\rm m} \end{bmatrix} = \begin{bmatrix} \overline{\overline{\widehat{Y}}}_{\rm ee} & \overline{\overline{\widehat{Y}}}_{\rm em} \\ \overline{\overline{\widehat{Y}}}_{\rm me} & \overline{\overline{\widehat{Y}}}_{\rm mm} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{E}_{\rm inc} \\ \mathbf{H}_{\rm inc} \end{bmatrix}$$

The same in terms of the dipole moments of unit cells $\mathbf{p}$ and $\mathbf{m}$:

$$\mathbf{J}_{\rm e} = \frac{j\omega\mathbf{p}}{S}, \qquad \mathbf{J}_{\rm m} = \frac{j\omega\mathbf{m}}{S}$$

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{m} \end{bmatrix} = \begin{bmatrix} \overline{\overline{\widehat{\alpha}}}_{\rm ee} & \overline{\overline{\widehat{\alpha}}}_{\rm em} \\ \overline{\overline{\widehat{\alpha}}}_{\rm me} & \overline{\overline{\widehat{\alpha}}}_{\rm mm} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{E}_{\rm inc} \\ \mathbf{H}_{\rm inc} \end{bmatrix}$$

Uniaxial symmetry

Electric and magnetic polarization:

$$\overline{\overline{\widehat{\alpha}}}_{\rm ee}=\widehat{\alpha}_{\rm ee}^{\rm co}\overline{\overline{I}}_{\rm t}+\widehat{\alpha}_{\rm ee}^{\rm cr}\overline{\overline{J}}_{\rm t},\qquad \overline{\overline{\widehat{\alpha}}}_{\rm mm}=\widehat{\alpha}_{\rm mm}^{\rm co}\overline{\overline{I}}_{\rm t}+\widehat{\alpha}_{\rm mm}^{\rm cr}\overline{\overline{J}}_{\rm t}$$

Magnetoelectric coupling:

$$\overline{\overline{\widehat{\alpha}}}_{\rm em}=\widehat{\alpha}_{\rm em}^{\rm co}\overline{\overline{I}}_{\rm t}+\widehat{\alpha}_{\rm em}^{\rm cr}\overline{\overline{J}}_{\rm t},\qquad \overline{\overline{\widehat{\alpha}}}_{\rm me}=\widehat{\alpha}_{\rm me}^{\rm co}\overline{\overline{I}}_{\rm t}+\widehat{\alpha}_{\rm me}^{\rm cr}\overline{\overline{J}}_{\rm t}$$

$\overline{\overline{I}}_{\rm t}=\overline{\overline{I}}-\mathbf{z}_0\mathbf{z}_0$ is the two-dimensional unit dyadic, and $\overline{\overline{J}}_{\rm t}=\mathbf{z}_0\times\overline{\overline{I}}_{\rm t}$ is the vector-product operator.

Reciprocal and nonreciprocal coupling:

$$\overline{\overline{\widehat{\alpha}}}_{\rm em}=(\widehat{\chi}+j\widehat{\kappa})\overline{\overline{I}}_{\rm t}+(\widehat{V}+j\widehat{\Omega})\overline{\overline{J}}_{\rm t},\qquad \overline{\overline{\widehat{\alpha}}}_{\rm me}=(\widehat{\chi}-j\widehat{\kappa})\overline{\overline{I}}_{\rm t}+(-\widehat{V}+j\widehat{\Omega})\overline{\overline{J}}_{\rm t}.$$

Red=non-reciprocal effect

Reflected and transmitted fields
Most general uniaxial sheets

Normally incident plane wave:

$$\mathbf{E}_{\mathrm{ref}} = -\frac{j\omega}{2S}\left[\left(\eta_0\widehat{\alpha}_{\mathrm{ee}}^{\mathrm{co}} \pm \widehat{\alpha}_{\mathrm{em}}^{\mathrm{cr}} \pm \widehat{\alpha}_{\mathrm{me}}^{\mathrm{cr}} - \frac{1}{\eta_0}\widehat{\alpha}_{\mathrm{mm}}^{\mathrm{co}}\right)\overline{\overline{\mathrm{I}}}_{\mathrm{t}} + \left(\eta_0\widehat{\alpha}_{\mathrm{ee}}^{\mathrm{cr}} \mp \widehat{\alpha}_{\mathrm{em}}^{\mathrm{co}} \mp \widehat{\alpha}_{\mathrm{me}}^{\mathrm{co}} - \frac{1}{\eta_0}\widehat{\alpha}_{\mathrm{mm}}^{\mathrm{cr}}\right)\overline{\overline{\mathrm{J}}}_{\mathrm{t}}\right]\cdot\mathbf{E}_{\mathrm{inc}}$$

$$\mathbf{E}_{\mathrm{tr}} = \left\{\left[1 - \frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\mathrm{ee}}^{\mathrm{co}} \pm \widehat{\alpha}_{\mathrm{em}}^{\mathrm{cr}} \mp \widehat{\alpha}_{\mathrm{me}}^{\mathrm{cr}} + \frac{1}{\eta_0}\widehat{\alpha}_{\mathrm{mm}}^{\mathrm{co}}\right)\right]\overline{\overline{\mathrm{I}}}_{\mathrm{t}} - \frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\mathrm{ee}}^{\mathrm{cr}} \mp \widehat{\alpha}_{\mathrm{em}}^{\mathrm{co}} \pm \widehat{\alpha}_{\mathrm{me}}^{\mathrm{co}} + \frac{1}{\eta_0}\widehat{\alpha}_{\mathrm{mm}}^{\mathrm{cr}}\right)\overline{\overline{\mathrm{J}}}_{\mathrm{t}}\right\}\cdot\mathbf{E}_{\mathrm{inc}}$$

This allows the most general device synthesis, within the physical optics approximation.

Reflected and transmitted fields
Reciprocal sheets

$$\widehat{\alpha}_{\rm em}^{\rm cr} + \widehat{\alpha}_{\rm me}^{\rm cr} = 2j\widehat{\Omega}, \qquad \widehat{\alpha}_{\rm me}^{\rm cr} - \widehat{\alpha}_{\rm em}^{\rm cr} = 2\widehat{\kappa}$$

$$\mathbf{E}_{\rm ref} = -\frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co} \pm 2j\widehat{\Omega} - \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right)\mathbf{E}_{\rm inc}$$

$$\mathbf{E}_{\rm tr} = \left[1 - \frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co} + \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right)\right]\mathbf{E}_{\rm inc} \mp \frac{\omega}{S}\widehat{\kappa}\,\mathbf{z}_0 \times \mathbf{E}_{\rm inc}$$

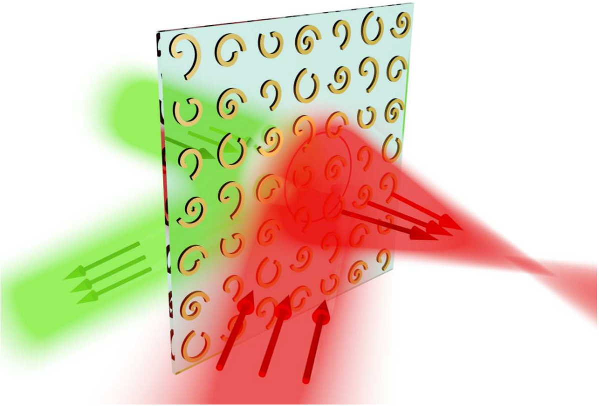
Magnetic response ($\alpha_{\rm mm}$) controls matching; omega coupling (Ω) controls asymmetry in reflections from two sides; chirality (κ) controls polarization transformation in transmission.

General bianisotropic sheets

Additional possible functionalities: All what was still impossible with magneto-dielectric sheets

Still impossible: Nothing (if not forbidden by basic physics)

Engineering reflections: Metamirrors



Y. Ra'di, V. S. Asadchy, S. A. Tretyakov, Tailoring reflections from thin composite metamirrors, IEEE Trans. Antennas Propag., vol. 62, no. 7, pp. 3749-3760, 2014.

Required collective polarizabilities

Desired performance: Transmitted field is zero; Lossless reflection; Reflection phase ϕ for one side and θ for the other.

No transmission:

$$1-\frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co} \pm \widehat{\alpha}_{\rm em}^{\rm cr} \mp \widehat{\alpha}_{\rm me}^{\rm cr} + \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right) = 0, \quad \eta_0\widehat{\alpha}_{\rm ee}^{\rm cr} \mp \widehat{\alpha}_{\rm em}^{\rm co} \pm \widehat{\alpha}_{\rm me}^{\rm co} + \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm cr} = 0$$

Desired reflection coefficients:

$$-\frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co} + \widehat{\alpha}_{\rm em}^{\rm cr} + \widehat{\alpha}_{\rm me}^{\rm cr} - \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right) = e^{j\phi}$$

$$-\frac{j\omega}{2S}\left(\eta_0\widehat{\alpha}_{\rm ee}^{\rm co} - \widehat{\alpha}_{\rm em}^{\rm cr} - \widehat{\alpha}_{\rm me}^{\rm cr} - \frac{1}{\eta_0}\widehat{\alpha}_{\rm mm}^{\rm co}\right) = e^{j\theta}$$

Several possible realizations. Assuming that we will use only reciprocal unit cells:

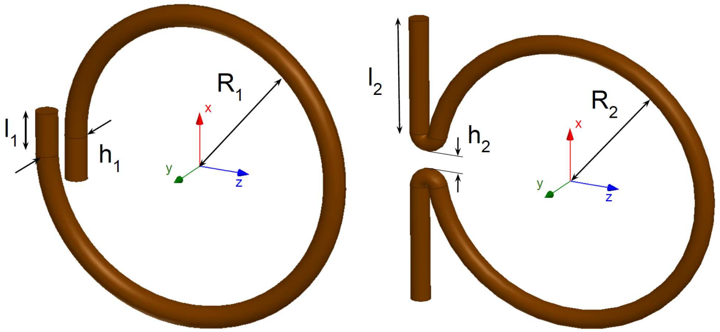
$$\eta_0 \widehat{\alpha}_{\rm ee}^{\rm co} = \frac{S}{j\omega}\left(1 - \frac{e^{j\phi} + e^{j\theta}}{2}\right)$$

$$\widehat{\alpha}_{\rm em}^{\rm cr} = \widehat{\alpha}_{\rm me}^{\rm cr} = \frac{-S}{j\omega}\left(\frac{e^{j\phi} - e^{j\theta}}{2}\right)$$

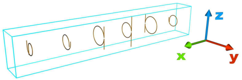
$$\frac{1}{\eta_0} \widehat{\alpha}_{\rm mm}^{\rm co} = \frac{S}{j\omega}\left(1 + \frac{e^{j\phi} + e^{j\theta}}{2}\right)$$

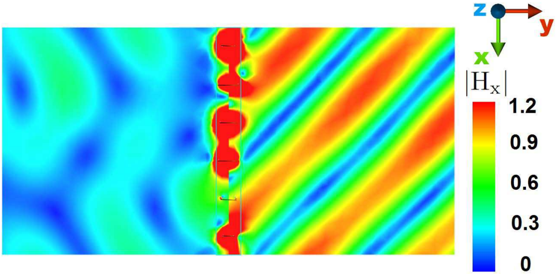
Need lossless bianisotropic unit cells (omega coupling, no chirality).

Shapes of wire “omega” particles



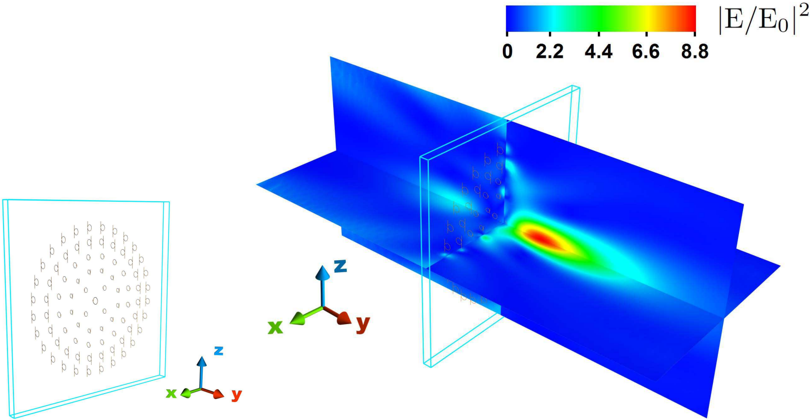
Example: Deflecting metamirror





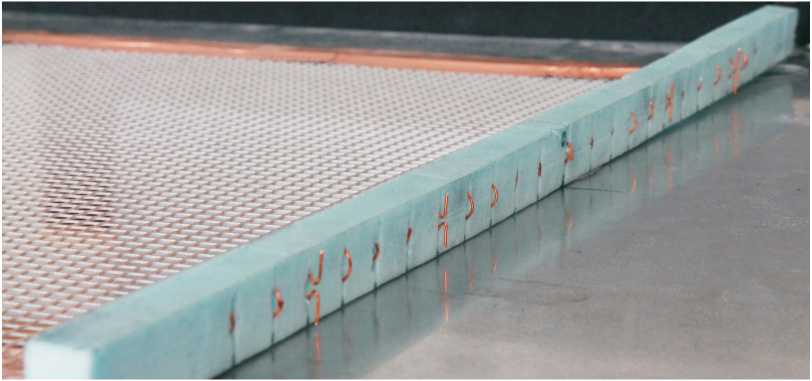
V.S. Asadchy, Y. Radi, J. Vehmas, and S.A. Tretyakov, Functional metamirrors using bianisotropic elements, Phys. Rev. Lett., vol. 114, p. 095503, 2015.

Example: Focusing metamirror

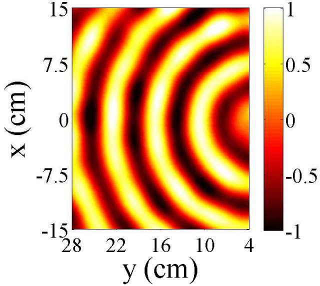


Focal length $\approx 0.65\lambda_0$

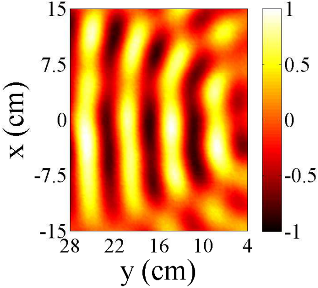
Experimental sample



Measured results: Focusing metamirror



Incident fields

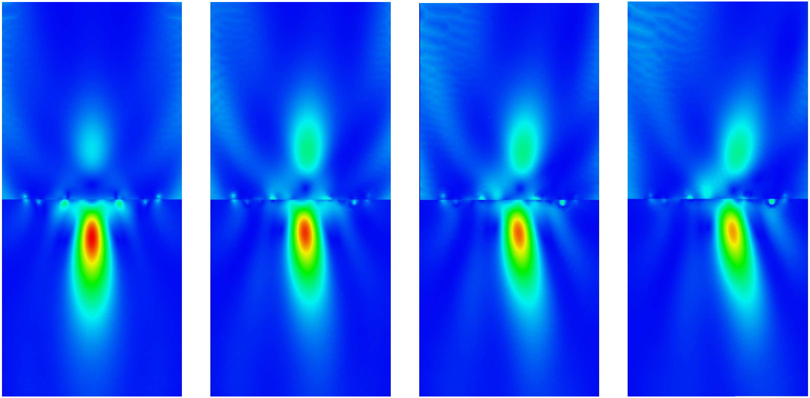


Reflected fields

Operating frequency 5 GHz, bandwidth ≈ 5%, reflectivity ≈ 86%, focal distance 0.65λ, f-number $f/D = 0.23$, focal spot size $2.8\lambda \times 0.9\lambda$, metasurface thickness $\lambda/7.6$, diameter 2.8λ, the focusing reflector is transparent outside of the resonant band.

Oblique incidence: Focal spot shift

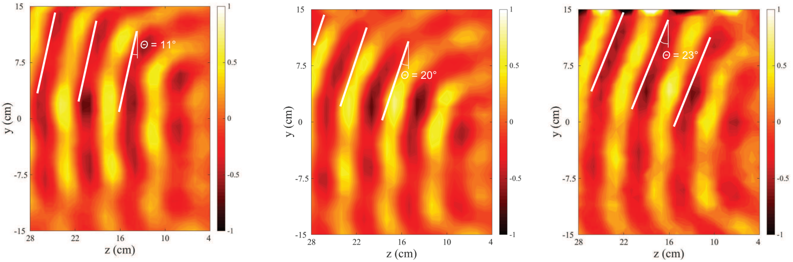
Simulation results



0° 5° 10° 15°

Obligue incidence / Low-profile multibeam antenna

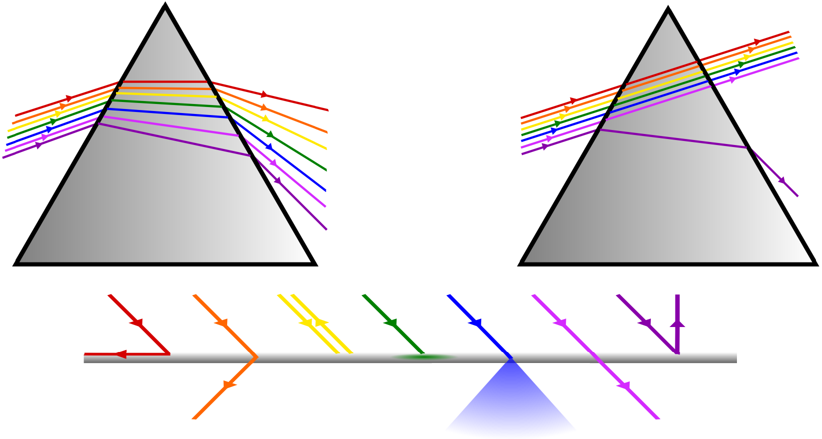
Experimental results



Three primary feeds at different positions create three beams in different directions.

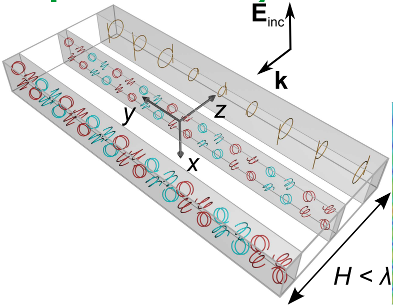
S.N. Tcvetkova, V.S. Asadchy, and S.A. Tretyakov, Scanning characteristics of metamirror antennas with sub-wavelength focal distance, IEEE Trans. Antennas Propag., vol. 64, no. 8, pp. 3656-3660, 2016.

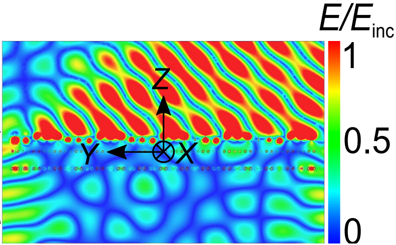
Multifunctional metasurfaces

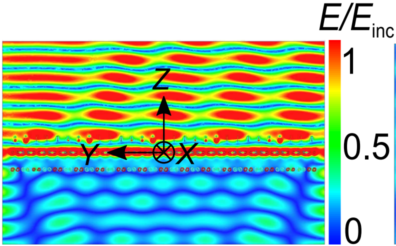


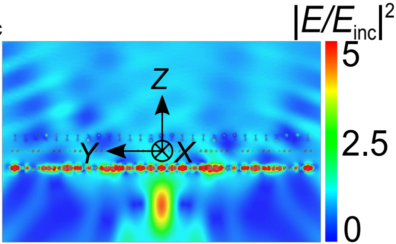
A.A. Elsakka, V.S. Asadchy, I.A. Faniayeu, S.N. Tcvetkova, and S.A. Tretyakov, Multifunctional cascaded metamaterials: Integrated transmitarrays, IEEE Trans. Antennas Propag., vol. 64, no. 10, pp. 4266-4276, 2016.

Example three-layer surface



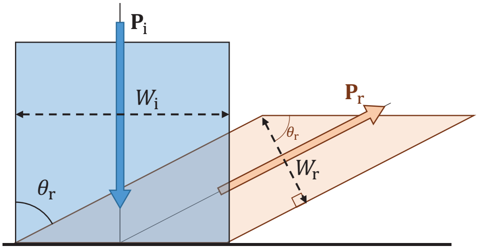


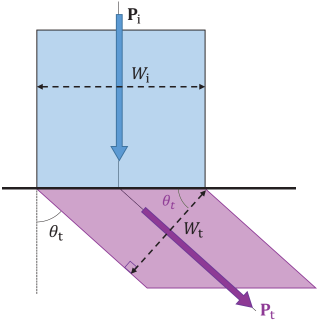




Physical optics is an approximation!

How one can create metasurfaces for *perfect* control of reflection and transmission?





Conventional design approach: Physical optics (the phased-array principle)

The incident wave is $e^{-jkx\sin\theta_\mathrm{i}}$ and the reflected wave is $e^{-jkx\sin\theta_\mathrm{r}}$.

The reflection coefficient is set to

$$R = \exp[jkx(\sin\theta_\mathrm{i} - \sin\theta_\mathrm{r})] = \exp(j\Phi_\mathrm{r}(x))$$

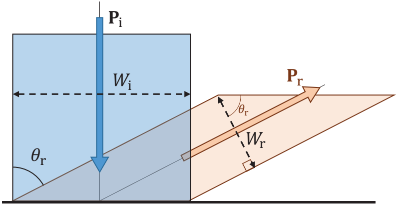
At every point we want to have full power reflection (or transmission):

$$|R| = 1 \qquad \text{or} \qquad |T| = 1$$

Gradient phase ("the generalized reflection law"):

$$\sin\theta_\mathrm{i} - \sin\theta_\mathrm{r} = \frac{1}{k}\frac{d\Phi_\mathrm{r}(x)}{dx}$$

But such reflectors do not produce the desired reflected fields!



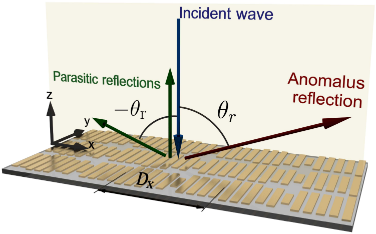
$$|R| \neq 1 \qquad \text{or} \qquad |T| \neq 1$$

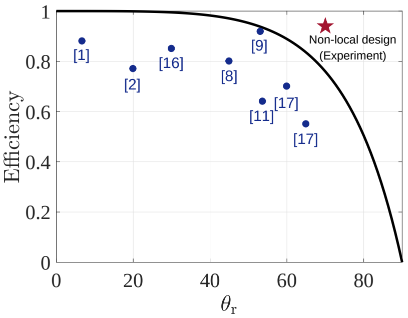
Power efficiency:

$$\zeta_r = 1 - \left(\frac{Z_r - Z_i}{Z_r + Z_i}\right)^2 = \frac{4\cos\theta_i \cos\theta_r}{(\cos\theta_i + \cos\theta_r)^2}$$

V.S. Asadchy, A. Wickberg, A. Díaz-Rubio, and M. Wegener, Eliminating scattering loss in anomalously reflecting optical metasurfaces, ACS Photonics, vol. 4, pp. 1264-1270, 2017.

Efficiency results

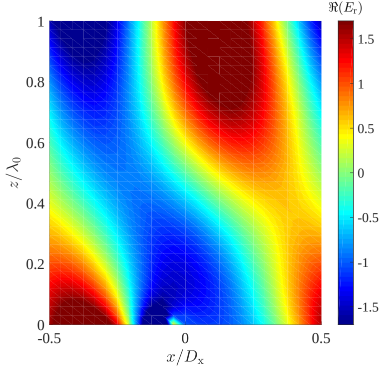


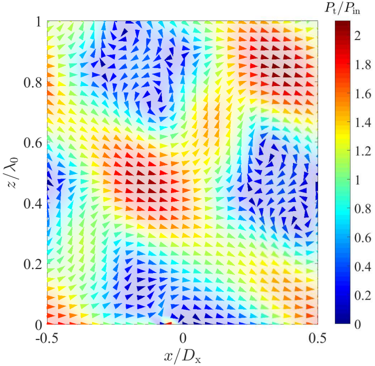


[1] M. Collischon, et al., App. Opt. 33, 16 (1994); [2] P. Lalanne, et al., Opt. Lett. 23, 14 (1998); [8] S. Sun, et al., Nano Lett. 12, 6223 (2012); [9] S. Sun, et al., Nat. Mat. 11, 3292 (2012); [11] A. Pors and S. I. Bozhevolnyi, Opt. Express 21, 27438 (2013); [16] Z. Li, et al., Nano Lett. 15, 3 (2015); [17] G. Zheng, et al., Nat. Nanotechnol. 10, 308 (2015).

Conventional design ("generalized reflection law")

$$\sin\theta_{\rm i} - \sin\theta_{\rm r} = \frac{1}{k_1}\frac{d\Phi_{\rm r}(x)}{dx}, \qquad Z_{\rm s}(x) = j\frac{\eta_1}{\cos\theta_{\rm i}}\cot[\Phi_{\rm r}(x)/2]$$

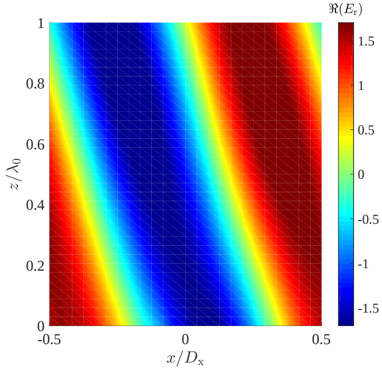


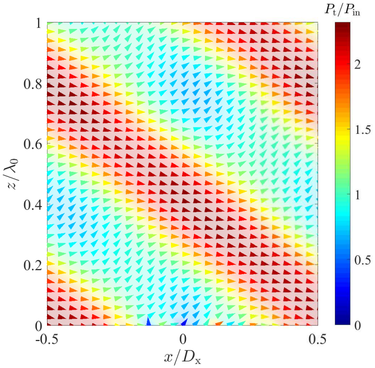


$\theta_{\rm i} = 0°, \quad \theta_{\rm r} = 70°$. Efficiency 75.7%.

“Active-lossy design”

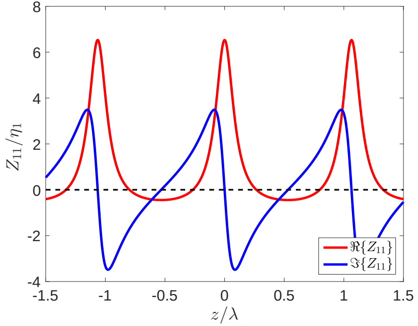
$$E_r = E_i \frac{\sqrt{\cos\theta_i}}{\sqrt{\cos\theta_r}}, \quad Z_s(x) = \frac{\eta_1}{\sqrt{\cos\theta_i \cos\theta_r}} \frac{\sqrt{\cos\theta_r} + \sqrt{\cos\theta_i}\, e^{j\Phi_r(x)}}{\sqrt{\cos\theta_i} - \sqrt{\cos\theta_r}\, e^{j\Phi_r(x)}}$$





Efficiency 100%

But how can we realize it?



We need either active elements (gain) or strong non-locality (receiving in “lossy regions” and radiating in “active regions”)

V.S. Asadchy, M. Albooyeh, S.N. Tcvetkova, A. Díaz-Rubio, Y. Ra’di, and S. A. Tretyakov, Perfect control of reflection and refraction using spatially dispersive metasurfaces, Phys. Rev. B, vol. 94, 075142, 2016.

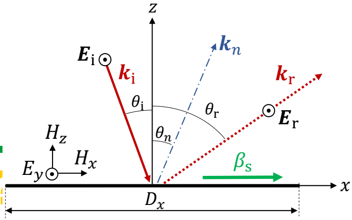
Design concept: Inhomogeneous leaky-wave antenna

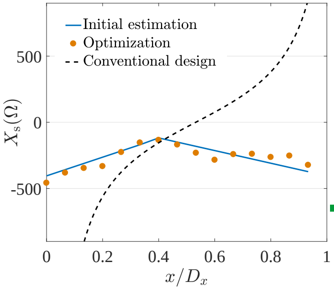
Select reactive surface impedance Z_{s0} such that a surface wave is supported: $\beta_{\mathrm{s}} = k_1 \sqrt{1 - \frac{Z_1^2}{Z_{\mathrm{s0}}^2}}$

Periodically modulating the surface, couple to plane waves with

$$\sin\theta_n = \frac{\beta_{\mathrm{s}}}{k_1} = \sqrt{1 - \frac{Z_1^2}{Z_{\mathrm{s0}}^2}} + n\sin\theta_{\mathrm{r}}$$

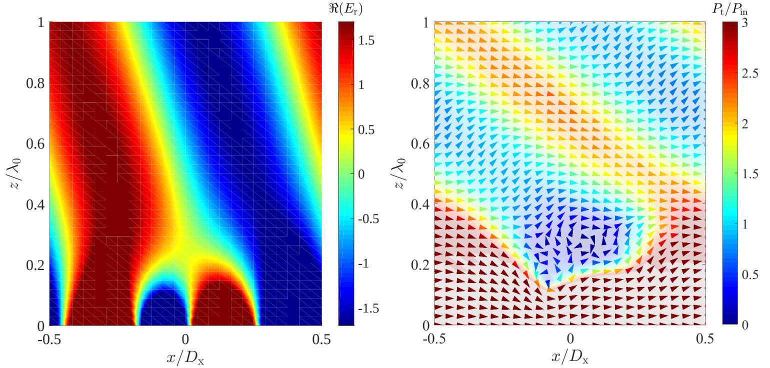
Next we LINEARLY modulate the reflection phase, to receive and radiate from/to the desired directions:





Numerical results

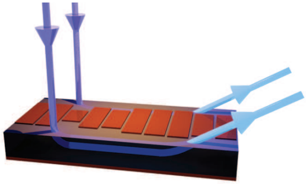
$$\Phi_{\rm r}(x) = \begin{cases} (\sin\theta_{\rm r} - \sin\theta_n)k_1 x - \Phi_0 & 0 \le x < x_1 \\ (\sin\theta_{\rm i} - \sin\theta_n)k_1(x - D) - \Phi_0 & x_1 \le x < D. \end{cases}$$

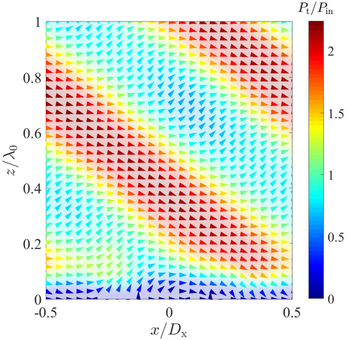


Efficiency 100% with a lossless reflector: an inhomogeneous reactive boundary.

Design for experimental realization

Array of rectangular patches on a grounded dielectric substrate.



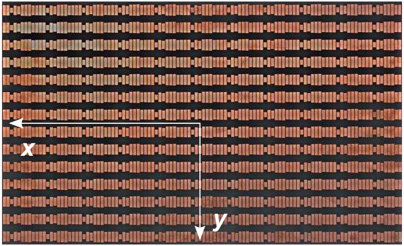


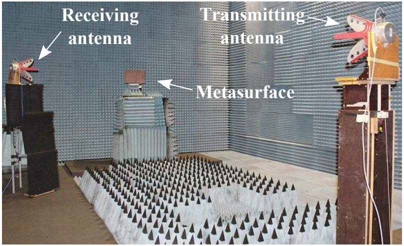
Numerically calculated efficiency 94%

(< 100% due to losses in copper and dielectric)

A. Díaz-Rubio, V.S. Asadchy, A. Elsakka, and S.A. Tretyakov, From the generalized reflection law to the realization of perfect anomalous reflectors, Science Advances, 3, e1602714, 2017.

Experimental sample for microwaves





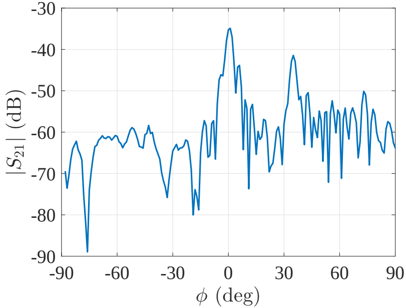
Target operational frequency 8 GHz.

1.575 mm thick Rogers 5880 substrate ($\epsilon_\mathrm{r} = 2.2$, $\tan\delta = 0.0009$), copper patches.

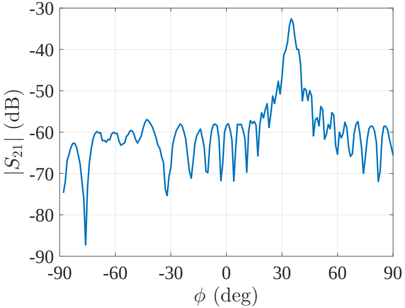
The sample size $11.7\lambda \times 7\lambda$ (440 mm × 262.5 mm).

Experimental results

Fixed bi-static antenna positions, at $0°$ and $70°$.
Rotating sample ($\phi = 0$ corresponds to the normal incidence).



Metamirror sample



Reference metal plate

Experimentally measured power efficiency 93.8% at 8.08 GHz (numerically simulated: 94% at 8 GHz).

Simple special case $\theta_\mathrm{r} = \pm\theta_\mathrm{i}$

General case: “active-lossy” surface

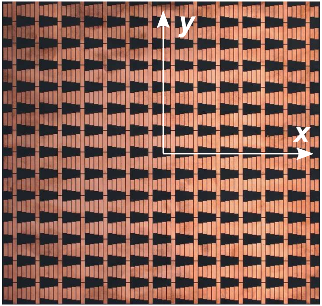
$$E_\mathrm{r} = E_\mathrm{i}\frac{\sqrt{\cos\theta_\mathrm{i}}}{\sqrt{\cos\theta_\mathrm{r}}}, \quad Z_\mathrm{s}(x) = \frac{\eta_1}{\sqrt{\cos\theta_\mathrm{i}\cos\theta_\mathrm{r}}}\frac{\sqrt{\cos\theta_\mathrm{r}} + \sqrt{\cos\theta_\mathrm{i}}\,e^{j\Phi_\mathrm{r}(x)}}{\sqrt{\cos\theta_\mathrm{i}} - \sqrt{\cos\theta_\mathrm{r}}\,e^{j\Phi_\mathrm{r}(x)}}$$

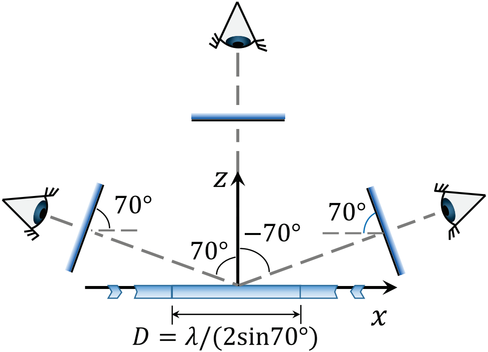
If $\theta_\mathrm{r} = \pm\theta_\mathrm{i}$, the surface is purely reactive at every point:

$$E_\mathrm{r} = E_\mathrm{i}, \qquad Z_\mathrm{s}(x) = j\frac{\eta_1}{\sqrt{\cos\theta_\mathrm{i}}}\cot\frac{\Phi_\mathrm{r}(x)}{2}$$

Specular reflection or *retroreflection*

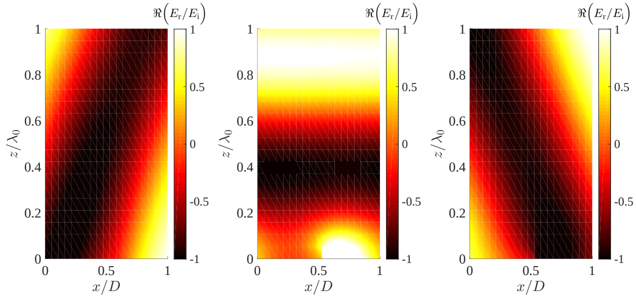
Three-channel mirror

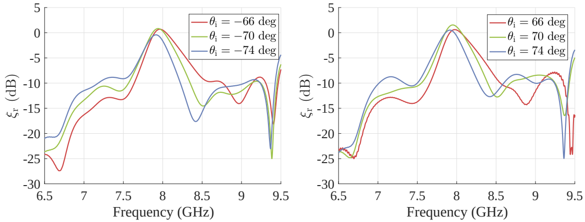




At $\theta_\mathrm{i} = 0°$ and $\theta_\mathrm{i} = \pm 70°$ angles, the observer sees himself as in a mirror normally oriented in respect to him.

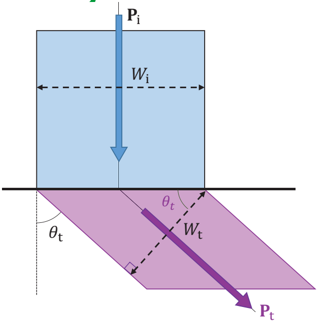
Results

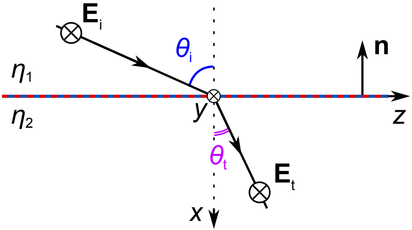




V.S. Asadchy, A. Díaz-Rubio, S.N. Tcvetkova, D.-H. Kwon, A. Elsakka, M. Albooyeh, and S.A. Tretyakov, Flat engineered multi-channel reflectors, to appear in Phys. Rev. X.

Exact synthesis of transmitting metasurfaces





$$\zeta_{\mathrm{r,TE}} = 1-\left(\frac{Z_\mathrm{t}-Z_\mathrm{i}}{Z_\mathrm{t}+Z_\mathrm{i}}\right)^2 = \frac{4\eta_1\eta_2\cos\theta_\mathrm{i}\cos\theta_\mathrm{t}}{(\eta_2\cos\theta_\mathrm{i}+\eta_1\cos\theta_\mathrm{t})^2},\ \zeta_{\mathrm{r,TM}} = \frac{4\eta_1\eta_2\cos\theta_\mathrm{i}\cos\theta_\mathrm{t}}{(\eta_2\cos\theta_\mathrm{t}+\eta_1\cos\theta_\mathrm{i})^2}$$

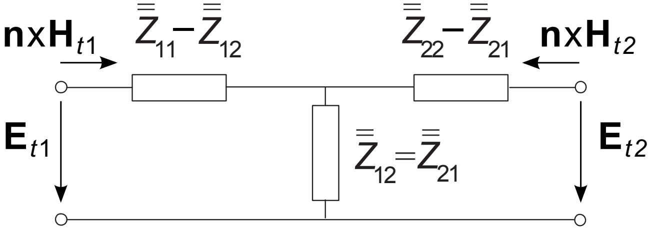
M. Selvanayagam and G.V. Eleftheriades, Discontinuous electromagnetic fields using orthogonal electric and magnetic currents for wavefront manipulation, Opt. Expr., 21, 14409, 2013; A. Epstein and G.V. Eleftheriades, Huygens' metasurfaces via the equivalence principle: design and applications, J. Opt. Soc. Am. B, 33, A31, 2016.

Homogenization model

Involving only *tangential* fields:

$$\mathbf{E}_{t1} = \overline{\overline{Z}}_{11} \cdot \mathbf{n} \times \mathbf{H}_{t1} + \overline{\overline{Z}}_{12} \cdot (-\mathbf{n} \times \mathbf{H}_{t2})$$

$$\mathbf{E}_{t2} = \overline{\overline{Z}}_{21} \cdot \mathbf{n} \times \mathbf{H}_{t1} + \overline{\overline{Z}}_{22} \cdot (-\mathbf{n} \times \mathbf{H}_{t2})$$



Equations for Z-parameters

Equating the normal components of the Poynting vector:

$$\mathbf{E}_{\rm t} = \mathbf{E}_{\rm i}\sqrt{\frac{\cos\theta_{\rm i}}{\cos\theta_{\rm t}}}\sqrt{\frac{\eta_2}{\eta_1}}\,e^{j\phi_{\rm t}}$$

Substituting the desired field values:

$$e^{-jk_1\sin\theta_{\rm i}z} = Z_{11}\frac{1}{\eta_1}\cos\theta_{\rm i}\,e^{-jk_1\sin\theta_{\rm i}z} - Z_{12}\frac{1}{\sqrt{\eta_1\eta_2}}\sqrt{\cos\theta_{\rm i}\cos\theta_{\rm t}}\,e^{-jk_2\sin\theta_{\rm t}z+j\phi_{\rm t}}$$

$$e^{-jk_2\sin\theta_{\rm t}z+j\phi_{\rm t}} = Z_{21}\frac{1}{\sqrt{\eta_1\eta_2}}\sqrt{\cos\theta_{\rm i}\cos\theta_{\rm t}}\,e^{-jk_1\sin\theta_{\rm i}z} - Z_{22}\frac{\cos\theta_{\rm t}}{\eta_2}\,e^{-jk_2\sin\theta_{\rm t}z+j\phi_{\rm t}}$$

Two equations, four design parameters.

V.S. Asadchy, M. Albooyeh, S.N. Tcvetkova, A. Díaz-Rubio, Y. Ra'di, and S. A. Tretyakov, Perfect control of reflection and refraction using spatially dispersive metasurfaces, Phys. Rev. B, vol. 94, 075142, 2016.

Perfect lossless design

$$e^{-jk_1 \sin\theta_\mathrm{i} z} = jX_{11}\,\frac{1}{\eta_1}\cos\theta_\mathrm{i}\,e^{-jk_1 \sin\theta_\mathrm{i} z} - jX_{12}\,\frac{1}{\sqrt{\eta_1\eta_2}}\,\sqrt{\cos\theta_\mathrm{i}\cos\theta_\mathrm{t}}\,e^{-jk_2 \sin\theta_\mathrm{t} z + j\phi_\mathrm{t}}$$

$$e^{-jk_2 \sin\theta_\mathrm{t} z + j\phi_\mathrm{t}} = jX_{21}\,\frac{1}{\sqrt{\eta_1\eta_2}}\,\sqrt{\cos\theta_\mathrm{i}\cos\theta_\mathrm{t}}\,e^{-jk_1 \sin\theta_\mathrm{i} z} - jX_{22}\,\frac{\cos\theta_\mathrm{t}}{\eta_2}\,e^{-jk_2 \sin\theta_\mathrm{t} z + j\phi_\mathrm{t}}$$

Unique solution [where $\Phi_\mathrm{t}(z) = -k_2 \sin\theta_\mathrm{t} z + \phi_\mathrm{t} + k_1 \sin\theta_\mathrm{i} z$]:

$$X_{11} = -\frac{\eta_1}{\cos\theta_\mathrm{i}}\cot\Phi_\mathrm{t}$$

$$X_{22} = -\frac{\eta_2}{\cos\theta_\mathrm{t}}\cot\Phi_\mathrm{t}$$

$$X_{12} = X_{21} = -\frac{\sqrt{\eta_1\eta_2}}{\sqrt{\cos\theta_\mathrm{i}\cos\theta_\mathrm{t}}}\,\frac{1}{\sin\Phi_\mathrm{t}}$$

Anomalous refraction requires bianisotropy (omega coupling)

Unit-cell polarizabilities

$$\widehat{\alpha}_{\rm ee}^{yy} = \frac{S}{j\omega}\frac{\cos\theta_{\rm i}\cos\theta_{\rm t}}{\eta_1\cos\theta_{\rm t}+\eta_2\cos\theta_{\rm i}}\left[2-\left(\sqrt{\frac{\eta_1\cos\theta_{\rm t}}{\eta_2\cos\theta_{\rm i}}}+\sqrt{\frac{\eta_2\cos\theta_{\rm i}}{\eta_1\cos\theta_{\rm t}}}\right)e^{-j\Phi_{\rm t}(z)}\right]$$

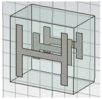
$$\widehat{\alpha}_{\rm mm}^{zz} = \frac{S}{j\omega}\frac{\eta_1\eta_2}{\eta_1\cos\theta_{\rm t}+\eta_2\cos\theta_{\rm i}}\left[2-\left(\sqrt{\frac{\eta_1\cos\theta_{\rm t}}{\eta_2\cos\theta_{\rm i}}}+\sqrt{\frac{\eta_2\cos\theta_{\rm i}}{\eta_1\cos\theta_{\rm t}}}\right)e^{-j\Phi_{\rm t}(z)}\right]$$

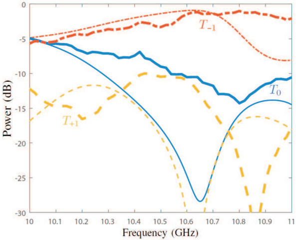
$$\widehat{\alpha}_{\rm em}^{yz} = -\widehat{\alpha}_{\rm me}^{yz} = \frac{S}{j\omega}\frac{\eta_2\cos\theta_{\rm i}-\eta_1\cos\theta_{\rm t}}{\eta_1\cos\theta_{\rm t}+\eta_2\cos\theta_{\rm i}}$$

where S is the unit-cell area.

Bianisotropic omega layers, e.g. arrays of Ω-shaped particles, arrays of split rings, double arrays of patches (different patches on the two sides of a thin dielectric substrate), asymmetric three-layer structures, …

Experimental realization





Ultimate efficiency for conventional Huygens' metasurfaces: 75.7%

Measured efficiency: 81%

G. Lavigne, K. Achouri, V. Asadchy, S. Tretyakov, C. Caloz, Refracting metasurfaces without spurious diffraction, arXiv:1705.09286v2, May-June 2017.

Another experiment: M. Chen, E. Abdo-Sánchez, A. Epstein, and G.V. Eleftheriades, Experimental verification of reflectionless wide-angle refraction via a bianisotropic Huygens metasurface, arXiv:1703.06669v1, March 2017 (presentation tomorrow).

Conclusions

- Metasurfaces are electrically thin composite material layers, designed and optimized to function as tools to control and transform electromagnetic waves
- Bianisotropic metasurfaces allow (nearly) full control of reflection and transmission properties, but advanced functionalities require strongly non-local structures

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