

Evidence for Single-Vortex Pinning and Unpinning Events in Superfluid ^4He

P. Hakonen* and O. Avenel

CEA-DRECAM, Service de Physique de l'État Condensé, Centre d'Études de Saclay, 91191 Gif-sur-Yvette Cedex, France

E. Varoquaux

CNRS-Laboratoire de Physique des Solides, Bâtiment 510, Université Paris-Sud, 91405 Orsay, France

(Received 30 March 1998)

We have observed two critical velocity levels for phase slips in superfluid ^4He at temperatures between 14 and 44 mK, due to microparticles formed in the cell at low temperature. We interpret these observations as evidence for pinning and unpinning events of single vortices of nanometric size. From a study of the lifetime and unpinning velocities of the pinned vortices, we are led to conclude that unpinning takes place by quantum tunneling. [S0031-9007(98)07420-1]

PACS numbers: 67.40.Vs, 47.32.-y

Nucleation and pinning of vortex lines, as well as their interplay, have been long-standing problems in superfluid ^4He . Progress has been made on the former subject during the recent years by studying phase slippage events of superfluid flow in submicron apertures [1]. A consistent picture of vortex nucleation, either thermally activated or via quantum tunneling, as a half-ring at the wall has emerged from these experiments. The purpose of this Letter is to shed light on the latter problem, the question of vortex pinning.

According to Feynman [2] the unpinning velocity for vortices is inversely proportional to the size of the flow geometry. *Ipsa facto*, unpinning events on nanometric structures are rather difficult to distinguish from real nucleation events. In the experiments described below, however, such a distinction happens to be very clear. This feature has allowed us to investigate quantitatively pinning/unpinning events of a single vortex line in superfluid ^4He .

Following Schwarz [3], the pinning potential of a vortex of core size a on a hemispherical asperity of radius b in a flow velocity v may be written as

$$\Delta E = \frac{1}{2} \rho_s \kappa_4^2 b \ln(8r_c/a) \left(1 - \frac{v}{v_{u0}}\right) = E_p \left(1 - \frac{v}{v_{u0}}\right), \quad (1)$$

where ρ_s is the superfluid density, κ_4 the quantum of circulation, and $r_c = b(1 - v/v_{u0})$ denotes the radius of a fluctuating wiggle on the vortex. Eq. (1) defines the pinning barrier parameter E_p . The unpinning velocity v_{u0} at which ΔE vanishes has been calculated numerically by Schwarz [4]. It is given by

$$v_{u0} = (\kappa_4/2\pi D) \ln(b/a), \quad (2)$$

where D is the distance between the two pinning sites on which the vortex is anchored.

On the experimental side, only a few hard-won facts are known about pinned vortices [5]. One of them is that the pinning strength depends markedly on the microscopic

roughness of the surface: Strong pinning of vortices is considerably enhanced when microscopic particles are purposely sprayed on the surface [6].

Our experiments on the pinning and unpinning of small vortex filaments make use of this property. They were performed in a Helmholtz-type resonator very similar to those used in previous investigations on vortex nucleation [1]. The operation of the double-hole resonator with one large channel and one microaperture is described in detail in [7] together with the phenomena of phase slips and of superflow collapses. Collapses are large phase slips which kill a sizable fraction, or all, of the superflow through the microaperture. Fluid motion is monitored by accurately tracking the position of the flexible membrane of the resonator which is electrostatically driven on resonance. The critical velocity v_c is obtained from the maximum amplitude of the Helmholtz oscillations. Velocity circulation can be trapped in the superfluid loop threading the two holes. The phase slips taking place in the microaperture change this trapped circulation by an integral number of circulation quanta. The time chart of the successive peak amplitudes of outward and inward half-cycles of the resonance (during which the resonator empties and fills, respectively) is analyzed to keep track of the successive slips and the quantum state of circulation of the loop q . The flow velocity in the microaperture is derived from the knowledge of this quantum state and of the membrane-driven flow. In these experiments, the ratio of the membrane-driven flow in the large channel to that in the microaperture is very nearly unity ($R = 1.053$).

The major distinctive feature of the present experiments is the formation of microscopic dust inside the resonator during the course of an experimental run. This was achieved by electrostatically driving the membrane across the stability limit [8], causing a small spark at the inner top part of the cell and the discharge of a 500 pF capacitor, charged to 200 V. In all likelihood, such a discharge resulted in the formation of microscopic dust particles, i.e., clusters of a few tens of atoms, due to

the sputtering reaction of charged ^4He ions hitting the counterelectrode [9]. Cluster formation was especially prominent when the walls of the cell were covered by ~ 10 atomic layers of H_2 . In the case of hydrogen, dust could be eliminated by thermally cycling up to ~ 10 K while in other cases a 77 K bakeout would be required, which does establish that the contaminants are the gases adsorbed on the resonator walls.

This microdust falls through the superfluid to the bottom of the resonator, where the microaperture is located. Its presence there is signaled by a profound alteration in the peak amplitude charts, with the frequent appearance of collapses. These features varied with changing dust deposits. Quite often, the flow properties of the aperture became completely irregular, with no well defined critical velocity. But in some instances, simple critical velocity patterns with two reproducible plateaus, as illustrated in Fig. 1, were found. At time "A" in Fig. 1, the Helmholtz oscillation undergoes, from a regular succession of phase slips, a strong collapse in the inward flow direction followed at the next half-cycle by a further reduction of the resonance amplitude level caused by a (outward) multiple slip of 14 ± 1 quanta. The resonator partially recovers from this collapse between A and "B" by outward slips only. Such sequences, which we call *recoveries* [7], represent the step-by-step adjustment of the circulation trapped in the superfluid loop back to a new steady-state value. After the recovery, the normal course of phase slippage resumes with a succession of alternating outward and inward slips between B and "C," but at a lower level v^* than the level at the start of the sequence v_c . After a random lapse of time, the upper plateau is regained at "D." In the recovery from C to D, which involves inward slips only, the circulation in the loop decreases by 14.2 (15.4) quanta

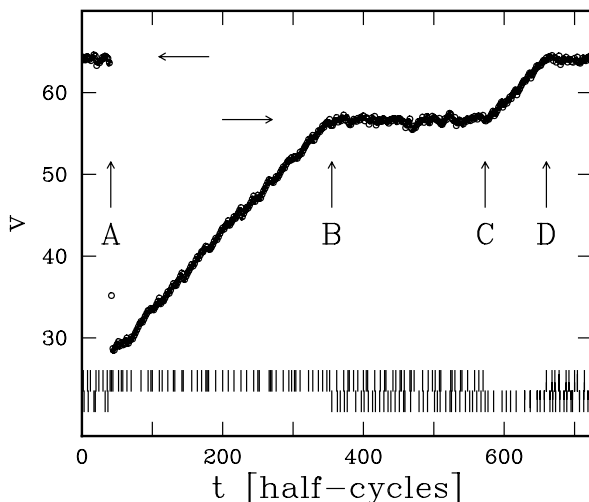


FIG. 1. Helmholtz resonance peak absolute amplitude, expressed as phase winding number in the aperture [10], vs time, at 1.0 bar and 14 mK. The ticks at the bottom of the graph indicate when and for which flow direction phase slips take place. The horizontal arrows mark the plateaus for v_c and v^* , the vertical ones, the break points discussed in the text.

on the average at 14 mK (20.5 mK). The whole sequence repeats itself randomly in time involving the same two levels v_c and v^* .

The upper plateau, with mostly single slips, corresponds to the ordinary behavior observed in our previous experiments. The shift from v_c to v^* is always preceded by a collapse. The presence of microdust in the cell increases the rate of occurrence of collapses by almost 3 orders of magnitude. Nearly all collapses take place during inward cycles for which the nucleated vortices are thought to move along the inner, contaminated side of the aperture. The present work firmly establishes the direct link between collapses and surface contamination which was suspected before [7].

The existence of the lower plateau and the detailed experimental features associated with it can be explained in their entirety by the following plausible mechanism. The velocity at the nucleation site appears to be subjected to a local influence of substantial magnitude. As it turns out, vortices are the only known long-lived defects in superfluid ^4He which can cause such a perturbing velocity, ions and electrons being short-lived. Therefore, a vortex line must, in our views, become trapped in the immediate vicinity of the vortex nucleation site at time A in Fig. 1, when the superfluid velocity has been considerably reduced immediately after the collapse.

The shape of the pinned filament adjusts to the imposed superflow. Its orientation is such that, in the case of outward-going flows, it bends toward the wall and toward the nucleation site on which it induces a velocity v_p^{\max} . This velocity adds to the membrane-driven flow. For inward-going flows the vortex moves away from the nucleation site on which it induces a reduced velocity v_p^{\min} which subtracts from the membrane-driven flow. The unbalance between v_p^{\max} and v_p^{\min} is compensated by a quantized change of the circulation in the loop, v_{loop} . Unpinning takes place at C, the start of the sequence of inward slips which forms the recovery from C to D and by which the resonator comes back to its previous regime of oscillation by releasing the 14 or so quanta of trapped circulation in the loop. At C, the vortex ceases to perturb the nucleation site. It resumes its motion in the direction in which it was moving before pinning took place, completing the interrupted phase slip.

All the qualitative features of the observations are accounted for by this model. We now proceed to draw quantitative information which will also fall within a quite reasonable range of values. Velocities v_p^{\max} and v_p^{\min} are estimated by noting (i) that the induced flow mostly lies either along or opposite to the external superflow as the pinned vortex tends to orient itself against the applied flow and (ii) that the local critical velocity for vortex nucleation is the same with and without the pinned vortex filament [11]. Then, the difference between the critical flow speed v_c and the apparent critical velocity v^* in the presence of a pinned vortex yields $v_p^{\max} - v_p^{\min} = 2(v_c - v^*)$. The

average flow $(v_p^{\max} + v_p^{\min})/2$ is balanced out by v_{loop} as stated earlier. At 14 mK, for instance, the trapped circulation q on the lower plateau is on average $14.2\kappa_4$, and the velocities, expressed as phase winding numbers [10], are $v_c = 64.3$ and $v^* = 56.9$. We thus deduce that v_p^{\min} is very small and that $v_p^{\max} = 0.23v_c$. This last value is large and the pinned vortex must lie quite close to the nucleation site. For example, for a local vortex nucleation velocity of 20 m/s, a typical value in these apertures [1], a pinned straight filament would come within 35 \AA of the nucleation site at the closest approach.

Between A and C , the fluctuations of v_c for outward slips are larger by a factor of nearly 3 than those for inward slips, which are the same as for both slip directions on the upper plateau. This observation reflects noise on v_p^{\max} , that is, random deformations of the pinned vortex. For the pinned straight vortex considered above, these random deformations correspond to an oscillation amplitude of 0.7 \AA , i.e., to the zero-point oscillations of the fundamental mode of a 30 \AA long vortex [12]. This value sets an estimate for the pinned vortex length.

At temperatures higher than 26 mK, the lower plateau disappears altogether but the transition between outward-slip recoveries to inward-slip recoveries can still clearly be seen. The quantity v^* can thus still be measured. It corresponds to the first inward slip when the recoveries reverse sign, that is, when v_p^{\max} disappears. It is shown in Fig. 2, which displays a summary of the ‘‘critical velocities’’ found in our experiments. The flow in the aperture at unpinning is the algebraic sum of v^* and v_{loop} . This last contribution is estimated by analyzing the loop quantum state as explained in [7]. The upper curves in Fig. 2 correspond to actual aperture flow for either unpinning (v_u) or vortex nucleation (v_c). The increase of v_c with temperature is due to ^3He impurities and agrees well with previous measurements [1].

The random nature of vortex unpinning, which is a stochastic process governed by a nucleation rate Γ , shows up on the probability for unpinning at velocity v . This probability, plotted in Fig. 3 for the 34 mK data, is obtained from the integral of the histogram of events occurring at v , shown in the inset. About 20% of the unpinning events are lying far out on the low velocity side of the probability distribution and have been rejected as being caused by different processes (of the same nature as collapses, for instance). In the thermal activation regime, Γ is given by the usual Arrhenius formula $\Gamma_t = (\omega_0/2\pi) \exp(-\Delta E/k_B T)$. As will turn out below however, the data favor quantum tunneling, in which case the unpinning rate is [13]

$$\Gamma_q = \frac{\omega_0}{2\pi} \left(864\pi \frac{\Delta E}{\hbar\omega_0} \right)^{1/2} \exp\left(-\frac{36}{5} \frac{\Delta E}{\hbar\omega_0}\right). \quad (3)$$

By performing a statistical analysis based on the same relations as those used in [14] for vortex nucleation processes, we obtain from a fit to the unpinning probability

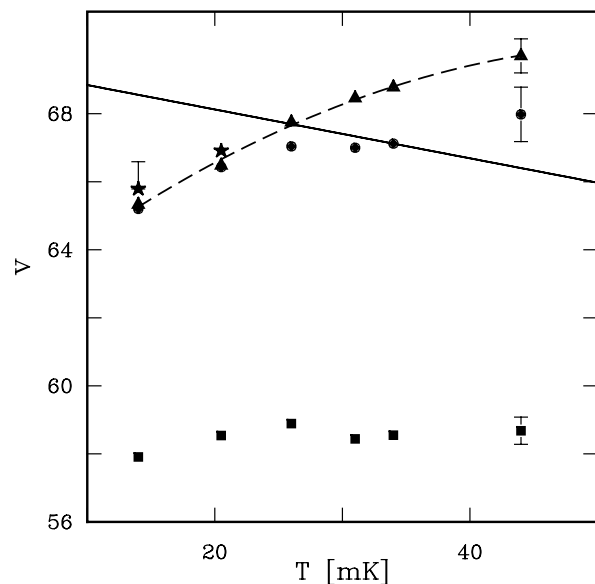


FIG. 2. Critical velocities vs temperature T at $P = 1$ bar. The lower branch (■) displays the membrane-induced velocity v^* at which the vortex unpins. The upper branches correspond to the critical velocities in the aperture for nucleation, v_c (▲), and for vortex unpinning v_u with (*) and without lifetime corrections (●). The dashed line is a fit to the nucleation velocity data. The plain line is the expected T dependence of v_u for thermal activation. Error bars, when not explicitly shown, are about twice the symbol size.

in Fig. 3 the values of the pinning barrier parameter for quantum nucleation, $E_p/\hbar\omega_0 = 76.0$. Similarly, for thermal activation at 34 mK, $E_p/k_B = 18.6$ K.

We now need an estimate of the attempt frequency in the quantum tunneling case which can only be crudely

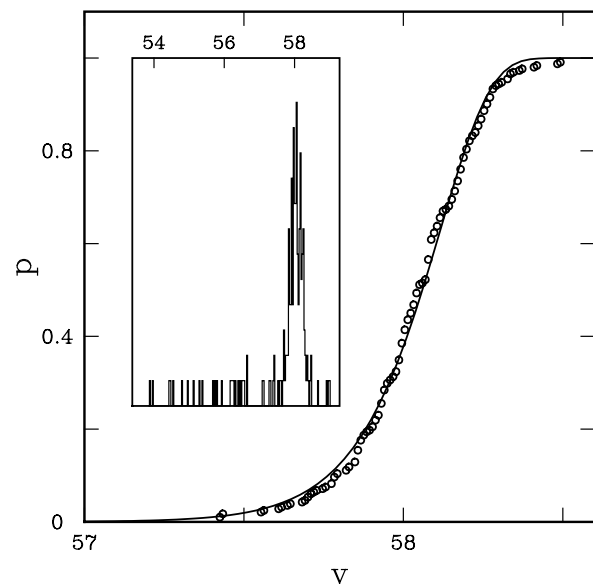


FIG. 3. Probability p for unpinning before velocity v at $T = 34$ mK, obtained from the histogram shown in the inset. The plain curve represents the fit for quantum tunneling mentioned in the text.

obtained as follows. If the crossover temperature T_q between the thermal and the quantum regimes had been experimentally identified, the attempt frequency would stem from the relation [13] $\omega_0/2\pi = k_B T_q/\hbar$. We know only that $T_q > 44$ mK, and hence that a lower bound for $\omega_0/2\pi$ is 5×10^9 Hz. An upper bound is fixed by the highest Kelvin mode sustained by the vortex filament [5], $\omega_c/2\pi = \kappa_4/2\pi^2 a^2 = 5 \times 10^{11}$ Hz, with $a = 1$ Å. Hence, within an order of magnitude, $\omega_0/2\pi = 5 \times 10^{10}$ Hz.

With the above estimate of ω_0 , we can analyze further the data at 14 and 20.5 mK, where nucleation is more probable than unpinning on the lower plateau, by constructing the statistical distribution of the pinned vortex lifetime. This distribution shows an exponential decay with a time constant τ of 49.5 resonance periods at 14 mK and 20 periods at 20.5 mK. Carrying out the full statistical analysis to disentangle phase slip nucleation from vortex unpinning [15] yields unpinning rates of 3.6 and 8.9 s^{-1} at 14 and 20.5 mK, respectively. These rates are very low, and the values for v_u derived from v^* at these temperatures have to be adjusted to the same probability per cycle as the higher temperature determinations [15]. The corrected values are shown by (\star) in Fig. 2. The experimentally determined values of v_u are found to be nearly T independent if we account for the fact that error bars at 44 mK are quite large, due to poor statistics, and that the pinned state lifetime at 14 mK is shortened by collapses.

If unpinning were thermally activated, the unpinning velocity would decrease when the temperature is raised and thermal fluctuations would become more and more important. The temperature dependence of v_u can be estimated from the knowledge of the energy barrier as in [16]:

$$v_u = v_{u0} \left\{ 1 - \frac{k_B T}{E_p} \ln \left[\frac{\omega_0}{\omega} \frac{1}{\ln 2} \sqrt{\frac{2 k_B T}{\pi E_p}} \frac{v_{u0}}{v_u} \right] \right\}, \quad (4)$$

$\omega/2\pi$ being the resonance frequency of the cell (9.4 Hz). The straight line representing this temperature dependence with the value of E_p/k_B found at 34 mK is shown in Fig. 2. Thermal activation seems ruled out as the underlying mechanism for the observed fluctuations of the pinning process while quantum tunneling is favored by both the constancy with T of v_u and of the fluctuations of both v_u and v_p^{\max} . Furthermore, the fundamental Kelvin mode of a pinned vortex 30 Å in length has an energy of 135 mK, which implies that T_q lies in the few 10^{-2} K range.

From our estimate for the activation energy parameter E_p/k_B for quantum tunneling, ~ 200 K, and from Eq. (1), we deduce a value of the local radius b of the asperities on which the vortex pins of ~ 20 Å. Such a value falls well within the expected range for clusters formed in

dense media. The barrier potential, ~ 5 K, turns out to be rather close to that relevant for vortex nucleation [17], in accordance with the idea put forward in the introduction that unpinning of small vortices, possibly a few tens of Å in length, and vortex nucleation have a definite kinship. But this work also clearly shows that these phenomena are different from one another and that they can coexist, one important distinction being that nucleation provides an inexhaustible supply of vortices.

*Present address: Low Temperature Laboratory, Helsinki University of Technology, 02150 Espoo, Finland.

- [1] See E. Varoquaux and O. Avenel, *Physica* (Amsterdam) **197B**, 306 (1994), and references therein.
- [2] R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland, Amsterdam, 1955), Vol. I, p. 15.
- [3] K. W. Schwarz, *Phys. Rev. Lett.* **59**, 1167 (1987).
- [4] K. W. Schwarz, *Phys. Rev. B* **31**, 5782 (1985); see also M. Tsubota and S. Maekawa, *Physica* (Amsterdam) **194B–196B**, 721 (1994); **194B–196B**, 723 (1994).
- [5] R. J. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, Cambridge, England, 1991).
- [6] P. W. Adams, M. Cieplak, and W. I. Glaberson, *Phys. Rev. B* **32**, 171 (1985), and references therein for bulk ^4He ; F. M. Ellis and L. Li, *Phys. Rev. Lett.* **71**, 1577 (1993) for films.
- [7] O. Avenel, M. Bernard, S. Burkhart, and E. Varoquaux, *Physica* (Amsterdam) **210B**, 215 (1995), and references therein.
- [8] P. J. Hakonen, O. Avenel, and E. Varoquaux, *J. Low Temp. Phys.* **110**, 503 (1998).
- [9] See, e.g., I. Brodie and J. J. Murray, *The Physics of Micro/Nano-Fabrication* (Plenum Press, New York, 1992), p. 83.
- [10] Phase winding numbers, in units of 2π , are obtained by dividing the membrane peak amplitudes A by $(1 + R)\Delta A_i$, ΔA_i being the amplitude drop due to a single slip; v_{loop} in the same units is $qR/(1 + R)$.
- [11] We have checked (by plugging the large channel with solid helium) that in normal operation the critical velocity is not influenced by any trapped vortices; i.e., the probabilities of outward and inward slips are equal.
- [12] A. L. Fetter, *Phys. Rev.* **138**, A429 (1965).
- [13] See, e.g., P. Hänggi, P. Talkner, and M. Borkovec, *Rev. Mod. Phys.* **62**, 250 (1990).
- [14] W. Zimmermann, Jr., O. Avenel, and E. Varoquaux, *Physica* (Amsterdam) **165B&166B**, 749 (1990).
- [15] E. Varoquaux, O. Avenel, P. Hakonen, and Yu. Mukharsky, *Physica B* (Amsterdam) (to be published).
- [16] E. Varoquaux, W. Zimmermann, Jr., and O. Avenel, in *Excitations in Two-Dimensional and Three-Dimensional Quantum Fluids*, edited by A. F. G. Wyatt and H. J. Lauter (Plenum, New York, 1991), p. 343.
- [17] G. G. Ihas, O. Avenel, R. Aarts, R. Salmelin, and E. Varoquaux, *Phys. Rev. Lett.* **69**, 327 (1992).