Two-Player D2D Interference Canceling Games

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***Abstract*—We investigate a set of non-cooperative radio resource management games in a Gaussian interference channel, where the receivers are equipped with two stage Successive Interference Cancellers (SIC). In these games users decide on their transmission power, rate and Interference Canceling (IC) strategy. A one-shot game, as well as two-stage variants, where either rate, or IC and rate, are decided in the second stage, are considered. We characterize the equilibria of the games and establish a relationship between the equilibria of the one shot and two-stage games. Postponing the rate decision to a second stage stabilizes the game in a region where no pure strategy Nash Equilibrium exists for the one-shot game. Further postponing the IC decision to a second stage stabilizes the game completely, an equilibrium exists in all network configurations. We simulate a 2-pair device-to-device network where these games are used for radio resource management. The regions where the one-shot game is unstable have a considerable probability, leading to a considerable outage probability. By staging the game, such outage can be mitigated, or removed altogether.**

I. Introduction

The best known transmission strategies in interference channels are known to be based on Interference Cancelation (IC) [1], [2] . These schemes involve complicated rate splitting mechanisms, and joint detection capabilities requiring a receiver to simultaneously handle three codewords. However, a simpler approach of opportunistic cancelation of interference with Serial Interference Cancelation (SIC) receivers has been shown to provide considerable gains in ad hoc [3], [4], [5] and cognitive [6] networks. When Power Control (PC) is mixed with IC, non-trivial optimizations may occur. In [5] it was shown that when users strategically determine their PC and IC decisions, they sometimes voluntarily lower their power to increase their payoff, increasing the potential gains of IC. Cooperative or planned use of SIC further improves the potential benefits of network IC [7], [8]. Rate regions of two-user Gaussian interference channels with SIC were explored in [9].

Despite the promise, network IC has not been used in existing wireless systems. The reason for this is twofold. First, in cellular systems, meticulous Radio Resource Management (RRM) is performed to guarantee that interfering signals are typically weaker than wanted signals, which makes network IC challenging. Second, Wireless Local Area Networks (WLANs) are based on carrier sensing, where transmission from different sources are time divided, and interfering signals are weak. Another reason for the absence of network IC is complexity—to perform IC with viable complexity, the wanted signal and interfering transmissions should be overlapping, which requires synchronization of transmitters. Conventionally, WLANs do not have synchronization functions, and in cellular networks, synchronization typically happens inside a cell.

Device-to-Device (D2D) communication underlaying cellular networks [10] has been intensively investigated recently, and has been specified as a part of the 4G LTE-A standard in 3rd Generation Partnership Project (3GPP) Release 12. D2D communication happening under an umbrella LTE cell may be naturally synchronized, whereas strong interference may occur, especially if multiple D2D users are allowed to use the same resources. Accordingly, strategies for applying IC in D2D networks have been discussed, and shown to have significant potential in increasing capacity [11], [12], [13]. In [11], a rate splitting approach was used, which has hyper-exponential complexity in transmission mode selection. The approach of [13], [14] is centralized; in [14], a greedy algorithm for grouping D2D users with SIC receivers was addressed, whereas in [13], a centralized scheduler performed Network Utility Maximization (NUM) in a convexified small instance.

There may be large numbers of D2D pairs coordinated by one cell. To reduce network management complexity, it is desirable to distribute decisions of receiver characteristics and transmission rates to the D2D nodes. Indeed, in [13] it was observed, that a greedy implementation where D2D pairs play the PC-IC game of [5] leads to user rate statistics that are closer to the proportionally fair NUM than to sum-rate maximizing NUM. However, compared to a centralized method, due to the underlying game, determining the resulting resource allocation is more problematic. There are regions in the network configuration space, where no equilibrium exists in a strategic game setting, and accordingly, a distributed RRM algorithm would not converge. Moreover, even when an equilibrium exists, simple best response dynamics are not always guaran-

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teed to converge to it.

In this paper, we address this problem. We consider a set of two-player D2D interference canceling games, with the objective of deriving converging distributed RRM for D2D networks. All players are equipped with a single-stage IC receiver. Each player can decide the transmit power (P), IC usage and transmission rate (R) strategies. We analyze and compare NE regions of the one-shot (P-IC-R) game, as well as two-stage (P-IC;R) and (P;IC-R) games. We show that the Nash Equilibria (NEs) of these games are connected, in such a way that the stability of the game increases in sequence from the one-shot game to the two-stage (P;IC-R) game with both IC and rate strategy decisions in the second stage. The latter has a pure strategy NE in all network configurations. A D2D network in a circular cell is investigated numerically. It is observed that the considerable outage probabilities caused by non-convergence in the one-shot game can be effectively removed by staging the strategy decisions. The two-stage (P;IC-R) game is thus a good candidate as a building block for a distributed RRM function in D2D networks with IC.

II. System Model

We consider a network of two pairs of transmitters (Tx) and receivers (Rx), in a D2D communication network. Each Rx is interested only in the message transmitted by the Tx belonging to the same pair. The transmitters do not cooperate in transmission, and the receivers do not cooperate in reception. The Tx-Rx pairs manage their use of radio resources based on a distributed optimization approach, where each Tx-Rx pair acts as a strategic player maximizing its own utility.

A. Network Model

In each pair, there is a designated transmitter (Tx) and receiver (Rx). The network is modeled as an interference channel, with the channel between Tx of pair i and Rx of pair j characterized by the channel gain g_{ij}. Note that as the Rx and Tx in the pairs are at different locations, g_{ij} is generically not equal to g_{ji}. We assume that the channel gains are constant within the time frame of performing RRM. The transmit power of transmitter j is $P_j \leq P_{\max}$, and its transmission rate is R_j.

The receivers are equipped with two-stage SIC receivers, so that they may cancel the interference from one source before attempting to decode the transmission of interest. The IC state of receiver i is characterized by the binary variable c_i. If $c_i = 1$, IC of the opponent transmission is attempted before payload decoding, otherwise not. Hence, there are 4 possible IC states in the network corresponding to the choice of c_i for each receiver i.

The Signal-to-Interference-plus-Noise Ratio (SINR) experienced by a transmission characterizes all uncancelled interference + noise. Thus with and IC without, the wanted signal transmission from transmitter i at receiver i has SINR

$$\gamma_{ii}^{\mathrm{IC}} = \frac{P_i g_{ii}}{N_0} \ ; \quad \gamma_{ii}^{\mathrm{no}} = \frac{P_i g_{ii}}{P_j g_{ji} + N_0} \ , \tag{1}$$

where N_0 is Additive Gaussian Noise, and $j \neq i$. The SINR of Tx $j \neq i$ at Rx i is

$$\gamma_{ji} = \frac{P_j g_{ji}}{P_i g_{ii} + N_0} \ . \tag{2}$$

The amount of information that can be reliable transmitted with a given SINR is given by the spectral efficiency function $f(\gamma)$. For simplicity we assume AWGN capacity achieving Gaussian codebooks, so that $f(\gamma) = \log_2(1+\gamma)$. A transmission from Tx j with SINR γ_{ji}^{c} at Rx i (with or without IC) can be successfully received at i if

$$f(\gamma_{ji}^{c}) \geq R_j \ . \tag{3}$$

We use the shorthand notations

$$\begin{array}{rcl} f_i^{\mathrm{IC}} & \equiv & f(\gamma_{ii}^{\mathrm{IC}}) \ ; \qquad f_i^{\mathrm{no}} \equiv f(\gamma_{ii}^{\mathrm{no}}) \\ f_{ji} & \equiv & f(\gamma_{ji}), \ i \neq j \end{array}$$

for the transmission efficiencies of the wanted signal with and without IC, and for the interference signals.

Any Gaussian interference channel may be transformed to an equivalent *standard form* [1], where $g_{ii} = g_{jj} = N_0 = 1$. In the analytical part of this paper we use standard-form channels.

B. Game Model

We shall treat D2D network RRM as a strategic game model, where the Tx-Rx pairs are players. The objective of this modeling is to design a simple distributed RRM mechanism which performs close to a network Pareto optimum.

1) Game Configuration and Player Strategy: There are four game configuration variables which are considered as constant during the game process. The configuration includes: maximum transmitter power of each player $P_1^{\max}$,$P_2^{\max}$ and propagation parameters g_{12}, g_{21}. When deciding how to transmit, Tx-Rx pair j considers three strategy variables: transmission power P_j, receiver IC state $\mathbf{c}_j$, and transmission rate R_j. The power and rate strategies are continuous, whereas the IC strategy is discrete. To simplify the analysis, we assume that the rate strategy sets with and without IC are non-overlapping, so that $R_j^{\mathrm{no}} \leq f_j^{no}$ and $R_j^{\mathrm{IC}} > f_j^{\mathrm{no}}$. Otherwise it is not necessary to consume extra power to implement IC if $R_j^{\mathrm{IC}} \leq f_j^{\mathrm{no}}$.

2) One Shot Game and Two-Stage Game: We study a one-shot game model, which we call (P-IC-R), as well as two-stage variants, where the strategy variables are grouped to a 1st stage subset and 2nd stage subset. To simplify the analysis we assume that all two-stage games are played with Subgame Perfect (SGP) strategies. Also we assume that each stage of a 2-stage game is played simultaneously by the players. That is, there is no sequential play within the stages.

A full strategy of player j in a two-stage game consists of a strategy $\mathcal{S}_{1,j}$ for the first stage, and a family of strategies $\mathcal{S}_{2,j}$ for the second stage which includes a conditional strategy for each possible strategy profile of the first stage.

Considering the possible grouping of strategy variables to stages, in principle, transmission power should be selected before transmission, and IC can be chosen at the receiver. However, rate has to be decided at the transmitter, and IC affects rate. To keep with the game interpretation, we assume IC to be a hard strategy. If IC is a 1st stage strategy, and player j chooses to cancel interference from opponent i in the first stage, the receiver has to use its SIC receiver against i. Accordingly, we concentrate on the two-stage games (P-IC;R) and (P;IC-R). In addition, we shall find that (IC;P-R) is useful for understanding the game solutions.

The payoff functions of the games can then be derived from (3). For the one-shot (P-IC-R) game, player j maximizes

$$u_j = R_j \left[(1-c_j)\,\theta\left(f_j^{\text{no}} - R_j\right) + c_j\,\theta\left(f_j^{\text{IC}} - R_j\right)\theta\left(f_{ij} - R_i\right)\right] . \quad (4)$$

Here

$$\theta(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases} \quad (5)$$

is the Heaviside step function. Note that the dependence of a player's payoff on the other player's action is via the f function. In the 2-stage games with SGP strategies, u_j is maximized conditioned on the first stage decisions. The second stage strategies thus become functions of the first stage strategies.

III. Nash Equilibrium Analysis

The Nash Equilibria in the one-stage game (P-IC-R), and the two-stage games (P-IC;R) and (P;IC-R) can be analyzed in closed form. We only consider pure strategy NEs.

When the two-stage games are played with SGP strategies, we find that there is a relationship between the NEs of these games. If there is a NE in (P-IC-R), there is a SGP NE in the 2-stage games. If there is a NE in (P-IC;R), there is a NE in (P;IC-R). These NEs can be understood as *(P-R) subgame NEs*, which are second stage NEs in the (IC;P-R) game, i.e. NEs in a game with fixed IC selections for the players.

A. One-shot (P-IC-R)

First we analyze the strategies of the players in the one-shot game. In (P-IC-R), each player chooses all strategies at the same time, and the strategy of i is $\mathcal{S}_i = (P_i, c_i, R_i)$. When interference cancelation is not applied, player j's best choice is transmitting with maximum power $P_j^{\max}$ at rate f_j^{no}. When player j is canceling interference from Tx i, player j should be able to decode the message of Tx i which requires that

$$f_{ij} = \log_2(1+\gamma_{ij}) \geq R_i \ . \quad (6)$$

This leads to the following power constraint of player j:

$$P_j \leq \frac{P_i g_{ij}}{2^{R_i}-1} - 1 \ . \quad (7)$$

The best power response for player j when playing IC is then

$$P_j^{\text{IC}} = \min\left(P_j^{\max}, \left[\frac{P_i g_{ij}}{2^{R_i}-1} - 1\right]_+\right) \ , \quad (8)$$

and the overall best response strategy of j is

$$\mathcal{B}_j = \begin{cases} (P_j^{\max}, 0, f(\frac{P_j^{\max}}{P_i g_{ij}+1})) & \text{if } \frac{P_j^{\max}}{P_i g_{ij}+1} \geq P_j^{\text{IC}} \\ (P_j^{\text{IC}}, 1, f(P_j^{\text{IC}})) & \text{otherwise.} \end{cases} \quad (9)$$

By matching the best response strategies of the players, we derive conditions for different types of NE. We denote a NE as $E^{(c_1,c_2)}$, according to the IC strategies of the players.

1) Equilibrium $E^{(0,0)}$: At this NE both players choose noIC strategy, Given the opponent's strategy $P_i = P_i^{\max}, R_i = f(\frac{P_j^{\max}}{P_i g_{ij}+1})$ as well as (8) and (9), the noIC condition for player j becomes

$$\frac{P_j^{\max}}{P_i^{\max} g_{ij} + 1} \geq g_{ij}(P_j^{\max} G_{ji} + 1) - 1 \ . \quad (10)$$

$E^{(0,0)}$ exist when this is fulfilled for both players simultaneously. The NE region for $E^{(0,0)}$ is depicted in Figure 1(a), for the case $P_1^{\max} = P_2^{\max} = 1$.

2) Equilibrium $E^{(1,0)}$: At this equilibrium noIC of player 2 and IC of player 1 are best responses to each other. From (8) and (9) we find the stability condition for $E^{(1,0)}$ for player 1 to achieve better rate with IC to be

$$\frac{P_j^{\max}}{P_i^{\max} g_{ij} + 1} \leq \tilde{P}_j^{IC} \ , \quad (11)$$

where $j=1, i=2$,

$$\tilde{P}_j^{IC} = \begin{cases} \min\left(\left[P_j^{\text{lim}}\right]_+, P_j^{\max}\right) & \text{when } g_{ij}g_{ji} < 1 \\ P_j^{\max}\,\theta\left(P_j^{\max} - P_j^{\text{lim}}\right) & \text{when } g_{ij}g_{ji} > 1 \\ P_j^{\max}\,\theta\left(1 - g_{ij}\right) & \text{else} \end{cases} \quad (12)$$

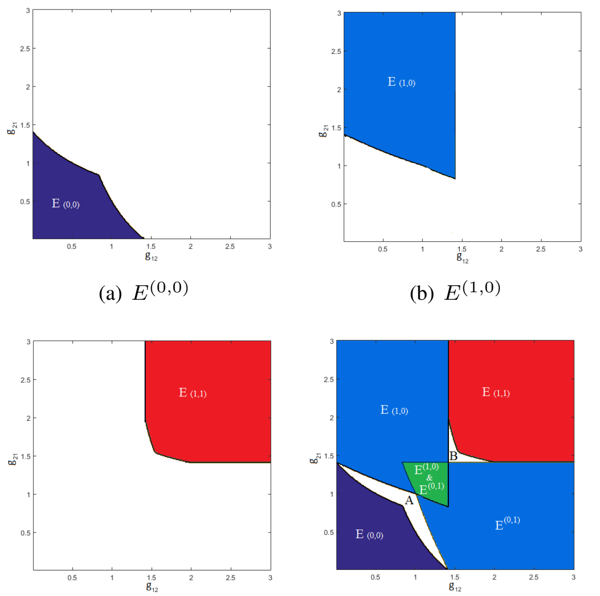
and

$$P_j^{\text{lim}} = \frac{1 - g_{ij}}{g_{ij}g_{ji} - 1} \ . \quad (13)$$

On the contrary, player i achieves better rate without IC, leading to

$$\frac{P_i^{\max}}{\tilde{P}_j^{IC} g_{ji} + 1} \geq g_{ji} - 1 \ . \quad (14)$$

The NE region for $E^{(1,0)}$ is depicted in Figure 1(b). The NE condition and NE region of $E^{(0,1)}$ is a mirror image of $E^{(1,0)}$ in the change $1 \leftrightarrow 2$.



(a) $E^{(0,0)}$ (b) $E^{(1,0)}$

(c) $E^{(1,1)}$ (d) Total NE region

Fig. 1. NE region for P-IC-R game while $P_1^{\max} = P_2^{\max} = 1$

3) Equilibrium $E^{(1,1)}$: While both players receive strong interference from the opponent, NE $E^{(1,1)}$ exists. This means that it is beneficial for both players to choose IC while the opponent is also applying IC. This leads to the condition

$$\frac{P_j^{\max}}{\hat{P}_i^{IC} g_{ij} + 1} \le \hat{P}_j^{IC} \tag{15}$$

where

$$\hat{P}_j^{IC} = \min\left([g_{ij} - 1]_+, P_j^{\max}\right). \tag{16}$$

The NE region of $E^{(1,1)}$ is depicted in Figure 1(c).

4) Region without NE: Figure 1(d) shows all NE regions. There is an intersection region where both $E^{(0,1)}$ and $E^{(1,0)}$ exist. In addition, there are two regions $\mathcal{A}$ and $\mathcal{B}$ where no pure strategy NE exists. In these regions, the game is thus always unstable against changing the IC-subspace. For simplicity, we denote player j's strategy as $\mathcal{S}_j(P_i)$ and $\mathcal{S}_j^{\mathrm{IC}}(P_j)$, where

$$\begin{aligned}
\mathcal{S}_j(P_i) &= (P_j^{\max},\ 0, f(\frac{P_j^{\max}}{P_i g_{ij} + 1})) \\
\mathcal{S}_j^{\mathrm{IC}}(P_j) &= (P_j,\ 1, f(P_j)) \\
\tilde{P}_j^{\lim} &= g_{ij}(P_j^{\max} g_{ji} + 1) - 1. \\
\hat{P}_j^{\lim} &= g_{ij} - 1.
\end{aligned}$$

In area $\mathcal{A}$ the condition (11) is not fulfilled for both players and condition (10) is not fulfilled for at least one player. When condition (10) is not fulfilled for one player, say j, we can derive the best response strategy chain as

$$\begin{aligned}
\mathcal{S}_i(P_j^{\max}) &\rightarrow \mathcal{S}_j^{\mathrm{IC}}(\tilde{P}_j^{\lim}) \rightarrow \mathcal{S}_i(\tilde{P}_j^{\lim}) \\
&\rightarrow \mathcal{S}_j(P_j^{\max}) \rightarrow \mathcal{S}_i(P_j^{\max})
\end{aligned} \tag{17}$$

where the arrow from left to right means the right side strategy is the best response of the left side strategy. The loop in (17) indicates that a NE does not exist. When condition (10) is not fulfilled for both players, a similar loop can be derived as

$$\begin{aligned}
\mathcal{S}_i(P_j^{\max}) \rightarrow \mathcal{S}_j^{\mathrm{IC}}(\tilde{P}_j^{\lim}) \rightarrow \mathcal{S}_i(\tilde{P}_j^{\lim}) \rightarrow \mathcal{S}_j(P_j^{\max}) \\
\rightarrow \mathcal{S}_i^{\mathrm{IC}}(\tilde{P}_i^{\lim}) \rightarrow \mathcal{S}_j(\tilde{P}_i^{\lim}) \rightarrow \mathcal{S}_i(P_j^{\max}).
\end{aligned} \tag{18}$$

Hence no pure NE exist in area $\mathcal{A}$.

In area $\mathcal{B}$ condition (14) is not fulfilled for both players and condition (15) is not fulfilled for at least one player. When condition (15) is not fulfilled for one player, say j, we can derive best response chain

$$\begin{aligned}
\mathcal{S}_i(P_j^{\max}) \rightarrow \mathcal{S}_j^{\mathrm{IC}}(P_j^{\max}) \rightarrow \mathcal{S}_i^{\mathrm{IC}}(\hat{P}_i^{\lim}) \rightarrow \mathcal{S}_j(\hat{P}_i^{\lim}) \\
\rightarrow \ldots \rightarrow \mathcal{S}_j^{\mathrm{IC}}(\hat{P}_j^{\lim}) \rightarrow \mathcal{S}_i^{\mathrm{IC}}(\hat{P}_i^{\lim}).
\end{aligned} \tag{19}$$

During omitted steps in (19), player i keep applying $\mathcal{S}_i^{\mathrm{IC}}(P_i)$ with increasing P_i and player j will keep applying $\mathcal{S}_j(P_i)$ until $P_i > \frac{1}{g_{ij}}(\frac{P_j^{\max}}{g_{ij}-1} - 1)$. When condition (15) is not fulfilled for both players, we can derive a similar loop

$$\begin{aligned}
\mathcal{S}_i(P_j^{\max}) \rightarrow \mathcal{S}_j^{\mathrm{IC}}(P_j^{\max}) \rightarrow \mathcal{S}_i^{\mathrm{IC}}(\hat{P}_i^{\lim}) \rightarrow \mathcal{S}_j(\hat{P}_i^{\lim}) \\
\rightarrow \ldots \rightarrow \mathcal{S}_j^{\mathrm{IC}}(\hat{P}_j^{\lim}) \rightarrow \mathcal{S}_i(P_j^{\lim}) \rightarrow \ldots \rightarrow \mathcal{S}_i^{\mathrm{IC}}(\hat{P}_i^{\lim}).
\end{aligned} \tag{20}$$

Hence, no pure NE exist in area $\mathcal{B}$. Figure 2 shows refined NE regions of (P-IC-R). Each $E^{(c_1,c_2)}$ region is further divided to smaller regions depending on whether maximum or limited power is used. The legend shows the NE strategies in these regions.

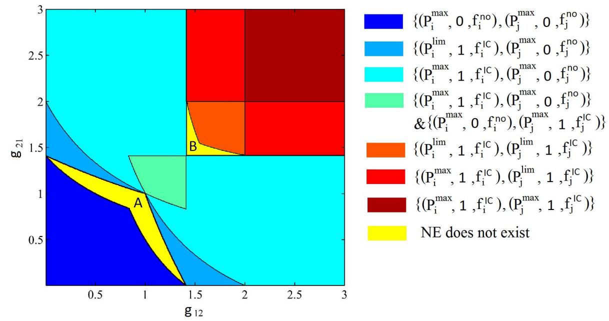


Fig. 2. Detailed NE regions for (P-IC-R) with $P_1^{\max} = P_2^{\max} = 1$. Subregions with $P^{\lim}$ and $P^{\max}$ for the players indicated.

B. Two-stage (P-IC;R)

The (P-IC;R) game is played in two sequential stages. In the 1st stage players choose power and IC strategy, in the 2nd stage players choose rate according to the 1st stage strategies of both players. As rate is selected in the 2nd stage, players can always select an achievable rate. As both players make a more informed rate decision, a possible source of instability is removed from the game.

Consider a NE of (P-IC-R) game $\{\mathcal{S}_1, \mathcal{S}_2\}$, where $\mathcal{S}_1 = (P_1, c_1, R_1)$, $\mathcal{S}_2 = (P_2, c_2, R_2)$. We define a derived strategy pair for the (P-IC;R) game as $\{(S_{1,1};S_{2,1}),(S_{1,2};S_{2,2})\}$ where for player i the strategies in the two stages are

$$S_{1,i} = (P_i, c_i)$$
$$S_{2,i} = \begin{cases} R_i & \text{if } S_{1,i} = (P_i, c_i) \\ & \quad S_{1,j} = (P_j, c_j) \\ \phi_i(S_{1,i}, S_{1,j}) & \text{else} \end{cases} .$$

Here $\phi_i(S_{1,i}, S_{1,j})$ are rate strategies in unrealized parts of the game which are subgame perfect. If a derived strategy pair forms a NE, we call it derived NE.

For (P-IC-R), being a NE guarantees that a player cannot gain utility by changing anyone or any combination of its strategies. On the other hand, the (P-IC;R) NE guarantees that each player cannot gain utility by changing its own strategies, when its opponent can adjust the rate according to both players' power and IC strategy. Postponing the rate decision to the second stage does not destabilize a NE. We have

Proposition 1. *Consider the one shot game (P-IC-R) and the two-stage game (P-IC;R). If there exist a NE of (P-IC-R), there exist a corresponding derived NE of (P-IC;R).*

Proof. Changing the second stage rate strategy only is counterproductive, as the derived strategy is SGP. For first stage strategy changes, we consider first stage strategy of player i in (P-IC;R). If the opponent plays $c_j = 1$, the opponent's rate does not depend on (P_i, c_i). Thus the best response of player i in the first stage of (P-IC;R) to $(P_j, 1)$ is the same as its response in (P-IC-R) to $(P_j, 1, R_j)$. If the opponent plays $c_j = 0$, the opponent's rate in the 2nd stage depends on P_i. Then, if player i plays $c_i = 1$ in the derived strategy, the power is given by (8). Increasing power will decrease the second stage R_j, but will decrease f_{ji} more, according to (11). Changing the IC-strategy to $c_i = 0$ makes the opponent's rate irrelevant. It is not beneficial, as it is not beneficial in (P-IC-R). Finally, if both players play $(P^{\max}, 0)$, changing to $(P_i^{\lim}, 1)$ is not beneficial in (P-IC-R), where R_j is fixed. Then it will be less beneficial in (P-IC;R), where the second stage response of the opponent would be a higher rate. The derived strategy pair is thus a NE of (P-IC;R). □

Using Proposition 1 we can derive NEs of (P-IC;R) from NEs of (P-IC-R). For all NE regions of (P-IC-R) game, the derived NE is unique.

If there were another (P-IC;R) NE, it must be in a different IC subspace. NE $E^{(1,c_2)}$ and $E^{(0,c_2)}$ cannot coexist since $E^{(1,c_2)}$ only exist when player 1 achieves better utility with IC. Thus the only way two NE exist at same time is $(c_i, c_j) = (1, 0)$ or $(0, 1)$, which is observed in the double NE region in Figure 2. Thus for each NE region of (P-IC-R) we have the same derived NE regions of (P-IC;R).

Furthermore, in (P-IC;R), a NE also exists in region $\mathcal{A}$ of Figure 1(d), where no NE exists for (P-IC-R). In the (P-IC;R) game, there is a corresponding rate strategy for each possible result of the 1st stage game. In region $\mathcal{A}$ both players will stick to a noIC strategy and loop in (17) or (18) cannot be formed. Thus a $E^{(0,0)}$ NE is stable. However, all strategy pairs in region $\mathcal{B}$ of Figure 1(d) still remains unstable, as the IC strategy response of the opponent is not considered in the (P-IC;R) game.

Thus the NE regions of (P-IC;R) are the same as in Figure 2 except that region $\mathcal{A}$ is also an $E^{(0,0)}$ type NE region.

C. Two-stage (P;IC-R)

In the (P;IC-R) game, IC is also decided in the 2nd stage. A player can decide the best IC and rate combination conditioned on power selections of the 1st stage. Given any first-stage choice of (P_i, P_j), the best response strategy in the second stage subgame for player j is

$$\mathcal{B}_{2,j} = \begin{cases} (0, f_j^{\text{no}}) & \text{if } \gamma_{ij} < \gamma_{ii}^{\text{no}} \\ (1, f_j^{\text{IC}}) & \text{if } \gamma_{ij} \geq \gamma_{ii}^{\text{IC}} \\ (1 - c_i, c_i f_j^{\text{no}} + (1 - c_i) f_j^{\text{IC}}) & \text{otherwise.} \end{cases}$$

By matching best response strategies of both players, it can proved that for two-stage (P;IC-R) game, given any result of first-stage subgame, there exists one or two NE for second-stage subgame. Hence, we can define a (P;IC-R) strategy derived from (P-IC;R) in a similar way as for (P-IC;R), i.e.,

$$S_{1,i} = P_i$$
$$S_{2,i} = \begin{cases} (c_i, R_i) & \text{if } S_{1,i} = P_i,\ S_{1,j} = P_j \\ \psi_i(S_{1,i}, S_{1,j}) & \text{else} \end{cases}$$

where $\psi_i(S_{1,i}, S_{1,j})$ are rate and IC strategies in unrealized parts of the game which are subgame perfect, and R_i is the rate in the realized second-stage of the (P-IC;R) NE. We have

Proposition 2. *Consider the two-stage games (P-IC;R) and (P;IC-R). If there exist a NE of (P-IC-R), there exists a corresponding derived NE of (P-IC;R).*

Proof. The proof proceeds by observing that in a derived NE, no change of the power of player 1 in first stage will lead to player 2 changing IC strategy in 2nd stage, player 2 IC-strategy is fixed by P_2. Thus P_1 has to be chosen as if c_2 were fixed, and the analysis in (P;IC-R) reduces to the one in (P-IC;R).

First, for a (P-IC;R) NE of $E^{(0,0)}$ type, both players apply $P^{\max}$ in the derived (P;IC-R) strategy. If player 1 changes its P_1 to achieve better R_1 and utility, the opponent cannot change its 2nd stage IC strategy since $c_2 = 1$ is not feasible with the derived rate R_1, and will be less possible with a lower P_1 and higher rate.

Second, for a NE of $E^{(1,1)}$ type, if player 1 changes its P_1 to achieve better utility, the opponent will not change its 2nd stage IC strategy since according to (15), its capability to perform IC increases with increasing opponent power.

Third, for a NE of $E^{(1,0)}$ type which is not located in the double NE region, if player 1 changes its power to achieve better utility, player 2 cannot apply IC since IC is not feasible with $P_2^{\max}$. Player 2 cannot achieve better utility with IC since IC is not feasible according to (14). The analysis for a NE of $E^{(0,1)}$ type which is not located in double the NE region has a similar proof.

Finally, consider a derived NE in the double NE region. Both players apply $P^{\max}$. In (P;IC-R), there are two 2nd stage SGP NEs, corresponding to $E^{(1,0)}$ and $E^{(0,1)}$, whereas in (P-IC;R) there are two first stage strategy pairs with unique 2nd stage strategies. The (P;IC-R) 2nd stage strategy pairs are derived from the (P-IC;R) strategies differing in first stage. □

As for the (P-IC;R) game, for all NE regions of (P-IC;R), the derived NEs are unique except for the double NE region with two derived NEs.

Furthermore, since in (P;IC-R) a player considering a change in power will have to face the opponent's best 2nd stage rate and IC response to the changed power, the loop in (19) or (20) cannot be formed. Hence there is a NE in $\mathcal{B}$ as well. NEs of $E^{(1,0)}$ and $E^{(0,1)}$ type are stable in this region, with both players using $P^{\max}$.

Accordingly, the NE regions of (P;IC-R) game are the game same as in Figure 2 except that the region $\mathcal{A}$ is an $E^{(0,0)}$ type NE region and region $\mathcal{B}$ is double NE region with $E^{(1,0)}$ and $E^{(0,1)}$ NEs. By extending the one-shot game (P-IC-R) to two-stage (P-IC;R) and (P;IC-R) games, we can gradually reduce the region without NEs, and eventually achieve a NE for all configurations.

IV. Simulation Results

A. Simulation model

To assess the performance of these games, we have performed simulations with a model of a D2D network. D2D players are located in a circular D2D area with a radius of $r = 100$ m. In this area the two Tx-RX pairs are randomly dropped uniformly and at random, with a predefined D2D distance D. When $D > r$, the users are uniformly distributed in an annulus with inner radius $D - r$. Statistics are collected from 10^5 instances for each D. The path loss between each Tx and Rx is modeled as

$$L_p = 40 \log_{10} d, \tag{21}$$

where d is distance measured in meters. The ratio of the transmit power and noise power spectral densities is 100 dB, which corresponds to maximum SNR of 100 dB, when the transmitter and receiver are co-located.

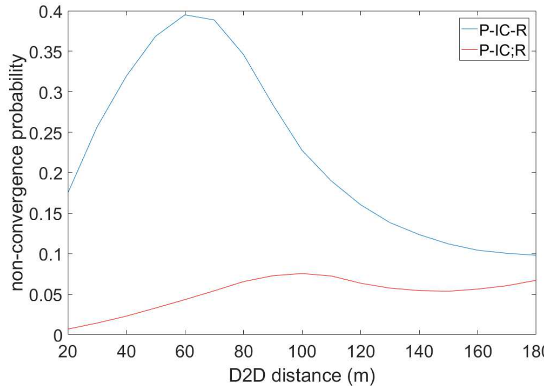
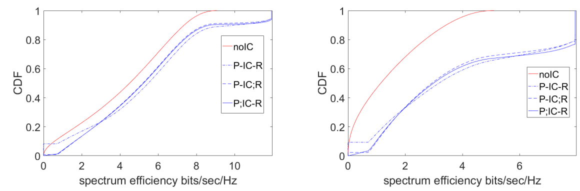


Fig. 3. Non-convergence probability as function of D2D distance (SNR=100dB).



(a) $D = 40$m, SNR=100dB (b) $D = 80$m, SNR=100dB

Fig. 4. Spectrum efficiency CDF of PCIC Rate game in a circular area with 100 meter radius

We assume that the Tx-Rx pairs are aware of all four channel gains g_{ij}. For $g_{ij}, i \neq j$, and the wanted signal channel g_{ii}, this can be arranged by measurements at the nodes, whereas for the opponent's gain, we assume overhearing of broadcast transmissions of the opponent pair. From this information, the Tx-Rx pairs can deduce a NE if it exists, and use the corresponding strategies. For (P-IC;R) game without NE, the first stage subgame is myopically played, with perfect information of the SGP 2nd stage selections.

B. Simulation result

Figure 3 illustrates the non-convergence probability of the (P-IC-R) and (P-IC;R) games, i.e., the probability that there is no NE in an instance. The horizontal axis represents the D2D distance D. As shown in Figure 3, the non-convergence probability of the one shot (P-IC-R) game first increases with growing D2D distance and reaches a maximum of 39% at $D = 50m$, then falls down to 28% at $D = 90m$. The non-convergence probability of (P-IC;R) is much smaller. Its has an ascending trend and reaches a maximum 7% when $D = 90m$. This reflects the probability of region $\mathcal{B}$, which is in the strong interference region. The larger the D2D distance, the more probable it is that interference is strong. The non-convergence probability of (P;IC-R) vanishes, as a NE always exists.

TABLE I
OUTAGE PROBABILITY OF THE IC-GAMES IN THE D2D NETWORK.

D(m)	40	60	80	100	120	140	160	180
P-IC-R(%)	8.2	10.3	9.3	6.5	4.6	3.6	3.2	3.1
P-IC;R(%)	0.8	1.5	2.2	2.6	2.2	1.9	2	2.3
P;IC-R(%)	0	0	0	0	0	0	0	0

The Cumulative Distribution Function (CDF) of the realized rate for the different games is depicted in Figure 4, with the rate without IC as a reference. This is given by the payoff function (4) with the strategy variables of the realized part of a NE. For no-NE regions, as the myopic play loops in similar way as in (17)-(20), the results of myopic repeated sequential play are collected from one corresponding loop in each drop.

First one observes that opportunistic use of IC considerably improves spectral efficiency, and the improvement grows with increasing D2D distance. The (P-IC-R) and (P-IC;R) games have high outage probabilities. When the channel gains are such that there is no NE, in myopic play the players sometimes transmit with a rate that is not realizable, leading to outage.

The outage probability for different D2D distances are shown in Table I. Although the non-convergence probability of (P-IC-R) is as high as about 40% when $D = 60m$, the corresponding outage probability is just about 10%. The outage probability is smaller than the non-convergence probability, as in sequences of myopic play, outage happens every third game or even more rare, depending on no-NE region.

V. CONCLUSION

We have analyzed a set of Interference Canceling Power Control Rate games, where a receiver is equipped with a two-stage Successive Interference Canceling receiver. We have found that by staging the decisions, one can increase the stability of the game. By postponing the rate decision to a second stage, one gets the game (P-IC;R). In this game, a medium-interference region where the one-stage game (P-IC-R) does not have a NE becomes stable. The (P-IC;R) game still has a high-interference region where no NE exists. By postponing the IC decision to the 2nd stage, one gets a (P;IC-R) game, which has a NE for all network configurations. We have demonstrated the gains from using IC, and the stability of using staged games, in a D2D network setting. In network configurations where no NE exist, D2D pairs following a protocol inspired by these games would be periodically in outage. The two-stage game (P;IC-R) removes all such outage instances, and thus provides a good candidate for performing distributed RRM in a reliable fashion.

Generalizing the multistage games to multiplayer settings is not straight forward. An ideal decision logic would require full channel state information (CSI) from the whole network at each node, and factorial complexity. We shall consider heuristic multiplayer protocols based on multi-stage games in future work.

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