

# CSI Quantization for FDD Massive MIMO Communication

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**Abstract**— We consider high-dimensional multiuser MIMO transmissions in Frequency Division Duplexing systems. For precoding, the frequency selective channel has to be measured, quantized and fed back to the base station by the users. In 5G New Radio (NR), a modular quantization approach has been applied for this, where first a low-dimensional subspace is identified for the whole frequency selective channel, and then subband channels are linearly mapped to this subspace and quantized. We analyze how the components in such a modular scheme contribute to the overall quantization distortion. Based on this analysis we improve the technology components in the modular approach. We compare the improved quantization scheme to the 5G NR standardized version by simulation in a scenario with a realistic spatial channel model. The improvements lead to a more than 25% improvement in spectral efficiency.

## I. INTRODUCTION

Using very large antenna arrays at the base station (BS) is highly beneficial when serving multiple users in downlink communication [1], and massive MIMO communication is one of the key components in the 5G New Radio (NR). However, the performance of Massive MIMO downlink depends heavily on high quality channel state information at the transmitter. In a Frequency Division Duplex (FDD) system the channel has to be measured by the users, quantized, and then fed back to the BS. FDD MIMO finite feedback has been studied intensively both in the single user [2], [3] and multiuser [4], [5] context. When the number of transmit antennas  $N_t$  at the BS is high, the complexity of the quantization and the amount of feedback needed can be prohibitively high. This problem becomes particularly difficult in a multiuser-MIMO (MU-MIMO) setting, where it is known that the amount of feedback should scale with the Signal-to-Noise Ratio (SNR) [4].

In [5] a modular approach was suggested to this problem, which benefits from the fact that while the channel can vary fast, the correlation between the antennas can stay stable for relatively many samples. If an individual  $N_t \times 1$  channel vector  $\mathbf{h}$  is correlated, the majority of channel energy lives in a low-dimensional subspace of  $\mathbb{C}^{N_t}$  and therefore it is sufficient to feed back the coordinates of the signal in this subspace. With  $K$  the subspace dimensionality, this can be done by selecting an  $N_t \times K$  unitary matrix  $\mathbf{U}_K$  based on the covariance matrix of  $\mathbf{h}$ , and creating a  $K$ -dimensional *effective channel* vector by  $\mathbf{c} = \mathbf{U}_K^H(\mathbf{h})$ . If  $\mathbf{U}_K$  is known to the BS, the user can simply quantize  $\mathbf{c}$  and feed back this data. The BS

can then approximate the channel vector as  $\mathbf{U}_K \mathbf{c}$ . If  $K \ll N_t$ , this considerably reduces both quantization complexity and feedback rate.

To use this approach the user also has to quantize and feed back the basis matrix  $\mathbf{U}_K$  using some codebook  $\mathcal{C}_W$ . MIMO covariance matrix, and covariance eigenspace quantization has been considered in [6]–[9]. In [6], it was shown that preserving orthogonality after feedback quantization is optimal. Accordingly, matrix codebooks consisting of a collection unitary matrices is considered. In contrast, [7]–[9] consider independent vector quantization of the columns of  $\mathbf{U}_K$ , i.e. codebooks of the form  $\mathcal{C}_W = \mathcal{C}_w^K$ , where  $\mathcal{C}_w$  is a codebook of  $N_t \times 1$  vectors. For a given quantization accuracy, the size of a vector codebook is only  $\sqrt{K}$  times the size of a matrix codebook, which significantly reduces quantization complexity. However, when a vector codebook is used, orthogonality of the matrices cannot be guaranteed. In [7], orthogonality is guaranteed by sequence design, limiting the vector codebook size to  $N_t$ .

In order to have high descriptive power with manageable complexity, high-resolution FDD feedback in 5G NR is based on overcomplete vector quantization codebooks [8], [9]. This enables high precision of describing the basis  $\mathbf{U}_K$ , at the cost of a potential loss of orthogonality. With a non-orthogonal basis, good effective channel quantization might not result into good overall quantization.

In this paper, we provide an implicit method for feeding back unitary matrices despite using high precision vector codebooks  $\mathcal{C}_w^K$ . We may use precisely the same vector codebook and the same number of feedback bits as [8], [9], but still guarantee unitarity of the fed back matrices. We prove how the overall quantization distortion of the channel decomposes into two independent parts. One describes the error of quantizing  $\mathbf{U}_K$ , while the other part is essentially the effective channel distortion. This analysis also provides a new criterion for quantizing  $\mathbf{U}_K$  in an optimal way. We also take a fresh look on effective channel quantization and suggest quantization bit allocation based on an *intrinsic order*. We simulate the resulting quantization scheme in a realistic multiuser setup and compare it to the standardized quantization scheme of [8]. The simulation results agree exactly with the developed theory. Applying the identified quantization principles leads to considerable throughput gains over [8]. Furthermore, if we increase effective channel quantization accuracy using the standardized quantization for the basis  $\mathbf{U}_K$ , the gains are only nominal. Based on our analysis this can be attributed to the

non-orthogonality of the used  $\mathbf{W}$ . In contrast, applying the identified quantization principles, improved effective channel quantization leads to considerable gain.

## II. SYSTEM MODEL AND PROBLEM SETTING

We consider a downlink MIMO channel, where a base station with  $N_t$  transmit antennas transmits to  $M$  users. For simplicity, we assume the users to have a single antenna. The channel is modeled as

$$\mathbf{y} = \mathbf{H}^H \mathbf{z} + \mathbf{n}, \quad (1)$$

where  $\mathbf{H}$  is an  $N_t \times M$  channel matrix, with the  $k$ th column  $\mathbf{h}_k$  the conjugate channel vector of user  $k$ ,  $\mathbf{z}$  is an  $N_t \times 1$  vector containing the data, and  $\mathbf{n}$  is an  $M \times 1$  vector of noise, modeled as being white with i.i.d  $\mathcal{CN}(0, 1)$  elements.

We assume that each of the users can measure their individual channels  $\mathbf{h}_k$  perfectly and have an error free feedback channel to the base station. The channels  $\mathbf{h}_k$  are assumed block fading, i.e., constant during a communication instance. There is a quantization scheme for  $\mathbf{h}_k$ , and a limited number of bits is used for communicating quantized Channel State Information (CSI) to the base station. Based on CSI, the BS will construct single-user precoders  $\hat{\mathbf{h}}_k$  which are quantized and normalized versions of  $\mathbf{h}_k$ , stacks them to a matrix  $\hat{\mathbf{H}}$ . Assuming that the BS applies equal power Zero Forcing (ZF) precoding with power  $P$ , the received signals become

$$\mathbf{y} = \sqrt{\frac{P}{M}} \mathbf{H}^H \hat{\mathbf{H}} (\hat{\mathbf{H}}^H \hat{\mathbf{H}})^{-1} \mathbf{A} \mathbf{x} + \mathbf{n},$$

where  $\mathbf{x}$  is a norm 1  $M \times 1$  vector of data symbols for the users, and  $\mathbf{A}$  is a diagonal normalization matrix that forces the columns of  $\hat{\mathbf{H}}^H (\hat{\mathbf{H}}^H \hat{\mathbf{H}})^{-1} \mathbf{A}$  to have norm 1. For simplicity we assume that  $\hat{\mathbf{H}}^H \hat{\mathbf{H}}$  is invertible. The ZF-beamforming vectors  $\mathbf{v}_j$  towards the users are the columns of the matrix  $\hat{\mathbf{H}}^H (\hat{\mathbf{H}}^H \hat{\mathbf{H}})^{-1} \mathbf{A}$ . User  $k$  then receives

$$y_k = x_k \sqrt{P/M} \mathbf{h}_k^H \mathbf{v}_k + \sqrt{P/M} \sum_{j \neq k} x_j \mathbf{h}_k^H \mathbf{v}_j + n_k.$$

From this we can calculate the Signal-to-Interference-and-Noise Ratio (SINR) of user  $k$  as

$$\gamma_k = \frac{\frac{P}{M} \|\mathbf{h}_k\|^2 |\tilde{\mathbf{h}}_k^H \mathbf{v}_k|^2}{\frac{P}{M} \|\mathbf{h}_k\|^2 \sum_{j \neq k} |\tilde{\mathbf{h}}_k^H \mathbf{v}_j|^2 + 1},$$

where we use the notation  $\tilde{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ . Assuming Gaussian single user codebooks, we get a spectral efficiency lower bound

$$R_k \geq \log_2(1 + \gamma_k).$$

This directly suggests that a good objective for quantization is to minimize the cross terms in the SINR expression and to maximize the term  $|\tilde{\mathbf{h}}_k^H \mathbf{v}_k|^2$  [4].

The *chordal distance* between two vectors  $\mathbf{x}$  and  $\mathbf{y}$  is

$$d(\mathbf{x}, \mathbf{y}) := \sqrt{1 - \frac{|\mathbf{x}^H \mathbf{y}|^2}{\|\mathbf{x}\|^2 \|\mathbf{y}\|^2}}. \quad (2)$$

It can be proven that  $|\tilde{\mathbf{h}}_k^H \mathbf{v}_k|^2 \leq 1 - d(\mathbf{h}_k, \hat{\mathbf{h}}_k)^2$ , and

$$|\tilde{\mathbf{h}}_k^H \mathbf{v}_j|^2 \leq 2d(\mathbf{h}_k, \hat{\mathbf{h}}_k) - d(\mathbf{h}_k, \hat{\mathbf{h}}_k)^2, \quad \text{when } k \neq j. \quad (3)$$

Thus if we manage to push  $d(\mathbf{h}_k, \hat{\mathbf{h}}_k)$  towards zero, then the spectral efficiency approaches  $\log_2(1 + P/M \|\mathbf{h}_k\|^2)$ , corresponding to the spectral efficiency with perfect feedback and zero forcing. Note that these bounds hold for any codebook and for any channel and are therefore rather pessimistic. For example, in [4] it is proven that with random quantization one can guarantee that  $|\tilde{\mathbf{h}}_k^H \mathbf{v}_j|^2 \leq d(\mathbf{h}_k, \hat{\mathbf{h}}_k)^2$ .

Accordingly a single-user codebook  $\hat{\mathcal{H}}$  for quantizing  $\mathbf{h}$  would be good if  $\min_{\hat{\mathbf{h}} \in \hat{\mathcal{H}}} \{d(\mathbf{h}_k, \hat{\mathbf{h}}_k)^2\}$  is small. The expected value of this is nothing but the quantization distortion with respect to the chordal distance:

$$D(\mathbf{h}_k, \hat{\mathbf{h}}_k) = E_{\mathbf{h}} \left[ \min_{\hat{\mathbf{h}} \in \hat{\mathcal{H}}} \{d(\mathbf{h}_k, \hat{\mathbf{h}}_k)^2\} \right]. \quad (4)$$

Our goal is now to develop quantization schemes that would minimize this distortion.

## III. MODULAR SINGLE USER QUANTIZATION

If the single user channels  $\mathbf{h}_k$  can come from an arbitrary distribution, their quantization and feedback can be highly complex. However, if we can assume that the channels from different antennas are correlated, it is possible to apply a modular quantization approach and reduce complexity considerably. The feedback method for 5G NR [8], [9] is based on such an approach. As in 5G NR, we consider a situation where multiple MU-MIMO communication channels (1) are operated in parallel on subbands in the frequency domain, and the the frequency selective channels of the users are correlated between subbands.

For simplicity, we assume that the single user channel on a subband is  $\mathbf{h} \sim \mathcal{CN}(0, \mathbf{R})$  with a symmetric positive semidefinite channel covariance matrix  $\mathbf{R}$ , describing the *wideband characteristics* of the channel. We have the singular value decomposition  $\mathbf{R} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H$ , where  $\mathbf{\Lambda}$  is an  $r \times r$  diagonal matrix whose elements are the  $r$  non-zero eigenvalues of  $\mathbf{R}$ , and  $\mathbf{U}$  is the tall unitary  $N_t \times r$  matrix of the eigenvectors of  $\mathbf{R}$  corresponding to the non-zero eigenvalues. According to the Karhunen-Loeve decomposition, the channel can be written as

$$\mathbf{h} = \mathbf{U} \mathbf{\Lambda}^{1/2} \mathbf{c}, \quad (5)$$

where  $\mathbf{c}$  is an  $r \times 1$  vector with i.i.d  $\mathcal{CN}(0, 1)$  coordinates.

The problem we are now considering is the following. Let us assume that the covariance matrix  $\mathbf{R}$  is fixed and we have a budget of bits for feeding back wideband statistical data of the channel pertaining to  $\mathbf{R}$ , and a separate budget for feeding back subband specific CSI pertaining to the coordinates  $\mathbf{c}$ . In the extreme case with unlimited wideband feedback, the BS knows  $\mathbf{U}$  and  $\mathbf{\Lambda}^{1/2}$  perfectly. We will denote with  $\mathbf{\Sigma}$  the  $K \times K$  diagonal matrix where on the diagonal are the  $K$  largest values of the diagonal of  $\mathbf{\Lambda}^{1/2}$  ordered from largest to smallest. Furthermore we denote with  $\mathbf{U}_K$  the  $N_t \times K$  matrix consisting of columns of  $\mathbf{U}$  ordered by the size of the corresponding eigenvalues.

The *modular channel quantization* [9] process can now be performed as follows. The user first feedbacks quantized CSI about  $\mathbf{U}_K$ , and then, using the fed back wideband CSI

as a basis, feeds back information about subband channels expanded in this basis.

Ideal subband quantization would proceed as follows. The user first calculates the *effective channel*

$$\tilde{\mathbf{c}} = \Sigma^{-1} \mathbf{U}_K^H \mathbf{h} \quad (6)$$

for each subband. According to Equation (5),  $\tilde{\mathbf{c}}$  is a  $K$ -dimensional i.i.d  $\mathcal{CN}(0,1)$  vector. The user quantizes  $\tilde{\mathbf{c}}$  to  $\hat{\tilde{\mathbf{c}}}$  using some Grassmannian quantization codebook  $\hat{\mathcal{C}}$  developed for  $K$ -dimensional i.i.d Gaussian vectors modulo norm and phase, and then sends this information to the BS. The BS can construct an estimate of  $\mathbf{h}$  as

$$\hat{\mathbf{h}} = \mathbf{U}_K \Sigma \hat{\tilde{\mathbf{c}}}. \quad (7)$$

The key point of this modular approach is that it reduces the dimension of the quantization problem considerably, as the dimensionality of the subband quantization problem has been reduced from  $N_t$  to  $K$  dimensions.

Note that the BS does not need to know either the overall channel phase or the channel norm—the channel phase is irrelevant, and information of the channel norm is subsumed by separately transmitted Channel Quality Indication feedback. Accordingly, the estimate  $\hat{\mathbf{h}}$  is up to phase and norm. Furthermore, due to the unitarity of  $\mathbf{U}_K$ , good quantization on the effective channel will result in good quantization on the actual channel, as  $\mathbf{U}_K$  captures most of the energy of  $\hat{\mathbf{h}}$ . We shall make this intuition explicit in Section IV.

#### A. Wideband Quantization of Prior Art

For wideband feedback we assume that the BS and the users share a vector codebook  $\hat{\mathcal{C}}_{\mathbf{w}}$  for quantizing the  $N_t \times 1$  columns in  $\mathbf{U}_K$ , and a scalar codebook for quantizing the elements in  $\Sigma$  [9]. After wideband CSI feedback, the BS has a matrix  $\mathbf{W}$ , which is a quantized version of  $\mathbf{U}_K$ , and  $\hat{\Sigma}$ , which is a quantized version of  $\Sigma$ .

The quantization process now proceeds as in (6,7) but replacing  $\mathbf{U}_K$  with  $\mathbf{W}$  and  $\Sigma$  with  $\hat{\Sigma}$ . However, if the codebook  $\hat{\mathcal{C}}_{\mathbf{w}}$  is overcomplete, as for example in the high resolution alternative of 5G NR [8], there is a high probability that  $\mathbf{W}$  is not unitary, and the connection between the channel and effective channel quantization is partially broken. In the following we will show how we can avoid this problem.

#### B. Orthogonalizing Vector Quantized Wideband Feedback

While conventionally wideband feedback is based on quantizing the covariance matrix  $\mathbf{R} = E_{\mathbf{h}}[\mathbf{h}\mathbf{h}^H]$  [7], or a normalized version  $E_{\mathbf{h}}[\mathbf{h}\mathbf{h}^H]/E_{\mathbf{h}}[\|\mathbf{h}\|^2]$  [9], we instead quantize

$$\tilde{\mathbf{R}} := E_{\mathbf{h}}[\mathbf{h}\mathbf{h}^H/\|\mathbf{h}\|^2] = \mathbf{U}^H \Lambda \mathbf{U}, \quad (8)$$

and its singular value decomposition. The motivation for this will be given in Section IV.

Let us now assume that we are aiming at  $K$ -dimensional effective channel feedback. We will later prove that in order to minimize the distortion, we should find an  $N_t \times K$  dimensional matrix  $\mathbf{W}$  that would minimize *projection distance*

$$d_p(\mathbf{W}, \tilde{\mathbf{R}}) := 1 - \text{Tr}(\mathbf{W}_O^H \tilde{\mathbf{R}} \mathbf{W}_O), \quad (9)$$

where  $\mathbf{W}_O$  is an  $N_t \times K$  matrix whose columns form an orthonormal basis of the space spanned by the columns of  $\mathbf{W}$ . As such, this matrix now satisfies  $\mathbf{W}_O^H \mathbf{W}_O = \mathbf{I}$ . The motivation for this criterion will be provided again in Section IV.

Using matrix quantization codebooks of  $N_t \times K$  matrices  $\mathbf{W}$  that would minimize (9) leads to typically high complexity quantization. Instead we proceed as previously by quantizing  $\mathbf{U}_K$  column by column using some vector quantization codebook  $\hat{\mathcal{C}}_{\mathbf{w}}$ . As a result the user has now an  $N_t \times K$  matrix  $\mathbf{W}$  consisting of quantized norm 1 vectors. These vectors are not necessarily orthonormal. We also assume that the data of  $\mathbf{W}$  will be fed back to the BS. However, now we assume that both the users and the base-station *have agreed on a method to orthonormalize* the vectors  $\mathbf{W}$ . This will produce a set of orthonormalized vectors which span the same space as the columns in  $\mathbf{W}$ . This method is independent of the structure of  $\mathbf{W}$  and is assumed to be shared at the same time as the codebook  $\hat{\mathcal{C}}_{\mathbf{w}}$ . Now the user and BS both perform this operation, after which they both have matrices  $\mathbf{W}_O$ . Exactly the same number of bits can be used to feed back this matrix as feeding back the original  $\mathbf{W}$ . The matrix  $\mathbf{W}_O$  can now be used in place of  $\mathbf{W}$  for the rest of the quantization. However, as  $\mathbf{W}$  is transformed to  $\mathbf{W}_O$  it does not make sense to use quantized singular values as wideband amplitudes. Instead we calculate wideband amplitudes as  $\sigma_i = \sqrt{\mathbf{w}_i^H \tilde{\mathbf{R}} \mathbf{w}_i}$  for each column  $\mathbf{w}_i$  in  $\mathbf{W}_O$ , and then feed back quantized versions of these elements by using the same quantization scheme designated for the original singular values. After this operation, BS and user share the matrix  $\mathbf{W}_O$  and the quantized wideband amplitude matrix  $\hat{\Sigma}$ . Using these matrices, the subband feedback can be performed as in Equations (6) and (7), replacing  $\mathbf{U}_K$  and  $\Sigma$  with  $\mathbf{W}_O$  and  $\hat{\Sigma}$ .

#### IV. QUANTIZATION DISTORTION WITH UNITARY $\mathbf{W}$

In this section we provide analytical derivations for the intuitive notions used. We consider modular quantization of a single user channel  $\mathbf{h}$  using the quantization codebook  $\hat{\mathcal{C}}$  for the  $K \times 1$  effective channels, a fixed covariance feedback matrix  $\mathbf{W}$  satisfying  $\mathbf{W}^H \mathbf{W} = \mathbf{I}$ , and a quantized  $K \times K$  wideband amplitude matrix  $\hat{\Sigma}$ . We denote the induced quantization codebook for the  $N_t \times 1$  vector  $\mathbf{h}$  by  $\hat{\mathcal{H}} = \mathbf{W} \hat{\Sigma} \hat{\mathcal{C}} := \{\mathbf{W} \hat{\Sigma} \hat{\mathbf{c}} \mid \hat{\mathbf{c}} \in \hat{\mathcal{C}}\}$ .

According to (2) and (4) we are interested in how well the elements of  $\hat{\mathcal{H}}$  quantize  $\mathbf{h}$  in terms of chordal distance. Hence, in what follows we assume that all the codewords  $\hat{\mathbf{h}} \in \hat{\mathcal{H}}$  satisfy  $\|\hat{\mathbf{h}}\| = 1$ .

Consider the projector map  $\Pi_{\mathbf{W}} = \mathbf{W} \mathbf{W}^H$  that maps the elements of  $\mathbb{C}^{N_t}$  to the space spanned by the columns of  $\mathbf{W}$ . We can now decompose  $\mathbf{h}$  to a component lying in the  $\mathbf{W}$ -subspace, and to a component in the perpendicular subspace;

$$\mathbf{h} = \Pi_{\mathbf{W}} \mathbf{h} + (\mathbf{I} - \Pi_{\mathbf{W}}) \mathbf{h} \equiv \mathbf{h}_{\parallel} + \mathbf{h}_{\perp}. \quad (10)$$

We use shorthand  $\mathbf{h}_{\perp}/\|\mathbf{h}\| = \tilde{\mathbf{h}}_{\perp}$ ,  $\mathbf{h}_{\parallel}/\|\mathbf{h}\| = \tilde{\mathbf{h}}_{\parallel}$  and  $\mathbf{h}/\|\mathbf{h}\| = \tilde{\mathbf{h}}$ . With this notation we have that  $\frac{\tilde{\mathbf{h}}}{\|\tilde{\mathbf{h}}\|} = \tilde{\mathbf{h}}_{\parallel} + \tilde{\mathbf{h}}_{\perp}$  and therefore

$$\langle \tilde{\mathbf{h}}_{\parallel} + \tilde{\mathbf{h}}_{\perp}, \tilde{\mathbf{h}}_{\parallel} + \tilde{\mathbf{h}}_{\perp} \rangle = \|\tilde{\mathbf{h}}_{\parallel}\|^2 + \|\tilde{\mathbf{h}}_{\perp}\|^2 = 1. \quad (11)$$

The proof of the following result is omitted due to space constraints.

*Lemma 1:* Assume that  $\mathbf{h}$  is a channel realization and that  $\hat{\mathbf{h}}$  is a quantized version selected from the codebook  $\hat{\mathcal{H}}$ . Then

$$1 - \left| \langle \tilde{\mathbf{h}}, \hat{\mathbf{h}} \rangle \right|^2 = \|\tilde{\mathbf{h}}_{\parallel}\|^2 \left( 1 - \left| \left\langle \frac{\mathbf{h}_{\parallel}}{\|\mathbf{h}_{\parallel}\|}, \hat{\mathbf{h}} \right\rangle \right|^2 \right) + \|\tilde{\mathbf{h}}_{\perp}\|^2$$

*Proposition 1:* Given a random vector  $\mathbf{h}$  and corresponding quantization codebook  $\hat{\mathcal{H}}$  we have

$$E_{\mathbf{h}} \left[ \min_{\hat{\mathbf{h}} \in \hat{\mathcal{H}}} \left\{ 1 - \left| \langle \tilde{\mathbf{h}}, \hat{\mathbf{h}} \rangle \right|^2 \right\} \right] = E_{\mathbf{h}} \left[ \min_{\hat{\mathbf{h}} \in \hat{\mathcal{H}}} \left\{ \|\tilde{\mathbf{h}}_{\parallel}\|^2 \left( 1 - \left| \left\langle \frac{\mathbf{h}_{\parallel}}{\|\mathbf{h}_{\parallel}\|}, \hat{\mathbf{h}} \right\rangle \right|^2 \right) \right\} \right] + d_p(\mathbf{W}, \tilde{\mathbf{R}}).$$

*Proof:* For simplicity, we will disregard the minimum term. By Lemma 1

$$E_{\mathbf{h}} \left[ 1 - \left| \langle \tilde{\mathbf{h}}, \hat{\mathbf{h}} \rangle \right|^2 \right] = E_{\mathbf{h}} \left[ \|\tilde{\mathbf{h}}_{\parallel}\|^2 \left( 1 - \left| \left\langle \frac{\mathbf{h}_{\parallel}}{\|\mathbf{h}_{\parallel}\|}, \hat{\mathbf{h}} \right\rangle \right|^2 \right) \right] + E_{\mathbf{h}} \left[ \|\tilde{\mathbf{h}}_{\perp}\|^2 \right].$$

It follows from (11) that

$$E_{\mathbf{h}} \left[ \|\tilde{\mathbf{h}}_{\perp}\|^2 \right] = 1 - E_{\mathbf{h}} \left[ \|\tilde{\mathbf{h}}_{\parallel}\|^2 \right] = 1 - E_{\mathbf{h}} \left[ \text{Tr} \left( \tilde{\mathbf{h}} \tilde{\mathbf{h}}^H \mathbf{\Pi}_W \right) \right] = 1 - E_{\mathbf{h}} \left[ \text{Tr} \left( \mathbf{W}^H \frac{\mathbf{h} \mathbf{h}^H}{\|\mathbf{h}\|^2} \mathbf{W} \right) \right]$$

Since the expected value commutes with trace and multiplication with constant matrices, we have that

$$E_{\mathbf{h}} \left[ \text{Tr} \left( \mathbf{W}^H \frac{\mathbf{h} \mathbf{h}^H}{\|\mathbf{h}\|^2} \mathbf{W} \right) \right] = \text{Tr} \left( \mathbf{W}^H \left( E_{\mathbf{h}} \left[ \frac{\mathbf{h} \mathbf{h}^H}{\|\mathbf{h}\|^2} \right] \right) \mathbf{W} \right).$$

The final result then follows straightforwardly. ■

*Corollary 1:* We have the following upper and lower bounds for the overall distortion

$$d_p(\mathbf{W}, \tilde{\mathbf{R}}) \leq E_{\mathbf{h}} [\min_{\hat{\mathbf{h}}} \{d(\hat{\mathbf{h}}, \mathbf{h})^2\}] \leq E_{\mathbf{c}} [\min_{\mathbf{c}^*} \{d(\mathbf{c}^*, \mathbf{c})^2\}] + d_p(\mathbf{W}, \tilde{\mathbf{R}}),$$

where  $\mathbf{c}^* \in \hat{\Sigma} \hat{\mathcal{C}}$  and  $\mathbf{c} = \mathbf{W}^H \mathbf{h}$ .

This result proves that the size of  $d_p(\mathbf{W}, \tilde{\mathbf{R}})$  provides an absolute lower bound for quantization distortion, when using the wideband feedback matrix  $\mathbf{W}$ . It also suggests that minimizing it is a good criterion for selecting the matrix  $\mathbf{W}$ . Furthermore we see that if we assume that  $\mathbf{W}$  is unitary and  $d_p(\mathbf{W}, \tilde{\mathbf{R}})$  is small, then good effective channel quantization results into good quantization in the actual channel.

## V. QUANTIZATION METHODS

It is natural, and shown in Corollary 1 that we should find a good quantization method for the effective channel  $\mathbf{c} = \mathbf{W}^H \mathbf{h}$ . Unfortunately the statistics of  $\mathbf{c}$  depend on  $\mathbf{W}$  and  $\hat{\Sigma}$ . However, according to Equation (5) a fair working hypothesis is  $\mathbf{c} \approx \hat{\Sigma} \tilde{\mathbf{c}}$  where  $\tilde{\mathbf{c}}$  is an i.i.d.  $K \times 1$   $\mathcal{CN}(0, 1)$  vector. Taking this as granted we have reduced the 32-dimensional quantization problem to a  $K$ -dimensional problem of quantizing vectors of independent Gaussian random

variables with different variances  $\hat{\sigma}_i^2$ . Quantization of i.i.d. channels is well under control [2], [3]. In the following we present a suggestion for overall low-complexity wideband and effective channel quantization. For comparison we present the method suggested in the 5G NR standard [8].

In order to highlight the differences between the approaches we present in both cases the whole modular quantization chain. As a starting point we assume a fixed vector quantization codebook  $\mathcal{C}_w$  for producing the covariance feedback matrix  $\mathbf{W}$ , and another fixed codebook for the wideband amplitudes  $\hat{\Sigma}$ . We can, for example, use options from [8]. For simplicity we assume that  $K = 8$ .

For the effective channels, quantization codebooks  $\hat{\mathcal{C}}$  that act on individual coordinates directly will be used. A general codeword of  $\hat{\mathcal{C}}$  can be written as  $\hat{\mathbf{c}} = (\hat{c}_1, \dots, \hat{c}_8)$ , where  $\hat{c}_i$  is freely selected from a coordinate codebook  $\mathcal{C}_i$ . The advantage of this approach is that we can use large codebooks, while quantization complexity stays limited as it can be performed coordinate by coordinate. Note that as overall phase is irrelevant, one may rotate the vector such that a chosen coordinate  $c_i$  is real positive.

A natural codebook for each coordinate is one where each  $\mathcal{C}_i$  is quantizing a  $\mathcal{CN}(0, 1)$  random variable. However when the BS reconstructs the channel by  $\mathbf{W} \hat{\Sigma} \tilde{\mathbf{c}}$ , coordinates corresponding to larger values of  $\hat{\sigma}_i$  have a larger effect on the overall distortion. Hence we should use variable-size codebooks  $\mathcal{C}_i$  for different coordinates. This leads to rather hard bit allocation problem. We consider three different granularities with 24, 26 or 28 bits for the quantization of the whole vector.

1) *Standardized Feedback [8]:* As a starting point the user finds the covariance matrix  $\mathbf{R}$  and then finds the covariance feedback  $\mathbf{W}$  and  $\hat{\Sigma}$  by quantizing  $\mathbf{U}_8$  and  $\Sigma_8$  from Equation (6). Then there are two possible options for finding the effective channel. This can be done either directly,  $\tilde{\mathbf{c}} = \hat{\Sigma}^{-1} \mathbf{W}^H \mathbf{h}$ , or by using pseudoinversion:

$$\tilde{\mathbf{c}} = \hat{\Sigma}^{-1} (\mathbf{W}^H \mathbf{W})^{-1} \mathbf{W}^H \mathbf{h}.$$

Irrespectively of the chosen method, the effective channel quantization then continues as follows.

Bit allocation for effective channel coordinate quantization is based on an *extrinsic order* of the coordinates, i.e., based on the size of the corresponding  $\hat{\sigma}_i$ . As we are not interested on feeding back constant multiplicative terms, we first divide all the other coordinates with the coordinate corresponding to the largest  $\hat{\sigma}_i$ . No bits are needed for quantizing this reference coordinate. Then for the next  $m = 5, 6, 7$  coordinates we quantize the amplitude with codebook  $\{1, \sqrt{0.5}\}$  and use 3-bit uniform phasing quantization. For the last  $8 - (m + 1)$  coordinates we allocate 0 amplitude bits and 2 bits for uniform phase quantization.

2) *Orthogonalized wideband precoding (OWP):* We start quantization from  $\tilde{\mathbf{R}}$  and find the matrix  $\mathbf{W}$  based on this matrix. We then use the shared orthogonalization method from Section III-B to produce  $\mathbf{W}_O$ , and find wideband amplitudes  $\sigma_i = \sqrt{\mathbf{w}_i^H \tilde{\mathbf{R}} \mathbf{w}_i}$  and quantizing these we get  $\hat{\Sigma}$ . The effective channel is then found by  $\tilde{\mathbf{c}} = \hat{\Sigma}^{-1} \mathbf{W}_O^H \mathbf{h}$ . For effective channel quantization we use an *intrinsic order*. First we normalize the

effective channel vector, and then quantize all coordinates with one bit amplitude quantization using codebook  $\{0.208, 0.462\}$ . Assuming perfect phase quantization, these provide optimal amplitude quantization for Rayleigh fading variables with average energy  $1/\sqrt{8}$ , see [10]. We denote the quantized amplitude value of the  $i$ th coordinate with  $a_i$ . The user then compares  $\hat{\sigma}_i a_i$  to each other and divides all the coordinates with the phase of the coordinate corresponding to the largest value of  $\hat{\sigma}_i a_i$ .

After this the user allocates phase bits for phase quantization of the coordinates based on the sizes of  $\hat{\sigma}_i a_i$ , not just based on  $\hat{\sigma}_i$  as when using extrinsic order. No bits are needed for quantizing the phase of the coordinate with largest  $\hat{\sigma}_i a_i$ . Then 4 phase bits are allocated to the  $m-1$  next largest coordinates, and 2 phase bits to the last  $8-m$  coordinates, where  $m = 3, 5, 7$ .

The user feeds back the amplitude information using extrinsic order, so that the base station will know which quantized amplitudes  $a_i$  correspond to which vector of  $\mathbf{W}_0$ . Then phase information is transmitted in any agreed order.

The basic idea underlying this bit allocation is that while typically the coordinates with largest  $\hat{\sigma}_i$  have the largest true amplitude  $\sigma_i |c_i|$ , this does not happen always. With the intrinsic order we allocate more phase bits for the coordinates having the largest impact on overall quantization distortion.

## VI. SIMULATIONS

We evaluate performance in a scenario where an  $N_t = 32$  antenna BS serves  $M = 4$  single antenna users. The channel between a user and the BS on subband  $s$  is  $\mathbf{h}_s$ . A user constructs the sample covariance  $\mathbf{R}$  or  $\hat{\mathbf{R}}$  by averaging over the subbands, quantizes this, and then quantizes  $\mathbf{h}_s$  on each subband. Based on the feedback, the BS performs ZF on each subband, as described in Section II. The two feedback schemes from Section V are compared; the standardized scheme was simulated with and without pseudo-inversion. The quantization codebooks for  $\mathbf{W}$  and  $\Sigma$  are from [8].

In the simulations, QuaDRiGa V2.0.0 [11] was used to generate MIMO channel correlations with 3GPP 38.901 UMa NLOS settings. The BS was assumed having Uniform Planar Array (UPA) with  $32 = 8 \times 2 \times 2$  antennas (horizontal x vertical x polarization). After frequency selective channel generation, user channels were normalized to a SNR.

Average single user spectral efficiency given an SNR is plotted in Figure 1. Simulations corroborate the theoretical principles discussed above. OWP provides a gain of more than 25% against the standardized versions. Using pseudo-inversion with the standardized approach gives nominal gain. Increasing the effective channel quantization granularity does improve the performance of OWP considerably, while doing so with the standardized version provides little gain. While we here demonstrate the results only with  $M = 4$  users, similar results can be observed with higher numbers of users.

## VII. CONCLUSIONS

We have addressed wideband and subband quantization in a modular quantization scheme. Analyzing the separation of

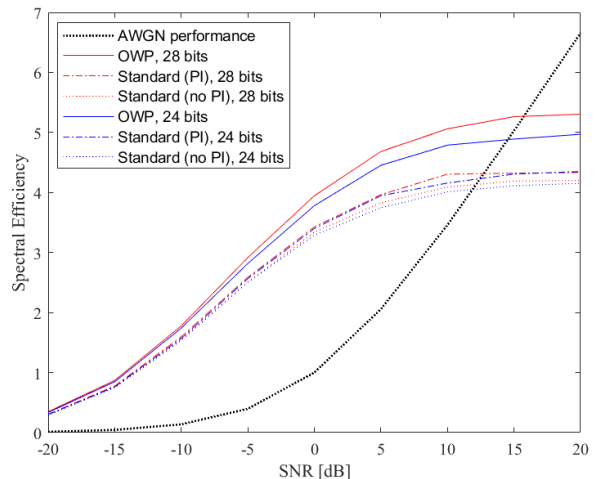


Fig. 1. Spectral efficiency with 4 users: Orthogonalized wideband precoding (OWP) versus Standard with pseudoinversion (PI) and without pseudoinversion (no PI) compared to single user SISO AWGN performance.

feedback to wideband and subband parts, we found quantization objectives for optimal modular quantization. We show considerable performance improvement in a MU-MIMO scenario with a high number of antennas, when these principles are applied. Orthogonalization of wideband feedback makes it possible to further increase quantization accuracy by improving subband quantization accuracy, while if orthogonalization is omitted, improving subband quantization seems useless. In future work we shall analyze the degree to which wideband feedback is a performance bottleneck, and investigate bit allocation between wideband and subband feedback.

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