Sum Rate Maximizing Zero Interference Linear Multiuser MIMO Transmission

Helka-Liina Määttänen* †
* Renesas Mobile Europe Ltd.
Systems Research and Standardization
Helsinki, Finland
Email: helka.maattanen@renesasmobile.com

Olav Tirkkonen†
† Aalto University School of Electrical Engineering
Department of Communications and Networking
Helsinki, Finland
Email: olav.tirkkonen@aalto.fi

***Abstract*—We consider sum rate maximizing linear zero interference precoder–receiver pairs for multiuser single stream transmission. We parametrize the transmit and receive filters with a unit norm vector per user, leading to a transparent interpretation of the tradeoff between aligning with the eigenspace of the user and avoiding loss of power due to multiuser zero forcing. With this formulation the sum rate may be numerically maximized. We investigate performance for two users that have two receive antennas in spatially correlated and uncorrelated channels. The presented scheme outperforms zero interference transmissions based on a maximum ratio combining receiver, especially in correlated channels.**

I. Introduction

For single-antenna receivers, zero forcing beamforming (ZFBF) with the combined channel of multiple users is the (asymptotically) optimum linear precoder under zero interference condition [1], [2]. When there are multiple receiver antennas, a generalized version of the channel inversion, called block diagonalization (BD), can be used [1], [3]. BD orthogonalizes the whole signal spaces of the users even if there is transmission only on part of the signal space. Thus, when the number of transmitted streams per user is less than the number of receive antennas, transmission schemes that take the receiver processing into account perform better than BD. In [4], a best combination of eigenmodes of the users are iteratively selected for zero forcing transmission and the receivers are then eigenvector combiners. Even though the degrees of freedom offered by multiple receive antennas are better exploited than in BD, the scheme is known suboptimal and thoroughly studied in [5]. In [6], [7], the transmit beamformers and receiver vectors are iteratively computed for single stream transmission per user in order to maximize the sum rate. In [8], joint optimization of the precoder and receiver for 2×2 multiple–input multiple–output (MIMO) with two users is considered. The receiver is selected to be a maximum ratio combining (MRC) receiver and the precoder is optimized numerically in a spatially uncorrelated channel. In [8], the MRC receiver is argued to be the optimum receiver when the transmission constraints force zero interference reception. In [9], [10] it is found that for two users, the generalized eigenvectors of $\mathbf{R}_1$ and $\mathbf{R}_2$, where $\mathbf{R}_k$ are the normalized instantaneous correlation matrices of the users, give the zero interference solution when the receiver is MRC. Also, for 2×2 MIMO, the generalized eigenvectors were found to reproduce the precoder of [8].

In this paper, we parametrize the transmitter and receivers with a unit norm vector s_k per user and consider a general receiver which covers all commonly used linear combiners such as MRC and the eigenvector combiner. Subject to the channels, and the s_k, a transmit precoder is selected that removes the interference from the combined signals of the users. The solutions in [8], [9], [10] for 2×2 MIMO are special cases of our formulation.

The s_k may be selected to maximize a suitable system utility. Here, we consider maximization of the sum rate, with equal power allocated to each user. We show results both in uncorrelated and correlated scenarios for two users with two receive antennas and two or four transmit antennas. The results show that the presented scheme outperforms the generalized eigenvector precoder especially in correlated scenarios, where the eigenvalue spread is larger. Also, our parametrization leads to a transparent interpretation of the tradeoff between aligning with the eigenspace of the user and avoiding loss of power due to multiuser zero forcing.

The rest of the paper is organized as follows. In Section 2, we give the system model. In Section 3, we give the general description of the receiver-precoder pair for single stream transmission and in Section 4 we consider the solution for the sum rate maximization. Section 5 presents the simulation results and in Section 6 we give the conclusions.

II. System model

We consider a single cell MIMO downlink system, where the base station has N_t transmit antennas and the users have N_r receive antennas each. It is natural to assume $N_\mathrm{r} \leq N_\mathrm{t}$ as the number of antennas at the UEs is considerably more limited by size than at the BS. There are K users, with $K \leq N_\mathrm{t}$, and we assume single stream transmission per user. The $N_\mathrm{r} \times 1$ signal vector $\mathbf{y}_k$ received by the k:th user reads

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W} \mathbf{A} \mathbf{x} + \mathbf{n}_k, \tag{1}$$

where $\mathbf{H}_k$ is the $N_\mathrm{r} \times N_\mathrm{t}$ MIMO channel between the base station and user k and $\mathbf{W} = [\mathbf{w}_1 \ , ..., \ \mathbf{w}_K]$ is the $N_\mathrm{t} \times K$ precoding matrix, where $\mathbf{w}_k$ is the precoding vector for user k. The diagonal matrix $\mathbf{A}$ controls the power division between users. A precoder that includes the power control is denoted as $\widehat{\mathbf{W}} = \mathbf{W}\mathbf{A}$. The $K \times 1$ vector $\mathbf{x} = [x_1 \ , ..., \ x_K]^{\mathrm{T}}$ contains the transmitted symbols for the users and $\mathbf{n}_k$ is the scaled noise vector whose entries are IID complex Gaussian distributed with zero mean and variance $\frac{\sigma^2}{P}$, where σ^2 is the variance of additive white Gaussian noise (AWGN) and P is transmitted signal power. The MIMO channel $\mathbf{H} \in \mathbb{C}^{N_{\mathrm{r}} \times N_{\mathrm{t}}}$ is a complex Gaussian matrix written as $\mathbf{H} = \widetilde{\mathbf{H}}\mathbf{R}_{\mathrm{T}}^{1/2}$ [11], where $\widetilde{\mathbf{H}}$ is an i.i.d. complex circular Gaussian matrix and $\mathbf{R}_{\mathrm{T}} = \mathrm{E}\left\{\mathbf{H}^{\mathrm{H}}\mathbf{H}\right\}$ is the transmitter end spatial correlation matrix. We consider both i.i.d. spatial fading and channels with transmit correlation. According to [12] the receiver correlation is typically small thus the receiver covariance matrix $\mathbf{R}_{\mathrm{R}}$ is set to identity. For transmit correlation, we use a simple exponential correlation model, a uniform linear array (ULA) [13], where the amplitude correlation between the signals in antennas m and n is given by $\rho^{|m-n|}$. To avoid favoring a particular direction of the signal or codewords, we define

$$[\mathbf{R}_{\mathrm{T}}]_{m,n} = \rho^{|m-n|} \exp(j(m-n)\phi), \tag{2}$$

where the direction of transmission ϕ is uniformly distributed across different channel realizations. The singular value decompositions (SVD) of the channels are $\mathbf{H}_k = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^{\mathrm{H}}$, where Σ_k is a $N_{\mathrm{r}} \times N_{\mathrm{r}}$ diagonal matrix of ordered non-zero singular values, and $\mathbf{U}_k$ and $\mathbf{V}_k$ are $N_{\mathrm{r}} \times N_{\mathrm{r}}$ and $N_{\mathrm{t}} \times N_{\mathrm{r}}$ unitary matrices, respectively.

III. General single stream Multiuser transmission

We consider single-stream transmission to each user and perfect channel state information (CSI) at the transmitter. The transmission requirements for interference free multiuser single stream transmission are written as

$$\mathbf{g}_k^{\mathrm{H}} \mathbf{H}_k \mathbf{W} \ = \ \mathbf{g}_k^{\mathrm{H}} \mathbf{H}_k \mathbf{w}_k \mathbf{e}_k^{\mathrm{H}}, \tag{3}$$

where $\mathbf{g}_k$ is the $N_{\mathrm{r}} \times 1$ receiver combining and $\mathbf{e}_k$ is the $N_{\mathrm{t}} \times K$ Euclidean basis vector. When $N_{\mathrm{r}} = 1$ and $N_{\mathrm{t}} = K$, there exists only one precoder matrix achieving zero-forcing transmission—the inverse of the square combined channel $\mathbf{H} = [\mathbf{h}_1^{\mathrm{H}}, ..., \mathbf{h}_K^{\mathrm{H}}]^{\mathrm{H}}$. In case $N_{\mathrm{t}} \geq K$, the inverse is not unique. A general inverse may be expressed as

$$\mathbf{W} = \mathbf{H}^{\dagger} + P_{\perp}\mathbf{B}, \tag{4}$$

where $\mathbf{H}^{\dagger}$ denotes the pseudo inverse, $P_{\perp} = \mathbf{I} - \mathbf{H}^{\dagger}\mathbf{H}$ is the orthogonal projection onto the null space of $\mathbf{H}$ and $\mathbf{B}$ is an arbitrary matrix. By choosing $\mathbf{B} = 0$, the non-normalized precoder equals the pseudo inverse of $\mathbf{H}$. The resulting precoder does not transmit energy to the null space of $\mathbf{H}$. In [14], it has been proven that pseudo inverse based precoders are optimal among the generalized inverses for maximizing any performance measure under a total power constraint assumption.

Here, we consider zero-interference single stream transmission to multiple users with more than one receive antenna. The users use linear receivers (vector combiners) when receiving a single stream transmission. Instead of restricting to a specific combiner, like MRC, we consider general combiners. The effective channel for each user is a $1 \times N_{\mathrm{t}}$ vector $\mathbf{h}_{\mathrm{eff},k} = \mathbf{g}_k^{\mathrm{H}}\mathbf{H}_k$, where $\mathbf{g}_k^{\mathrm{H}}$ is the normalized combiner. The combined effective channel is a $K \times N_{\mathrm{t}}$ matrix

$$\mathbf{H}_{\mathrm{eff}} = [\mathbf{h}_{\mathrm{eff},1}^{\mathrm{H}}, ..., \mathbf{h}_{\mathrm{eff},K}^{\mathrm{H}}]^{\mathrm{H}}. \tag{5}$$

Interference-free multiuser transmission-reception is achieved if and only if

$$\mathbf{H}_{\mathrm{eff}}\widehat{\mathbf{W}} = \widetilde{\mathbf{A}}, \tag{6}$$

where $\widetilde{\mathbf{A}} = \mathrm{diag}(\tilde{\mathbf{a}})$ is a diagonal matrix defined by a vector $\tilde{\mathbf{a}}$ of K arbitrary non-zero real positive numbers.[1] Consider the SVD of the effective channel, $\mathbf{H}_{\mathrm{eff}} = \mathbf{U}_{\mathrm{eff}}\mathbf{\Sigma}_{\mathrm{eff}}\mathbf{V}_{\mathrm{eff}}^{\mathrm{H}}$. The matrix $\mathbf{V}_{\mathrm{eff}}^{\mathrm{H}}$ spans the space seen by the users equipped with the general combiners $\mathbf{g}_k^{\mathrm{H}}$. Consequently, a general precoder which does not send energy to the part of the transmission subspace that is not visible to any of the receivers can be expressed as a linear combination of the vectors in $\mathbf{V}_{\mathrm{eff}}^{\mathrm{H}}$. We have

$$\widehat{\mathbf{W}} = \mathbf{V}_{\mathrm{eff}}\widetilde{\mathbf{W}}, \tag{7}$$

where $\widetilde{\mathbf{W}}$ is a $K \times K$ matrix of complex numbers. Applying the general precoder of (7) to (6) we have $\mathbf{U}_{\mathrm{eff}}\mathbf{\Sigma}_{\mathrm{eff}}\widetilde{\mathbf{W}} = \widetilde{\mathbf{A}}$. If we fix the vector $\tilde{\mathbf{a}}$ and assume $\mathbf{\Sigma}$ invertible, $\widetilde{\mathbf{W}}$ has an unambiguous solution $\widetilde{\mathbf{W}} = \mathbf{\Sigma}_{\mathrm{eff}}^{-1}\mathbf{U}_{\mathrm{eff}}^{\mathrm{H}}\widetilde{\mathbf{A}}$. It follows that any solution to (6) can be written as

$$\widehat{\mathbf{W}} = \mathbf{V}_{\mathrm{eff}}\mathbf{\Sigma}_{\mathrm{eff}}^{-1}\mathbf{U}_{\mathrm{eff}}^{\mathrm{H}}\widetilde{\mathbf{A}}, \tag{8}$$

which is an inverse of the combined effective channels. One of these solutions is the pseudo inverse $\widehat{\mathbf{W}} = \mathbf{H}_{\mathrm{eff}}^{\mathrm{H}}(\mathbf{H}_{\mathrm{eff}}\mathbf{H}_{\mathrm{eff}}^{\mathrm{H}})^{-1}$.

The combined effective channel and therefore the precoder is a function of the receiver combiners. Finding receiver combiners to maximize a performance metric is a nonconvex optimization problem. We approach the problem by parameterizing the receiver combiners by unit norm vectors $\mathbf{s}_k$ from which the overall phase can be removed. Without loss of generality, the $\mathbf{g}_k^{\mathrm{H}}$ can be written as

$$\mathbf{g}_k^{\mathrm{H}} = \frac{\mathbf{s}_k^{\mathrm{H}} \ \mathbf{\Sigma}_k^{-1}\mathbf{U}_k^{\mathrm{H}}}{||\mathbf{s}_k^{\mathrm{H}} \ \mathbf{\Sigma}_k^{-1}||}, \tag{9}$$

where we assume $\mathbf{\Sigma}_k$ to be invertible. With this combiner the effective channel for each user becomes

$$\mathbf{h}_{\mathrm{eff},k} = \frac{(\mathbf{V}_k\mathbf{s}_k)^{\mathrm{H}}}{||\mathbf{s}_k^{\mathrm{H}} \ \mathbf{\Sigma}_k^{-1}||}. \tag{10}$$

The combining vectors $\mathbf{s}_k$ select linear combinations of the N_{r} eigendirections of each user, which form the effective channels. Next define a $N_{\mathrm{t}} \times K$ matrix

$$\mathbf{C} = [\mathbf{V}_1, ..., \mathbf{V}_K]\,\mathbf{S} \ , \tag{11}$$

where $\mathbf{S}$ is a $KN_{\mathrm{r}} \times K$-matrix with the vectors $\mathbf{s}_k$ on the block diagonal. The columns of $\mathbf{C}$ are the effective channels without

[1] Note that the phase of these numbers is irrelevant, as information is packed into the phases of the symbols x_k.

normalization, $\mathbf{V}_k\mathbf{s}_k$, and the columns have unit norm. Now we can rewrite the precoder as

$$\mathbf{W} = \mathbf{C}(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1}\mathbf{A}, \tag{12}$$

where $\mathbf{A}$ has the diagonal entries $a_k = \tilde{a}_k||\mathbf{s}_k^{\mathrm{H}}\Sigma_k^{-1}||$ and the matrix $\mathbf{C}^{\mathrm{H}}\mathbf{C}$ represents the instantaneous correlation matrix of the effective channels of the users. With this precoder and the receiver of (9), the received signal for user k after receiver processing is

$$r_k = \frac{a_k}{||\mathbf{s}_k^{\mathrm{H}}\ \mathbf{\Sigma}_k^{-1}||}x_k + n_k, \tag{13}$$

where n_k is receiver processed noise. Due to the normalization of the combiner (9), the covariance of n_k remains σ^2. The post processing SINR for user k is

$$\gamma_k = \frac{a_k^2}{\sigma^2\mathbf{s}_k^{\mathrm{H}}\mathbf{\Sigma}_k^{-2}\mathbf{s}_k}, \tag{14}$$

which depends on the power normalization factor a_k and the vector s_k that defines the use of the eigenmodes of the users.

The vector $\mathbf{s}_k$ is needed at the reception and a dedicated pilot transmission may be arranged so that each user gets information about his pertinent $\mathbf{s}_k$. The overhead from such a pilot transmission is similar to a dedicated pilot transmission needed in the techniques proposed in [8], [9], [10].

A. Power constraint

We assume a total power constraint and normalize the precoder (12), so that

$$\mathrm{Tr}\left(\mathbf{W}^{\mathrm{H}}\mathbf{W}\right) = \mathrm{Tr}\left(\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)^{-1}\mathbf{A}^2\right) = 1. \tag{15}$$

The normalization is thus effected by the instantaneous correlation matrix $\mathbf{C}^{\mathrm{H}}\mathbf{C}$ between the effective channels of the users, and the power allocation matrix $\mathbf{A}^2$, where $\sum a_k^2\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)_{k,k}^{-1} = 1$. With equal power allocation to users we have $a_k^2 = a^2$ and

$$a^2 = \frac{1}{\mathrm{Tr}\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)^{-1}} = \frac{\det\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)}{\mathrm{Tr}[\mathrm{Adj}\left(\mathbf{C}^{\mathrm{H}}\mathbf{C}\right)]}. \tag{16}$$

It can be seen that when the effective channels of the users are orthogonal, $a^2 = \frac{1}{K}$. For two users, the instantaneous correlation matrix between the effective channels reads

$$\mathbf{C}^{\mathrm{H}}\mathbf{C} = \begin{pmatrix} 1 & \mathbf{s}_1^{\mathrm{H}}\mathbf{E}\mathbf{s}_2 \\ \mathbf{s}_2^{\mathrm{H}}\mathbf{E}^{\mathrm{H}}\mathbf{s}_1 & 1 \end{pmatrix}, \tag{17}$$

where $\mathbf{E} = \mathbf{V}_1^{\mathrm{H}}\mathbf{V}_2$ is the $N_{\mathrm{r}} \times N_{\mathrm{r}}$ inner product matrix of the N_{r} eigenvectors of the two users. Denoting the instantaneous correlation of the effective channels of the two users by $\beta = \mathbf{s}_1^{\mathrm{H}}\mathbf{E}\mathbf{s}_2$, the normalization factor becomes

$$a = \sqrt{\frac{1-|\beta|^2}{2}}. \tag{18}$$

Due to the forced orthogonalization, the transmission for one user is not steered towards the optimum single user direction for that user—it is steered towards the best possible interference free direction. The normalization loss i.e. the penalty of forced orthogonal transmission between the users depends only on the non-orthogonality β of the effective channels of the users.

B. Special cases as sub-solutions

Our general design for the precoder and combiner includes solutions where the combiner is fixed as special cases. The scheme presented in [4], where a best combination of eigenmodes of the users are iteratively selected for zero forcing transmission refers in our formulation as the case where the $\mathbf{s}_k$ are binary selection vectors. Applying binary selection vectors to (10) and (9) the effective channels become eigenmodes of the users and the receivers become eigenvector combining receiver $\mathbf{u}^{\mathrm{H}}$.

In [8], [9], [10], MRC processing is used at the receiver, and the number of users is restricted to two. For the special case of two users with $N_r = 2$ and $N_t = 2$, the generalized eigenvectors $\mathbf{f}_1$ and $\mathbf{f}_2$ of $\mathbf{R}_1$ and $\mathbf{R}_2$, where $\mathbf{R}_k = \mathbf{H}_k^{\mathrm{H}}\mathbf{H}_k$ are the $N_{\mathrm{t}} \times N_{\mathrm{t}}$ normalized instantaneous correlation matrices of the users, give the optimum precoding selection for MRC. For GE precoders the MRC receiver is

$$\mathbf{g}_{\mathrm{mrc}}^{\mathrm{H}} = \mathbf{f}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}. \tag{19}$$

If these are applied to the criteria of the zero interference transmission we get

$$\begin{aligned} \mathbf{f}_1^{\mathrm{H}}\mathbf{R}_1\mathbf{w}_2 &= 0, \\ \mathbf{f}_2^{\mathrm{H}}\mathbf{R}_2\mathbf{w}_1 &= 0. \end{aligned} \tag{20}$$

From the definition, if $\mathbf{f}_1$ and $\mathbf{f}_2$ are generalized eigenvectors of $\mathbf{R}_1$ and $\mathbf{R}_2$, then selecting $\mathbf{w}_2 = \mathbf{f}_2$ and $\mathbf{w}_1 = \mathbf{f}_1$ fulfills the requirements of (21).

The effective channel $\mathbf{h}_{\mathrm{eff},k}$ for user k assuming a GE precoder and the corresponding MRC combiner is

$$\mathbf{h}_{\mathrm{eff},k} = \mathbf{f}_k^{\mathrm{H}}\mathbf{R}_k. \tag{21}$$

By making a pseudo inverse of the resulting combined effective channel, the transmission is redirected such that no power is transmitted to the common null space. Thus the precoders are formed as

$$\mathbf{W} = \mathbf{H}_{\mathrm{eff}}^{\mathrm{H}}(\mathbf{H}_{\mathrm{eff}}\mathbf{H}_{\mathrm{eff}}^{\mathrm{H}})^{-1}, \tag{22}$$

where $\mathbf{H}_{\mathrm{eff}} = [\mathbf{h}_{\mathrm{eff},1}^{\mathrm{H}}\mathbf{h}_{\mathrm{eff},2}^{\mathrm{H}}]^{\mathrm{H}}$. In this special case, the precoders of (22) are the generalized eigenvectors $\mathbf{f}_1$ and $\mathbf{f}_2$. This coincides with the result of [9], [10] that the generalized eigenvectors give the optimum precoding selection for MRC for two users for 2×2 MIMO.

IV. Sum rate maximizing linear single stream transmission for two users

In this section, we concentrate on sum rate maximizing linear precoding for two users with single stream transmission. The sum rate to be maximized is

$$C = \sum_{k=1}^{2} C_k = \sum_{k=1}^{2} \log_2(1+\gamma_k), \tag{23}$$

where γ_k is the post processing signal-to-noise ratio (SINR) for user k. When $K = 2$, the post processing SINR for user k is

$$\gamma_k = \frac{1-|\beta|^2}{2\sigma^2\mathbf{s}_k^{\mathrm{H}}\mathbf{\Sigma}_k^{-2}\mathbf{s}_k}. \tag{24}$$

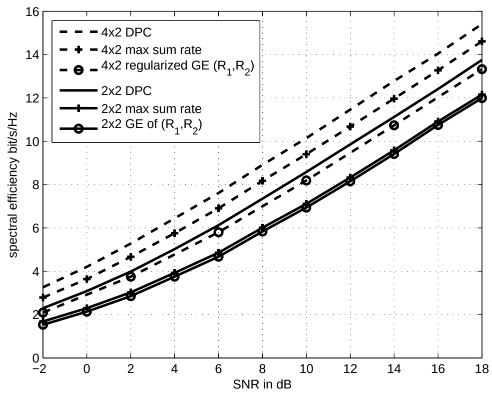

Fig. 1. Sum rate of two paired users in IID channel.

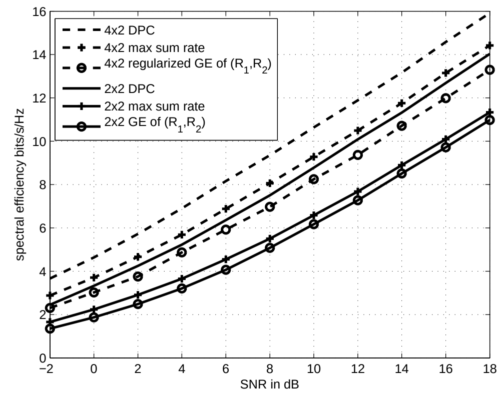

Fig. 2. Sum rate of two paired users in ULA correlated channel with spatial correlation $\rho = 0.9$.

The SINR optimization is a tradeoff between the penalty of forced orthogonalization and the effective utilization of the eigenmodes. This tradeoff is controlled by the combiners $\mathbf{s}_k$. The degrees of freedom are characterized by the set $\{s_k\}$. Each s_k is a complex unit norm N_r-dimensional vector. It can be unambiguously defined in terms of a real N_r-dimensional vector $\mathbf{b}_k$ with non-negative entries, and a vector $\mathbf{p}_k$ of phases. The $\mathbf{p}_k$ affect only the term $|\beta|^2$ in the numerator of (24). Thus the optimization problem splits into two parts; an outer and inner optimization. In the inner optimization, for any values of $\mathbf{b}_k$, optimal $\mathbf{p}_k$ exist, which minimize $|\beta|^2$. The outer optimization finds the optimal $\mathbf{b}_k$, subject to a solution of the inner problem.

For the special case of two receive antennas, we parametrize $\mathbf{b}_k = [\sin\theta_k \quad \cos\theta_k]^\mathrm{H}$ with $\theta \in [0, \pi/2]$, so that

$$\gamma_k = \frac{(1-|\beta|^2)}{2\sigma^2\left(\frac{\sin^2\theta_k}{\lambda_{1,k}} + \frac{\cos^2\theta_k}{\lambda_{2,k}}\right)}, \tag{25}$$

where $\lambda_{l,k}$ is the l:th eigenvalue of $\mathbf{H}_k$. It can be seen that the more equal the eigenvalues of the user k are, the less influence the angle θ has. The extreme is that $\lambda_{1,k} = \lambda_{2,k}$ and all θ values result in same SINR.

To get initial values for the $\mathbf{s}_k$, or $\mathbf{b}_k$ and $\mathbf{p}_k$, N random pairs of $\mathbf{s}_1$ and $\mathbf{s}_2$ are generated and the $\mathbf{b}_k$ and $\mathbf{p}_k$ are selected as starting point that result in maximum sum rate (23) out of the N samples. Then $|\beta|$ is minimized to find $\mathbf{p}_k$. With these $\mathbf{p}_k$ the sum rate in (23) is maximized to find $\mathbf{b}_k$.

V. Simulation results

In [9], the Generalized Eigenvector (GE) precoder is found to perform at least as well as the numerically optimized precoder scheme in [8] and the iterative schemes of [6], [7]. Thus here we compare our scheme to the GE solution, and to Dirty Paper Coding [15], where the interference is assumed to be pre-canceled without altering the optimum single stream transmission to each user. For rank deficient channels the GE problem is ill posed due to common null spaces of the matrices [16] and is not considered in [9]. For example, for 4×2 MIMO, the null-space of each $\mathbf{R}_k$ is 2-dimensional, and there are two GEs with ∞ eigenvalue and two with eigenvalue 0. The GEs of $(\mathbf{R}_1, \mathbf{R}_2)$ with eigenvalue ∞ are in the null-space of $\mathbf{R}_2$, and the ones with eigenvalue 0 are in the null-space of $\mathbf{R}_1$. Thus the ∞-eigenvalue vectors of $\mathbf{q}_1, \mathbf{q}_2$ can be used to transmit to user 1, and the 0-eigenvalue vectors $\mathbf{q}_3, \mathbf{q}_4$ to user 2. Any linear combination of the two vectors per user may be used, and an optimum linear combination may be sought for. In addition of a brute force search, the optimum linear combination can be found by finding regularized generalized eigenvectors, which means that $\mathbf{R}_k$ are first regularized by adding a small constant to the diagonal. Out of the regularized GEs $\mathbf{q}_{\mathrm{reg},j}, j = 1, \ldots, 4$ of $(\mathbf{R}_{\mathrm{reg},1}, \mathbf{R}_{\mathrm{reg},2})$, the best GE transmission to user 1 is $\mathbf{q}_{\mathrm{reg},1}$ and to user 2 $\mathbf{q}_{\mathrm{reg},4}$. Here, for 4-Tx, we have regularized the instantaneous correlation matrices $\mathbf{R}_k$ with $\mathrm{diag}(0.01)$.

In the simulations two users are generated with flat Rayleigh fading channels. Then precoders and receive filters according to each specific scheme are generated and the SINRs are calculated. From the SINRs the sum rates are evaluated. Simulations are link level Monte Carlo simutions where the SNR of the users are controlled by the noise level.

Figures 1 and 2 show the sum rate versus SNR for $N_\mathrm{r} = 2$ and $N_\mathrm{t} = 2, 4$ in uncorrelated and correlated flat Rayleigh fading channel with antenna correlation $\rho = 0.9$, respectively. Figure 3 compares the schemes as a function of antenna correlation ρ at SNR=10 dB. First, it can be seen that the single stream DPC performance increases for higher antenna correlation as the stronger eigenvalue of the channel increases. When comparing Figures 1 and 2, the optimized linear precoding performs better in uncorrelated channel as here the eigenmodes have more equal weights which gives more flexibility in balancing between orthogonality and power streering. Also, in uncorrelated channels the optimal solution is closer to the MRC solution than in correlated channels. It can be seen that especially in correlated channels, where the eigenvalue spread is larger, our receiver–transmitter pair outperforms the precoder optimized for MRC in 2×2 MIMO.

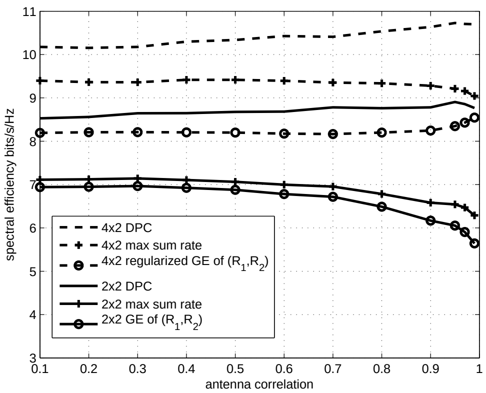

Fig. 3. Sum rate of two paired users as a funtion of ρ at SNR=10dB.

VI. Conclusion

We have formulated a general sum rate maximizing linear precoder and receiver pair for zero interference multiuser single stream transmission. The formulation is independent of the number of transmit/receive antennas and users. The analysis is simplified by first taking two users, then two receive antennas. Monte Carlo simulations are performed for two users with two receive antennas both in correlated and uncorrelated scenarios with two and four transmit antennas. The proposed maximization outperforms outperforms zero interference transmissions based on maximum ratio combining receiver, especially in correlated channels. Both the proposed scheme and those in [8], [9], [10] are subject to further optimization by water-filling power allocation.

References

[1] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO," *IEEE Trans. Sign. Proc.*, vol. 52, no. 2, pp. 461–471, Feb. 2004.

[2] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Sel. Areas Comm.*, vol. 24, no. 3, pp. 528–541, Mar. 2006.

[3] R. Chen, R. W. Heath, and J. G. Andrews, "Transmit selection diversity for unitary precoded multiuser spatial multiplexing systems with linear receivers," *IEEE Trans. Sign. Proc.*, vol. 55, no. 3, pp. 1159–1170, Mar. 2007.

[4] F. Boccardi and H. Huang, "A near optimum technique using linear precoding for the MIMO broadcast channel," in *Proc. IEEE ICC*, 2007.

[5] A. Bayesteh and A. K. Khandani, "On the user selection for MIMO broadcast channels," *IEEE Trans. Inf. Th.*, vol. 54, no. 3, pp. 1086–1107, Mar. 2008.

[6] Z. Pan, K.-K. Wong, and T.-S. Ng, "Generalized multiuser orthogonal space-division multiplexing," *IEEE Trans. Wireless Comm.*, vol. 3, no. 6, pp. 1969–1973, Nov. 2004.

[7] C.-B. Chae, J. R. W. Heath, and D. Mazzarese, "Achievable sum rate bound of zero-forcing based linear multi-user MIMO systems," in *Proc. Allerton Conf.*, Sept. 2006, pp. 1134–1140.

[8] K.-K. Wong, "Maximizing the sum-rate and minimizing the sum-power of a broadcast 2-user 2-input multiple-output antenna system using a generalized zeroforcing approach," *IEEE Trans. Wireless Comm.*, vol. 5, no. 12, pp. 3406–3412, Dec. 2006.

[9] C.-B. Chae, D. Mazzarese, N. Jindal, and J. R. W. Heath, "Coordinated beamforming with limited feedback in the MIMO broadcast channel," *IEEE J. Sel. Areas Comm.*, vol. 26, no. 8, pp. 1505—1515, Oct. 2008.

[10] C.-B. Chae and J. R. W. Heath, "On the optimality of linear multiuser MIMO beamforming for a two-user two-input multiple-output broadcast system," *IEEE Sign. Proc. Lett.*, vol. 16, no. 2, pp. 117–120, Feb. 2009.

[11] D.-S. Shiu, G. Foschini, M. Gans, and J. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Trans. Comm.*, vol. 48, no. 3, pp. 502–513, Mar. 2000.

[12] Lucent, Nokia, Siemens, Ericsson, "A standardized set of MIMO radio propagation channels," 3GPP: TSG RAN 1 contribution #23 R1-01-1179, Nov. 2001.

[13] S. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Comm. Lett.*, vol. 5, pp. 369 – 371, Oct. 2001.

[14] A. Wiesel, Y. C. Eldar, and S. Shamai, "Zero-forcing precoding and generalized inverses," *IEEE Trans. Sign. Proc.*, vol. 56, no. 9, pp. 4409–4418, Sept. 2008.

[15] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," in *Proc. IEEE ISIT*, June 2004.

[16] J. Demmel and B. Kgstrm, "The generalized schur decomposition of an arbitrary pencil $a - \lambda b$: Robust software with error bounds and applications. part i: Theory and algorithms," *AMC Trans. Math. Software*, vol. 19, no. 2, pp. 160–174, Mar. 1993.