# Outage Probability over Composite $\eta-\mu$ Fading -Shadowing Radio Channels 

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#### Abstract

We analyse the outage probability over compound $\eta-\mu$ fading - log-normal shadowing radio channels. We use the gamma distribution as a substitute to the log-normal shadowing model and obtain new finite-integral expressions for the probability density and cumulative distribution functions of the composite $\eta-\mu$ - gamma distribution. We also present approximate estimates obtained by reducing the considered problem to that over generalized K - fading.


## 1 Introduction

Recently, the $\eta-\mu$ fading distribution was proposed for modelling small-scale fading in non-line-of-sight scenarios [1]. It was shown in [1], that the $\eta-\mu$ distribution fits better to experimental data than other frequently used fading models (for example, the Nakagami- $m$ distribution), and it involves some of them as particular cases. The $\eta-\mu$ model considers the fading signal as the composition of multipath clusters where either the in-phase and quadrature Gaussian components within each cluster have different powers, or these components are correlated. It was shown in [1] that under both scenarios (formats), the probability density function (PDF) is expressed by the same formula but with different definition of the parameter $\eta$. Analysis of the outage probability over $\eta-\mu$ fading is given in [1]-[2].

In a wireless environment, large-scale fading (shadowing) is often superimposed on small-scale fading, and the log-normal distribution based on experimental data is commonly used for modelling shadowing effects [3]. The compound fadingshadowing PDF can be obtained by averaging the small-scale fading PDF conditioned on the mean signal power over the log-normal distribution [3]. Such an approach does not result, however, in closed-form PDF expressions even for the Rayleigh small-scale fading model. This fact makes any analytical evaluations over the compound fading-shadowing channels difficult. Different substitutes to the log-normal distributions have been proposed to overcome this problem. Such are approximations by the gamma and inverse Gaussian distributions [4]-[5]. These approximations lead to closed-form expressions for the composite PDF for Rayleigh and Nakagami-m small-scale fading models [4]-[7]. The compound gamma - gamma
distribution is the generalized K-distribution [6]-[9], and the gamma - inverse Gaussian distribution is the $G$-distribution [5]. The outage probability over $G$ - and K - fading is evaluated in [5] and [7]-[9], respectively. The composite Weibull-gamma distribution is introduced and analysed in [10], and a generalized fading composite model termed the extended generalized K- fading distribution is proposed and studied in [11].

The compound $\eta-\mu-$ gamma distribution is considered in [12]. But the analysis in [12] is restricted by obtaining formulas for the probability density function (PDF) only. Those are an approximate formula for arbitrary values of the fading parameters and a closed-form expression for integer values of $\mu$. Meanwhile, shadowing channels are often not ergodic because the mean signal power varies slowly. In this case, the probability of the outage is a very important characteristic of such channels. The problem of the definition of the outage probability is not addressed in [10] at all. Additionally, no numerical estimates were given to show accuracy of the applied approximation and applicability of the derived results to the lognormal shadowing model.

In this paper, in contrast to [10], we analyse the outage probability $P_{\text {out-comp }}\left(\Omega_{0}\right)$ over the compound $\eta-\mu$ fading - gamma shadowing radio channels. We obtain a new finite-integral expression for $P_{\text {out-comp }}\left(\Omega_{0}\right)$ \#for arbitrary values of the fading parameters and a novel simplified formula for integer values of the fading parameter $\mu$. The latter result corresponds to the case of an even number of the multipath clusters in the small-scale fading model [1]. Both formulas are derived on the basis of new PDF formulas of the compound $\eta-\mu$ - gamma distribution for the arbitrary fading parameters and for integer values of the fading parameter $\mu$. Additionally to the exact formulas, we present approximate estimates obtained by reducing composite $\eta-\mu-$ gamma fading to generalized $\mathrm{K}-$ fading. We present numerical estimates confirming applicability of the obtained results for analysing the outage probability over $\eta-\mu$ fading - log-normal shadowing radio channels. To the best of our knowledge, no results on $P_{\text {out-comp }}\left(\Omega_{0}\right)$ have been reported yet.

## 2 Fading and shadowing models

### 2.1 The $\eta-\mu$ fading distribution

The PDF of the $\eta-\mu$ power variable $\gamma$ with the average $E\{\gamma\}=\Omega$ is [1]:

$$
\begin{align*}
& f_{\gamma_{\eta-\mu}}(x, \Omega)=\frac{2 \sqrt{\pi} \mu^{\mu+\frac{1}{2}} h^{\mu} x^{\mu-\frac{1}{2}}}{\Gamma(\mu) H^{\mu-\frac{1}{2}} \Omega^{\mu+\frac{1}{2}}} \exp \left(-\frac{2 \mu h}{\Omega} x\right)  \tag{1}\\
& \times \mathrm{I}_{\mu-\frac{1}{2}}\left(\frac{2 \mu H}{\Omega} x\right)
\end{align*}
$$

where $2 \mu$ is the number of multipath clusters, $\mu=\frac{\Omega^{2}}{2 \operatorname{var}\{\gamma\}}\left[1+\left(\frac{H}{h}\right)^{2}\right]$ (where $\operatorname{var}\}$ denotes the variance), $\Gamma($.$) is the gamma function, and \mathrm{I}_{\alpha}($.$) is the$ modified Bessel function of the first kind of the order $\alpha$. For the format $1,0<\eta<\infty$ is the power ratio of the inphase and quadrature scattered waves in each multipath cluster; $H=\left(\eta^{-1}-\eta\right) / 4$ and $h=\left(2+\eta^{-1}+\eta\right) / 4$. For the format $2,-1<\eta<1$ is the correlation coefficient between the in-phase and quadrature scattered waves; $H=\eta /\left(1-\eta^{2}\right)$ and $h=1 /\left(1-\eta^{2}\right)$.

### 2.2 Gamma approximation to log-normal shadowing model

Under shadowing (large-scale fading), the average power $\Omega$ is a random variable that is commonly modeled by the log-normal distribution with the PDF [3]:

$$
\begin{equation*}
f_{\text {log-norm }}(\Omega)=\frac{1}{\sqrt{2 \pi} \Omega \sigma} \exp \left(-\frac{(\ln \Omega-v)^{2}}{2 \sigma^{2}}\right) \tag{2}
\end{equation*}
$$

where $v$ and $\sigma$ are the respective mean and standard shadow deviations expressed in nepers. The corresponding values in dB are: $v(\sigma)[\mathrm{dB}]$
$\approx 8.686 v(\sigma)[\mathrm{Np}]$.
The gamma distribution was proposed in [4] as an approximation to (2):

$$
\begin{equation*}
f_{\text {gamma }}(\Omega)=\frac{1}{\Gamma(M)}\left(\frac{M}{\Omega_{\mathrm{s}}}\right)^{M} \Omega^{M-1} \exp \left(-\Omega \frac{M}{\Omega_{\mathrm{s}}}\right) \tag{3}
\end{equation*}
$$

where $\Omega_{\mathrm{s}}$ is a measure of the mean power, and $M>0$ is the shape parameter of the gamma distribution. Different approaches for specifying the parameters of the approximating distribution (3) have been proposed. In our numerical evaluations, we follow
a widely used method presented in [4] that is based on an approximate equality of all raw moments of the distributions (2) and (3):

$$
\begin{equation*}
\Psi^{\prime}(M)=\sigma^{2} \text { and } \Omega_{\mathrm{s}}=M \exp (v-\Psi(M)) \tag{4}
\end{equation*}
$$

where $\Psi(x)=\frac{d \ln \Gamma(x)}{d x}$ is the digamma function [11, vol. 3, Section II. 4]. The value of the parameter $M$ is calculated numerically from the first equation in (4), and then it is used for evaluation of $\Omega_{\mathrm{s}}$ on the basis of the second equation.

## 3 Outage probability over compound $\eta-\mu$ gamma channels

### 3.1 The PDF of the composite $\eta-\mu-$ gamma distribution

The compound $\eta-\mu$ - gamma PDF is defined by averaging the distribution (1) over the gamma PDF (3), i.e.

$$
\begin{equation*}
f_{\text {comp }}(x)=\int_{0}^{\infty} f_{\gamma_{\eta-\mu}}(x, \Omega) f_{\text {gamma }}(\Omega) d \Omega \tag{5}
\end{equation*}
$$

Proposition 1: The compound $\eta-\mu-$ gamma PDF can be expressed as

$$
\begin{align*}
& f_{\text {comp }}(x)=2 \frac{\left(\alpha_{1} \alpha_{2}\right)^{\mu}}{[\Gamma(\mu)]^{2} \Gamma(M)}\left(\frac{M}{\Omega_{\mathrm{s}}}\right)^{M / 2+\mu} x^{\mu+M / 2-1} \\
& \times \int_{0}^{1} t^{\mu-1}(1-t)^{\mu-1}\left[\alpha_{2}-t\left(\alpha_{2}-\alpha_{1}\right)\right]^{M / 2-\mu} \\
& \times K_{2 \mu-M}\left(2 \sqrt{\frac{M}{\Omega_{\mathrm{s}}} x\left[\alpha_{2}-t\left(\alpha_{2}-\alpha_{1}\right)\right]}\right) d t \tag{6}
\end{align*}
$$

where $\alpha_{1}<\alpha_{2}, \quad \alpha_{1(2)}=2 \mu[h+(-) H]$, and $K_{\lambda}()$ is the modified Bessel function of the second kind of the order $\lambda$ [11, vol.2, Section 2.16].

Proof: We use a representation of the $\eta-\mu$ power variable as the sum of two gamma variables with the same shape parameter $\mu$ and different scale parameters $\quad \beta_{1}=\Omega /[2 \mu(h+H)] \quad$ and $\beta_{2}=\Omega /[2 \mu(h-H)][12]$. Then the PDF (1) can be represented as the convolution of two gamma PDFs [13]. Thus the compound PDF is:

$$
f_{\text {comp }}(x)=\frac{\left(\alpha_{1} \alpha_{2}\right)^{\mu}}{[\Gamma(\mu)]^{2} \Gamma(M)}\left(\frac{M}{\Omega_{\mathrm{s}}}\right)^{M} \int_{0}^{\infty} \int_{0}^{x} t^{\mu-1}(x-t)^{\mu-1}
$$

$\times y^{M-2 \mu-1} \exp \left\{-\left[\frac{(x-t) \alpha_{2}+t \alpha_{1}}{y}\right]\right\}$ $\times \exp \left(-y \frac{M}{\Omega_{\mathrm{s}}}\right) d t d y$.

Changing the order of integration in (7) and applying [11, vol.1, eq. (2.3.16.1)] (valid for $\alpha_{1}<\alpha_{2}$ ) we obtain (6).

Eq. (6) is given in the form of a finite-range integral. This is a proper integral that can be easily evaluated via any modern software.

### 3.2 The outage probability over $\eta-\mu-$ gamma fading

We use the derived formula (6) for evaluation of the outage probability.

Proposition 2: The outage probability $P_{\text {out-comp }}\left(\Omega_{0}\right)=\int_{0}^{\Omega_{0}} f_{\text {comp }}(x) d x$ over the compound $\eta-\mu$ fading - gamma shadowing radio channel is:
$P_{\text {out-comp }}\left(\Omega_{0}\right)=\frac{\left(\alpha_{1} \alpha_{2}\right)^{\mu}}{[\Gamma(\mu)]^{2} \Gamma(M)}$
$\times\left\{\Omega_{0}^{M} \frac{\Gamma(2 \mu-M)}{M}\left(\frac{M}{\Omega_{\mathrm{s}}}\right)^{M}\right.$
$\times \int_{0}^{1}\left[\alpha_{2}-t\left(\alpha_{2}-\alpha_{1}\right)\right]^{M-2 \mu} \cdot t^{\mu-1}(1-t)^{\mu-1}$
$\times_{1} F_{2}\left(M ; 1+M-2 \mu, M+1 ; \Omega_{0}\right.$
$\left.\times \frac{M}{\Omega_{\mathrm{s}}}\left[\alpha_{2}-t\left(\alpha_{2}-\alpha_{1}\right)\right] d t\right)$
$+\Omega_{0}^{2 \mu} \cdot \frac{\Gamma(M-2 \mu)}{\mu}\left(\frac{M}{\Omega_{\mathrm{s}}}\right)^{2 \mu}$
$\times \int_{0}^{1} t^{\mu-1}(1-t)^{\mu-1}{ }_{1} F_{2}\left(2 \mu ; 1+2 \mu-M, 2 \mu+1 ; \Omega_{0}\right.$
$\left.\times \frac{M}{\Omega_{\mathrm{s}}}\left[\alpha_{2}-t\left(\alpha_{2}-\alpha_{1}\right)\right] d t\right\}$
where ${ }_{1} F_{2}($.$) is a hypergeometric function [11, vol.$ 3, Section 7.2.3].

Proof: Eq. (8) is obtained by application of an integration formula [11, vol. 2, eq. (1.12.1.2) ] to (6)
after changing the order of integration in the double integral.

The integrals in (8) are proper too and they can be easily evaluated via any modern software.

### 3.3 The PDF of composite $\eta-\mu-$

 gamma distribution for integer values of $\mu$Simplified formulas for $f_{\text {comp }}(x)$ and $P_{\text {out-comp }}(x)$ can be obtained for integer values of the fading parameter $\mu$, i.e. for the small-scale fading model with the even number of the multipath clusters [1].

Proposition 3: For integer values of $\mu$, the PDF of the compound $\eta-\mu-$ gamma distribution can be expressed as:
-for $x>0$,

$$
\begin{align*}
& f_{\text {comp }}(x)=\frac{h^{\mu} \cdot \mu}{2^{2 \mu-3} \cdot[(\mu-1)!]^{2} H^{2 \mu-1} \cdot \Gamma(M)} \cdot\left(\frac{M}{\Omega_{\mathrm{s}}}\right)^{M} \\
& \times \sum_{k=0}^{\mu-1}\binom{\mu-1}{k}(-1)^{\mu-1-k}(2 \mu-2 k-2)! \\
& \times \sum_{m=0}^{2 \mu-2 k-2}(2 x)^{k+\frac{M+m-1}{2} \cdot \frac{(\mu H)^{2 k+m}}{m!}} \\
& \times\left[(-1)^{m}\left(\frac{\mu(h-H) \Omega_{\mathrm{s}}}{M}\right)^{\frac{M-m-1}{2}-k}\right. \\
& \times K_{M-2 k-m-1}\left(2 \sqrt{\frac{2 \mu \cdot M(h-H)_{X}}{\Omega_{\mathrm{s}}}}\right) \\
& -\left(\frac{\mu(h+H) \Omega_{\mathrm{s}}}{M}\right)^{\frac{M-m-1}{2}-k} \\
& \left.\times K_{M-2 k-m-1}\left(2 \sqrt{\frac{2 \mu \cdot M(h+H) X}{\Omega_{\mathrm{s}}}}\right)\right] \\
& f_{\text {comp }}(0)=0 \tag{9}
\end{align*}
$$

Proof: First we derive a new PDF formula for the $\eta-\mu$ distribution with integer values of $\mu$.
By using an integration formula [11, vol. 1, eq.
(2.3.5.1)], $\int_{-\left(\frac{2 \mu H}{\Omega}\right)}^{\frac{2 \mu H}{\Omega}}\left(\frac{4 \mu^{2} H^{2}}{\Omega^{2}}-t^{2}\right)^{\mu-1} \exp (-x t) d t$
$=2^{\mu-1 / 2} \sqrt{\pi} \Gamma(\mu)\left(\frac{x}{2 \mu H} \Omega\right)^{1 / 2-\mu} I_{\mu-1 / 2}\left(\frac{2 \mu H}{\Omega} x\right)$, we
express (1) in the integral form:

$$
\begin{align*}
& f_{\gamma_{\eta-\mu}}(x, \Omega)= \\
& \times \underbrace{\frac{\mu \cdot h^{\mu}}{[\Gamma(\mu)]^{2} 2^{2 \mu-2} H^{2 \mu-1} \Omega}}_{C} \exp \left(-\frac{2 \mu h}{\Omega} x\right) .  \tag{10}\\
& x_{-\frac{2 \mu H}{\Omega}}^{\int^{\mu-1}} \int_{C}^{\frac{2 \mu H}{\Omega}}\left(\frac{4 \mu^{2} H^{2}}{\Omega^{2}}-t^{2}\right)^{\mu-1} \exp (-x t) d t . \quad \text { (10) }
\end{align*}
$$

Taking into account the definition of the lower incomplete gamma-function $\quad \gamma(a, x)=\int_{0}^{x} t^{a-1}$
$\times \exp (-t) d t$ [11, vol. 1, eq. (1.3.2.3)], we obtain that the integral in (10) is:

Using the representation of $\gamma(a, x)=(a-1)!\left[1-\exp (-x) \sum_{n=0}^{a-1} \frac{x^{n}}{n!}\right]$ for integer values of $a$ [11, vol. 1, eq. (4.1.7.10)], we obtain that

$$
f_{\gamma_{\eta-\mu}}(x, \Omega)=C \sum_{k=0}^{\mu-1}\binom{\mu-1}{k}(-1)^{\mu-1-k}\left(\frac{2 \mu H}{\Omega} x\right)^{2 k}
$$

$$
\left[\gamma\left(2 \mu-2 k-1, \frac{2 \mu H}{\Omega} x\right)-\gamma\left(2 \mu-2 k-1,-\frac{2 \mu H}{\Omega} x\right)\right]
$$

$$
f_{\gamma_{\eta-\mu}}(x, \Omega)=\frac{h^{\mu} \cdot \mu}{2^{2 \mu-2} \cdot[(\mu-1)!]^{2} H^{2 \mu-1} \Omega}
$$

$$
\times \sum_{k=0}^{\mu-1}\binom{\mu-1}{k}(-1)^{\mu-1-k}\left(\frac{2 \mu H}{\Omega} x\right)^{2 k}
$$

$$
\times(2 \mu-2 k-2)!\sum_{m=0}^{2 \mu-2 k-2} \frac{1}{m!}\left(\frac{2 \mu H}{\Omega} x\right)^{m}
$$

$$
\begin{equation*}
\times\left[(-1)^{m} \exp \left(-\frac{2 \mu(h-H)}{\Omega} x\right)\right. \tag{12}
\end{equation*}
$$

Eq. (12) is a new PDF formula of the $\eta-\mu$ distribution with integer values of $\mu$.

Applying an integration formula [11, vol. 1, eq. (2.3.16. 1)] that is valid for $x>0$, we obtain the expression for $x>0$ given in (9). By direct evaluation of (5) we find that $f_{\text {comp }}(0)=0$.

### 3.4 The outage probability for integer values of $\mu$

From (9), we obtain a simplified expression for the outage probability.

Proposition 4: The outage probability for integer values of $\mu$ is:

$$
\begin{aligned}
& P_{\text {out-comp }}\left(\Omega_{0}\right)=\frac{h^{\mu}}{(2 H)^{2 \mu-1} \cdot[(\mu-1) \cdot]^{2} \cdot \Gamma(M)} \\
& \times \sum_{k=0}^{\mu-1}\binom{\mu-1}{k}(-1)^{\mu-1-k}(2 \mu-2 k-2)
\end{aligned}
$$

$$
\begin{equation*}
\times \sum_{m=0}^{2 \mu-2 k-2} \frac{H^{2 k+m}}{m!} \cdot\left[(-1)^{m} \cdot G(h-H)-G(h+H)\right] \tag{13}
\end{equation*}
$$

where

$$
\begin{align*}
& G(z)={ }_{1} F_{2}(2 k+m+1 ; 2 k+m-M+2,2 k+m+2 \\
& \left.2 k+m+1 ; 2 k+m-M+2,2 k+m+2 ; \frac{2 \mu \cdot M \cdot z}{\Omega_{s}} \Omega_{0}\right) \\
& \times \frac{\Gamma(M-2 k-m-1)}{2 k+m+1}\left(\frac{2 \mu \cdot M}{\Omega_{s}} \Omega_{0}\right)^{2 k+m+1} \\
& +\frac{\Gamma(2 k+m+1-M)}{M}\left(\frac{2 \mu \cdot M}{\Omega_{s}} \Omega_{0}\right)^{M} z^{M-2 k-m-1} \\
& \times{ }_{1} F_{2}\left(M ; M-2 k-m, M+1 ; \frac{2 \mu \cdot M \cdot z}{\Omega_{s}} \Omega_{0}\right) \tag{14}
\end{align*}
$$

Proof: Eq. (13) - (14) directly follow from the integration formula [11, vol. 2, eq. (1.12.1.2)] after some algebra.

Eq. (8) is not defined for integer values of $(2 \mu-M$ ) and (13) is not defined for integer values of the shadowing parameter $M$. But the outage probability is continuous with respect to these parameters. Thus $P_{\text {out-comp }}\left(\Omega_{0}\right)$ can be found numerically from (8) or
(13) by setting the values of the above parameters close to the integers.

### 3.5 Approximate expression for the outage probability

Along with the exact expressions for the outage probability (8) and (13), we present also approximate formulas based on reducing the compound $\eta-\mu-$ gamma fading to the generalized K-fading.

Proposition 5: The outage probability $P_{\text {out-comp }}\left(\Omega_{0}\right)$ is expressed approximately as:
$P_{\text {out-comp }}\left(\Omega_{0}\right) \approx \frac{\Gamma(M-m)}{\Gamma(M) \Gamma(1+m)}\left(\Xi \Omega_{0}\right)^{m}$
$\times_{1} F_{2}\left(m ; 1-M+m, 1+m ; \Xi \Omega_{0}\right)$
$+\frac{\Gamma(m-M)}{\Gamma(m) \Gamma(1+M)}\left(\Xi \Omega_{0}\right)^{M}$
$\times_{1} F_{2}\left(M ; 1-m+M, 1+M ; \Xi \Omega_{0}\right)$
where
$m=\left\{\frac{2 \mu+1}{2 \mu \cdot h^{\mu+2}} F_{2}\left(\mu+1.5, \mu+1 ; \mu+0.5 ;(H / h)^{2}\right)\right.$
$-1\}^{-1}$,
and ${ }_{2} F_{1}($.$) is the Gauss hypergeometric function [11,$ vo. 3, Section 7.2.1], and $\Xi=m \cdot M / \Omega_{\mathrm{s}}$.

Proof: We use the representation of the $\eta-\mu$ power variable in the form of the sum of two independent gamma variables (see the proof of Proposition 1) and then approximate the sum of the independent gamma variables by the gamma distribution [14]. In such a way we obtain that $f_{\gamma_{\eta-\mu}}(x, \Omega) \approx \frac{m^{m}}{\Gamma(m) \Omega^{m}} \cdot z^{m-1} \exp \left(-\frac{m}{\Omega} z\right) \quad, \quad$ where $m=\frac{E^{2}\{\gamma\}}{E\left\{\gamma^{2}\right\}-E^{2}\{\gamma\}}$ [14]. Using [1, eq. (21)] we obtain the formula given below (15). Thus the generalized K (gamma-gamma) - distribution can be viewed as an approximation to the compound $\eta-\mu$ fading - gamma distribution. Eq. (15) represents the outage probability over the generalized K - distribution [7, eq. (13)].

Numerical aspects of the proposed approximation are discussed in the next Section.

## 4 Numerical results

We emphasize that all the derived expressions show very good agreement with the corresponding integrals evaluated numerically for various combinations of the parameters of (1) and (3).

In Fig. 1, we present estimates of the outage probability versus the shadowing severity $\sigma$ for the format 1 of small-scale fading, and in Fig. 2, we show estimates of the outage probability against the mean SNR $\boldsymbol{V}$ for the format 2 . These results were obtained via (8) and (13) and via numerical averaging over composite $\eta-\mu-$ log-normal fading. It is seen that the presented technique provides an acceptable accuracy of the approximation in all cases considered.

We also check the accuracy of the approximate formula (15). Under all scenarios tested, we observe very slight differences between the estimates obtained on the basis of (8) and (15), which are not distinguished in Fig. 1-2. This is due to a good accuracy of the approximation of the compound $\eta-\mu-$ gamma distribution by the generalized Kdistribution. This fact is confirmed by curves presented in Fig. 3 where the PDFs of both distributions are shown for a few sets of the parameters.

## 5 Conclusion

In this paper, we analyse the outage probability over the compound $\eta-\mu$ fading - log-normal shadowing radio channels. Our approach is based on applying the gamma distribution as a substitute to the log-normal distribution. We present finite-integral expressions for the PDF and CDF of the composite $\eta-\mu-$ gamma distribution. The obtained formulas contain proper finite-range integrals that can be easily evaluated via any modern software. We also derive simplified formulas for the case of integer values of the small-scale fading parameter $\mu$. The PDF formulas obtained in this paper are given in the form of linear transforms of the products of the modified Bessel functions $K_{\lambda}()$ and power functions. Thus, many theoretical results derived for the generalized K -fading via integral transforms of the PDF of the K-distribution (e.g. those given in [6]-[9]) can be applied to performance evaluation of communication systems operating over $\eta-\mu$ fading - shadowing radio channels. As a by-product, we derive a new formula for the PDF of the $\eta-\mu$ distribution with integer values of $\mu$.

Along with the exact formulas for the outage probability, we present approximate expressions obtained by application of the generalized Kdistribution as a substitute to the composite $\eta-\mu-$ gamma distribution. Our numerical estimates confirm good accuracy of the used approximation and
applicability of all presented results to the analysis of the outage probability over the $\eta-\mu$ fading - lognormal shadowing radio channels.

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Fig. 1. Outage probability versus shadowing severity; format 1 of small-scale fading; $v=0 \mathrm{~dB}, \Omega_{0}=-7 \mathrm{~dB}$. Lines represent numerical estimates for $\eta-\mu$ - lognormal fading, and single points report analytical results (8), (13), and (15).


Fig. 2. Outage probability versus mean SNR $v$; format 2 of small scale fading; $\sigma=7.77 \mathrm{~dB}, \Omega_{0}=-7 \mathrm{~dB}$. Lines represent numerical estimates for $\eta-\mu$ - lognormal fading, and single points report analytical results (8), (13), and (15).


Fig. 3 Comparison of PDFs of $\eta-\mu-$ gamma (lines) and generalized K-distributions (circles).

