On the Time-Frequency Localisation of 5G Candidate Waveforms

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***Abstract*—Among the requirements of the 5th generation (5G) radio access technology (RAT) is that the associated waveform be well-localised in time and frequency. Localisation characteristics of waveforms are important when considering multiplexing and duplexing of transmissions. This paper proposes a measure of time-frequency localisation (TFL) for sets of orthogonal waveforms, based on Heisenberg's well-known Uncertainty Principle, allowing for the containment of OFDM and FBMC multicarrier signals over the entire bandwidth to be determined. A theoretical bound on TFL for sets of orthogonal waveforms sharper than the general uncertainty principle is provided. Several multicarrier waveforms generated using well-defined filters are shown by simulation to approach the derived bound as the number of subcarriers increases.**

I. Introduction

The selection of the radio access technology (RAT), and the associated radio waveform, is a contentious issue in the ongoing conceptualisation and eventual standardisation of the fifth generation (5G) of mobile communications technologies. The 5G RAT will play an integral role in meeting the performance targets for 5G technologies. These targets, which include data rates in excess of 10 Gbps and sub-ms latencies, as well as near-optimal performance for machine-type and device-to-device communications, will be collectively facilitated by the proliferation of small cells, frequency spectrum re-farming, and advanced technologies such as multiple-input multiple-output (MIMO) techniques, and inter-cell interference coordination. However, the optimal waveform upon which to design the 5G RAT remains a matter of some debate [1], [2], [3], [4].

The 5G RAT waveform should, ideally, have the following properties: limited generation/detection complexity, limited time/frequency overhead, good localisation in time, good spectral containment, a straightforward extension to MIMO, and a robustness to hardware impairments [4]. Candidate multicarrier waveforms for 5G are the incumbent Orthogonal Frequency Division Multiplexing (OFDM), utilised in the current generation of mobile technologies, and Filter Bank Multicarrier (FBMC). Rigorous comparisons between the two modulations can be found in recent literature, and FBMC often ousts OFDM with regard to overhead and spectral containment, at the cost of an increased implementation complexity [4], [2].

In an environment where there are simultaneous radio transmissions conducted on adjacent frequency carriers or time slots, the leakage of power and consequential interference between the different transmissions is of prime interest. Conventionally, measures of this interference include the Adjacent Channel Leakage Ratio (ACLR), Adjacent Channel Selectivity (ACS), and Inter-Symbol Interference (ISI). In addition for the transmission of one user, both ISI and Inter-Carrier Interference (ICI) are essential, and related to equalization performance. The Heisenberg parameter is a measure of time-frequency localisation (TFL) related to Heisenberg's well-known Uncertainty Principle from quantum mechanics [5], [6]. In this context, the principle states that a given waveform cannot be infinitely localised or perfectly contained in both time and frequency, which follows from Fourier transform theory. For this reason, the Heisenberg parameter is adopted in the literature on multicarrier waveform as a convenient catch-all parameter describing the overall interference leakage properties of waveforms [2], [7], [8], [9].

In discussions involving the Heisenberg parameter, the time-frequency localization of a single waveform, the prototype filter is commonly addressed [2]. This paper is based on the assumption that for Time or Frequency Division Multiplexing (TDM/FDM) of transmissions of different users, the Heisenberg parameter is a good tool to capture the amount of resources needed for one transmission, when looking at the transmission from outside. When looking at a transmission from the perspective of the intended receiver, detailed characteristics related to orthogonality, cyclicity and equalization are important, and not captured by the Heisenberg parameter.

For this reason, we consider overall localization of a transmission consisting of a number of orthogonal waveforms. For each component waveform/subcarrier, a portion of the time-frequency non-locality resides in the domains of the other waveforms and can be dismissed, as a consequence of orthogonality. We provide a TFL measure for families of orthogonal waveforms, along with a novel bound. Using this measure we show that both OFDM and FBMC waveforms are well localised as the number of subcarriers increases.

The paper is structured as follows. Section II includes a short review of multicarrier communications, time-frequency localization, and a brief overview of OFDM and FBMC. Section III follows this with the presentation of a novel measure of TFL for multicarrier waveforms. The respective localisation of OFDM and FBMC multicarrier waveforms, along with other relevant waveforms, are compared with regard to this measure in Section IV. Finally, Section V offers

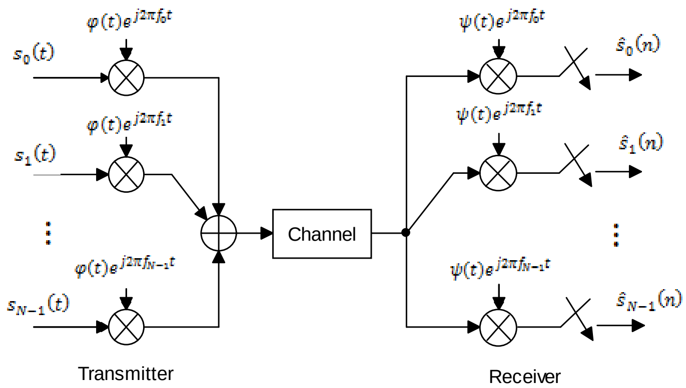

Figure 1. Block diagram of a multicarrier transceiver.

conclusions.

II. Multicarrier Communications and TFL

Multicarrier schemes allow for data symbols to be transmitted simultaneously using multiple frequency subcarriers. The block diagram in Figure 1 depicts a multicarrier transceiver, whose inputs are the data signals

$$s_k(t) = \sum_n s_k[n]\,\delta(t - nT), \tag{1}$$

where $s_k[n]$ is the real- or complex-valued data symbol located on the kth of N subcarriers at time index n, and T is the symbol duration. Each of the N input signals is subject to a corresponding transmission function, a modulation and translation of a pulse shape $\varphi(t)$, known as the synthesis filter or transmitter prototype filter. The transmission functions are given by

$$\varphi_{n,k}(t) = \varphi(t - nT)\,e^{j2\pi kFt}, \tag{2}$$

where F is the subcarrier spacing. For each n, the transmitted multicarrier signal is the sum of N time-limited and complex-valued signals, whose magnitude and phase are modulated by the symbols $s_k[n]$, and can be written as

$$x(t) = \sum_n \sum_{k=0}^{N-1} s_k[n]\,\varphi_{n,k}(t). \tag{3}$$

The effect of the channel is to modify each of the component signals by the channel gains at the respective frequencies, after the transient period of the channel impulse response [2]. The receiver estimates the transmitted symbols $\hat{s}_k[n]$ by projecting the received signal $y(t)$ onto time- and frequency-shifted versions of another pulse shape $\psi(t)$, the receiver prototype filter.

A. Heisenberg Uncertainty Parameter

In a multisymbol transmission, the distance between data symbol $s_k[n]$ and its neighbours is increased as the TFL of $\varphi_{n,k}(t)$ improves, at the cost of a reduced spectral efficiency [7]. Ideally, the waveforms $\varphi_{n,k}(t)$ would be mutually orthogonal and well-localised, and spectral efficiency would be maximised by selecting T and F such that $TF = 1$. However, the Balian-Low theorem prohibits these conditions from being satisfied simultaneously, and it becomes necessary for spectral efficiency to be sacrificed $(TF > 1)$ in order mitigate interference caused by dispersion [7], [10].

Time-frequency localisation of a filter is measured by the Heisenberg uncertainty parameter ξ. For a function $f \in L_2(\mathbb{R})$, the parameter is given by [7], [11]

$$\xi = \frac{\|f\|^2}{4\pi\sigma_t\sigma_f} \leq 1, \tag{4}$$

where

$$\sigma_t = \sqrt{\int_{\mathbb{R}} (t - \bar{t})^2 |f(t)|^2\,dt}, \tag{5}$$

is the standard deviation of the signal energy around the mean time $\bar{t}$, characterizing the time dispersion of the signal, and

$$\sigma_f = \sqrt{\int_{\mathbb{R}} (\xi - \bar{\xi})^2 \left|\hat{f}(\xi)\right|^2 d\xi}, \tag{6}$$

is the respective dispersion in frequency around $\bar{\xi}$. Here $\hat{f}$ denotes the Fourier transform of f. A filter that is well-localised has a Heisenberg parameter close to 1, and equality holds in (14) if and only if $f(t)$ is Gaussian [7], [11].

B. OFDM and FBMC

OFDM and FBMC are established multicarrier techniques vying for supremacy and inclusion in the next generation of mobile communication technologies. The schemes differ primarily in the choice of symbol duration T and the transmitter and receiver prototype filters $\varphi(t)$ and $\psi(t)$. In conventional OFDM, $\varphi(t)$ is a simple square window or rectangular pulse of height one and width T and, as a result, the filter has a sinc shape in the frequency domain. This allows for an efficient implementation based on fast Fourier transform (FFT) techniques. However, it has the consequence that the OFDM subcarriers are not well-localised in frequency From (3), the transmitted OFDM signal can be written as

$$x(t) = \sum_n \sum_{k=0}^{N-1} s_k[n]\,\mathrm{rect}(t - nT)\,e^{j2\pi kFt}, \tag{7}$$

where $s_k[n]$ is the complex data symbol (e.g. QAM) on the kth of N subcarriers at time index n.

IFFT processing at the transmitter for OFDM ensures that the frequency subcarriers are initially orthogonal (i.e., the spectral peaks of each subcarrier are located at the nulls of their neighbours). In a time-dispersive channel, orthogonality can be maintained by extending the duration T of the transmit pulse $\varphi(t)$ by a interval greater than the duration of the channel impulse response [2]. This is achieved by appending a cyclic prefix (CP) to each OFDM symbol. Including a CP ensures that the transmitted multicarrier signal appears cyclic to the receiver, which may employ efficient one-tap per-subcarrier equalisation in the frequency domain [4].

One of the major limitations of conventional OFDM is significant out-of-band (OOB) emissions caused by the relatively large side lobes of the pulse in the frequency domain; the first lobe of the sinc pulse is only 13dB below the main lobe. These detrimental emissions can by reduced by an appropriate windowing function, or filtering, typically a raised root cosine filter [12]. In any implementation of OFDM, there is a realisable windowing function capable of mitigating the sharp changes at symbol boundaries, in such a way as to reduce the power in the spectral side lobes at the cost of spreading symbols in time [2].

FBMC systems are designed to overcome some of the limitations of OFDM, at the cost of increasing implementation complexity. Rectangular windows are replaced with more advanced transmitter and receiver prototype filters, which are well localised in frequency and reduce OOB emissions [13]. Figure 2 compares the frequency responses of an OFDM and FBMC prototype filter, from which the repressed side lobes of the latter are clearly observable. Spectral efficiency is maximised for FBMC such that $T_{FFT} = T = 1/F$. However, the respective prototype filters span a period larger than the symbol duration T, typically an integer K (called the overlapping factor) multiple of T, which has the consequence that successive data symbols overlap in time. An FBMC signal can be written as

$$x(t) = \sum_n \sum_{k=0}^{N-1} j^{(n+k)} s_k[n]\, g\left(t - n\frac{T}{2}\right) e^{j2\pi kFt}, \qquad (8)$$

where $g(t)$ is a suitable prototype filter with good time and frequency properties, and $s_k[n]$ is the *real-valued* data symbol on the kth of N subcarriers at time index n. FBMC typically employs an offset QAM (OQAM) scheme, where the real and imaginary parts of each data symbol are transmitted on alternating subcarriers, resulting in a data rate being reduced by a factor of two when compared with OFDM [1]. To compensate for this rate loss, real and imaginary parts are inserted with half symbol intervals. The use of a well-localised pulse shape negates the need for a cyclic prefix to maintain orthogonality in dispersive channels. The absence of a CP precludes the use of one-tap per-subcarrier equalisation and therefore increases the computational complexity [4].

III. TFL Measure for Multicarrier Transmissions

In this section, we present a parameter to evaluate the time-frequency localisation of multicarrier signals. This measure follows from the theoretical bound on the TFL of a signal which is a linear combination of orthogonal waveforms.

A. Average TFL of Multicarrier Transmissions

We first consider a multicarrier transmission of N complex symbols $\{s_k\} \in \mathbb{C}$, written as

$$f(t) = \sum_{k=0}^{N-1} s_k g_k(t), \qquad (9)$$

where $g_k(t) \in L^2(\mathbb{R})$ are unit-norm waveforms which are not necessarily orthogonal. With linear modulation, this is a sum

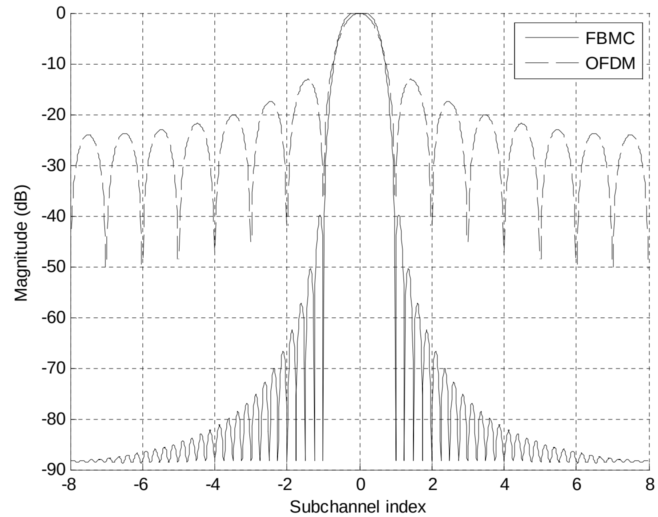

Figure 2. Frequency response of OFDM and FBMC prototype filters, with $T = 64$, $N = 16$ and overlapping factor $K = 4$.

of N random processes, where the randomness comes from the random symbol selection. We assume that the transmitted symbols are independent and identically distributed (i.i.d) with zero mean and variance $\mathrm{E}[|s_k|^2] = 1$. The Power Spectral Density (PSD) of this process can be calculated from the auto-correlation function. Using the i.i.d and zero mean assumptions on the symbols, the PSD is simply the sum of the component waveform PSDs

$$\mathrm{E}[\|f\|^2] = \sum_{k=0}^{N-1} \|g_k\|^2. \qquad (10)$$

Accordingly, the average time and frequency dispersion of f are given by

$$\tilde{\sigma}_t^2 = \mathrm{E}[\sigma_t^2] = \sum_{k=0}^{N-1} \int_{\mathbb{R}} (t - \bar{t})^2 |g_k(t)|^2 \, dt, \qquad (11)$$

$$\tilde{\sigma}_f^2 = \mathrm{E}[\sigma_f^2] = \sum_{k=0}^{N-1} \int_{\mathbb{R}} \left(\xi - \bar{\xi}\right)^2 |\hat{g_k}(\xi)|^2 \, d\xi. \qquad (12)$$

The conventional Heisenberg parameter of this stochastic process is thus

$$\tilde{\xi} = \frac{\mathrm{E}[\|f\|^2]}{4\pi\tilde{\sigma}_t\tilde{\sigma}_f} \leq 1. \qquad (13)$$

Note that for this, no assumptions on orthogonality of the component waveforms were made. The time-frequency localization $\tilde{\xi}$ of the stochastic process matches the classical Heisenberg parameter ξ of a sample realization. A multicarrier transmission has, on average, a TFL similar to considering the synthesis waveforms orthogonal.

B. TFL Measure for Sets of Orthogonal Waveforms

We now show that imposing an orthogonality constraint on the component waveforms modifies the maximum achievable

TFL (4), and by extension the statistical time-localisation (13) for multicarrier transmissions.

Proposition 1. *The Heisenberg parameter of a function $f \in L^2(\mathbb{R})$ being a linear combination of N orthogonal waveforms $g_k \in L^2(\mathbb{R})$ is upper bounded as*

$$\xi = \frac{\|f\|^2}{4\pi\sigma_t\sigma_f} \leq \frac{1}{N}, \tag{14}$$

where the time and frequency dispersions are defined as (5) and (6), respectively.

Proof: Due to a lack of space, we only give an outline of the proof. The mean-dispersion principle (sum of the time and frequency dispersions) for orthonormal sequences is lower bounded in [14]. Based on this, one can upper bound the TFL as above using time-frequency dispersion optimization. ∎

C. Normalised TFL Parameters

To compare different multicarrier transmissions, we further normalise the Heisenberg parameter by the upper bound given in Proposition 1

$$\xi_N = \frac{N\|f\|^2}{4\pi\sigma_t\sigma_f} \leq 1, \tag{15}$$

and respectively its statistical equivalent as $\tilde{\xi}_N = N\tilde{\xi} \leq 1$. This captures the intuitive notion of the time-frequency resources required by one transmitter transmitting N orthogonal waveforms, and the essential frequency occupied by such a transmission from the point of view of another transmitter. For $N = 1$, the conventional Heisenberg parameter effectively considers each transmission of a symbol as coming from a different transmitter. Due to the uncertainty relation, any waveform has an unavoidable spread in time and frequency. With multiple orthogonal waveforms, part of the unavoidable spread of one waveform is in the domain occupied by another and, as long as these two are orthogonal, this dispersion is not harmful. The proposed measure is capturing the amount of time-frequency spread of the family of orthogonal waveforms, outside of the fundamental time-frequency resources occupied by the orthogonal transmissions. Fundamentally, this measure shows in which sense N orthogonal waveforms occupy at least N units of time×frequency.

IV. Numerical Comparisons

To compare the localisation characteristics of multicarrier waveforms, we first generate prototype filters for both. It is a trivial matter to design a rectangular pulse with a given ramp up factor R to serve as the prototype filter for windowed OFDM. For FBMC, the so-called frequency sampling technique may be utilised to generate an appropriate filter waveform. We use a discrete time prototype filter of length $L = KN$, where K is the overlapping factor and N is the number of subchannels, of the form [15]

$$h(t) = 1 + 2\sum_{k=1}^{K-1} (-1)^k H(k/L)\cos(2\pi kt/L), \tag{16}$$

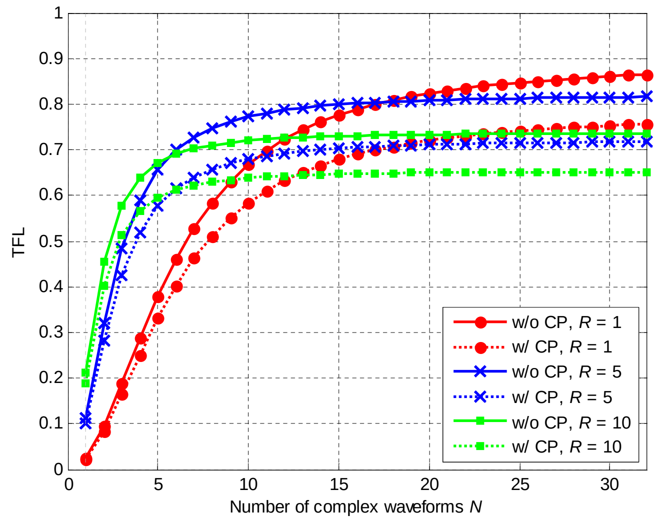

Figure 3. Time-frequency localisation $\tilde{\xi}_N$ of OFDM with and without CP of 7%, and a ramp-up/down filter of variable length.

for $1 \leq t \leq L-1$, where the samples of the filter's frequency response are

$$\begin{aligned} H(0) &= 1, \\ H(1/L) &= 0.971960, \\ H(2/L) &= 1/\sqrt{2}, \\ H(3/L) &= 0.235147, \\ H(k/L) &= 0, \quad \text{for } 4 \leq k \leq L-1. \end{aligned} \tag{17}$$

The FBMC prototype filter can be obtained by windowing the periodic function $h(t)$ appropriately.

The OFDM and FBMC multicarrier signals are then generated by translating N copies of the prototype waveforms in time and frequency. The dispersion of the multicarrier signal in both domains is determined by computing the second moment of the combined signal's energy in time and power spectral density (PSD), respectfully. Finally, the localisation of the multicarrier waveform is given by Equation (15).

Figure 3 depicts the TFL of OFDM waveforms with FFT-length 64, and a simple ramp-up, ramp-down window of 1, 5, or 10 OFDM samples. The number of waveforms used varies between 1 and 32, and a complex information symbol is assumed to be transmitted on each waveform. An OFDM transmission with and without a CP is considered. the CP overhead is assumed to be 7%, as in LTE. We clearly see how TFL improves with the number of orthogonal waveforms used. For small numbers of subcarriers, the cost of increased time dispersion from longer ramp-up/down times is overcompensated by gains in frequency dispersion. With more subcarriers, frequency localization improves automatically, and a small ramp-up/down time is optimal. We also see how the CP induces a constant cost due to increased time dispersion. Here, the ramp-up/down is taken after the addition of CP.

Figure 4 compares the TFL of OFDM and FBMC multicarrier signals as a function of the number of complex

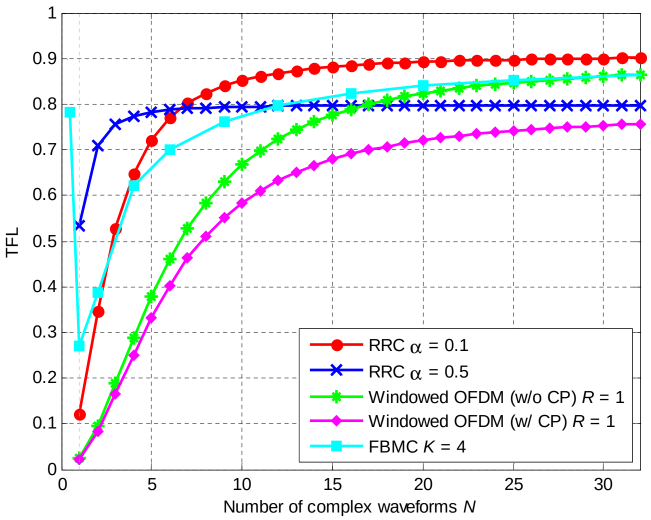

Figure 4. Time-frequency localisation of OFDM, FBMC and RRC multicarrier waveforms, as measured by the multicarrier Heisenberg Uncertainty parameter $\tilde{\xi}_N$.

baseband waveforms. The prototype filter for FBMC thus represents half a complex waveform. The TFL of the prototype filter is 0.78. Note that even though the FBMC waveforms are not orthogonal, the TFL measure(15) captures their behavior well. In addition to the multicarrier waveforms, the figure also includes families of single-carrier raised-root cosine (RRC) waveforms. RRC pulses are ideally localised in frequency, and orthogonal in time shifts of a symbol period. The roll-off factor α trades off dispersion in frequency against dispersion in time. In the plot, N time-shifted RRC pulses are considered.

It can be observed in the figure that the respective TFL of all waveforms increases with the number of orthogonal components, and tend, at varying rates, towards maximum values. This can be attributed to the fact that, as the number of component waveforms increases, the shape of the multicarrier waveforms becomes increasingly rectangular in time and frequency.

The observable upper limit corresponds to the localisation of a non-realisable waveform which is perfectly rectangular in both domains, for which we would have $\xi = 9/\pi^2 \approx 0.91$. This would asymptotically be achieved by the FBMC, the OFDM w/o CP, and RRC with infinitesimal α. Interestingly, single carrier RRC transmission result in the best localisation properties of those compared. Compared to the cost of a rectangular 7% extension by CP, the cost from 10% roll-off in RRC seems negligible. The advantage of FBMC over OFDM is also evident, though they get arbitrarily close when the number of subcarriers is sufficiently large, when the CP is not considered.

V. Conclusions

It is well understood that FBMC prototype filters are better localised than those utilised in OFDM. Even with appropriate windowing such that the waveform of the latter ramps up and down at the symbol boundaries, the waveform exhibits excessive variance in the frequency domain and consequently is poorly localised. In this paper, we have proposed a novel measure of time-frequency localisation, allowing us to show that both OFDM and FBMC multicarrier signals, and other multicarrier signals generated with a well-defined filter, become optimally-localised as the number of subcarriers increases. This can be attributed to the fact that some of the time-frequency non-locality for each orthogonal subcarrier resides in the domains of its neighbouring waveforms, and that the ratio of useful signal to out-of-band emissions increases with the number of subcarriers. The results presented lead us to conclude that, from the perspective of comparing the candidate waveforms for the 5G radio access technology, OFDM and FBMC waveforms are both equally well-localised for large numbers of subcarriers.

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