Distributed Algorithm for Downlink Resource Allocation in Multicarrier Small Cell Networks

Furqan Ahmed, Alexis A. Dowhuszko, and Olav Tirkkonen
Department of Communications and Networking (Comnet), Aalto University
P.O. Box 13000, FI-00076 Aalto, Finland
E-mail: {furqan.ahmed, alexis.dowhuszko, olav.tirkkonen}@aalto.fi

***Abstract*—In *small cell networks* (SCNs) co-channel interference is an important issue, and necessitates the use of interference mitigation strategies that allocate resources efficiently. This work discusses a distributed utility-based algorithm for downlink resource allocation (i.e., power and scheduling weights per carrier) in multicarrier SCNs. The proposed *distributed downlink resource allocation* (DDRA) algorithm aims to maximize the sum utility of the whole system. To achieve this goal, each base station (BS) selects the resource allocation strategy to maximize a surplus function comprising both, own cell utility and interference prices (that reflect the interference that is caused to neighboring cells). Two different utility functions are considered: max-rate and proportional fair-rate. For performance evaluation, a SCN deployed in a single story WINNER office building is considered. Simulation results show that the proposed algorithm is effective in enhancing not only the sum data rate of a SCN, but also the degree of fairness in resource sharing among users.**

I. Introduction

The evergrowing demand for higher data rates, lower power consumption, and better coverage imposes new challenges in the design of future mobile networks. It is well known that conventional macrocellular approach is not able to provide adequate coverage indoors, due to the high penetration losses that indoor *mobile stations* (MSs) usually experience. Deployment of low-cost and low-power *base stations* (BSs), to serve small areas in the interior of buildings, has been proposed as a solution [1], [2]. The transmission range of small cell BSs enables superior signal reception at indoor MSs. In addition, the use of small cell BSs reduces the load of the macrocellular network, thereby decreasing the *capital expenses* (CAPEXs) and *operational expenses* (OPEXs) for the network service provider. The promising aspects of low-power BSs motivate the paradigm of *small cell networks* (SCNs) for improvement of coverage and capacity in indoor environments.

Although SCNs can provide significant benefits, their use in practice comes with several challenges. A major factor that may compromise the performance of a SCN is the possibility of having severe (co-channel) interference, emanating from macrocell layer (i.e. cross-layer interference) [3] or other small cells deployed nearby (co-layer interference) [4]. Moreover, the use of closed-access modes further aggravates the interference problem, since handover operations are not allowed to those MSs that are not identified as member of the *closed subscriber group* (CSG) [5]. The cross-layer interference can be kept under control, to some extent, by orthogonalization of resources between macro- and small cell layer [6]. On the contrary, more sophisticated interference management techniques are required to alleviate the co-layer interference in SCN.

In SCNs, due to potentially limited backhaul, self-organizing interference management techniques are important. For example, SCNs are able to adapt the operating parameters (e.g., allocation of resources) to improve the situation of MSs that experience low *signal-to-interference plus noise* power ratios (SINRs). With a minimal exchange of control information among neighboring cells, the resource allocation of BSs can be adapted to mitigate the co-layer interference [7], [8].

Power allocation problems in cellular networks are usually non-convex optimization problems, as the received SINRs at the MSs are coupled. The complexity of the problem is even higher in a multicarrier case, making it difficult to find methods that converge to a global optimal solution, even in a centralized way. The problem has been addressed in the literature in the context of ad hoc networks, see [9]–[14]. In most of these works, the system model is composed of multiple transmitter-receiver pairs randomly placed in a certain area, and the aim is to distributively allocate the total transmit power (of each node) over the available channels. For example, the authors of [9], [10] develop a distributed algorithm for a multi-channel ad hoc network, based on finding a (local optimal) power profile solution that satisfies the *Karush-Kuhn-Tucker* (KKT) conditions of the problem. A different approach is proposed in [11], where a simulated annealing-based method is used for power allocation in a single-channel network. In general, the non-convex resource allocation problem can also be solved using the decomposition-based methods that were discussed in [12], [13] for distributed optimization of coupled systems.

The downlink resource allocation problem for a cellular network has been addressed in a number of works, such as [15]–[17] and references therein. These works focus on intra-cell resource allocation, aiming to optimize the power levels and the allocation of sub-carriers (for the OFDM case) to serve the users within a single cell. In a multicellular context, autonomous carrier selection has been discussed in e.g. [7]. There, a carrier is either used or not, and there is no joint power constraint on the carriers.

Here we consider both inter- and intra-cell resource allocation in a cellular network, with a joint power constraint over multiple carriers. We and propose a joint optimization framework, optimizing power and scheduling weights per carrier, to improve the downlink system utility. Both maximum sum rate and proportionally fair utility functions are considered. We present a *distributed downlink resource allocation* (DDRA) algorithm that can be used to mitigate the co-layer interference in SCNs. The approach extends the work presented in [9], [10] (originally for ad hoc networks) to the downlink of a cellular systems scenario. The proposed DDRA algorithm gives a solution of the underlying resource allocation problem in a distributed way, taking into account the interference pricing information that the neighboring BSs report. Interference pricing information represents low-rate control signaling that is exchanged among neighboring cells, and reflects the way in which power allocation decisions in certain BS affects the local utility of its neighboring BSs.

The rest of paper is organized as follows: Section II discusses the system model and presents the utility functions considered for resource allocation. Section III introduces the interference pricing concept and explains the DDRA algorithm for a multicarrier cellular system in detail. The convergence proof for the DDRA algorithm when using small adaptation steps per iteration is presented in this section as well. Section IV shows the simulation results for a single story WINNER office building and analyzes its performance. Finally, conclusions are drawn in Section V.

II. System Model

We consider a cellular system comprising of BSs deployed in a small cell environment (following a pre-defined pattern), with indices $\mathcal{I} = \{1, \ldots, i, \ldots, I\}$. The total bandwidth is divided into K equal-size carriers with indices $\mathcal{K} = \{1, \ldots, k, \ldots, K\}$. It is assumed that the maximum *transmit* (Tx) power $P_{\max}$ of all BSs is identical, and can be distributed over all carriers such that

$$\sum_{k \in \mathcal{K}} p_i^k = P_{\max} \qquad i \in \mathcal{I} \tag{1}$$

is verified, where p_i^k is the power that BS i uses on carrier k. During the resource allocation procedure, a BS allocates its Tx power (across the multiple carriers) taking into account the effect of its strategy on the local utility of the neighboring cells. In this paper, neighboring cells are all those cells that experience a *significant* impairment/improvement in their own utility, whenever the original cell changes its power profile across the different carriers [1]. Without loss of generality, we assume that the set of neighboring cells for BS i is denoted as $\mathcal{J}_i$ (with $\mathcal{J}_i \subseteq \mathcal{I}$).

The intra-cell scheduling decisions per carrier $k \in \mathcal{K}$, are reflected in the selection of the scheduling weights w_l^k that BS i performs for each associated user in set $\mathcal{L}_i \subseteq \mathcal{L}$. Note that $\{\mathcal{L}_i : i \in \mathcal{I}\}$ represents a partition of set $\mathcal{L} = \{1, \ldots, L\}$, which contains the (global) indices of all MSs in the system. To simplify the analysis, we consider that carriers are infinitely divisible and shared orthogonally among the users of each cell. In addition, we assume that each BS distributes its resources of $\mathcal{K}$ carriers among its active users, verifying

$$\sum_{l \in \mathcal{L}_i} w_l^k = 1 \qquad i \in \mathcal{I}, k \in \mathcal{K}, \tag{2}$$

at each iteration of the resource allocation algorithm. The analysis carried out in this paper focuses on the normalized data rate per user (i.e., spectral efficiency in [bps/Hz]). Nevertheless, results can be easily extended to actual data rates scaling them by the corresponding bandwidth of the system.

The aim is to maximize the sum-utility of the cellular system in the downlink, i.e.,

$$U_{\text{sum}}(\mathbf{P}, \mathbf{W}) = \sum_{i \in \mathcal{I}} U_i(\mathbf{P}, \mathbf{W}), \tag{3}$$

where $\mathbf{P} \in \mathbb{R}^{I \times K}$ and $\mathbf{W} \in \mathbb{R}^{L \times K}$ contain the power levels and scheduling weights that are applied in transmission by all BSs in the system, respectively, while

$$U_i(\mathbf{P}, \mathbf{W}) = \sum_{l \in \mathcal{L}_i} u_l(\mathbf{P}, \mathbf{W}) \tag{4}$$

represents the sum-utility for the users served by BS i.

In the system model, the SINR that MS l (served by BS i) experiences on channel k is given by

$$\gamma_l^k(\mathbf{P}) = \frac{p_i^k \, h_{i,l}^k}{I_l^k + N_0} \qquad l \in \mathcal{L}_i, k \in \mathcal{K}, \tag{5}$$

where $h_{i,l}^k$ is the channel gain between BS i and MS l on carrier k, N_0 is the (flat) backgnoise power, and

$$I_l^k = \sum_{j \neq i} p_j^k \, h_{j,l}^k \qquad l \in \mathcal{L}_i, k \in \mathcal{K}, \tag{6}$$

denotes the interference level of user l on carrier k.

The utility function for the users should be selected according to the performance metric to be maximized. Two different types of utility functions are considered in this paper: *max-rate* utility and *proportional fair-rate* utility [18], [19].

The max-rate utility function seeks the maximization of the sum rate of each individual cell, leaving aside fairness issues on the individual rates that each user perceives. The max-rate utility function for user l (served by BS i) is then given by

$$u_l(\mathbf{P}, \mathbf{W}_i) = \sum_{k \in \mathcal{K}} w_l^k \log_2 \left[1 + \gamma_l^k(\mathbf{P})\right] \quad l \in \mathcal{L}_i, \tag{7}$$

where $w_l^k \in \mathbf{W}_i$ is the scheduling weight that BS i allocates to user l on carrier k. It should be noted that all the interference is caused by the neighboring cells as users within cell are sharing resources orthogonally.

As discussed in references like [20], [21], the proportional fair-rate utility function can be employed to enhance the fairness in sharing common resources (i.e., power and scheduling weights per carrier) among the active users of each cell. To

[1] In a practical system implementation, a generic MS may identify the IDs of neighboring cells measuring the received strength of pilot signals and decoding the broadcast messages of the surrounding BSs.

achieve this goal, the proportional fair-rate utility function can be formalized as

$$u_l\left(\mathbf{P},\mathbf{W}_i\right)=\log_e\left\{\sum_{k\in\mathcal{K}} w_l^k\,\log_2\left[1+\gamma_l^k(\mathbf{P})\right]\right\}\quad l\in\mathcal{L}_i. \quad (8)$$

Note that the $\log_e\{\cdot\}$ function that appears in (8) allows to improve the situation of those users that are experiencing low data rates (due to high co-layer interference situations in all channels). Nevertheless, this logarithmic function makes the proportional fair-rate utility a non-separable one, increasing the complexity of DDRA.

III. Utility-Based Distributed Resource Allocation

In this section, we discuss the concept of interference pricing and derive the pricing equations for both utility functions.

A. Interference Pricing Information

The essential observation when designing an interference coordination algorithm for a downlink cellular system is that, in each cell, co-channel interference originated in neighboring cells is experienced by all active users. In addition, it should be noted that a change in the scheduling weights $\mathbf{W}_i$ does not affect the interference caused to the users in the neighboring cells $\mathcal{J}_i$, but may change the price of interference that BS i reports to the neighbors. Here, we design a distributed scheduling and power control algorithm, which attempts to maximize the total utility of network for the utility functions presented in Section II.

The centralized optimization problem is given by

$$\begin{array}{ll}\underset{\mathbf{P},\mathbf{W}}{\text{maximize}} & \sum_{i\in\mathcal{I}} U_i\left(\mathbf{P},\mathbf{W}_i\right)\\ \text{subject to} & \sum_{k\in\mathcal{K}} p_i^k \le P_{\max},\quad p_i^k\ge 0,\\ & \sum_{l\in\mathcal{L}_i} w_l^k = 1,\qquad w_{i,l}^k\ge 0,\end{array}\quad (9)$$

where the sum utility for BS i is given by the sum of the utilities of the users that belong to $\mathcal{L}_i$, see (4). Note that this problem is in general non-concave, even in presence of concave utility functions. A local optimal solution could be found using non-convex optimization algorithms. However, such an approach is not appropriate for our system model since the optimization needs to be carried out in a distributed way.

Any local optimum of optimization problem (9) must satisfy the KKT conditions of the problem. Then, following the analysis presented in [9], [10], it is possible to see that the KKT conditions of the centralized problem (9) are equivalent to the KKT conditions of the distributed problem

$$\begin{array}{ll}\underset{\mathbf{p}_i,\mathbf{W}_i}{\text{maximize}} & U_i\left(\mathbf{p}_i,\mathbf{W}_i|\mathbf{P}_{-i}\right)\\ & -\sum_{k\in\mathcal{K}} p_i^k\left\{\sum_{j\in\mathcal{J}_i}\pi_{j,i}^k(\mathbf{P},\mathbf{W}_j)\right\}\\ \text{subject to} & \sum_{k\in\mathcal{K}} p_i^k\le P_{\max},\quad p_i^k\ge 0,\\ & \sum_{l\in\mathcal{L}_i} w_{i,l}^k = 1,\qquad w_{i,l}^k\ge 0,\end{array}\quad (10)$$

where $\mathbf{P}_{-i}$ denote the fixed power profiles for all users rather than i, and

$$\pi_{j,i}^k(\mathbf{P},\mathbf{W}_j) = -\frac{\partial U_j(\mathbf{P},\mathbf{W}_j)}{\partial p_i^k}\qquad j\in\mathcal{J}_i, k\in\mathcal{K}, \quad (11)$$

is the interference price that neighboring BS j reports to BS i. This interference price can be interpreted as a marginal cost that BS i needs to afford per unit of power it uses on the k-th carrier (due to the additional co-layer interference that is generating in the network). It is important to note that the objective in the distributed optimization problem (10) is still non-concave, and therefore may have multiple local optima that satisfy the KKT conditions.

The procedure can be interpreted as follows: The original problem (9) is decomposed in I sub-problems that are solved locally by each BS. During each iteration of the distributed algorithm, only user i adjusts its power profile $\mathbf{p}_i$ and scheduling weights $\mathbf{W}_i$ (taking into account the interference prices of neighbors). As expected, this procedure modify the objective functions of the neighboring BSs, that are going to be updated according to the new interference prices that BS i reports after its power and scheduling weight update.

In case of max-rate utilities, the price of interference that BS i reports to BS j on carrier k is

$$\pi_{i,j}^k\left(\mathbf{p}^k,\mathbf{W}_i^k\right)=\sum_{l\in\mathcal{L}_i}\frac{w_{l,k}\,p_i^k\,h_{i,l}^k\,h_{j,l}^k}{\left[1+\gamma_l^k(\mathbf{p}^k)\right]\left[I_l^k(\mathbf{p}_{-i}^k)+N_0\right]^2}, \quad (12)$$

where $\mathbf{p}^k$ denote the power profile of all BSs on channel k.

On the other hand, for the proportional fair-rate utility, the price of interference that BS i reports to BS j on carrier k can be expressed as

$$\pi_{i,j}^k(\mathbf{P},\mathbf{W}_i)=\sum_{l\in\mathcal{L}_i}\frac{w_{l,k}\,p_i^k\,h_{i,l}^k\,h_{j,l}^k}{u_l(\mathbf{P},\mathbf{W}_i)\left[1+\gamma_l^k(\mathbf{p}^k)\right]\left[I_l^k(\mathbf{p}_{-i}^k)+N_0\right]^2}. \quad (13)$$

It is important to note that, according to the previous equations, the sum-rate utility attains the form of a separable utility function, while the proportional fair-rate utility represents a non-separable utility case.

B. Interference Pricing-Based Algorithm

Our approach is an extension of [9], where each BS only maximized a surplus function over its power profile (followed by the calculation and announcement of new interference prices). In DDRA, however, the surplus is maximized over both powers and scheduling weights. After the update of powers and scheduling weights, separate prices are calculated and communicated to neighboring BSs. The two main steps of power update and weight update can be expressed as follows:

1) Power update part of DDRA algorithm: In the first part, a randomly selected BS i determines its best response according to

$$\begin{array}{ll}\widetilde{\mathbf{p}}_i(n+1) = \underset{\mathbf{p}_i}{\arg\max} & s_i\left[\mathbf{p}_i|\mathbf{P}_{-i}(n),\mathbf{W}(n)\right]\\ \text{subject to} & \sum_{k\in\mathcal{K}} p_i^k\le P_{\max},\quad p_i^k\ge 0,\end{array}\quad (14)$$

where

$$s_i\left[\mathbf{p}_i|\mathbf{P}_{-i}(n),\mathbf{W}(n)\right] = U_i\left[\mathbf{p}_i|\mathbf{P}_{-i}(n),\mathbf{W}_i(n)\right] - \sum_{k\in\mathcal{K}} p_i^k \Big\{ \sum_{j\,\in\,\mathcal{J}_i} \pi_{j,i}^k(\mathbf{P}(n),\mathbf{W}_j(n))\Big\}. \quad (15)$$

Then, each BS chooses its new power profile by forming a convex combination between its best response $\widetilde{\mathbf{p}}_i(n+1)$ and the current power profile, i.e.,

$$\mathbf{p}_i(n+1) = (1-\alpha_i)\,\mathbf{p}_i(n) + \alpha_i\,\widetilde{\mathbf{p}}_i(n+1), \quad (16)$$

where $\alpha_i \in [0,1]$ is a fixed parameter that represents the step-size of the power update part of our DDRA algorithm.

2) Scheduling weight update part of DDRA algorithm: In the second part, the randomly selected BS i of the previous part determines its best response according to

$$\widetilde{\mathbf{W}}_i(n+1) = \underset{\mathbf{W}_i}{\arg\max} \; s_i\left[\mathbf{W}_i|\mathbf{p}_i(n+1),\mathbf{P}_{-i}(n)\right] \quad \text{subject to} \quad \textstyle\sum_{l\,\in\,\mathcal{L}_i} w_l^k = 1, \quad w_l^k \geq 0, \quad (17)$$

where

$$s_i\left[\mathbf{W}_i|\mathbf{p}_i(n+1),\mathbf{P}_{-i}(n)\right] = U_i\left[\mathbf{W}_i|\mathbf{p}_i(n+1),\mathbf{P}_{-i}(n)\right]. \quad (18)$$

Note that in the latter expression, the prices of interference from neighboring users do not affect the objective function. As in the previous case, the scheduling weights are determined by taking a convex combination of the best response and the current values, according to

$$\mathbf{W}_i(n+1) = (1-\beta_i)\,\mathbf{W}_i(n) + \beta_i\,\widetilde{\mathbf{W}}_i(n+1), \quad (19)$$

where $\beta_i \in [0,1]$ is represents the step-size of the scheduling weight update part of our DDRA algorithm.

The proper selection of the step sizes α_i and β_i allow us to control the speed of convergence of the algorithm. An optimal value for the step sizes should be determined in practice, to ensure convergence of DDRA algorithm. The complete DDRA algorithm is summarized as Algorithm 1.

C. Convergence Analysis

In this section we show that for small enough values of step-size α_i (and β_i), the DDRA algorithm converges monotonically to a fixed point. The convergence proof for the DDRA algorithm consists of two parts: power update part and the scheduling weight update part. Since the convergence analysis is similar in both situations, in this section we focus on showing convergence for the former one.

Let us first define $U_{\text{sum}}^{(\text{p})}(n)$ as the sum-utility of the cellular system at the n-th iteration after power update has been performed. Based on the algorithm presented in Section III-B, it is possible to see that

$$U_{\text{sum}}^{(\text{p})}(n+1) = \max_{\mathbf{p}_i} \textstyle\sum_{l\,\in\,\mathcal{L}_i} u_l\left[\mathbf{p}_i|\mathbf{P}_{-i}(n),\mathbf{W}_i(n)\right] + \sum_{j\neq i}\Big\{\sum_{l\in\mathcal{L}_j} u_l\left[\mathbf{p}_i|\mathbf{P}_{-i}(n),\mathbf{W}_j(n)\right]\Big\}$$
$$s.t. \quad \textstyle\sum_{k\,\in\,\mathcal{K}} p_i^k \leq P_{\max},\, p_i^k \geq 0 \quad \forall\, k. \quad (20)$$

Algorithm 1 Distributed Downlink Resource Allocation

1: Each BS $i \in \mathcal{I}$ chooses the initial power profile $\mathbf{p}_i(0)$ and scheduling weights $\mathbf{W}_i(0)$. Then, it calculates the prices $\pi_{i,j}^k(0)$ that should be reported to neighbors $j \in \mathcal{J}_i$

2: At iteration n, only BS i updates its power profile and scheduling weights according to

$$\begin{aligned}\mathbf{p}_i(n+1) &\leftarrow (1-\alpha_i)\mathbf{p}_i(n) \\ &+ \alpha_i \arg\max_{\mathbf{p}_i} s_i(\mathbf{p_i}|\mathbf{P_{-i}(n)},\mathbf{W(n)}) \\ \mathbf{W}_i(n+1) &\leftarrow (1-\beta_i)\mathbf{W}_i(n) \\ &+ \beta_i \arg\max_{\mathbf{W}_i} s_i(\mathbf{W_i}|\mathbf{p_i(n+1)},\mathbf{P_{-i}(n)})\end{aligned}$$

$\mathbf{p}_{-i}(n+1) \leftarrow \mathbf{p}_{-i}(n)$
$\mathbf{W}_{-i}(n+1) \leftarrow \mathbf{W}_{-i}(n)$

3: BS i updates and communicates its new interference prices $\pi_{i,j}^k(n+1) \leftarrow -\frac{\partial}{\partial p_j^k}\left\{U_i\left[\mathbf{P}(n+1),\mathbf{W}_i(n+1)\right]\right\}$

4: Go to 2, and repeat until convergence is achieved

Let us assume that the step-size α_i is set to a very low value. In this situation, the linearized version of the utility function

$$\begin{aligned}\widetilde{u}_l\left[\mathbf{p}_i|\mathbf{P}_{-i}(n),\mathbf{W}_i(n)\right] &= u_l\left[\mathbf{P}(n),\mathbf{W}_i(n)\right] \\ &+ \left[\mathbf{p}_i-\mathbf{p}_i(n)\right]^T \nabla_{\mathbf{p}_i}\left\{u_l\left[\mathbf{P}(n),\mathbf{W}(n)\right]\right\} \\ &= u_l\left[\mathbf{P}(n),\mathbf{W}_i(n)\right] \\ &+ \sum_{k\in\mathcal{K}}\left[p_i^k-p_i^k(n)\right]\frac{\partial}{\partial p_i^k}\left\{u_l\left[\mathbf{P}(n),\mathbf{W}_i(n)\right]\right\}, \quad (21)\end{aligned}$$

around the current power profile $\mathbf{p}_i(n)$ becomes accurate enough, where $\nabla_{\mathbf{p}_i}\{u_l\left[\mathbf{P}(n),\mathbf{W}(n)\right]\}$ is the gradient of utility u_l at iteration n with respect to power profile $\mathbf{p}_i$. Then, combining (20) with (21), we have that

$$U_{\text{sum}}^{(\text{p})}(n+1) = U_{\text{sum}}^{(\text{p})}(n) + \Delta_{U_{\text{sum}}^{(\text{p})}}(n+1), \quad (22)$$

where

$$\Delta_{U_{\text{sum}}^{(\text{p})}}(n+1) = \max_{\mathbf{p}_i} \textstyle\sum_{k\,\in\,\mathcal{K}}\left[p_i^k-p_i^k(n)\right]\left\{-\pi_{ii}^k(n)\right\} + \sum_{k\in\mathcal{K}}\sum_{j\neq i}\left[p_i^k-p_i^k(n)\right]\left\{-\pi_{ji}^k(n)\right\}$$
$$\text{s.t.} \quad \textstyle\sum_{k\,\in\,\mathcal{K}} p_i^k \leq P_{\max},\, p_i^k \geq 0 \quad \forall\, k, \quad (23)$$

with

$$\pi_{ii}^k(n) = \sum_{l\in\mathcal{L}_i} -\frac{\partial}{\partial p_i^k}\left\{u_l\left[\mathbf{P}(n),\mathbf{W}_i(n)\right]\right\} \leq 0, \quad (24)$$

$$\pi_{ji}^k(n) = \sum_{l\in\mathcal{L}_j} -\frac{\partial}{\partial p_i^k}\left\{u_l\left[\mathbf{P}(n),\mathbf{W}_j(n)\right]\right\} \geq 0. \quad (25)$$

As expected, objective function (23) equals 0 when $\mathbf{p}_i = \mathbf{p}_i(n)$. Then, it is straightforward to conclude that $\Delta_{U_{\text{sum}}^{(\text{p})}}(n+1) \geq 0$ for all n, and that the sum-utility function $U_{\text{sum}}^{(\text{p})}(n+1)$ will be monotonically increasing if the power update is done with small step-sizes (i.e., when $\alpha_i \to 0^+$).

We note that the convergence proof for the scheduling weight update part of the DDRA algorithm (when step-sizes $\beta_i \to 0^+$) can be derived following the same steps.

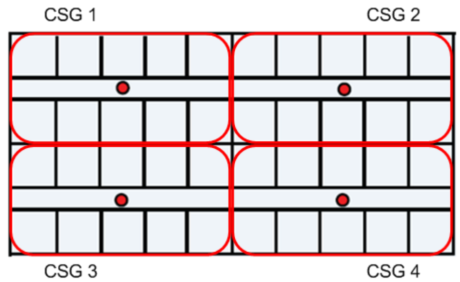

Fig. 1. Layout for the single-story WINNER office building that is used in this paper. The dots in the figure represent small-cell BSs, and the (distance dependent) range of each closed-access group is demarcated using red lines.

IV. Numerical Results

In this section we analyze the performance of DDRA algorithm for both, max-rate and proportional fair-rate utility functions. We first present the simulation scenario, followed by the analysis of results.

A. Simulation Scenario

We consider a small-cell network comprising of 4 BSs with 2 users each, deployed in a single story building with WINNER office characteristics. The total bandwidth of the system is equally divided into 5 carriers. For sake of simplicity, we assume that the coherence bandwidth of the wireless channel is larger than the bandwidth of each carrier. In addition, the frequency selectivity across carriers is modeled as an exponentially distributed random variable with unitary mean. Note that this approach is reasonable, e.g., when we consider that the total available bandwidth is composed by non-contiguous carriers. We assume that each BS serves it users in a CSG configuration, such that we may have visiting MSs within the coverage region that are not allowed to perform a handover operation to the corresponding BS (due to they are not identified as members of the CSG). The probability of having a visiting MS within the coverage region is set to 20 %. The BS locations and CSG coverage areas are illustrated in Fig. 1.

The propagation characteristics inside the buildings are modeled according to Winner A1 office model; see [22] for more details. The distance dependent path loss attenuation is calculated according to the following formula:

$$\text{PL [dB]} = A\,\log_{10}(d) + B + C\log_{10}\left(\frac{f_c}{5}\right), +X \qquad (26)$$

where d [m] is the distance between the transmitter and receiver, f_c [GHz] is the carrier frequency of the system in, X [dB] identifies the (discrete) loss that is produced by walls and windows (see Table I for more details). The modeled small cell system is a three-dimensional one, with wrap-around boundary conditions in all directions. The shadow fading effect is also considered in the model. Details of system parameters are given in Table II.

TABLE I
WINNER 2 Path Loss Model

Building dimensions	100 [m] × 50 [m]
Room dimensions	10 [m] × 10 [m]
Corridor width	5 [m]
Room height	3 [m]
BS antenna height	2 [m]
MS antenna height	1 [m]
Number of floors	1
Antenna patterns	omni directional
Carrier frequency	2.6 [GHz]
Line-of-sight	in same room/corridor
Path loss coefficients	$A = 18.7$, $B = 46.8$, $C = 20$
Inner wall loss	5 [dB] per wall

TABLE II
Simulation Parameters

Number of BSs	4
Number of UEs	8
Number of Carriers	5
Bandwidth	1.25 [MHz]
Bandwidth Efficiency	0.85
Maximum Transmit Power	20 [dBm]
Noise Figure	9 [dB]
Thermal Noise	−174 [dBm/Hz]
Shadow Fading Correlation	0.5
Shadow Fading Standard Deviation	3 [dB]

B. Performance Analysis

The results are presented for 1000 random network instantiations, generated according to the aforementioned parameters. The data rates that are experienced by individual users are collected and used to plot the rate *cumulative distribution functions* (CDFs) for max-rate and proportional fair-rate utility functions. Four cases are analyzed:

1) Fixed power allocation (i.e., equal power in all carriers) with fixed scheduling (i.e., identical scheduling weights),
2) Adaptive power allocation (i.e., power control part of DDRA algorithm) with fixed scheduling,
3) Fixed power allocation with adaptive scheduling (i.e., scheduling control part of DDRA algorithm), and
4) Adaptive power allocation and adaptive scheduling (i.e. both, power control part and scheduling control part of DDRA algorithm).

Figure 2 shows the performance curves when the different combinations of DDRA algorithm are applied with max-rate utility functions. As expected, the best mean rate performance is obtained when all BSs in the system apply both, power control part and scheduling control part of DDRA algorithm (i.e., A-PA + A-SCH case). Note that in terms of mean rate, the implementation of only one of those features (i.e., A-PA + F-SCH and F-PA + A-SCH cases) provides almost the same performance. Nevertheless, it is important to observe that the use of adaptive scheduling with max-rate utility functions increases the probability of having users equipments in outage, but improves considerably the data rate of those users that are receiving resources effectively in the system (i.e., users that are not in outage). Note that the use of power control in presence of fixed scheduling provides an improvement of 30 % in the

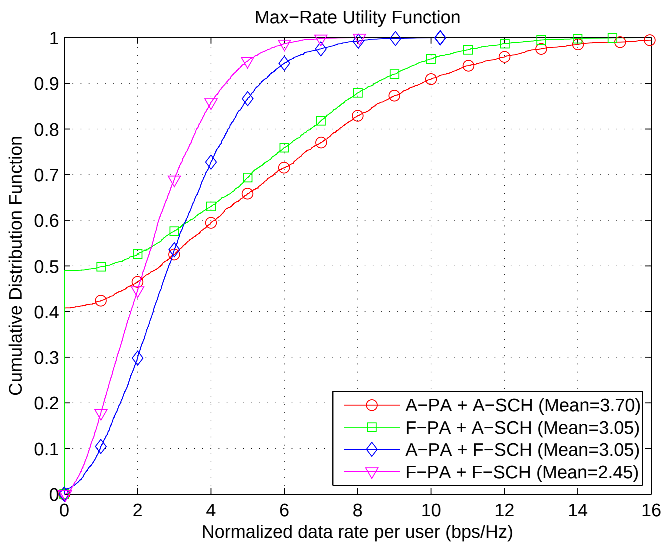

Fig. 2. Cumulative distribution function for normalized data rates of users when different versions of DDRA algorithm are used with max-rate utility functions. A/F-PA: Adaptive/Fixed Power Allocation. A/F-SCH: Adaptive/Fixed Scheduling (full carrier allocation).

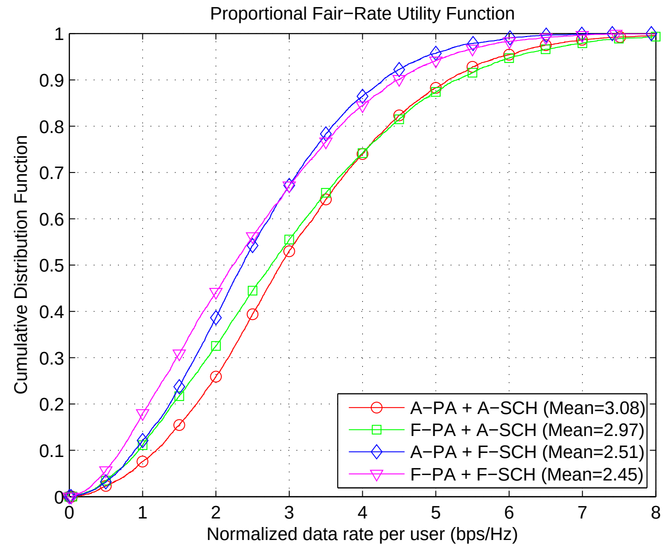

Fig. 3. Cumulative distribution function for normalizeddata rates of users when different versions of DDRA algorithm are used with proportional fair-rate utility functions. A/F-PA: Adaptive/Fixed Power Allocation. A/F-SCH: Adaptive/Fixed Scheduling (full carrier allocation).

5 %-tile outage capacity of the system (i.e., A-PA + F-SCH versus F-PA + F-SCH).

Figure 3 shows the performance curves when the different combinations of DDRA algorithm are applied with proportional fair-rate utility functions. Again, the use of both control features of DDRA algorithm (i.e., A-PA + A-SCH) provides the best performance in terms of mean data rate and outage capacity, followed by the scheme that implements the adaptive scheduling part of DDRA algorithm with fixed power allocation in all carriers (i.e., F-PA + A-SCH). The scheme that implements the power control part of DDRA algorithm with fixed scheduling weights lies in the third position (i.e., A-PA + F-SCH), and the fourth place corresponds to the scheme that do not perform resource allocation at all (i.e., F-PA + F-SCH). Since in this case we are using proportional fair-rate utility functions, there are no users in outage in any case. Note that the use of power control in presence of fixed scheduling does not provide a significant improvement in the mean data rate, but results in a gain of 40 % in the 5 %-tile outage capacity of the system (i.e., A-PA + F-SCH versus F-PA + F-SCH). On the other hand, the use of power control in presence of adaptive scheduling increases slightly the mean data rate of the system, but improving considerably the situation of those users that experience the lower data rates (e.g., the 5 %-tile outage capacity improves around 30 % when comparing A-PA + A-SCH with F-PA + A-SCH).

V. Conclusion

We have proposed a utility-based distributed algorithm to allocate resources (i.e., transmit power and spectral bandwidth) in the downlink of a multicarrier small cell networks (SCNs), and proved its convergence for small adaptation steps. Different utility functions and resource allocation strategies are considered for performance evaluation under a practical SCN scenario. The simulation results show that with an appropriate utility function, the proposed algorithm can allocate resources efficiently to improve the sum data rate as well as fairness among users. It is important to highlight that both, transmit power and spectral bandwidth should be assigned jointly to maximize the utility function of the system in a proper way.

Acknowledgment

This work was prepared in NETS2020 project framework and supported in part by Nokia Siemens Networks, Ericsson Finland, Nokia, Renesas Mobile, Elektrobit and Nethawk.

References

[1] D. Knisely, T. Yoshizawa, and F. Favichia, “Standardization of femtocells in 3GPP,” *IEEE Commun. Mag.*, vol. 47, no. 9, pp. 68–75, Sep. 2009.

[2] D. Knisely and F. Favichia, “Standardization of femtocells in 3GPP2,” *IEEE Commun. Mag.*, vol. 47, no. 9, pp. 76–82, Sep. 2009.

[3] V. Chandrasekhar, J. Andrews, T. Muharemovic, Z. Shen, and A. Gatherer, “Power control in two-tier femtocell networks,” *Wireless Communications, IEEE Transactions on*, vol. 8, no. 8, pp. 4316 –4328, august 2009.

[4] V. Chandrasekhar, J. Andrews, and A. Gatherer, “Femtocell networks: a survey,” *Communications Magazine, IEEE*, vol. 46, no. 9, pp. 59 –67, september 2008.

[5] H. Mahmoud and I. Guvenc, “A comparative study of different deployment modes for femtocell networks,” in *Personal, Indoor and Mobile Radio Communications, 2009 IEEE 20th International Symposium on*, sept. 2009, pp. 1 –5.

[6] D. Lopez-Perez, I. Guvenc, G. de la Roche, M. Kountouris, T. Quek, and J. Zhang, “Enhanced intercell interference coordination challenges in heterogeneous networks,” *Wireless Communications, IEEE*, vol. 18, no. 3, pp. 22 –30, june 2011.

[7] L. Garcia, K. Pedersen, and P. Mogensen, “Autonomous component carrier selection: Interference management in local area environments for LTE-Advanced,” *IEEE Commun. Mag.*, vol. 47, no. 9, pp. 110–116, Sep. 2009.

[8] H. Claussen, L. T. W. Ho, and L. G. Samuel, "An overview of the femtocell concept," *Bell Labs Technical Journal*, vol. 13, no. 1, pp. 221–245, 2008. [Online]. Available: http://dx.doi.org/10.1002/bltj.20292

[9] J. Huang, R. Berry, and M. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Sel. Areas Comm.*, vol. 24, no. 5, pp. 1074–1084, May 2006.

[10] C. Shi, R. Berry, and M. Honig, "Distributed interference pricing for OFDM wireless networks with non-separable utilities," in *Proc. Conf. on Inform. Sciences and Systems*, Mar. 2008, pp. 755–760.

[11] L. Yang, Y. Sagduyu, J. Zhang, and J. Li, "Distributed power control for ad-hoc communications via stochastic nonconvex utility optimization," in *Proc. IEEE Int. Conf. on Commun.*, Jun. 2011, pp. 1–5.

[12] D. Palomar and M. Chiang, "Alternative distributed algorithms for network utility maximization: Framework and applications," *IEEE Trans. Autom. Control*, vol. 52, no. 12, pp. 2254–2269, Dec. 2007.

[13] C. W. Tan, D. Palomar, and M. Chiang, "Distributed optimization of coupled systems with applications to network utility maximization," in *Proc. IEEE Int. Conf. on Acoust., Speech and Signal Process.*, vol. 5, May 2006, p. V.

[14] F. Ahmed and O. Tirkkonen, "Local optimum based power allocation approach for spectrum sharing in unlicensed bands," in *Self-Organizing Systems*, ser. Lecture Notes in Computer Science, T. Spyropoulos and K. Hummel, Eds. Springer Berlin / Heidelberg, 2009, pp. 238–243.

[15] J. Huang, V. Subramanian, R. Agrawal, and R. Berry, "Downlink scheduling and resource allocation for OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 288–296, Jan. 2009.

[16] K. Seong, M. Mohseni, and J. Cioffi, "Optimal resource allocation for OFDMA downlink systems," in *Proc. IEEE Int. Symp. on Inform. Theory*, Jul. 2006, pp. 1394–1398.

[17] P. Marbach and R. Berry, "Downlink resource allocation and pricing for wireless networks," in *Proc. IEEE Joint Conf. of Computer and Commun. Societies*, vol. 3, 2002, pp. 1470– 1479.

[18] F. Kelly, "Charging and rate control for elastic traffic," *European Trans. Telecommun.*, vol. 8, pp. 33–37, 1997.

[19] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Trans. Netw.*, vol. 8, no. 5, pp. 556–567, Oct. 2000.

[20] H. Kim, K. Kim, Y. Han, and S. Yun, "A proportional fair scheduling for multicarrier transmission systems," in *Proc. IEEE Veh. Tech. Conf. Fall*, vol. 1, Sep. 2004, pp. 409–413.

[21] C. Wengerter, J. Ohlhorst, and A. von Elbwart, "Fairness and throughput analysis for generalized proportional fair frequency scheduling in OFDMA," in *Proc. IEEE Veh. Tech. Conf. Spring*, vol. 3, May 2005, pp. 1903–1907.

[22] P. Kyösti et al., "Winner II channel models," Tech. Rep. D1.1.2 V1.2, 2007, available: http://www.ist-winner.org.