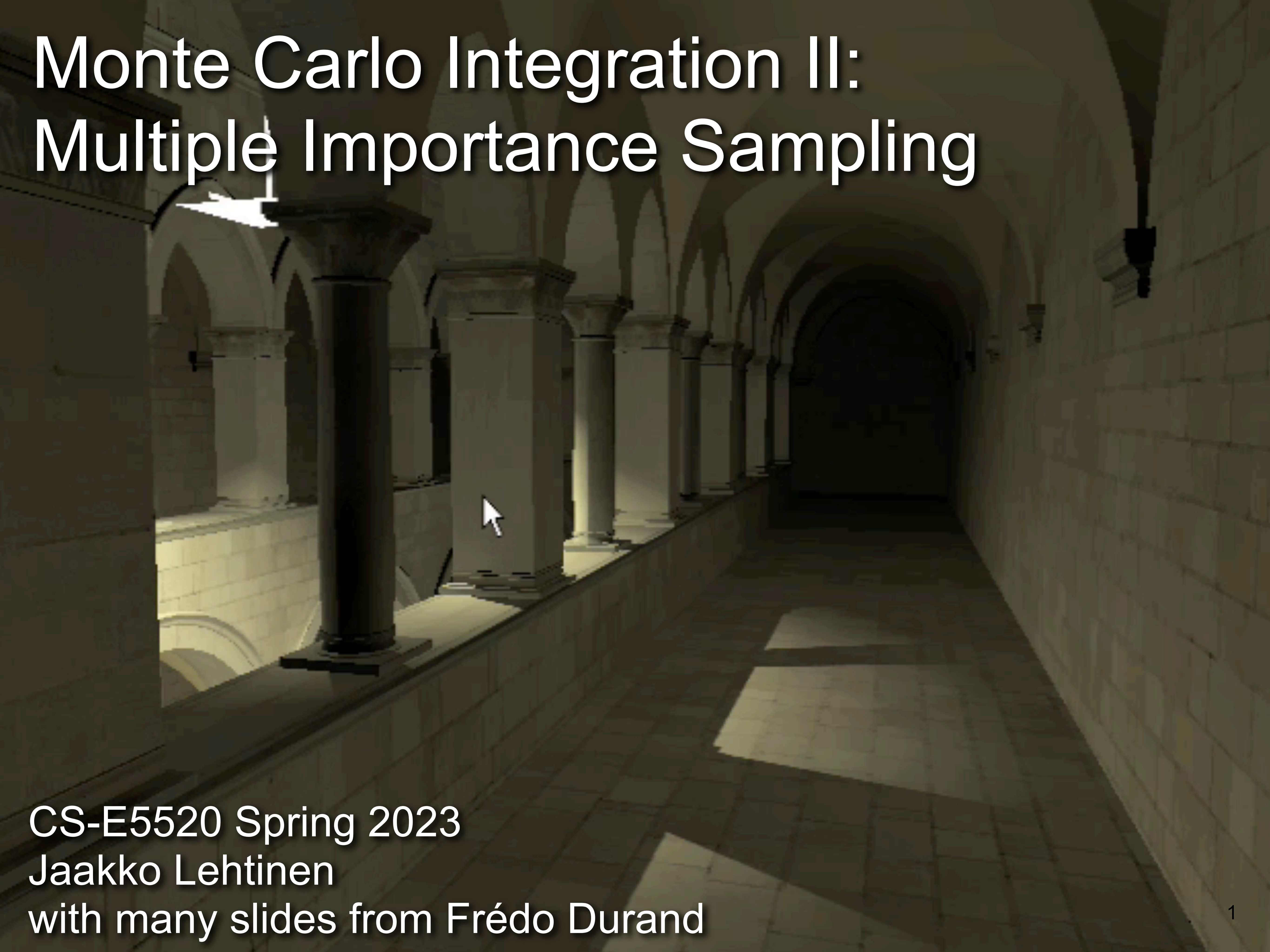
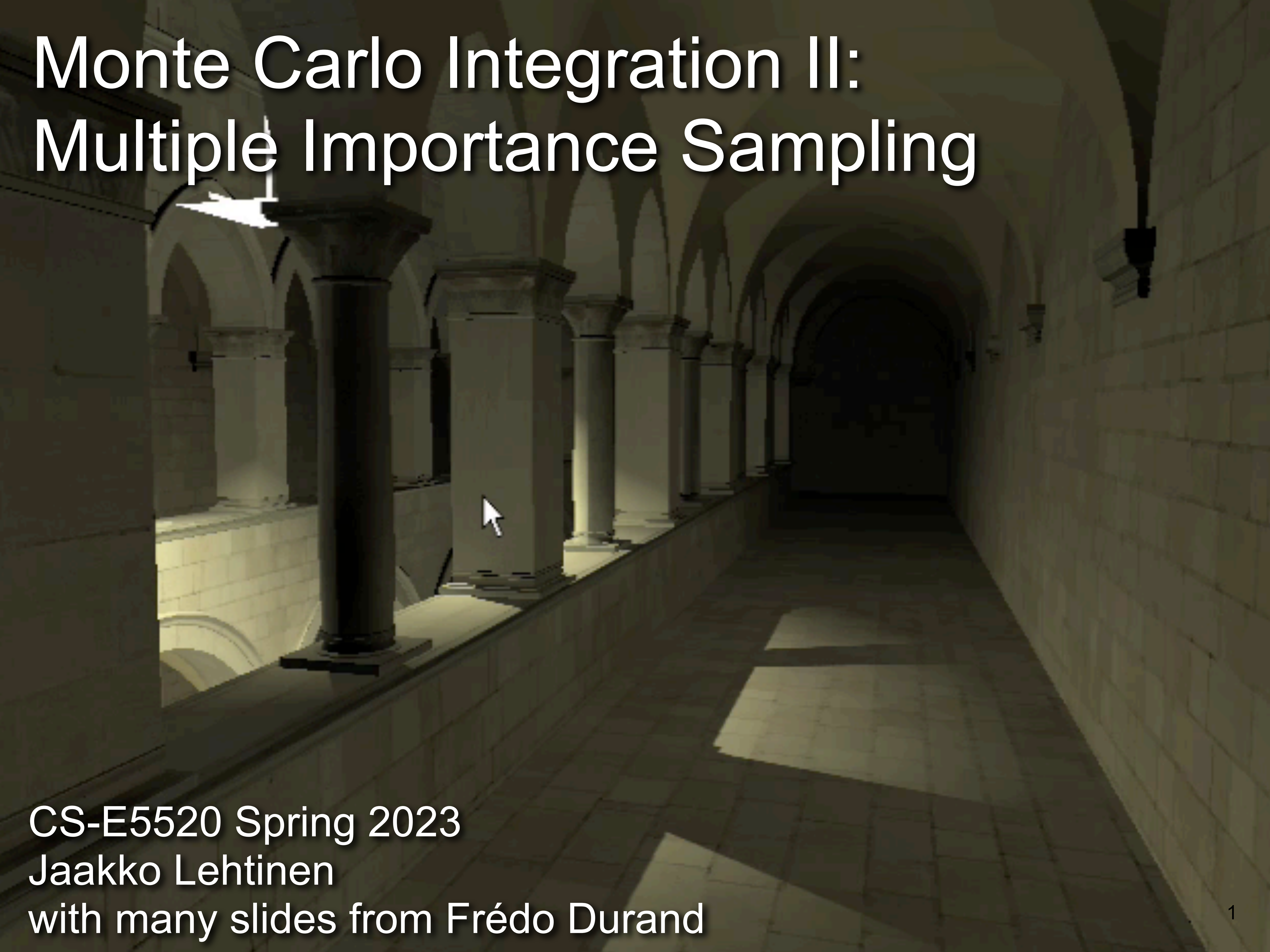
Monte Carlo Integration II: Multiple Importance Sampling



CS-E5520 Spring 2023
Jaakko Lehtinen
with many slides from Frédo Durand



Monte Carlo Integration II: Multiple Importance Sampling

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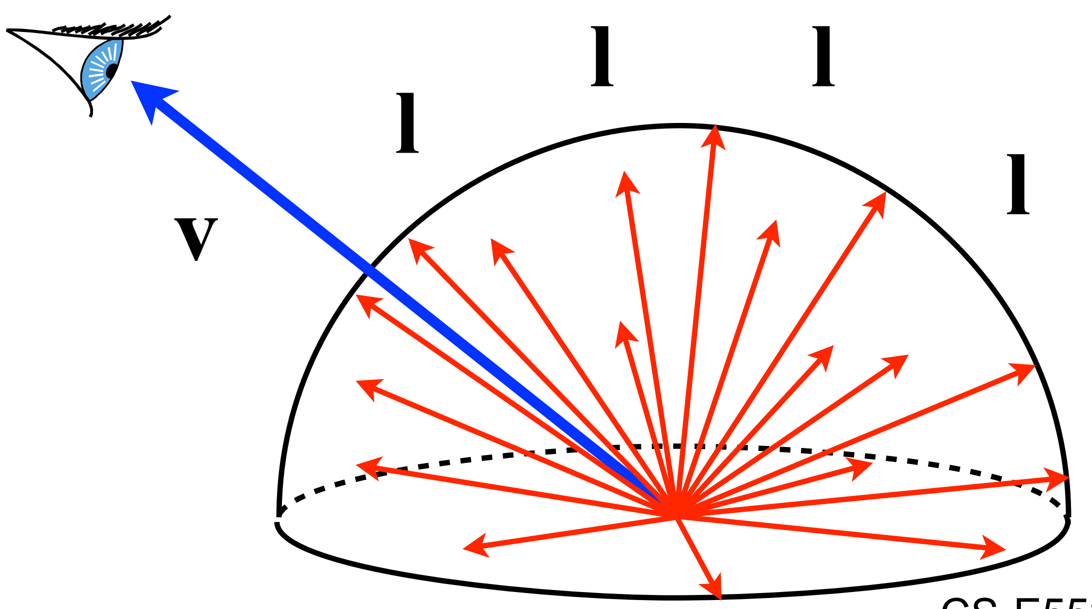
Recap: Reflectance Equation

$$L(x \rightarrow \mathbf{v}) =$$

$$\int_\Omega f_r(x, \mathbf{l} \rightarrow \mathbf{v}) \, L(x \leftarrow \mathbf{l}) \, \cos\theta \, \mathrm{d}\mathbf{l}$$

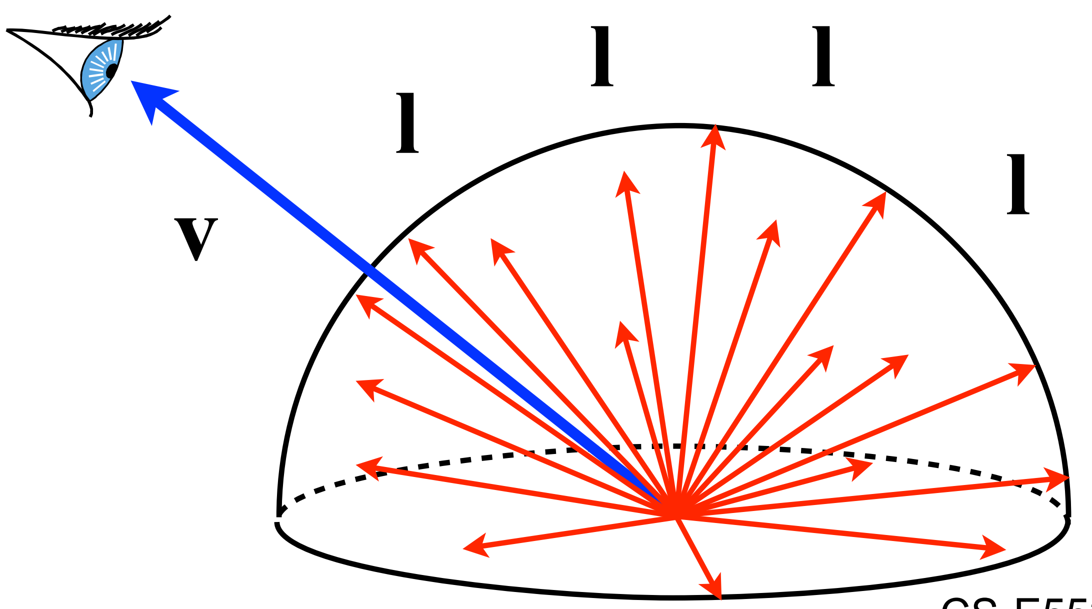


Recap: Reflectance Equation

$$L(x \rightarrow \mathbf{v}) =$$

outgoing radiance

$$\int_\Omega f_r(x, \mathbf{l} \rightarrow \mathbf{v})\, L(x \leftarrow \mathbf{l})\; \cos\theta\, \mathrm{d}\mathbf{l}$$



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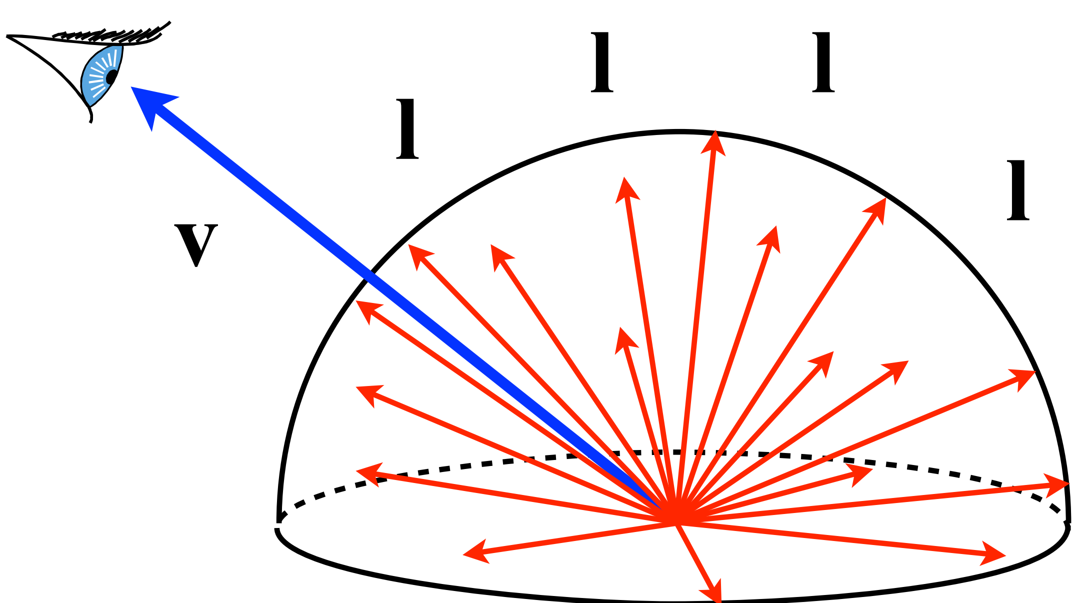
integral over hemisphere



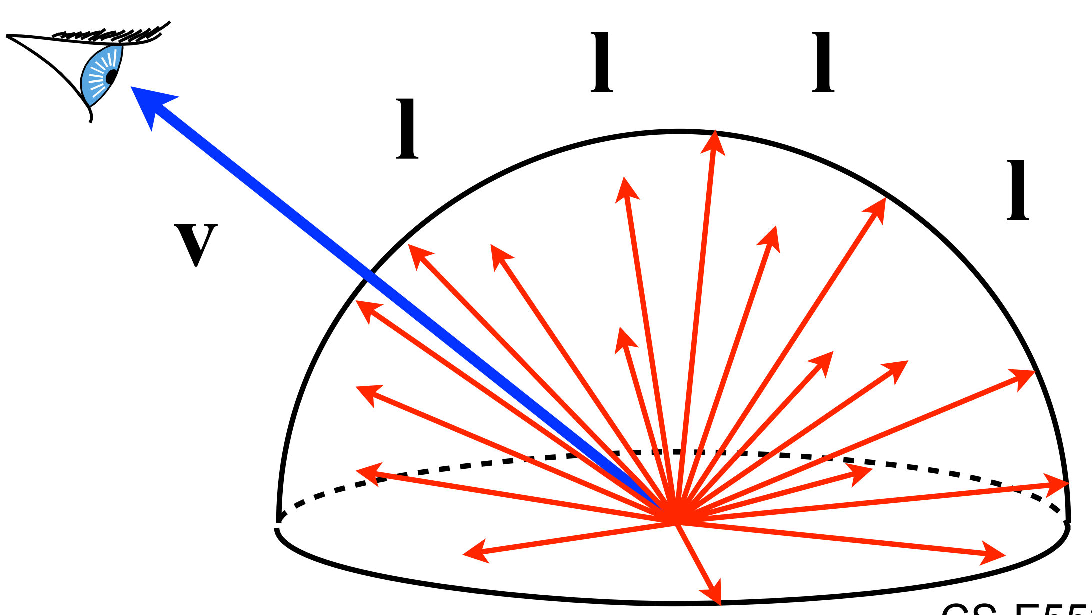
Recap: Reflectance Equation

$$L(x \rightarrow \mathbf{v}) =$$ ⇐ **outgoing radiance**

⇒ $$\int_\Omega f_r(x, \mathbf{l} \rightarrow \mathbf{v})\, L(x \leftarrow \mathbf{l})\, \cos\theta \,\mathrm{d}\mathbf{l}$$

integral over hemisphere

incoming radiance



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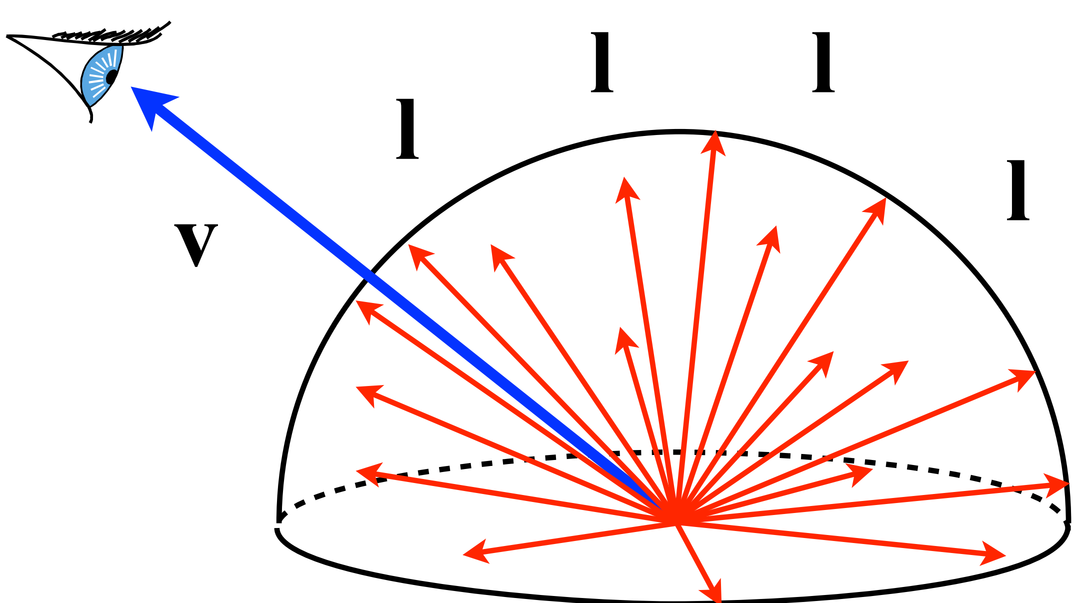
$$L(x \to \mathbf{v}) = \int_\Omega f_r(x, \mathbf{l} \to \mathbf{v})\, L(x \leftarrow \mathbf{l})\, \cos\theta \,\mathrm{d}\mathbf{l}$$

outgoing radiance

integral over hemisphere

incoming radiance

cosine of incident angle



Recap: Reflectance Equation

$$L(x \to \mathbf{v}) =$$

outgoing radiance

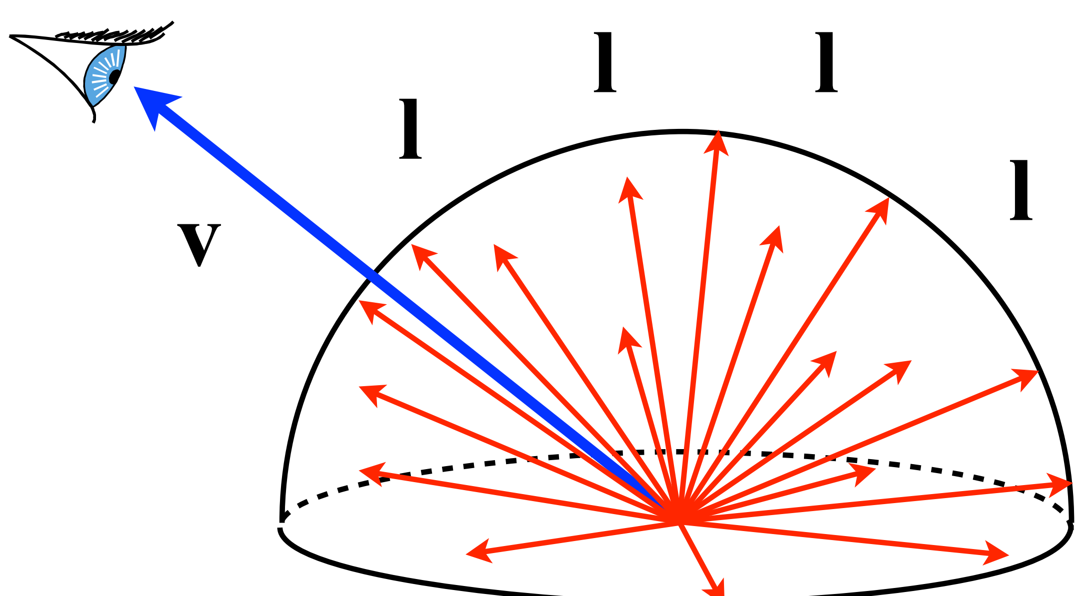
$$\int_\Omega f_r(x, \mathbf{l} \to \mathbf{v})\, L(x \leftarrow \mathbf{l})\, \cos\theta \, \mathrm{d}\mathbf{l}$$

integral over hemisphere

BRDF

incoming radiance

cosine of incident angle



Recap: Reflectance Equation

$$L(x \rightarrow \mathbf{v}) =$$

outgoing radiance

$$\int_\Omega f_r(x, \mathbf{l} \rightarrow \mathbf{v}) \, L(x \leftarrow \mathbf{l}) \, \cos\theta \, \mathrm{d}\mathbf{l}$$

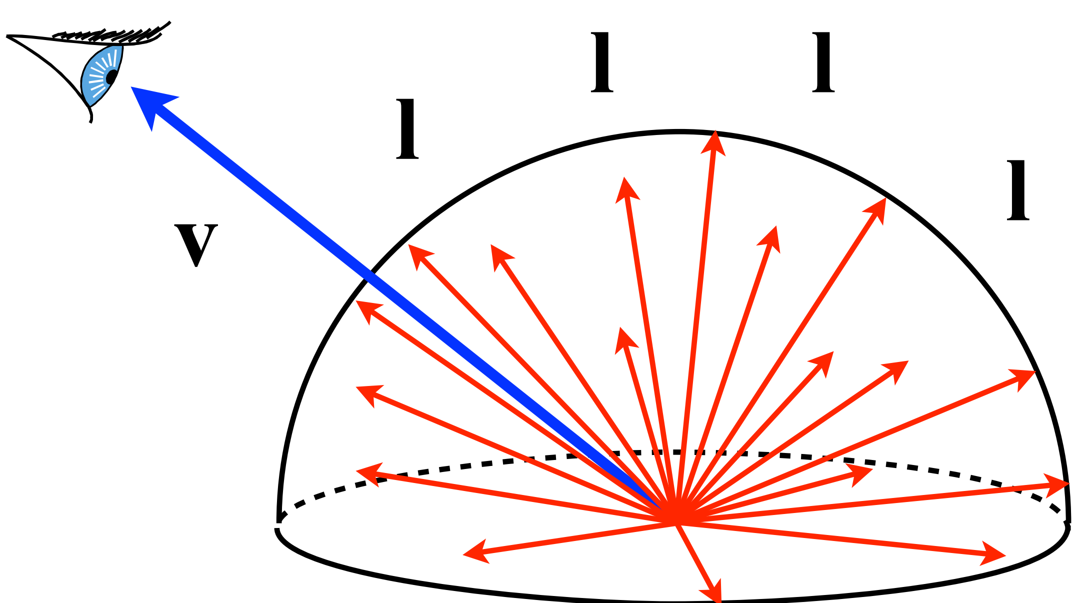
L_in * cos = incident differential irradiance

integral over hemisphere

BRDF

incoming radiance

cosine of incident angle



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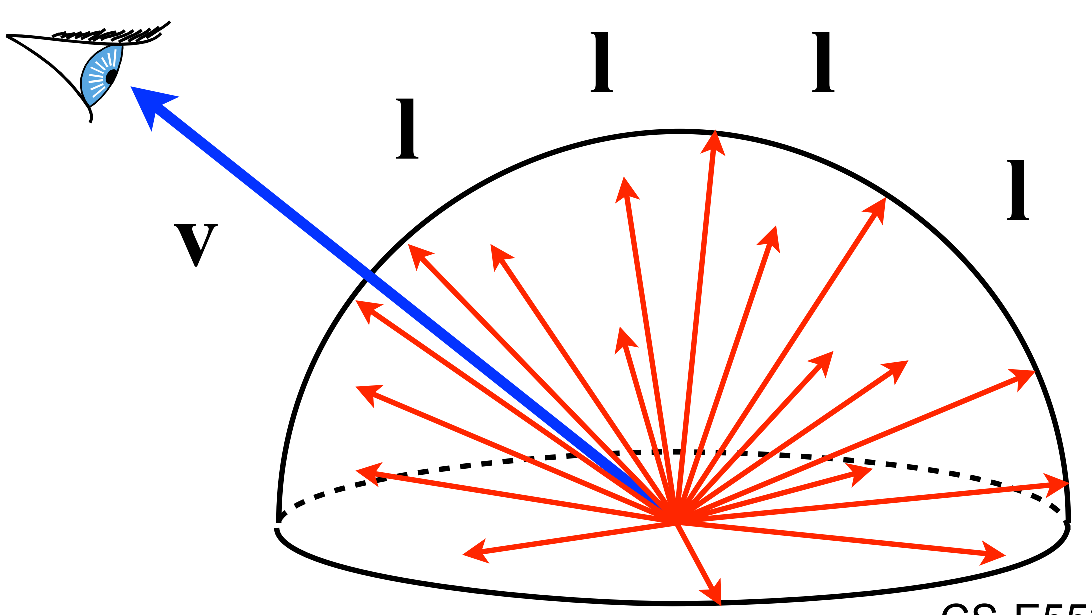
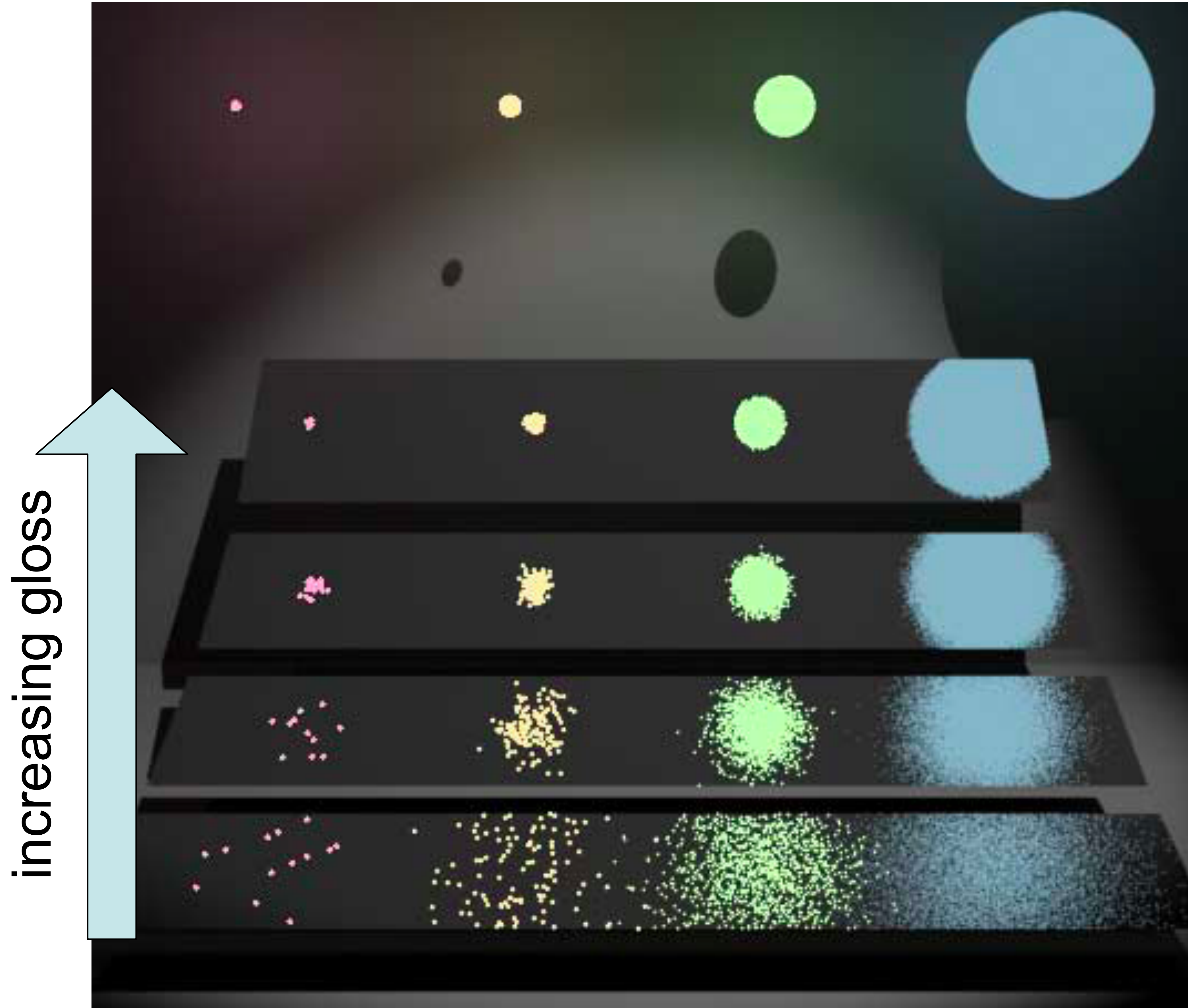
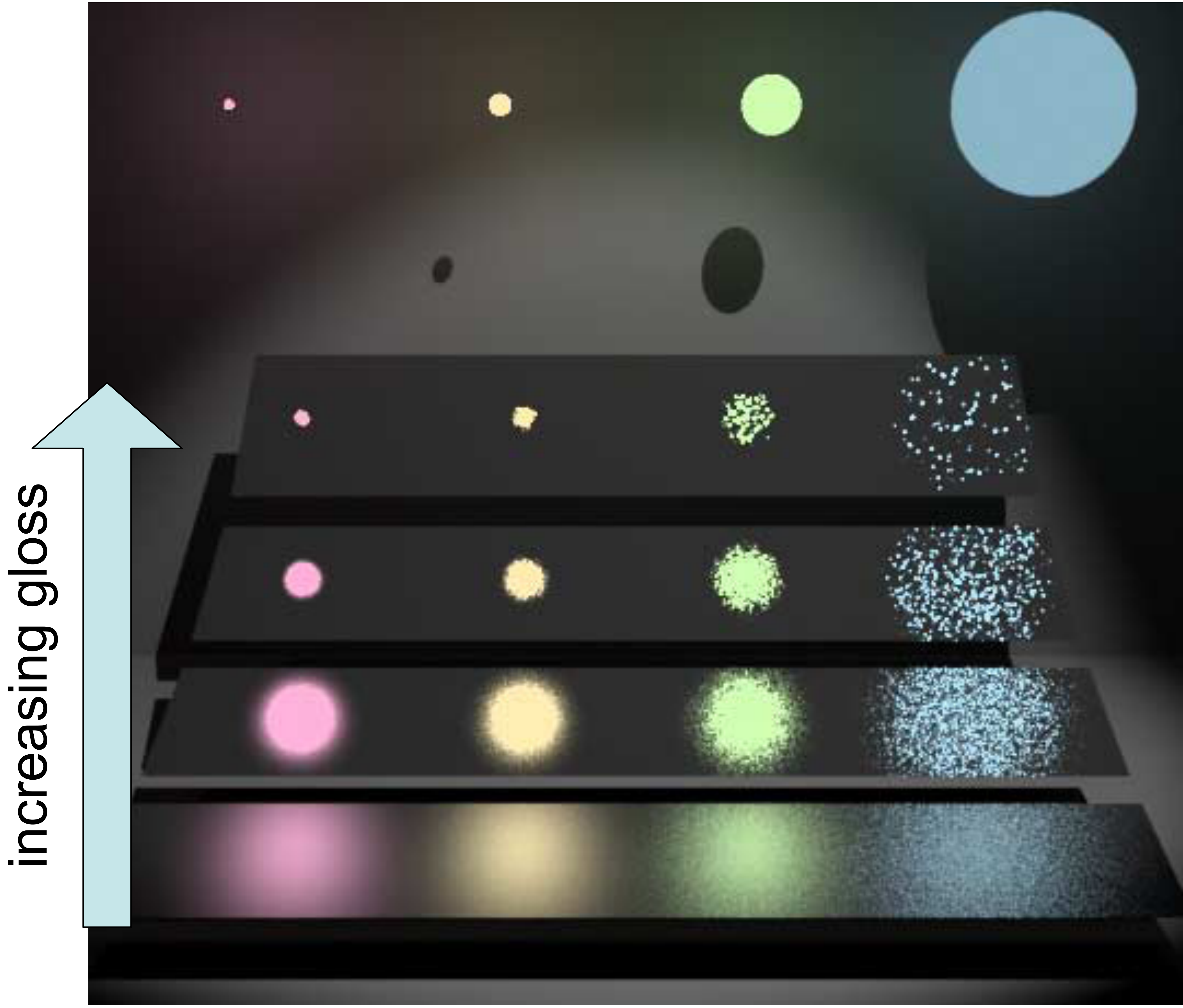


Imp. Sampling According to BRDF



Imp. Sampling According to Light

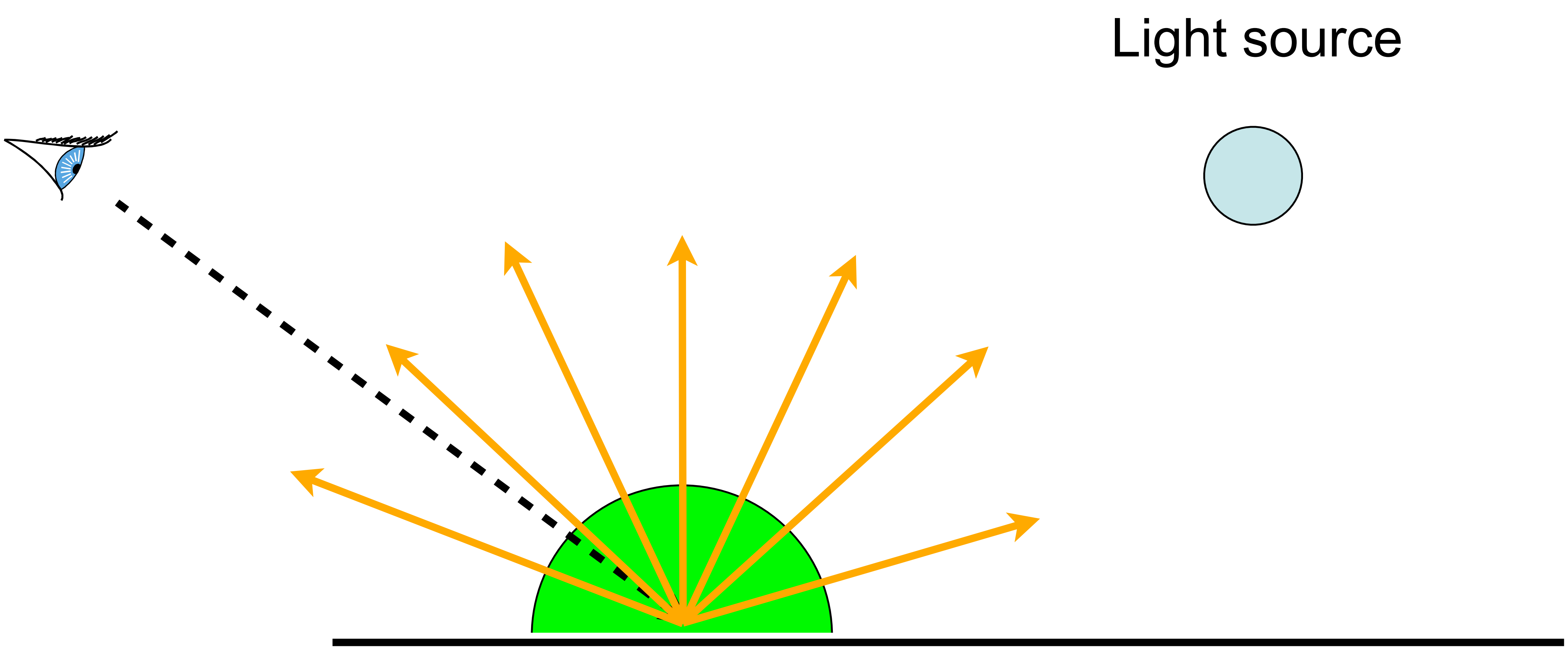


Multiple Importance Sampling (MIS)

- If integrand *f* has a complex shape that consists of distinct features that are easy to sample from individually, we can use multiple PDFs and combine them in a nice way so that we got lower variance
  - See Veach and Guibas 1995

What's Going on Here?

- Dull gloss/diffuse surface, importance sample BRDF



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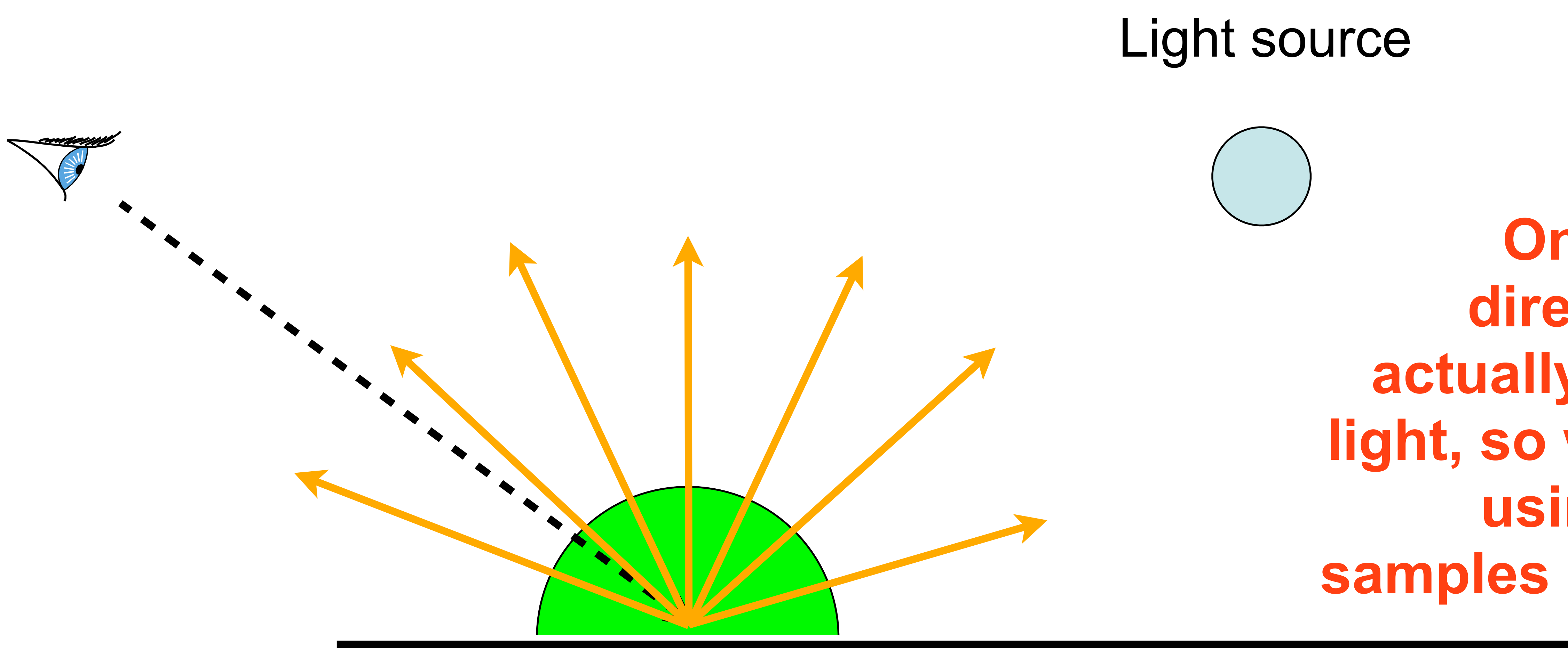
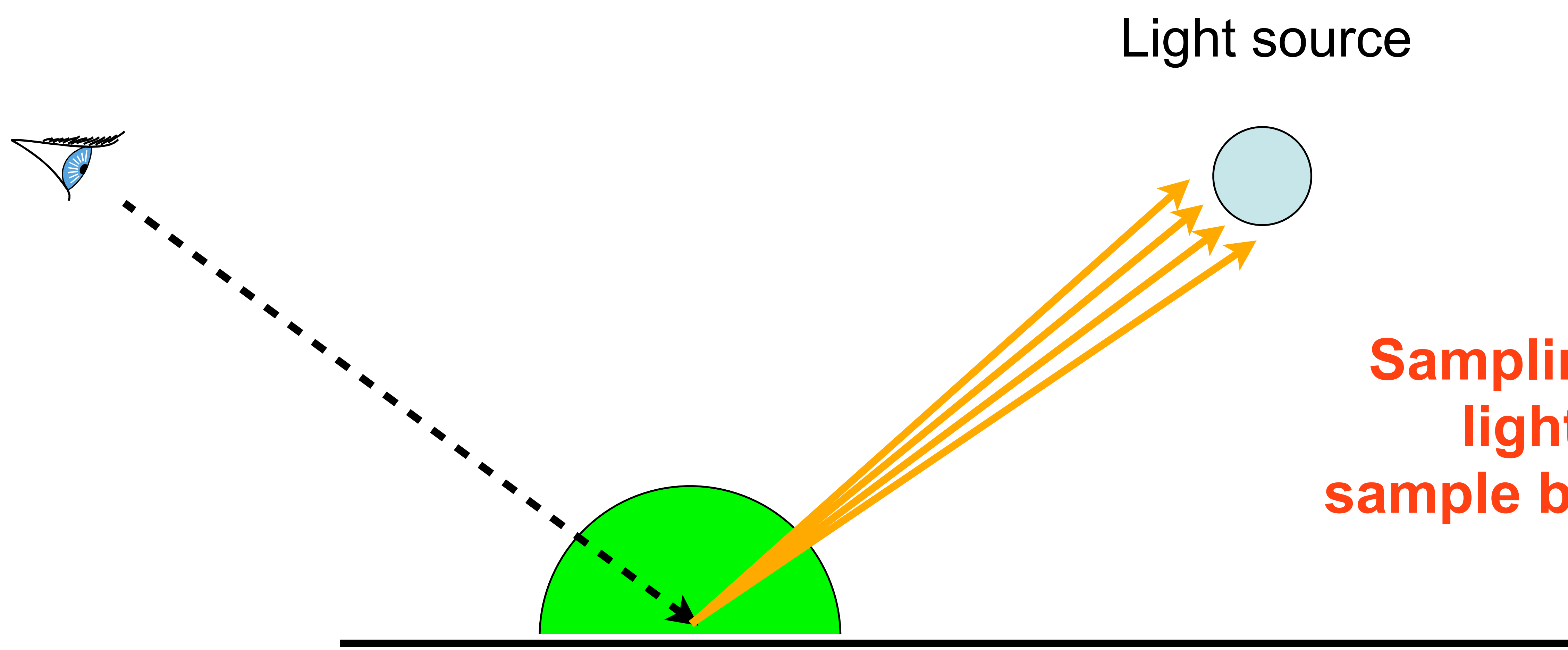


Only few directions actually carry light, so we are using our samples poorly

Here Makes Sense to Sample Light

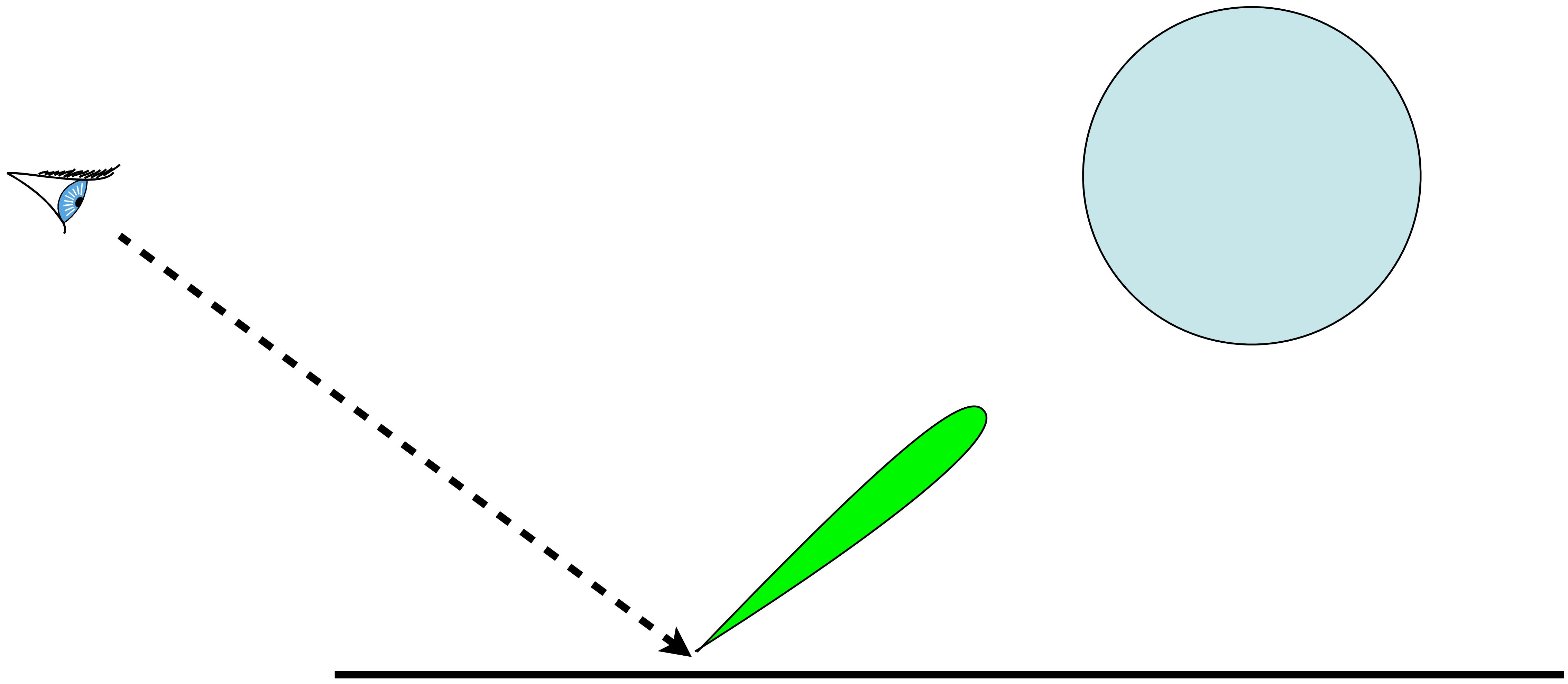
- Dull gloss/diffuse surface, importance sample light



Sampling the light uses sample budget better

What's Going on Here?

- Highly glossy surface, narrow lobe, large light source, importance sample light



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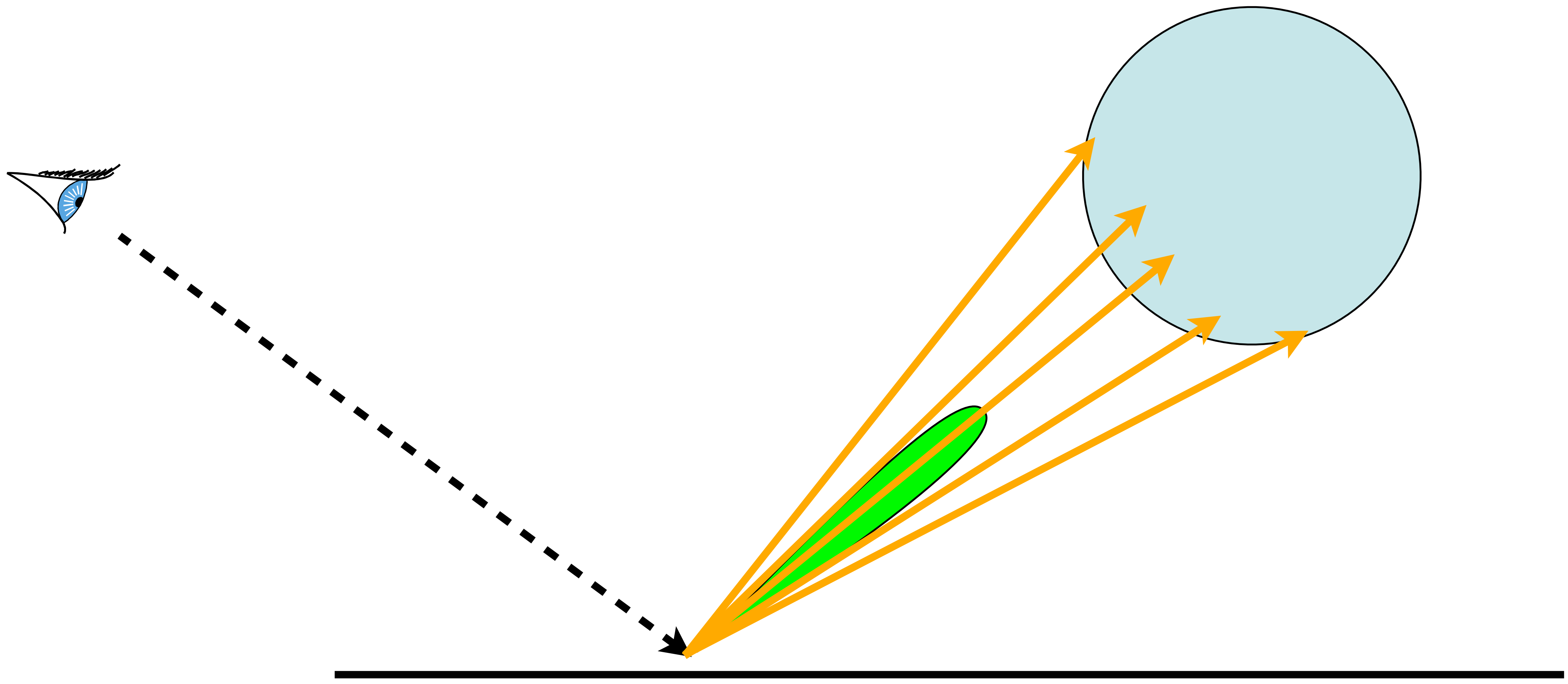

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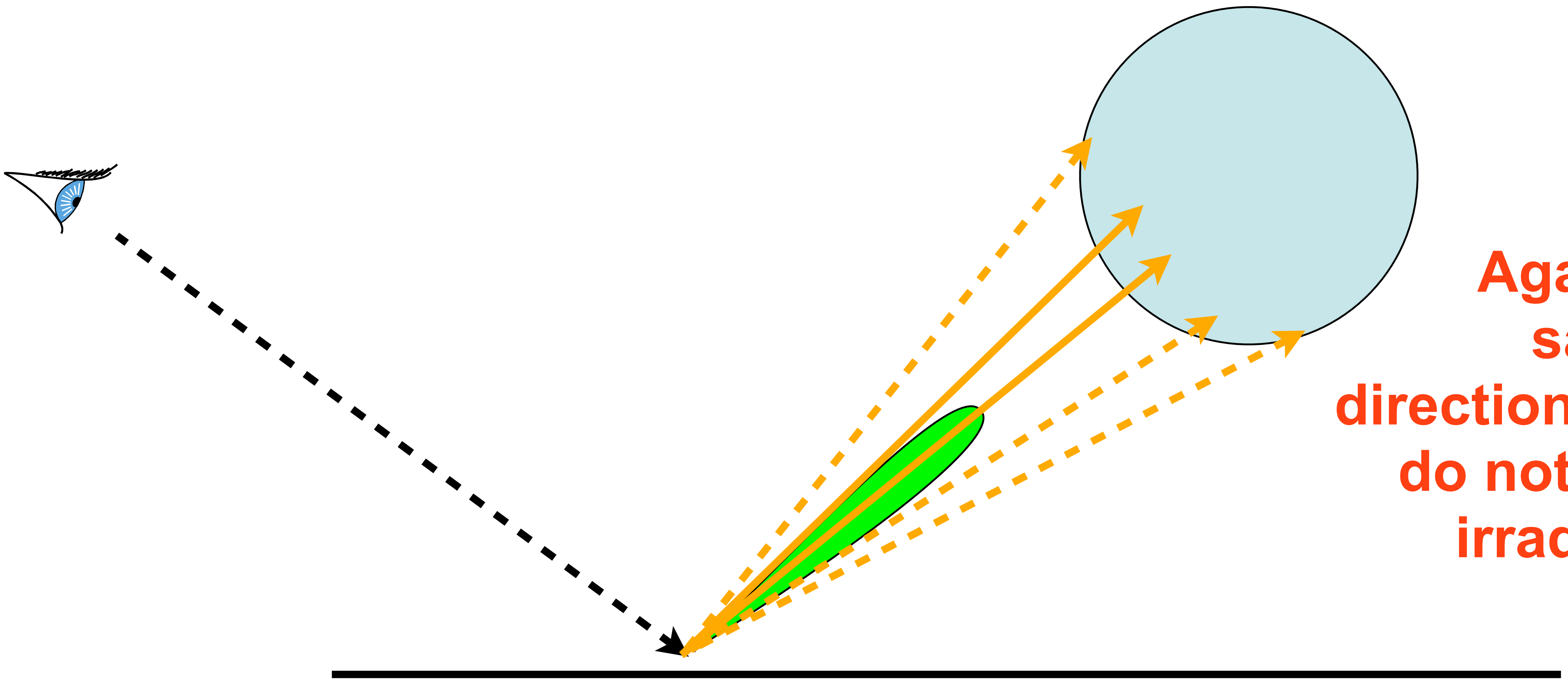
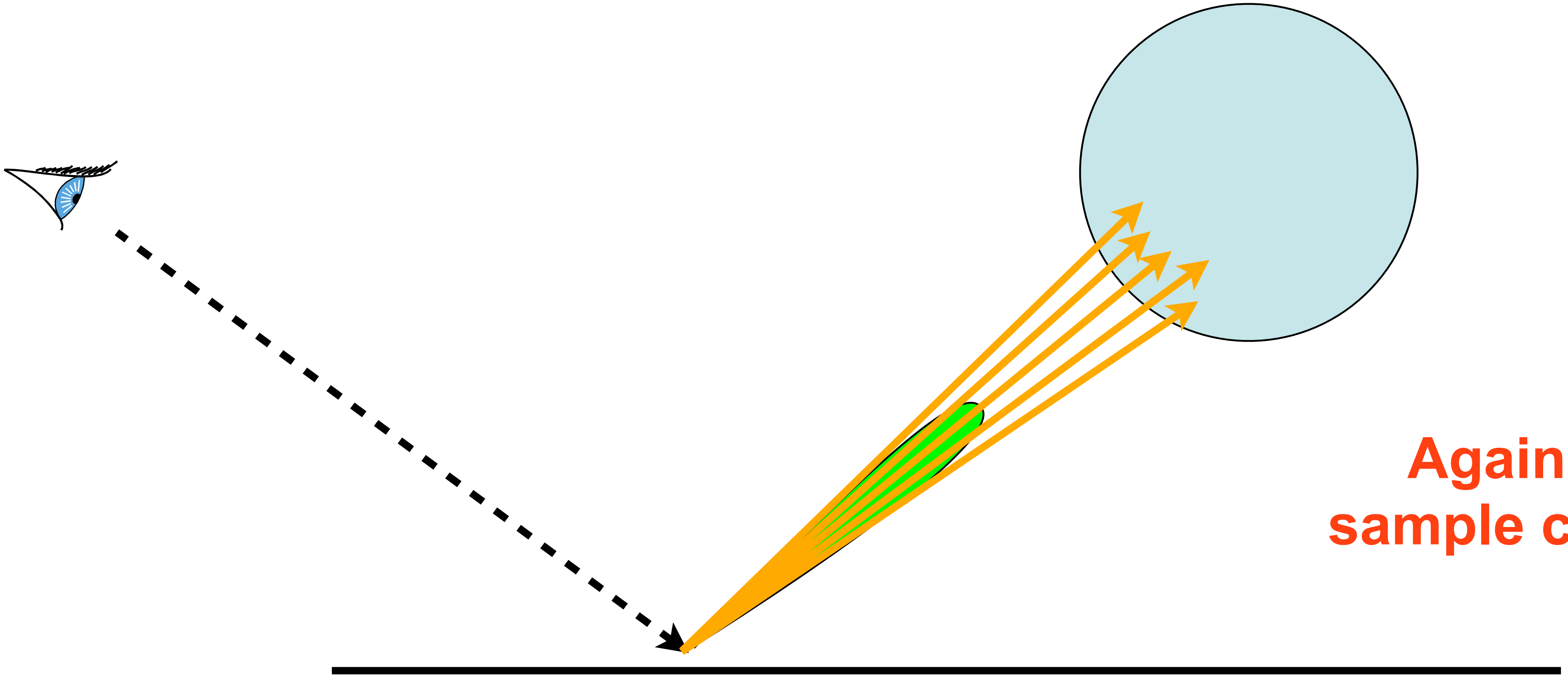


Again we sample directions that do not carry irradiance

Here, Better to Sample BRDF

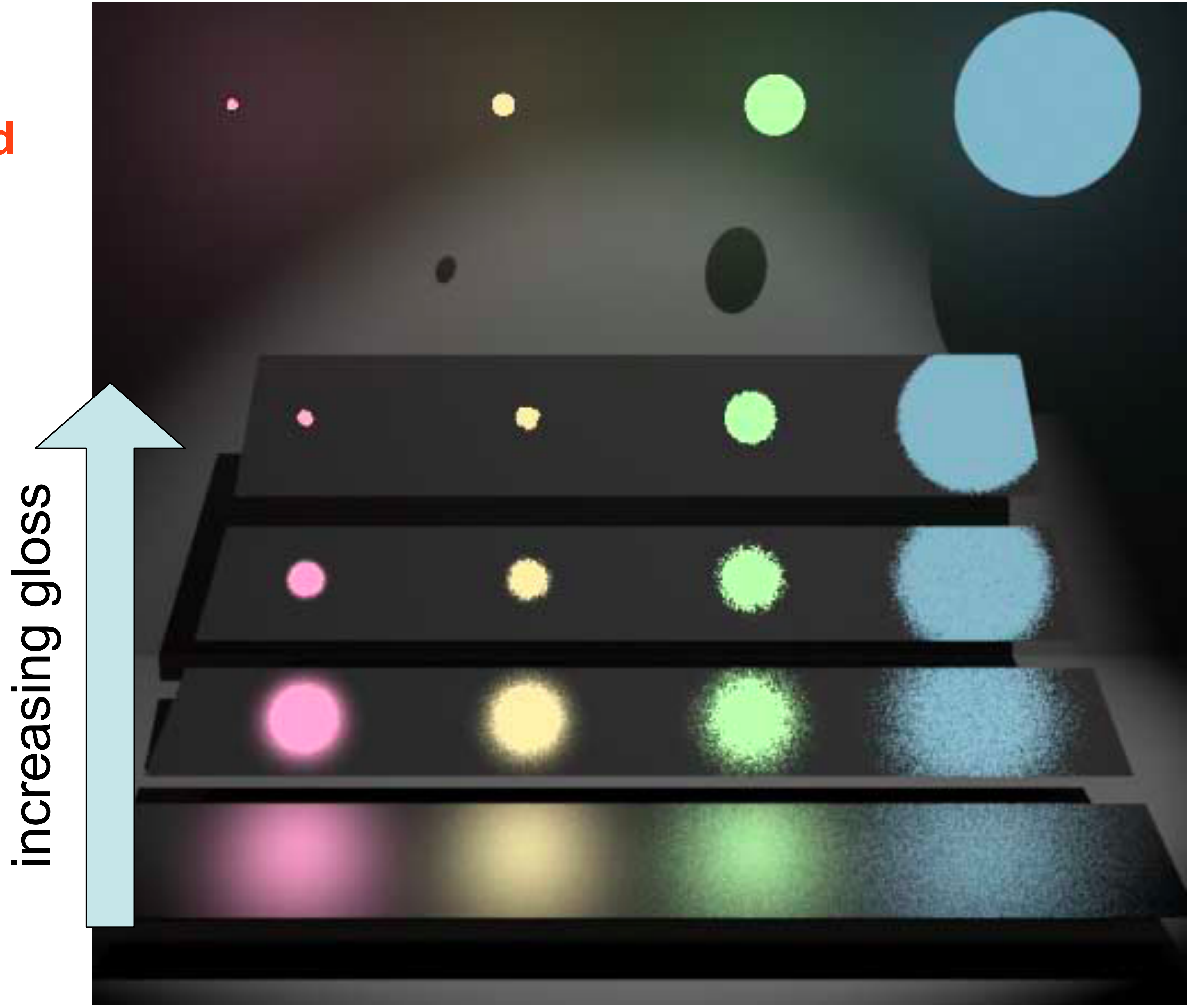
- Highly glossy surface, narrow lobe, large light source, importance sample light



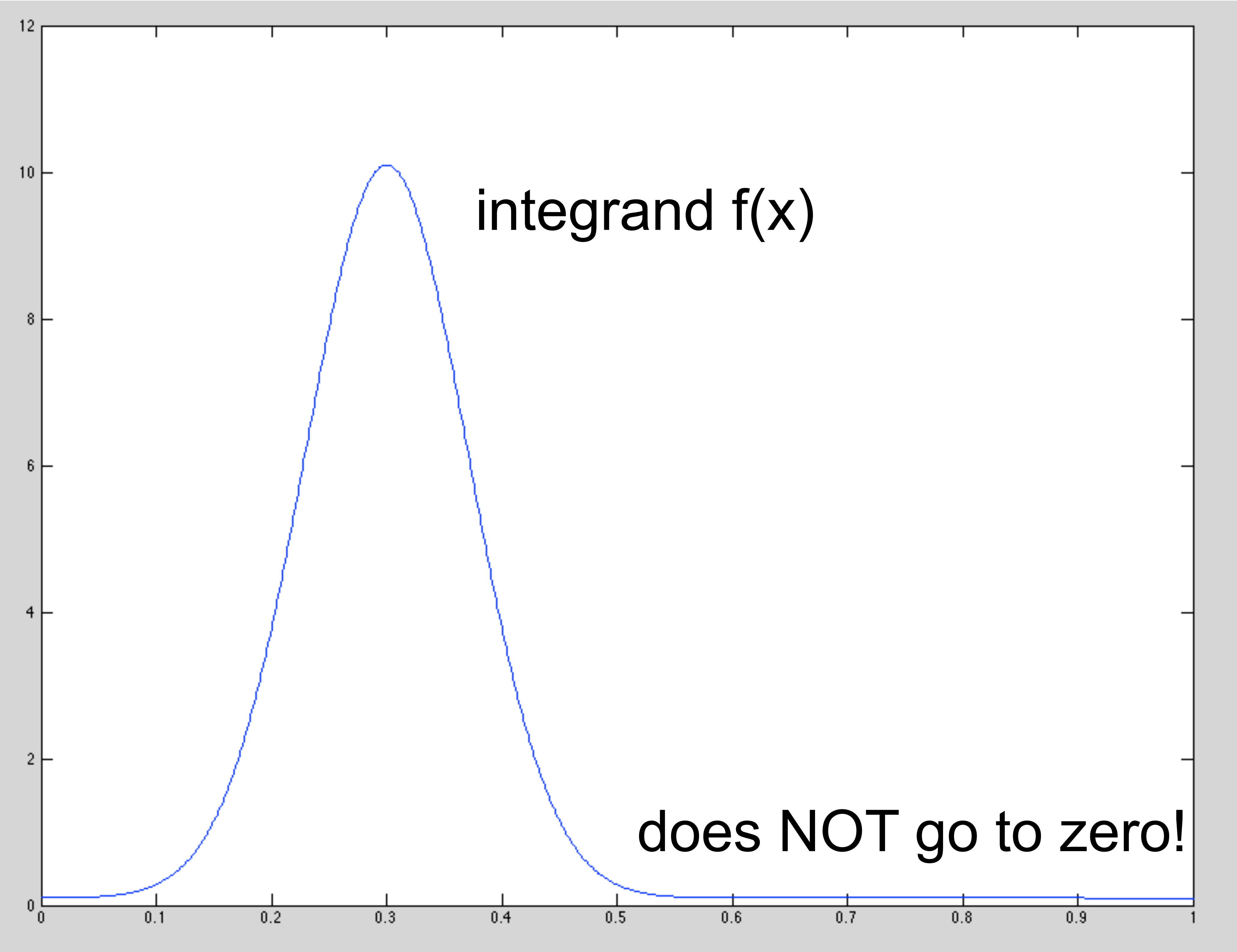
Again, each sample counts more

Multiple Importance Sampling ✔

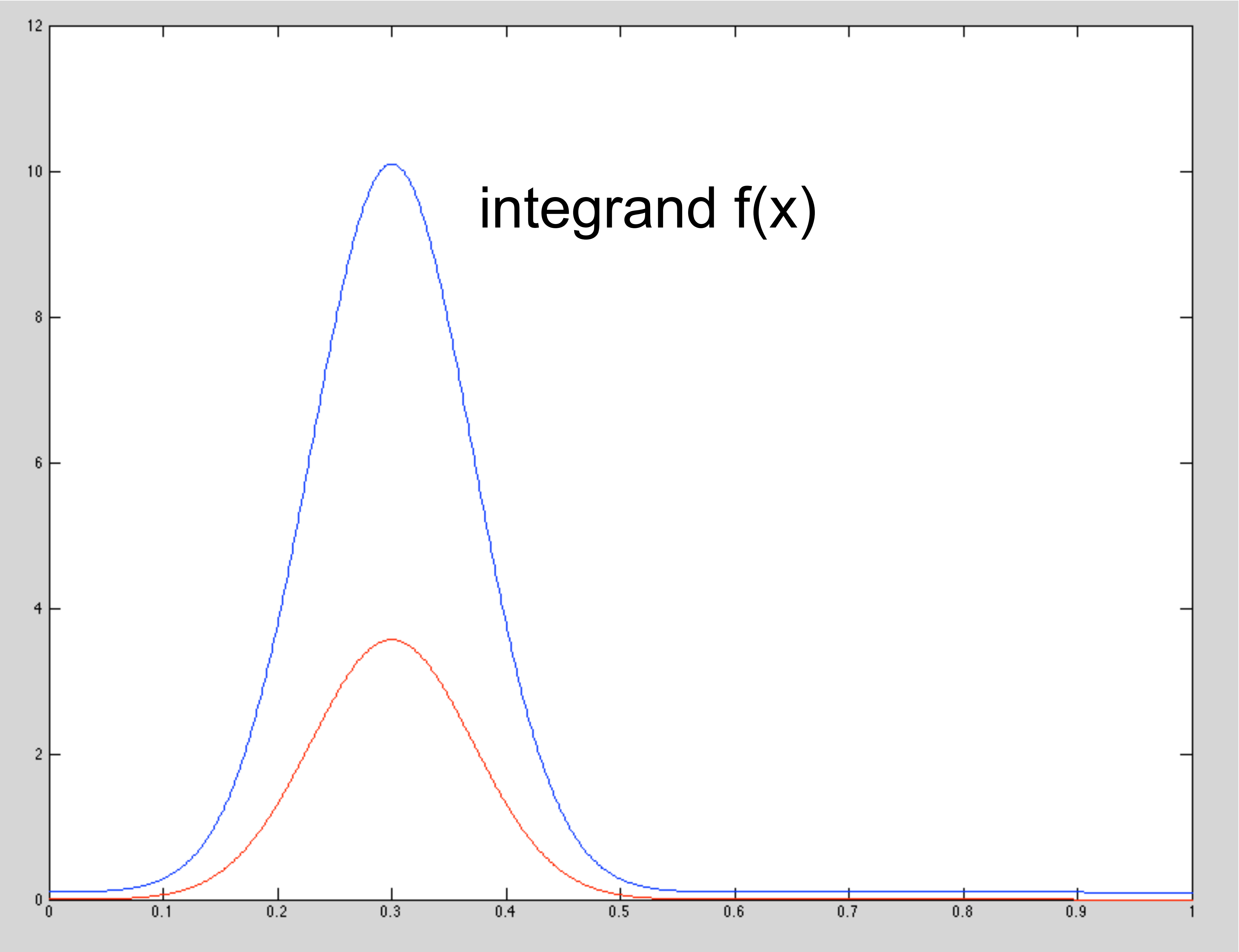
MIS = Sample both ways and optimally combine the samples



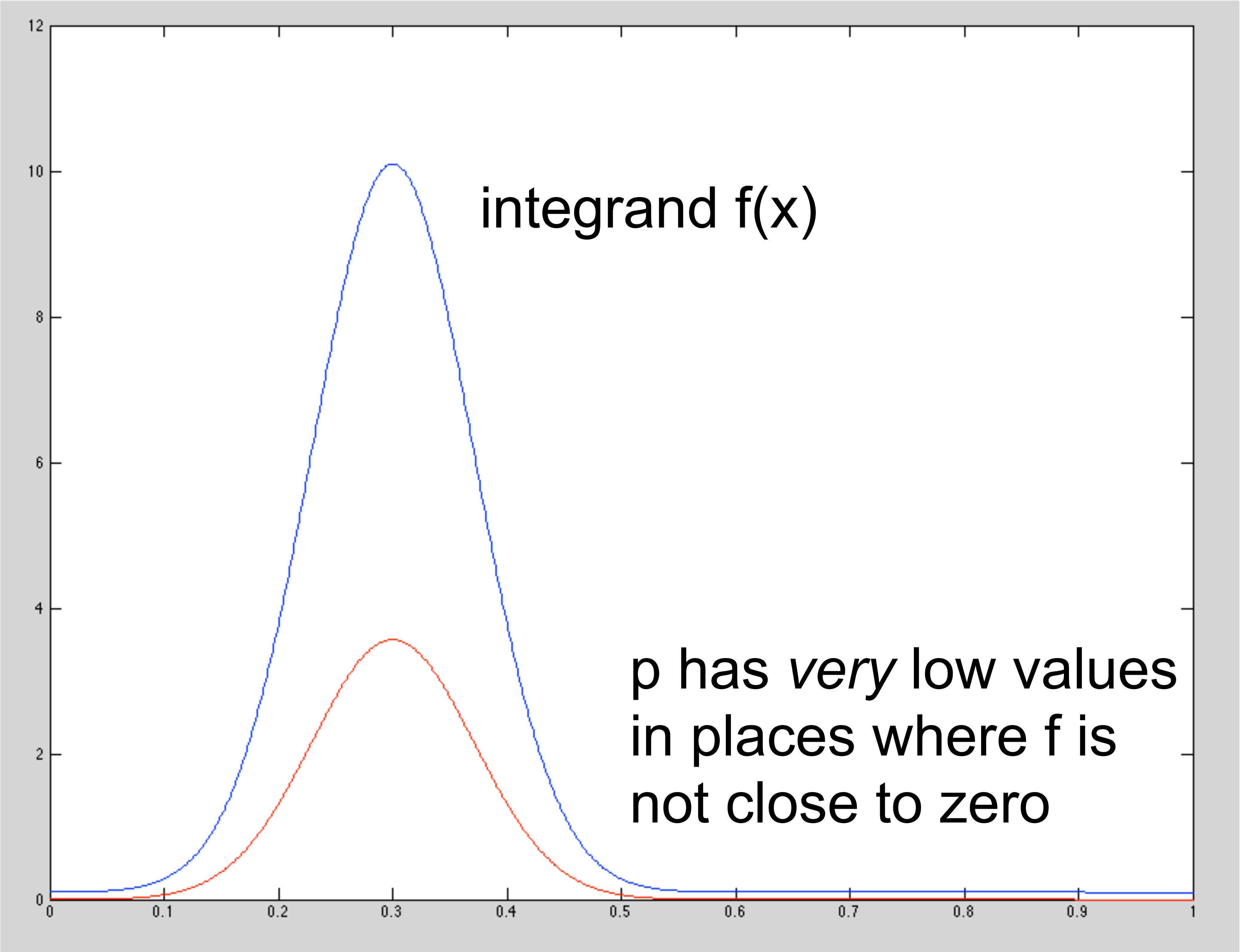
Ok, how do you do it?



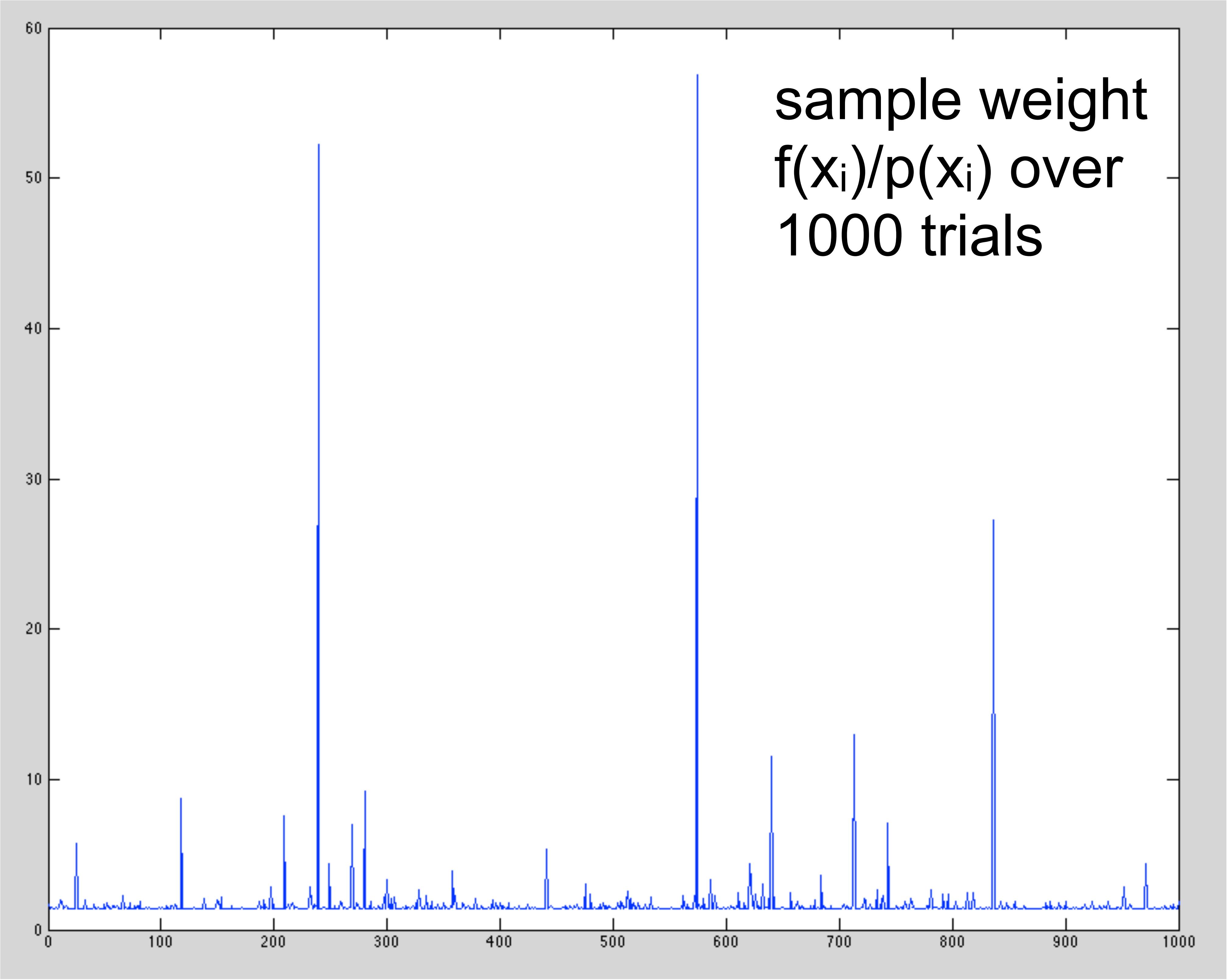
Why is the Red Gaussian bad for IS?



Why the Red Gaussian is *bad* for IS



Why This Matters



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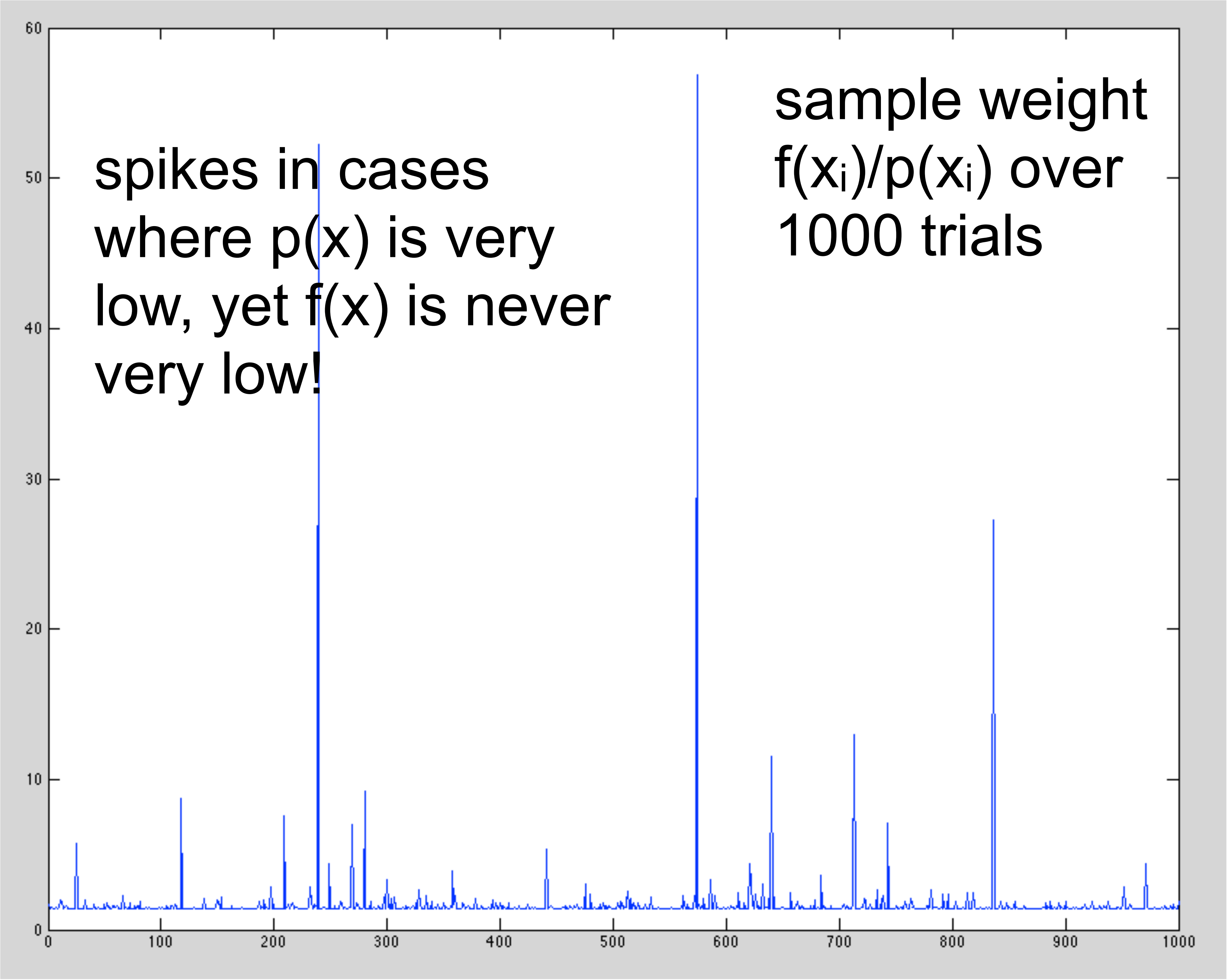

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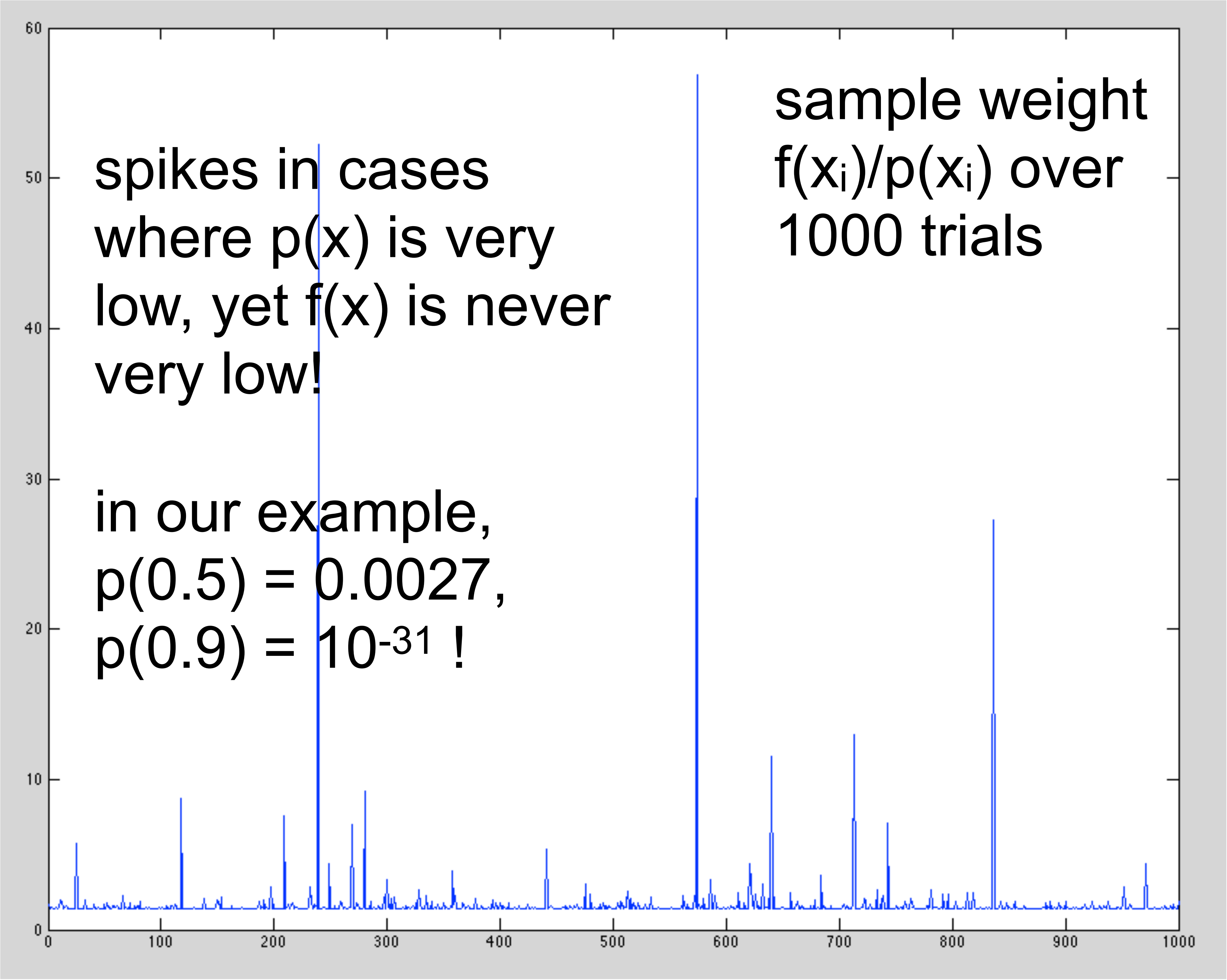
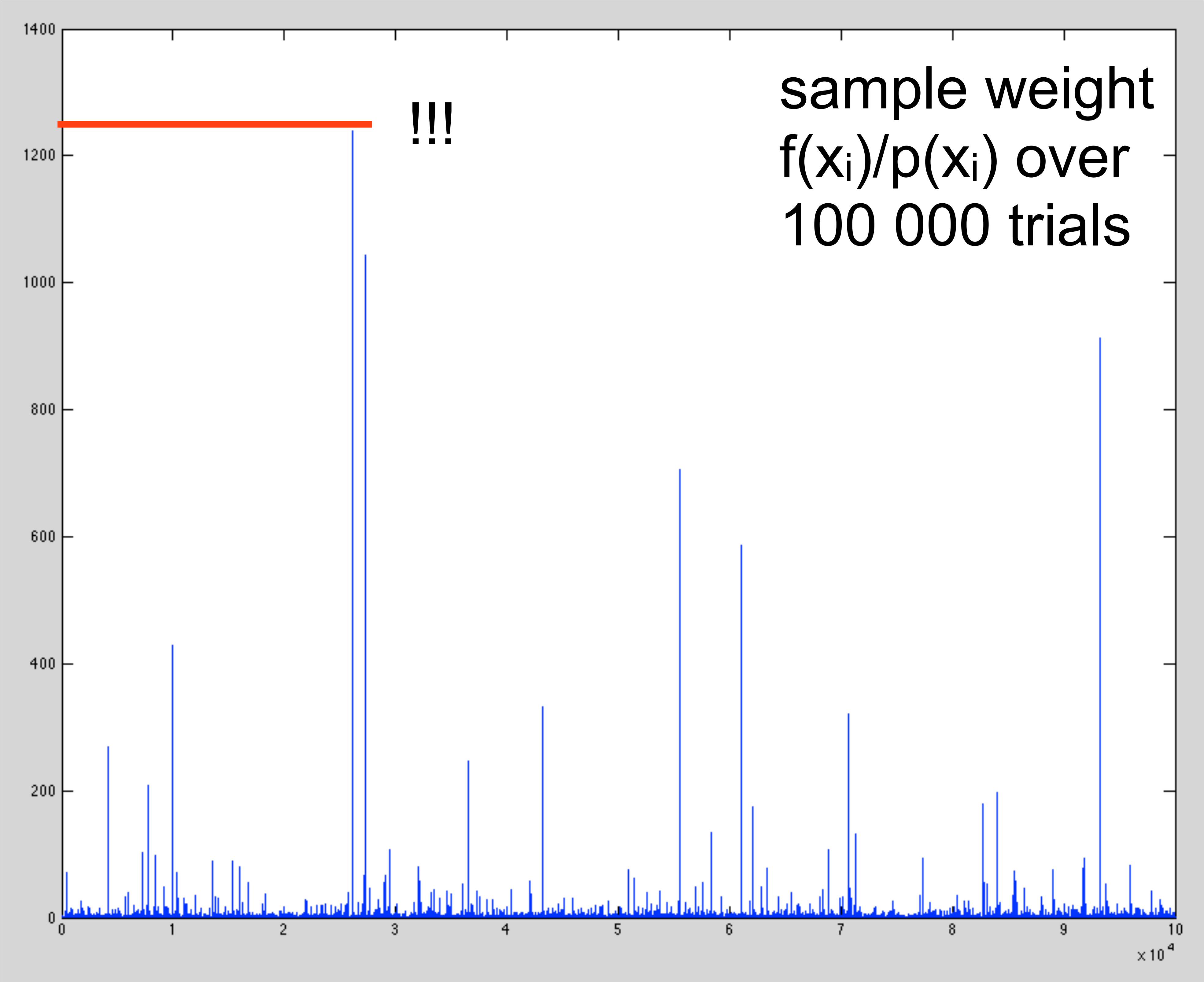
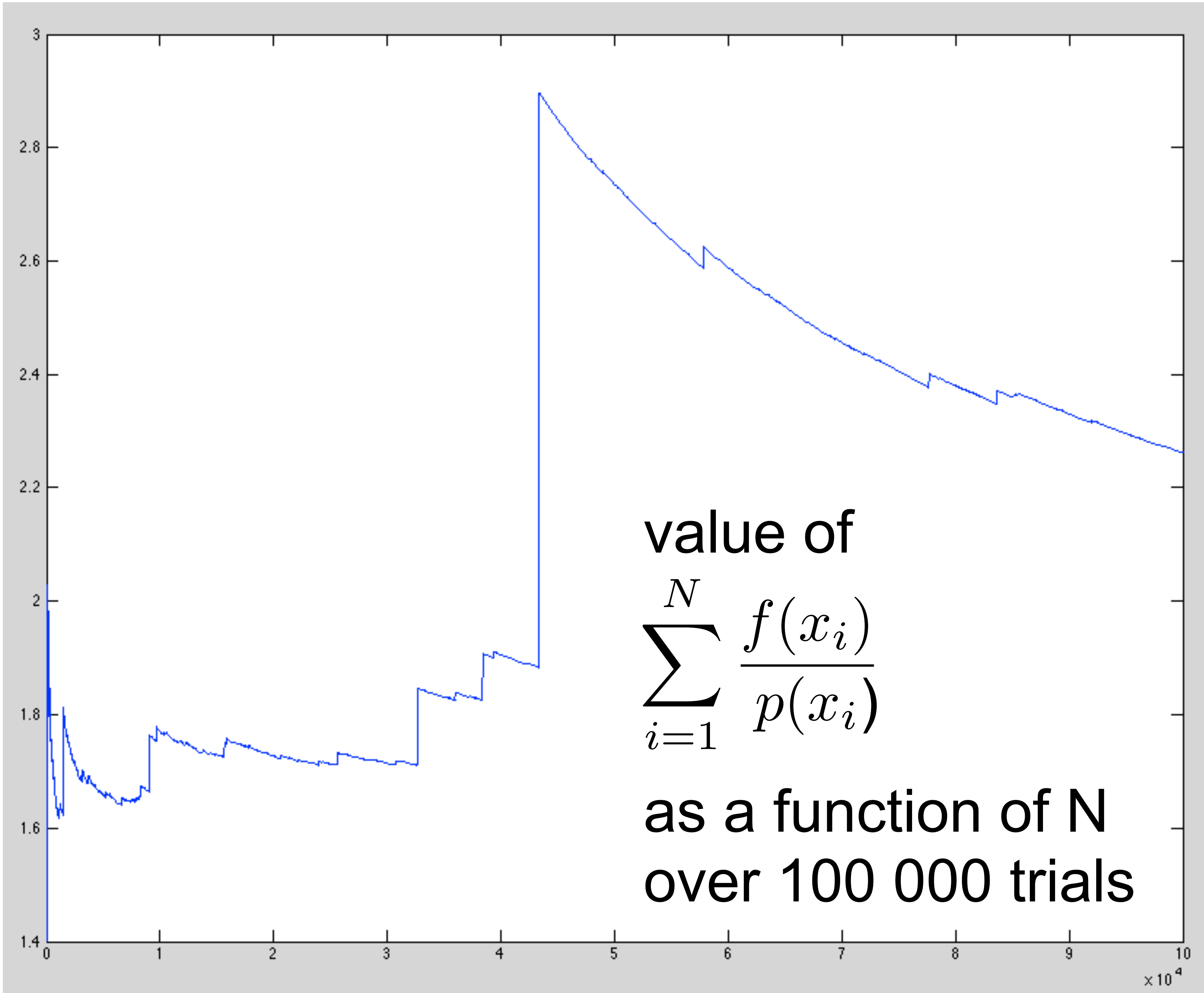


Spikes get worse with higher N



Effect of Spikes on Integral Estimate

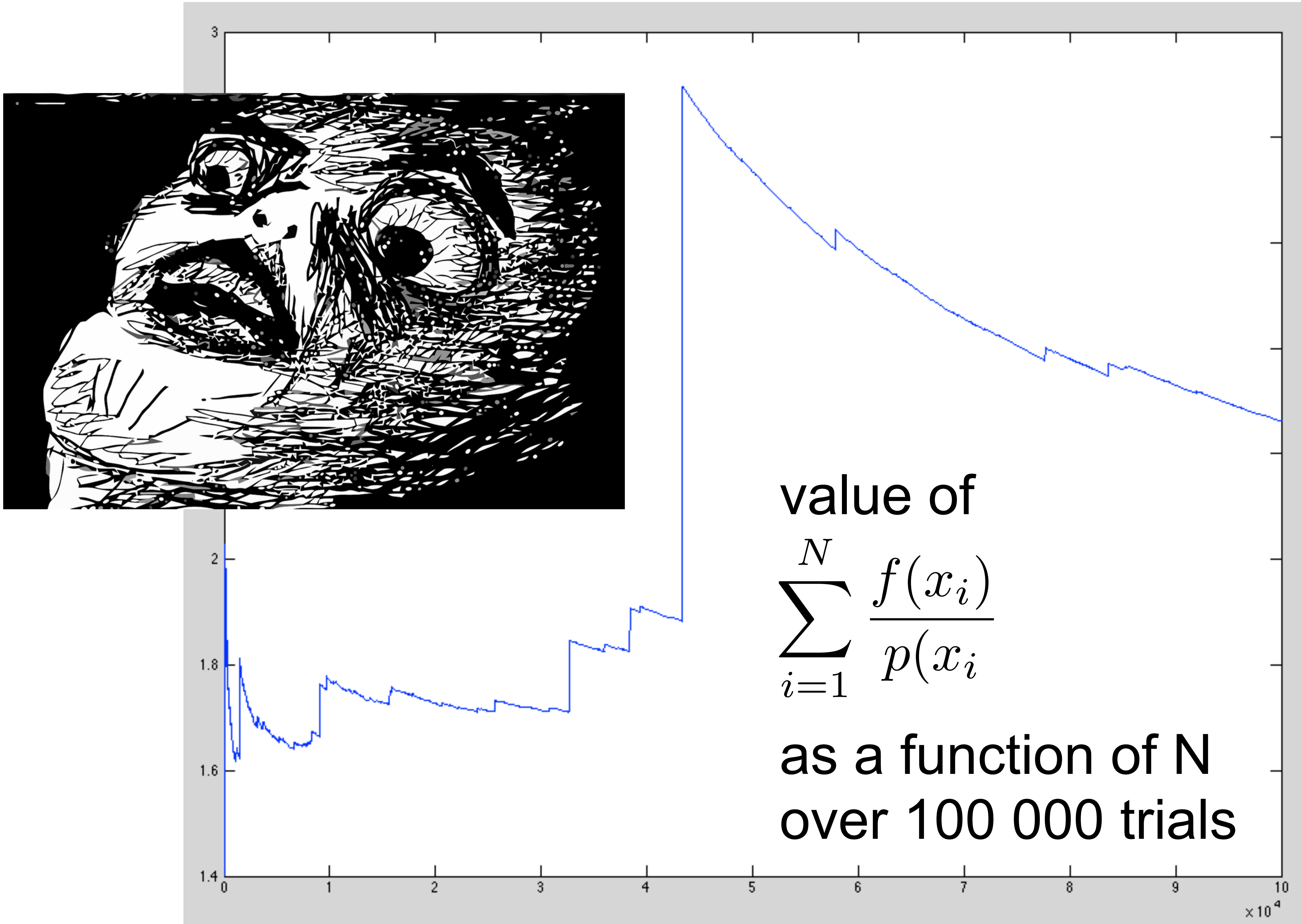


value of

$$\sum_{i=1}^{N} \frac{f(x_i)}{p(x_i)}$$

as a function of N over 100 000 trials

Effect of Spikes on Integral Estimate



value of

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as a function of N
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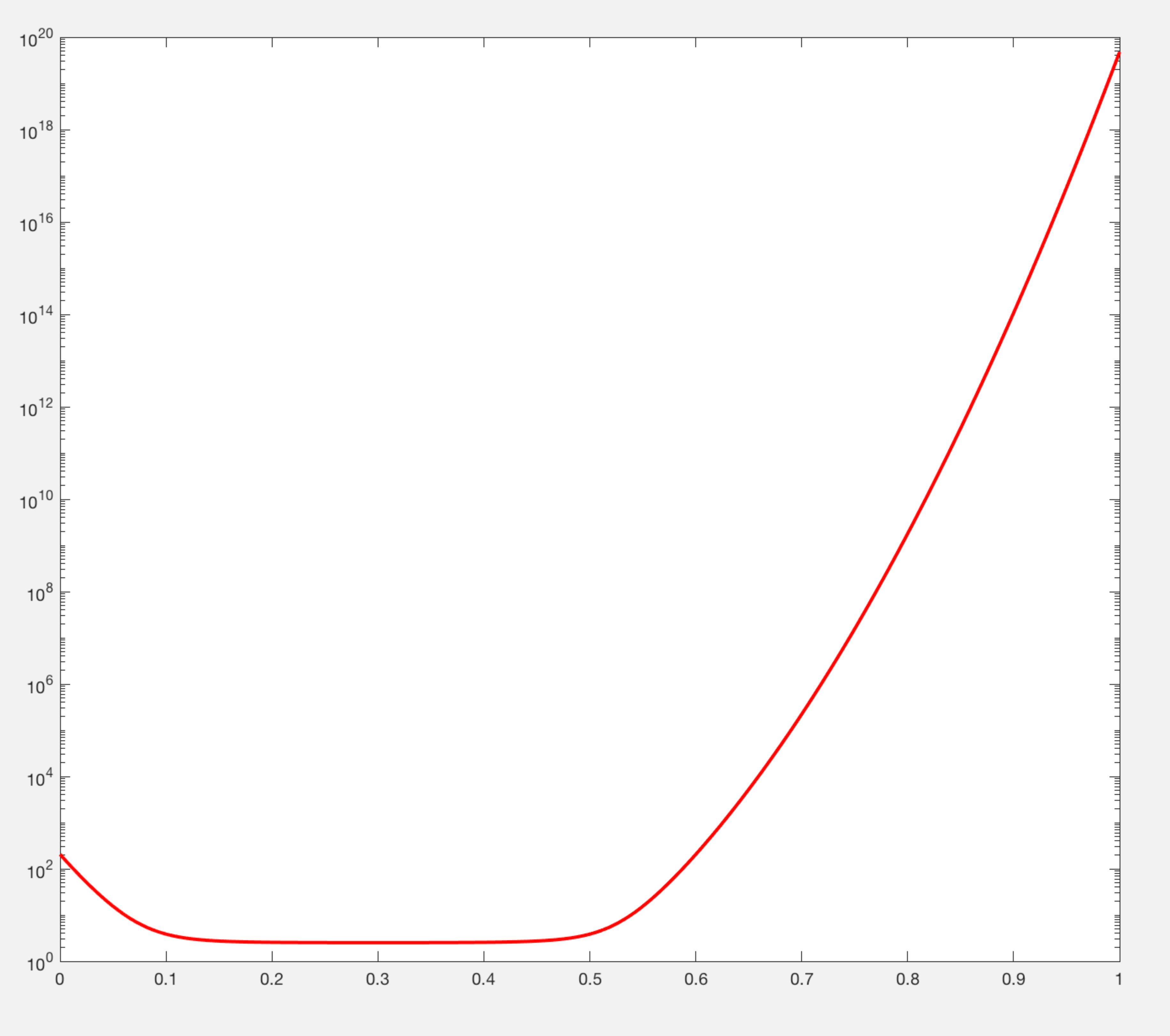
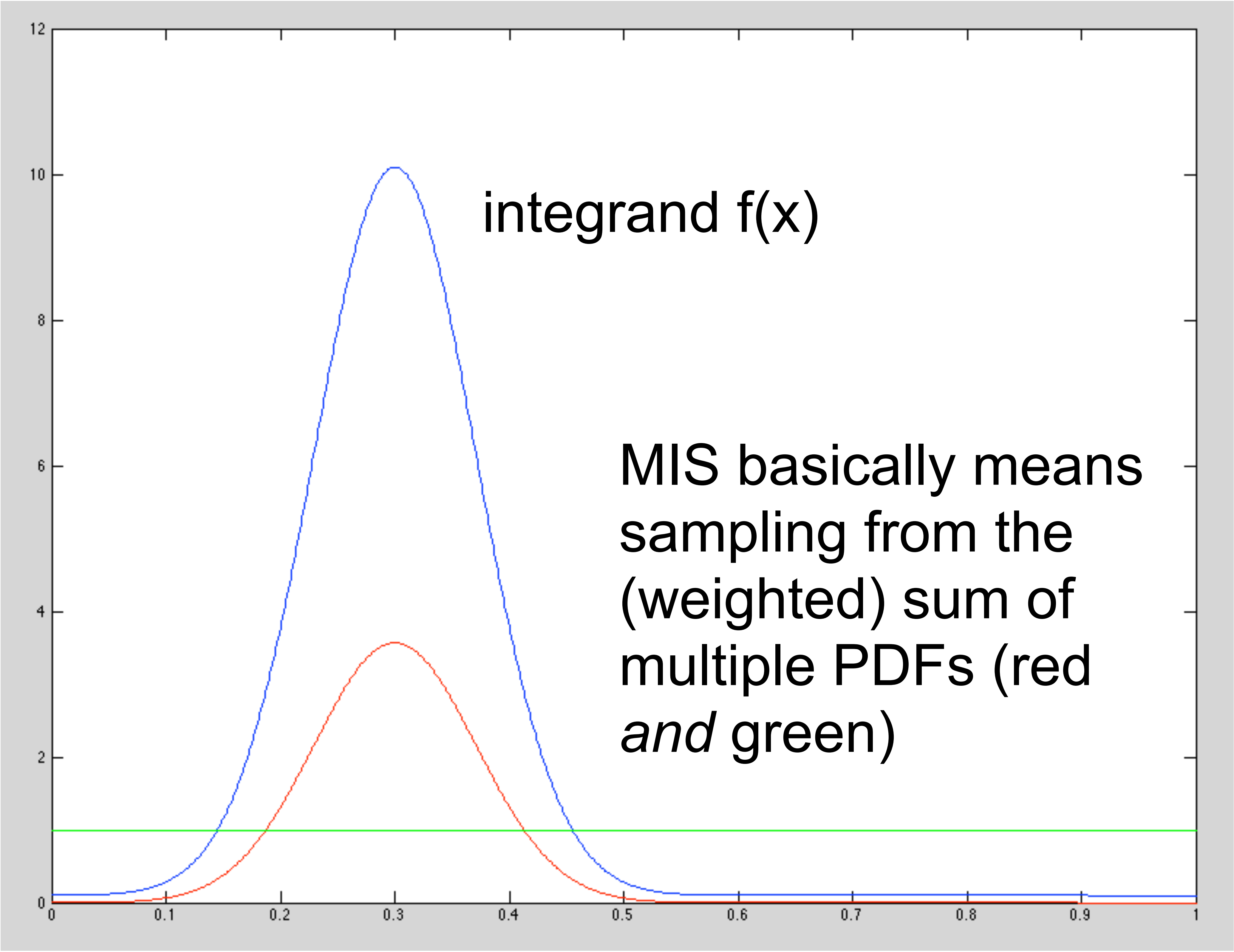
Graph of f/p (note log scale in y!)



Better: Let's mix in a constant PDF



Basic MIS Recipe

- You have M sampling distributions.
- For each sample *i*
  - Pick one distribution at random, let's say it's the *j*th one
    - You can't do much better than equal chances, i.e. using probability p(*j*) = 1/M for all *j* (Veach 1995, Sec. 5.2) (I assume this below.)
  - Draw a sample x_i from the *j*th distribution
  - Compute

$$W_i = \frac{f(x_i)}{\sum_{j=1}^{M} p(j) p_j(x_i)}$$

  - Take the average of the W_i
  - Done!

What's Going On?

- The above process generates samples with the joint distribution

$$\bar{p}(x) = \sum_{j=1}^{M} p(j) p_j(x)$$

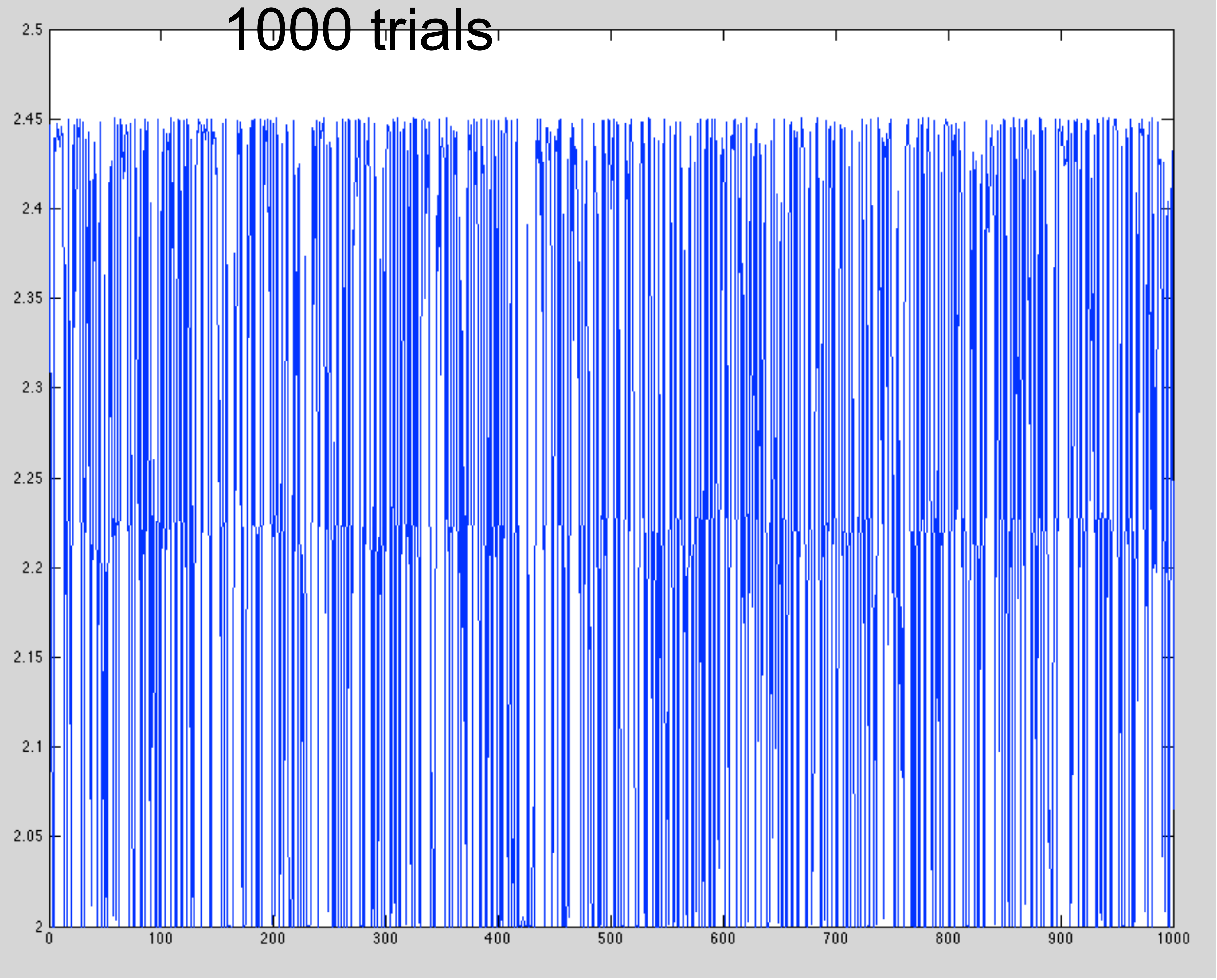
- Hence, we're just computing f/p with this new PDF.
  - Note that the p(j)'s are a discrete distribution, their sum must be 1!

- *This is an unbiased estimate, just like regular MC.*

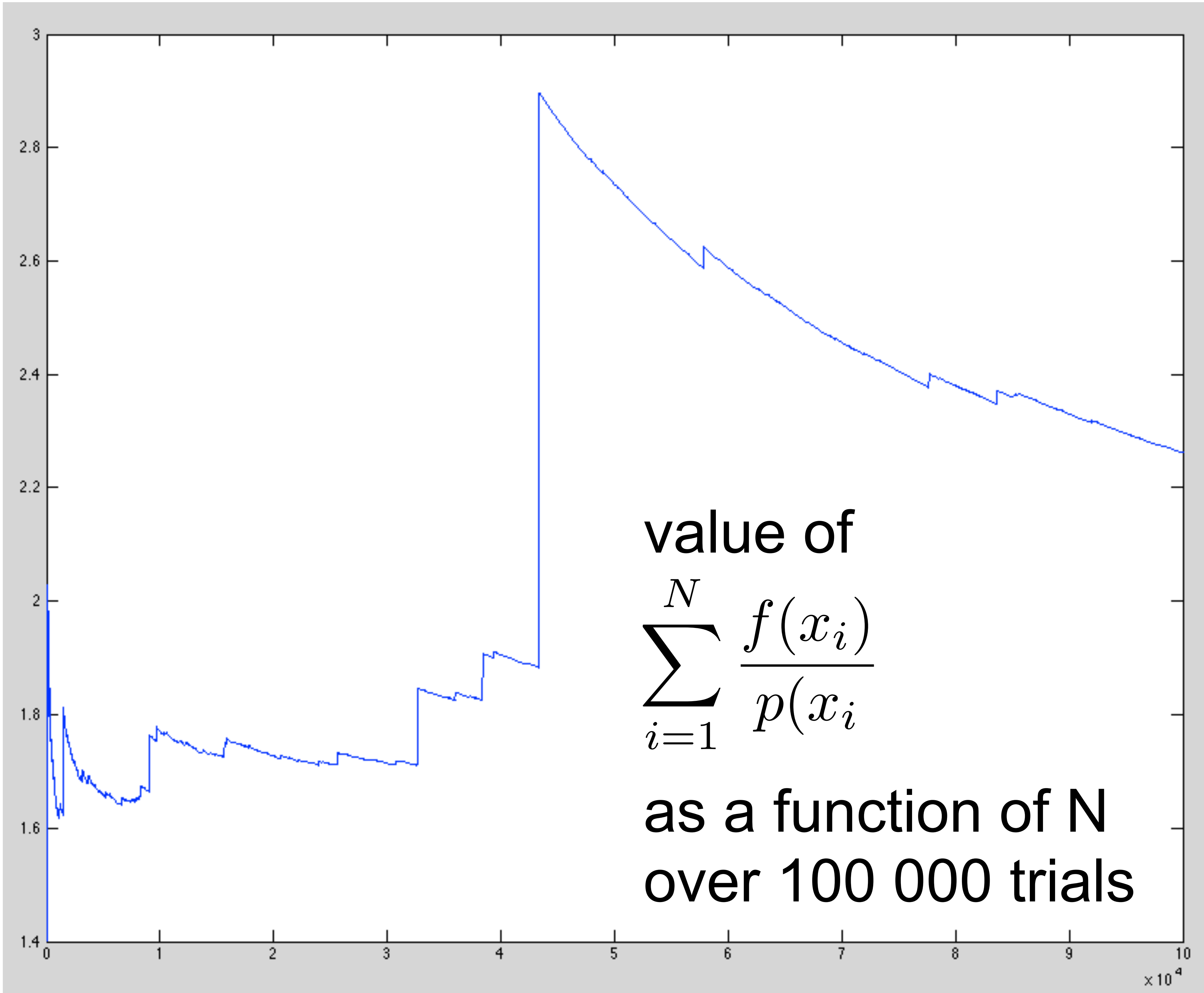
Ha!

sample weight $f(x_i)/p(x_i)$ over 1000 trials

–

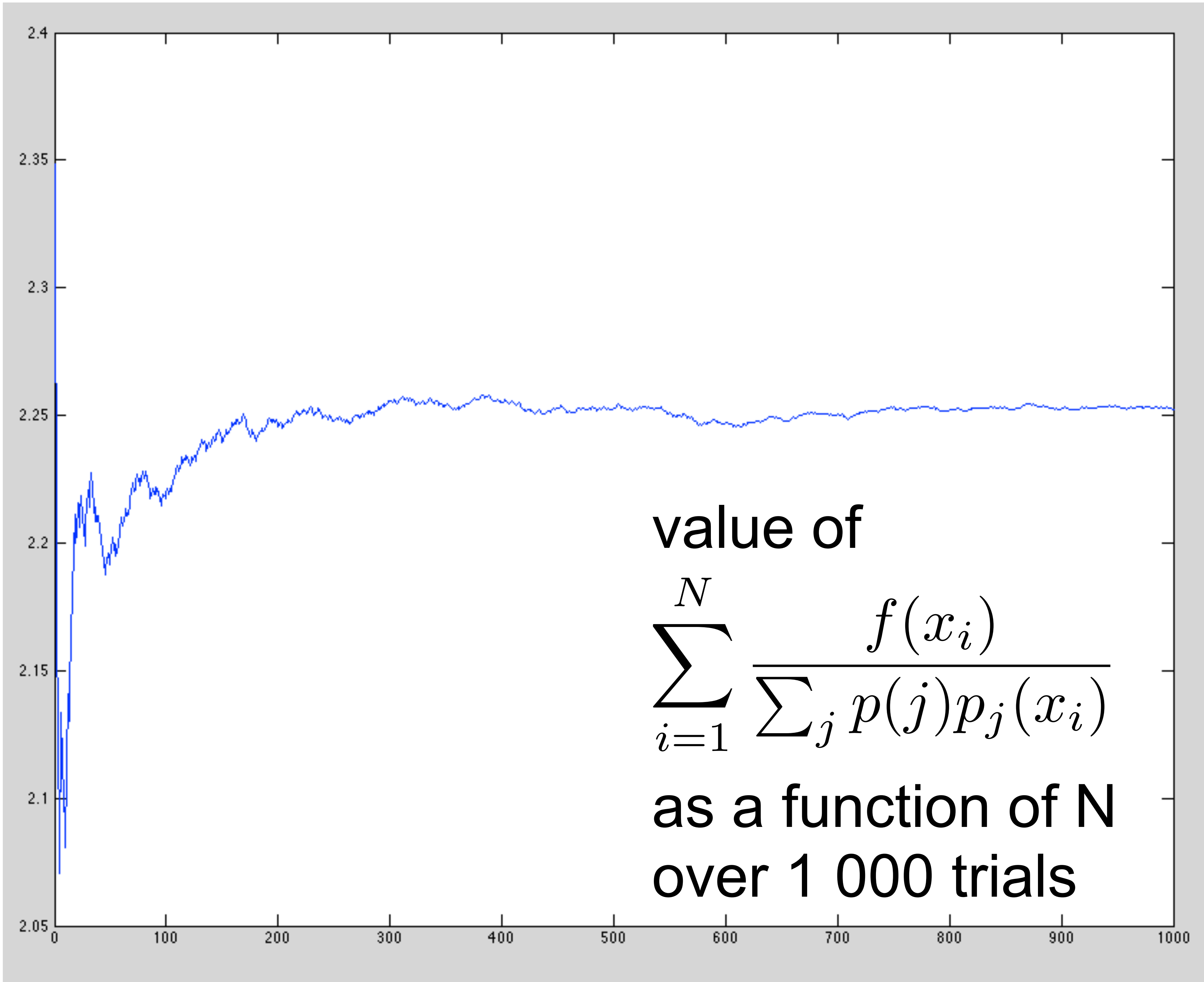


Integral Estimate, No MIS, 100k samples



Integral Estimate, MIS, 1k samples

(100x fewer than previous terrible non-MIS result)



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Integral Estimate, MIS, 1k samples

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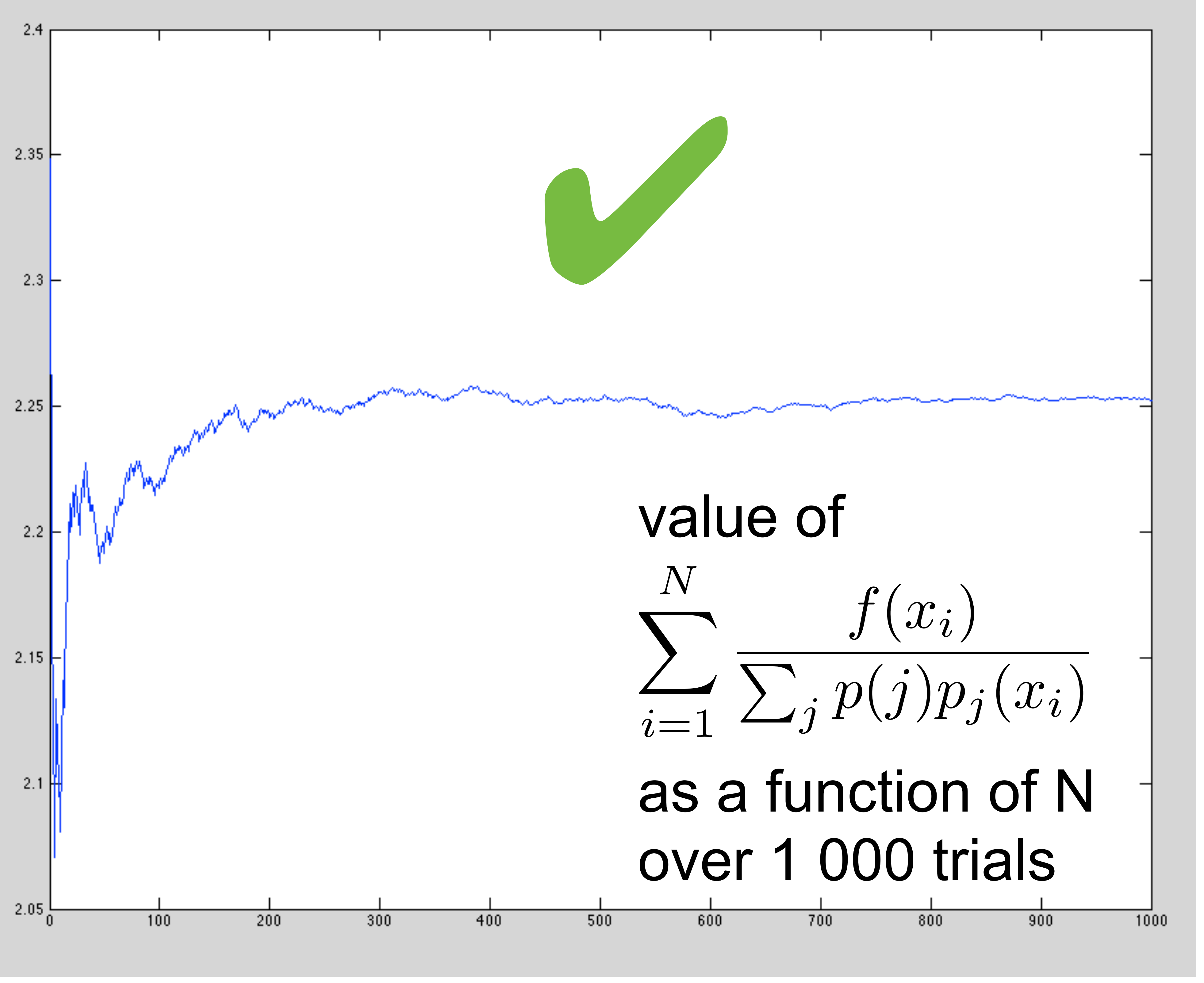


Bells And Whistles

- This is the basic intuition and approach.
- Veach's 1995 paper contains a long treatment on how to choose the relative weighting between the PDFs and more general ways of constructing $\bar{p}(x)$ based on the individual distributions.
- However, we won't go into this. This process is really general and applies wherever MC can be applied.

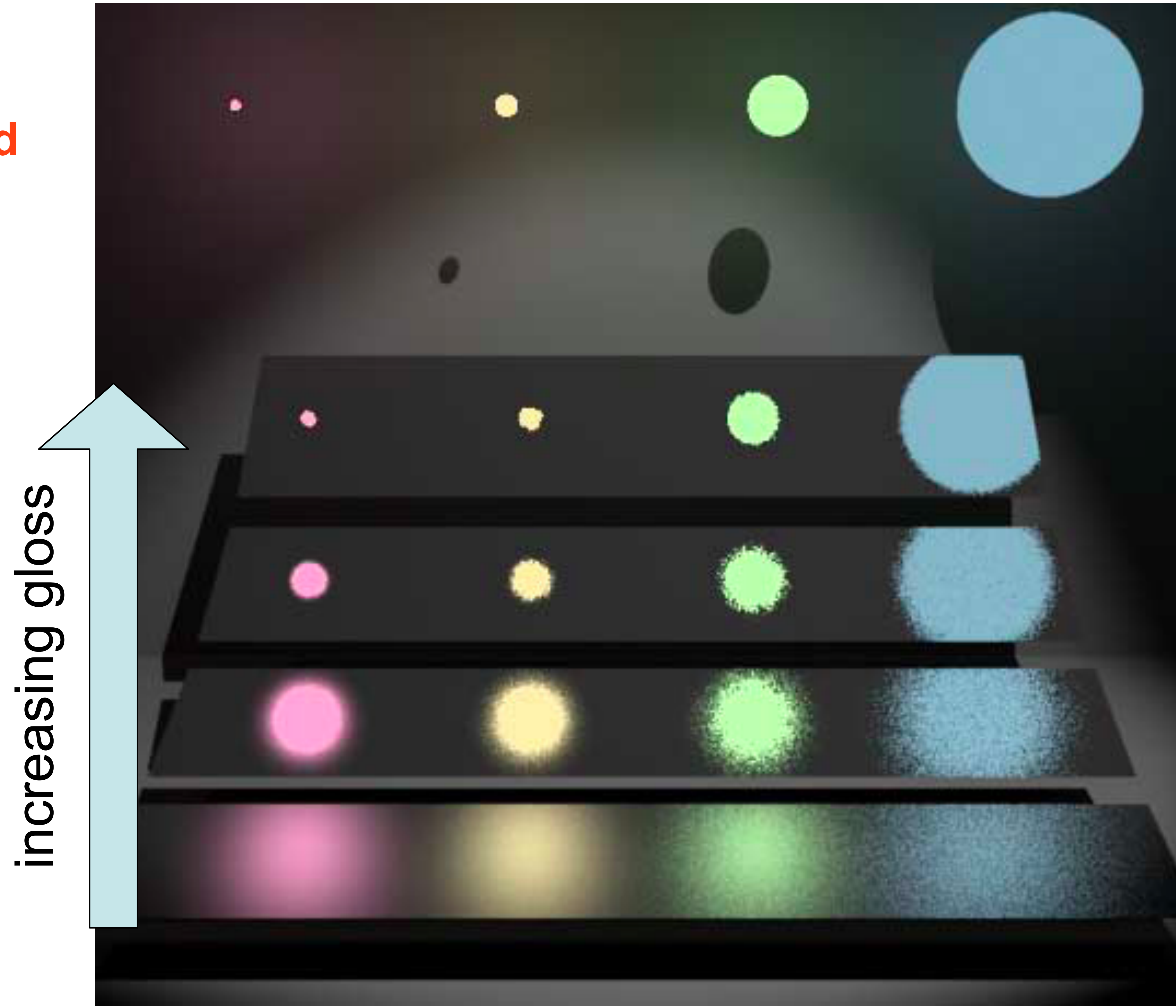
Example: Use in a Path Tracer

- Apart from the direct eye ray, our basic path tracer only accounts for light through shadow rays
  - If the extension ray, which is sampled from the BRDF, hits a light source, we set its contribution to zero.
  - Is this the best we can do?

- Indeed, we can repurpose the extension ray for another purpose: we'll try to make the light connection by both light sampling and BRDF sampling.
  - However we deterministically use both samplers, no random picking.

Multiple Importance Sampling ✔

MIS = Sample both ways and optimally combine the samples



Questions?

estudibasic, Rendered using Maxwell