

# Monte Carlo Integration II: Multiple Importance Sampling



CS-E5520 Spring 2022  
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with many slides from Frédo Durand

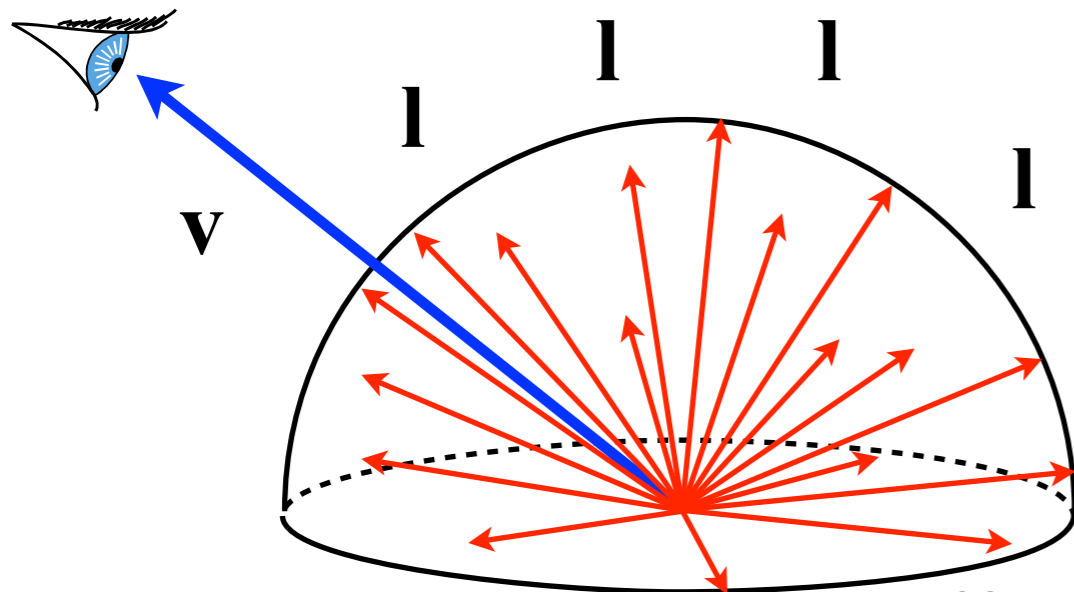
# Recap: Reflectance Equation

$$L(x \rightarrow \mathbf{v}) = \leftarrow \text{outgoing radiance}$$

$$\int_{\Omega} f_r(x, \mathbf{l} \rightarrow \mathbf{v}) L(x \leftarrow \mathbf{l}) \cos \theta \, d\mathbf{l}$$

integral over hemisphere  
 BRDF  
 incoming radiance  
 cosine of incident angle

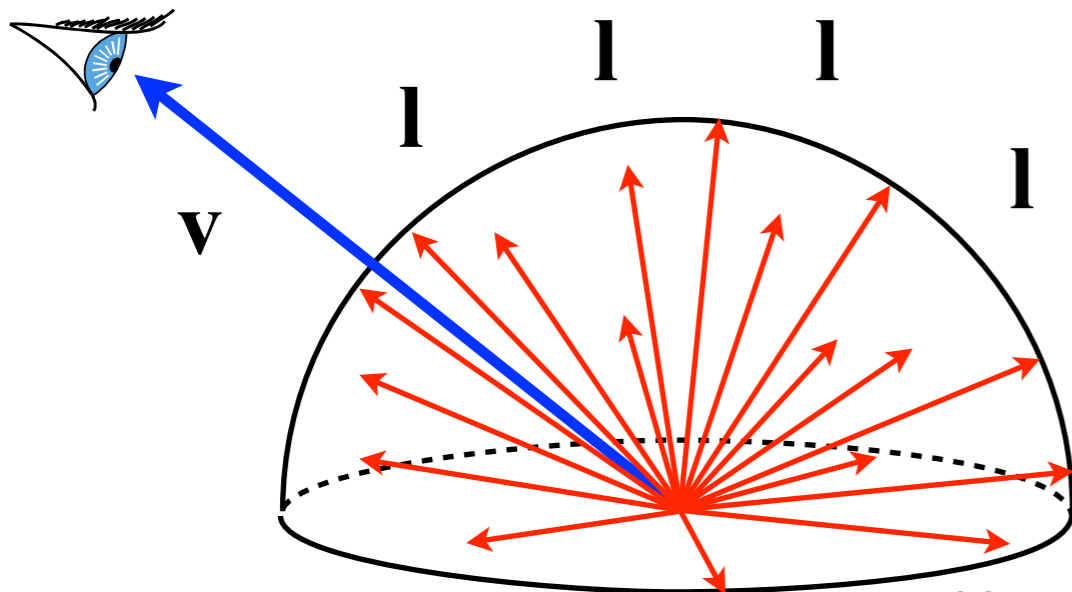
$L_{in} * \cos =$   
 incident differential irradiance



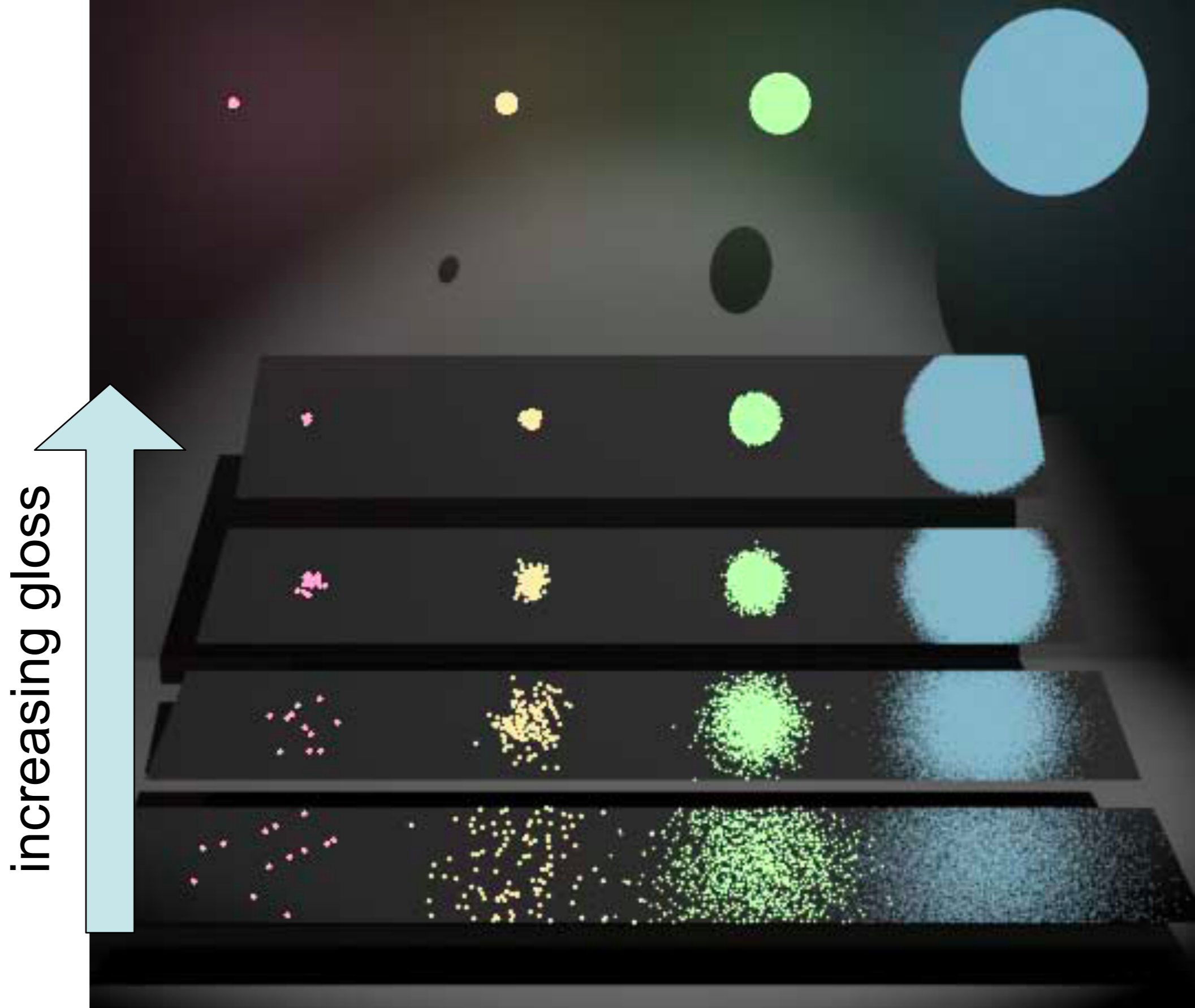
# Recap: Reflectance Equation

$$L(x \rightarrow \mathbf{v}) =$$

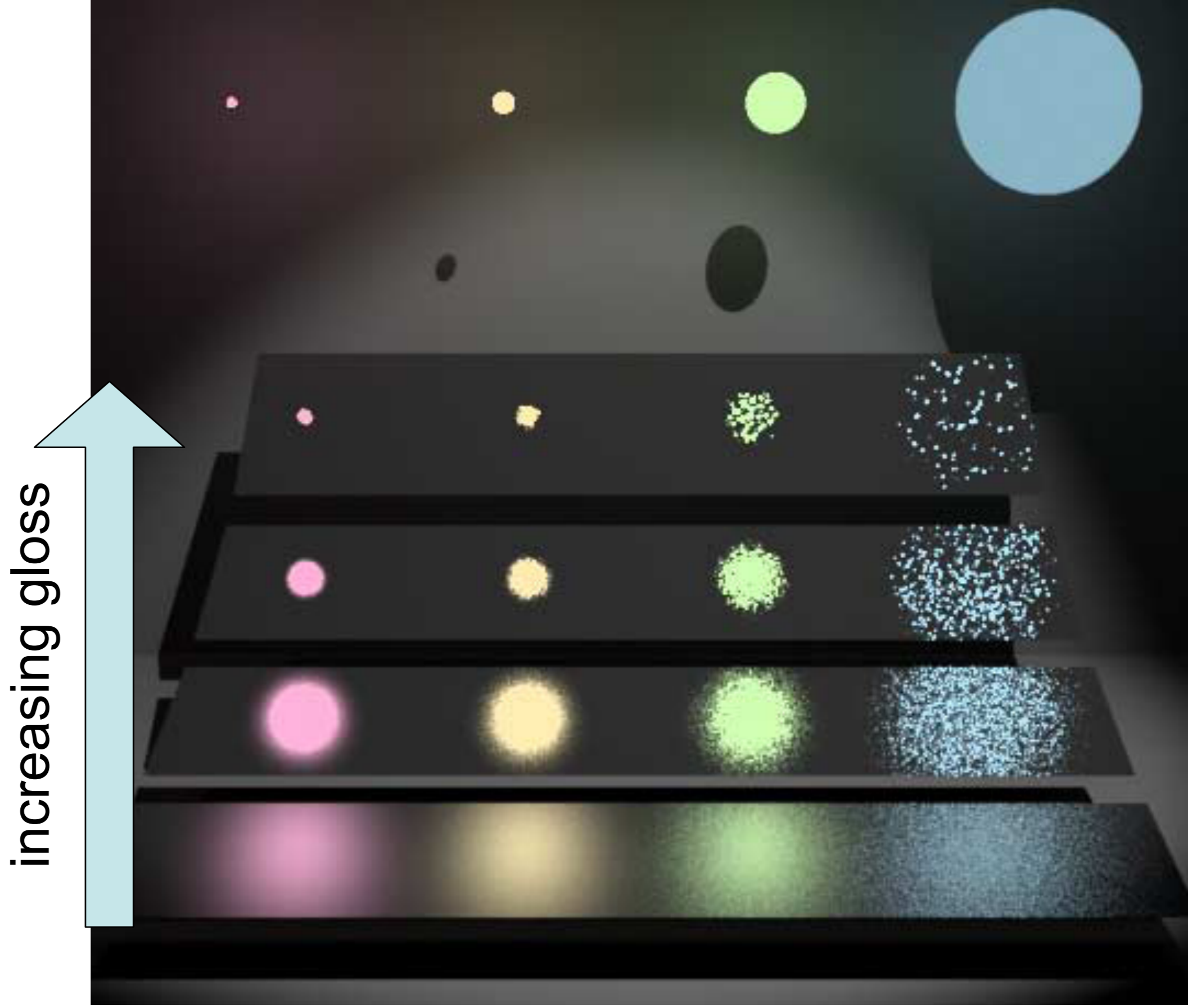
$$\int_{\Omega} f_r(x, \mathbf{l} \rightarrow \mathbf{v}) L(x \leftarrow \mathbf{l}) \cos \theta \, d\mathbf{l}$$



# Imp. Sampling According to BRDF



# Imp. Sampling According to Light

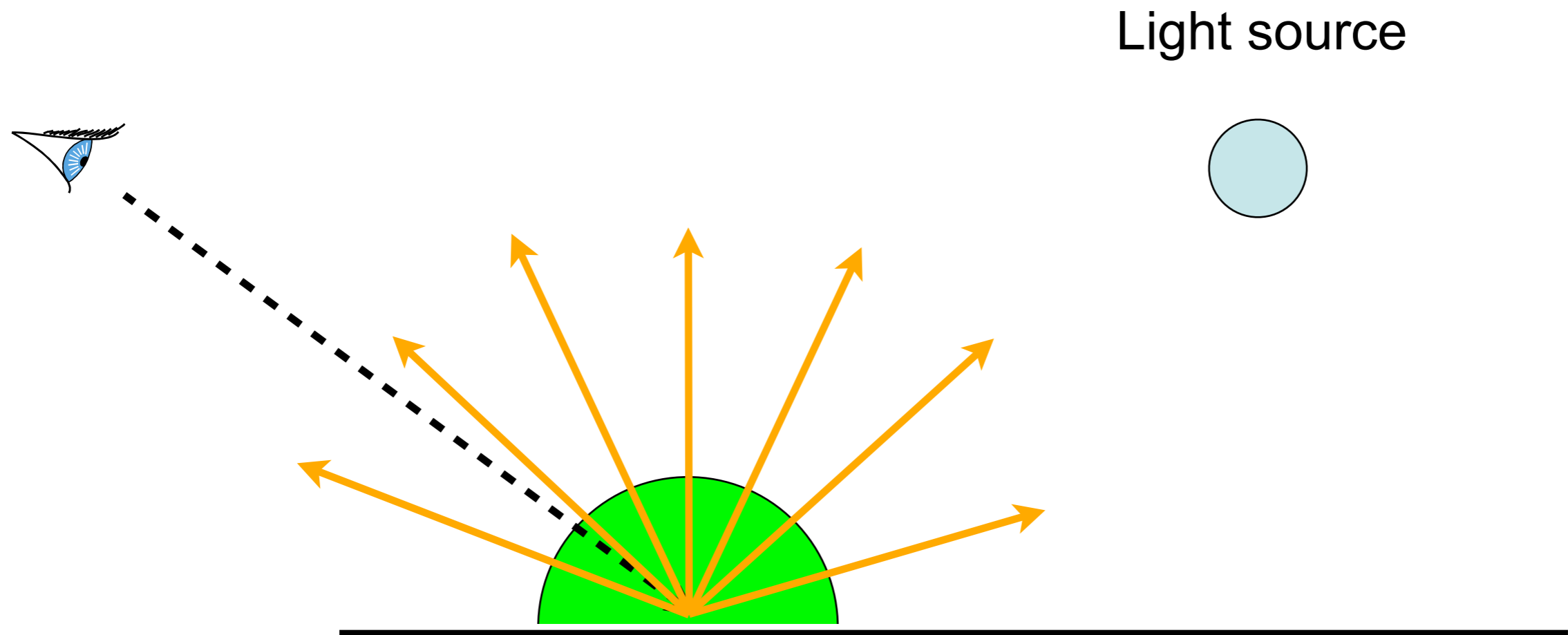


# Multiple Importance Sampling (MIS)

- If integrand  $f$  has a complex shape that consists of distinct features that are easy to sample from individually, we can use multiple PDFs and combine them in a nice way so that we got lower variance
  - See Veach and Guibas 1995

# What's Going on Here?

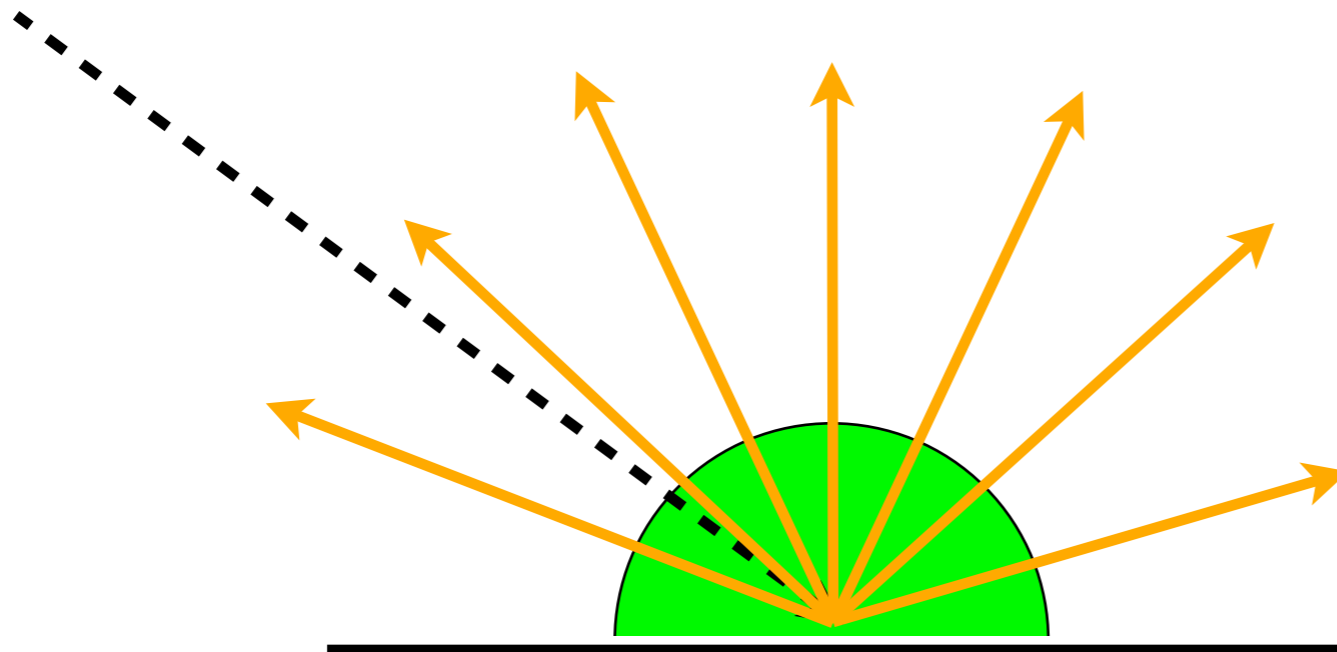
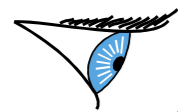
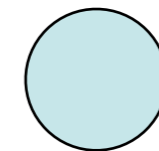
- Dull gloss/diffuse surface, importance sample BRDF



# What's Going on Here?

- Dull gloss/diffuse surface, importance sample BRDF

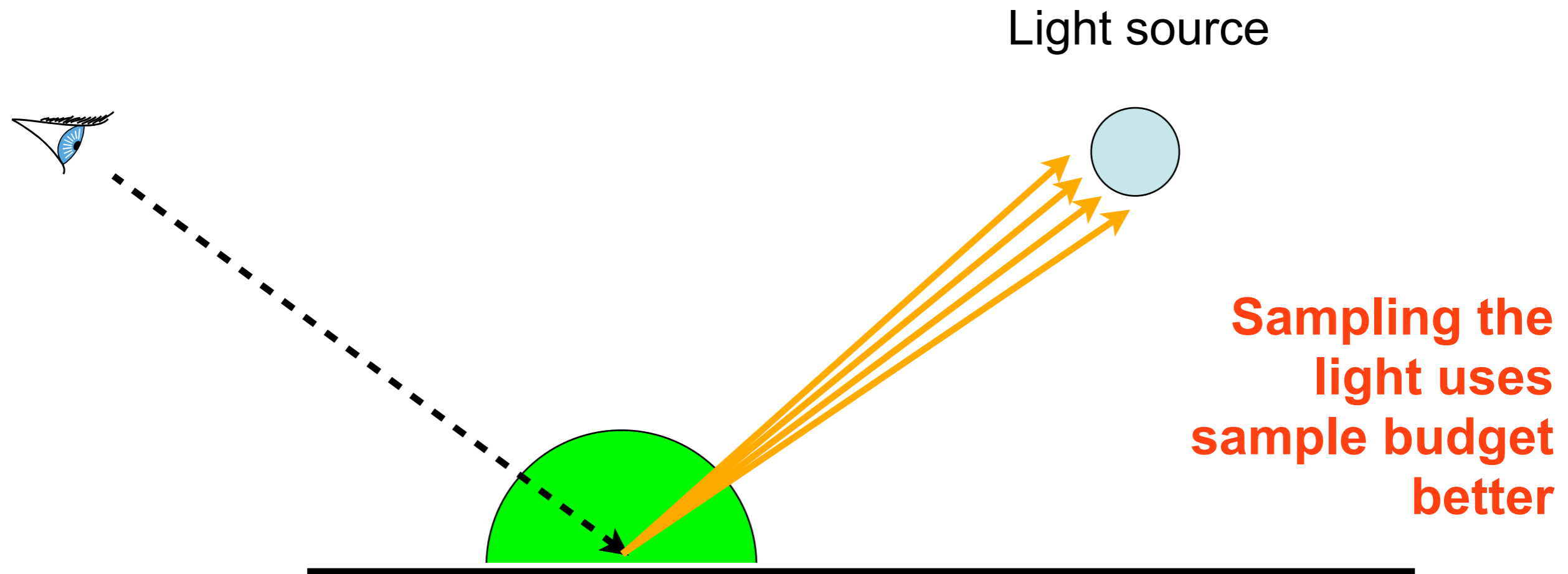
Light source



**Only few directions actually carry light, so we are using our samples poorly**

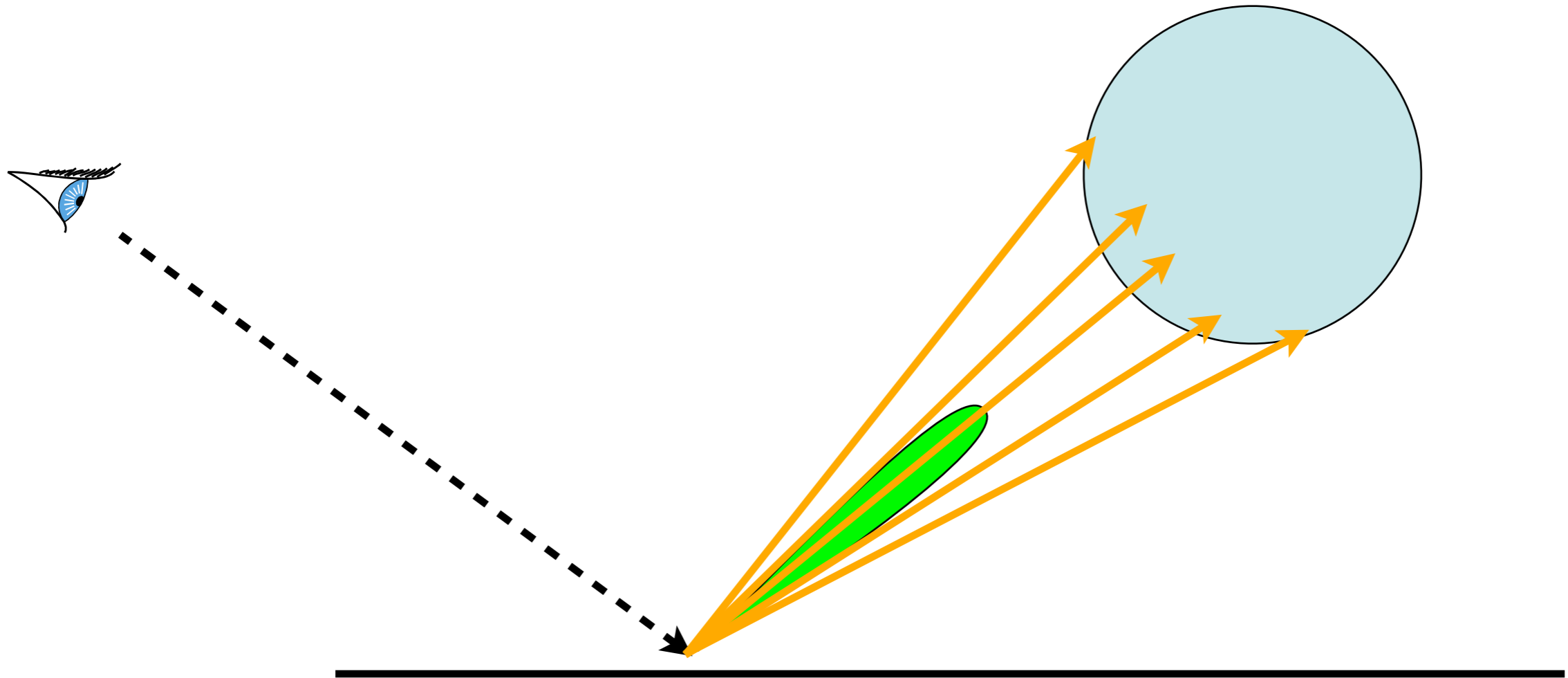
# Here Makes Sense to Sample Light

- Dull gloss/diffuse surface, importance sample light



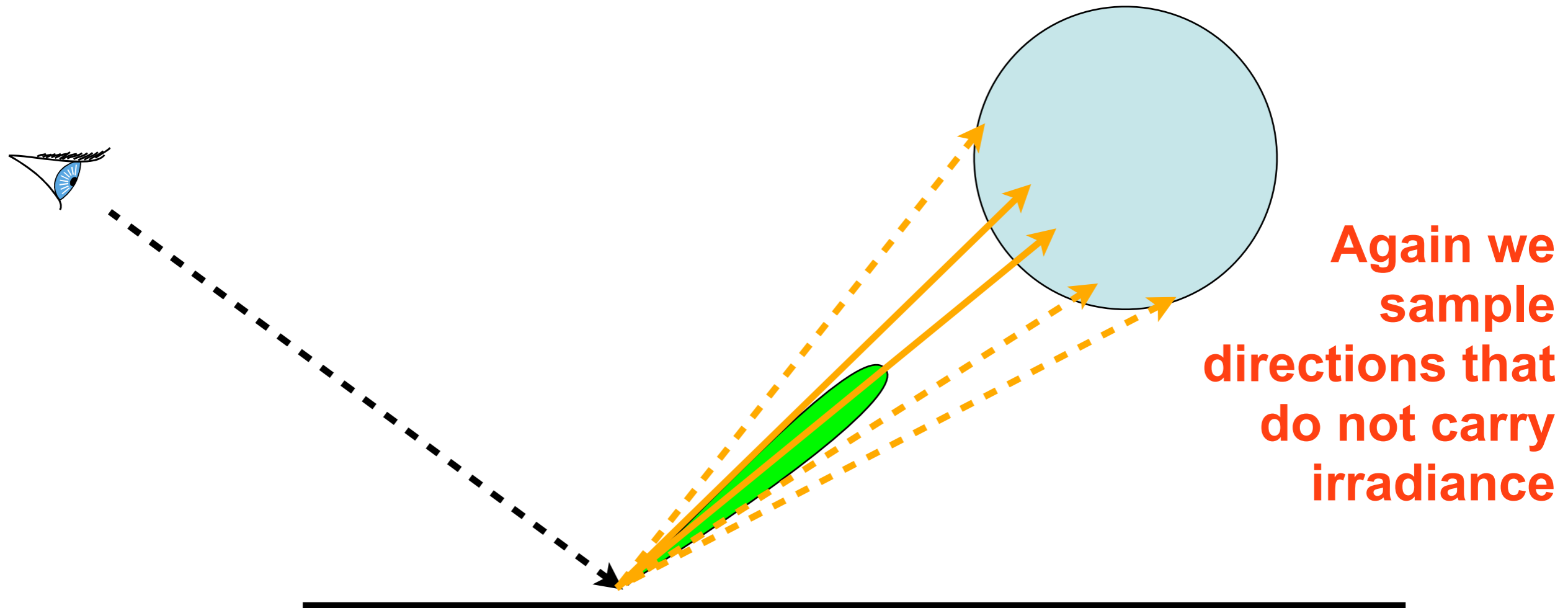
# What's Going on Here?

- Highly glossy surface, narrow lobe, large light source, importance sample light



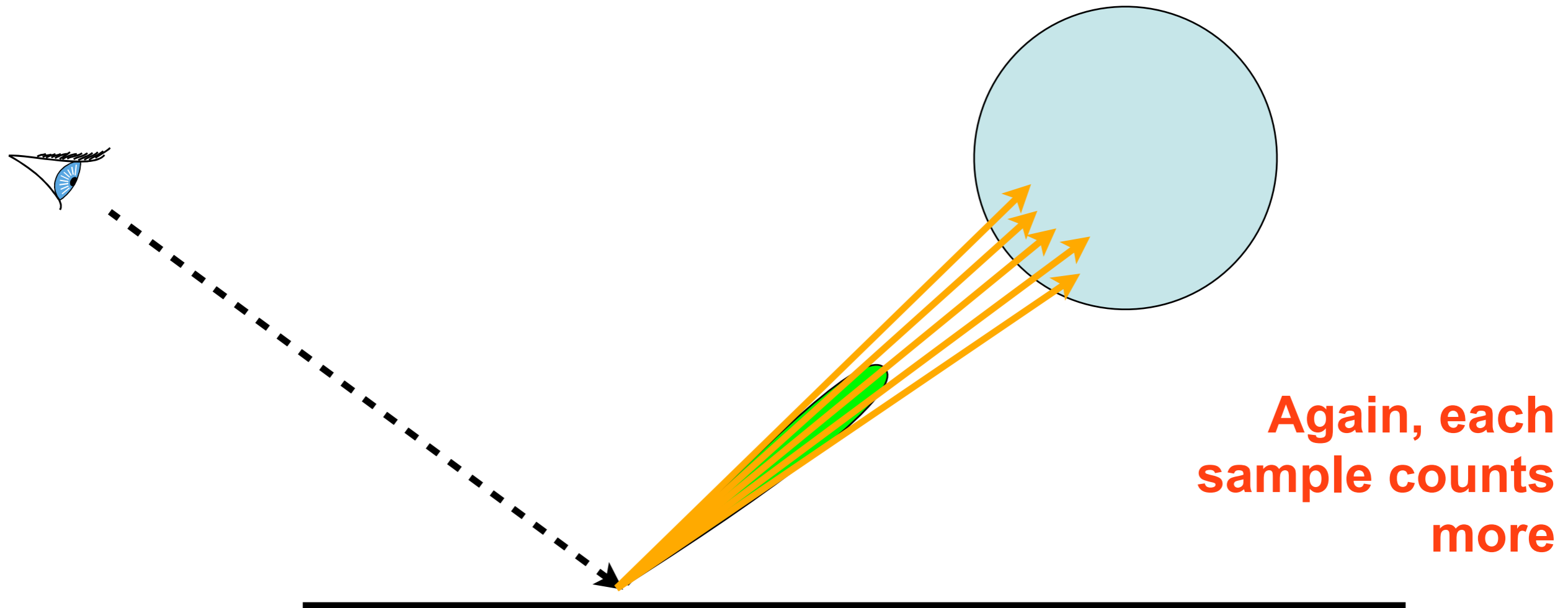
# What's Going on Here?

- Highly glossy surface, narrow lobe, large light source, importance sample light



# Here, Better to Sample BRDF

- Highly glossy surface, narrow lobe, large light source, importance sample light

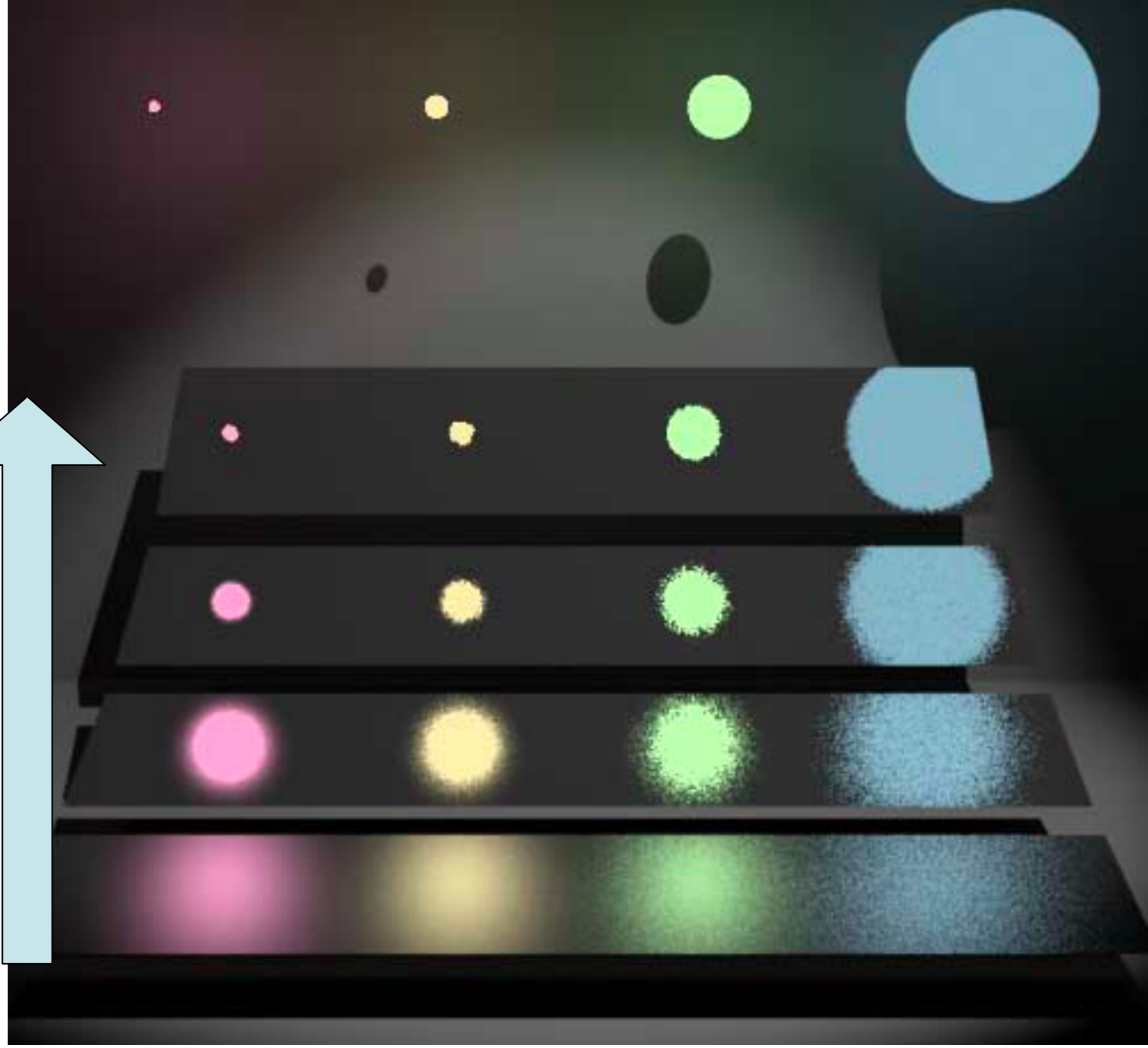


# Multiple Importance Sampling

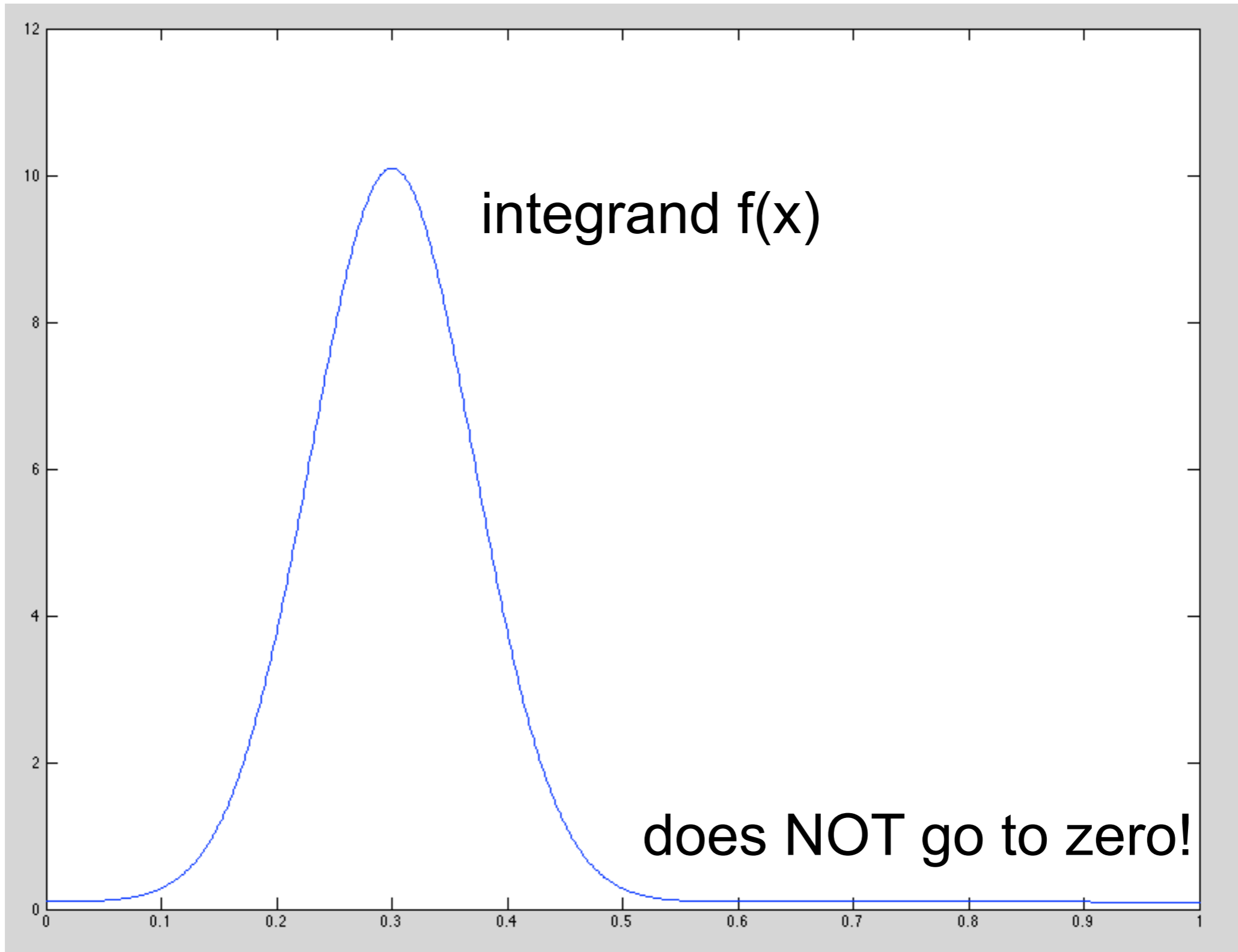


**MIS = Sample both ways and optimally combine the samples**

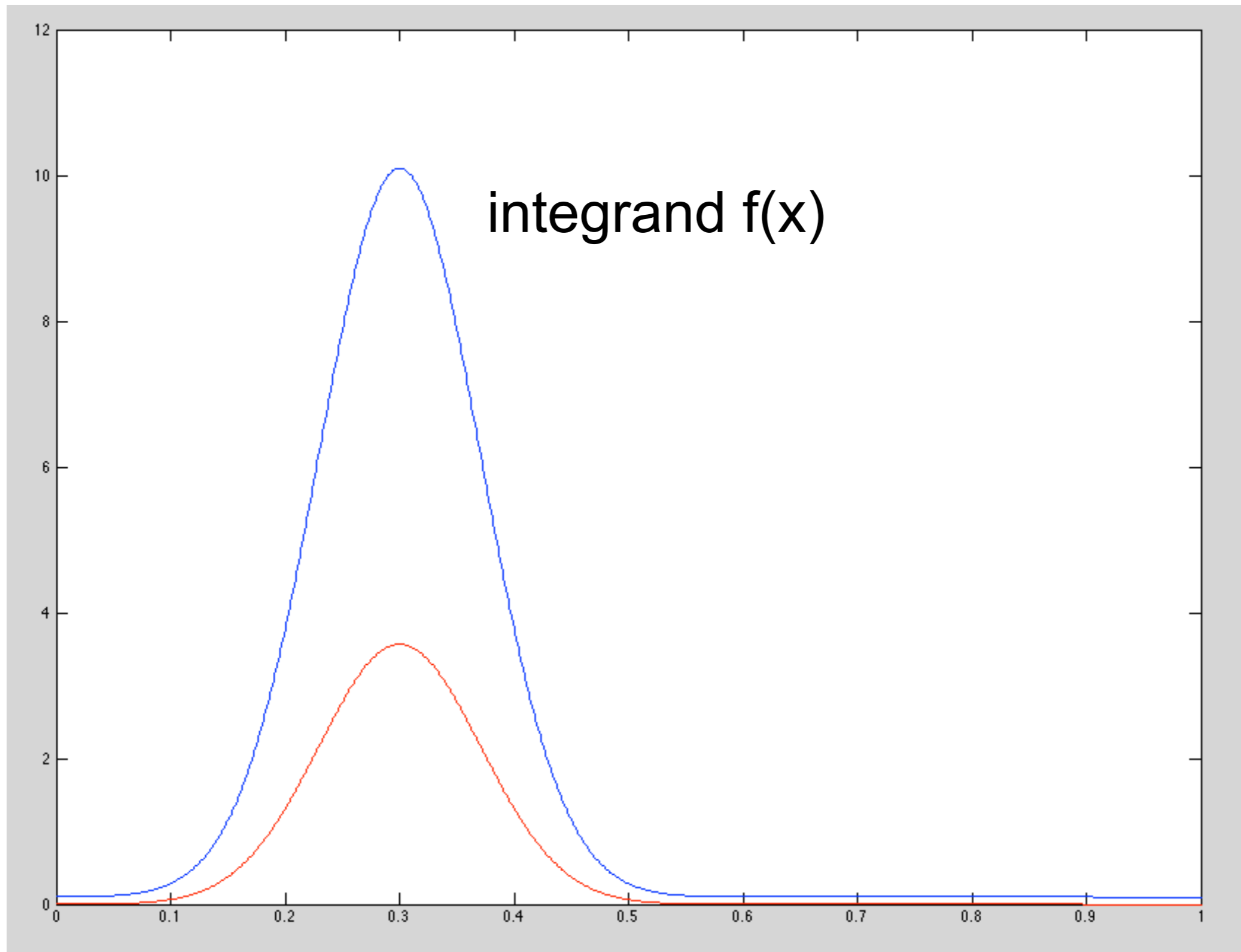
increasing gloss



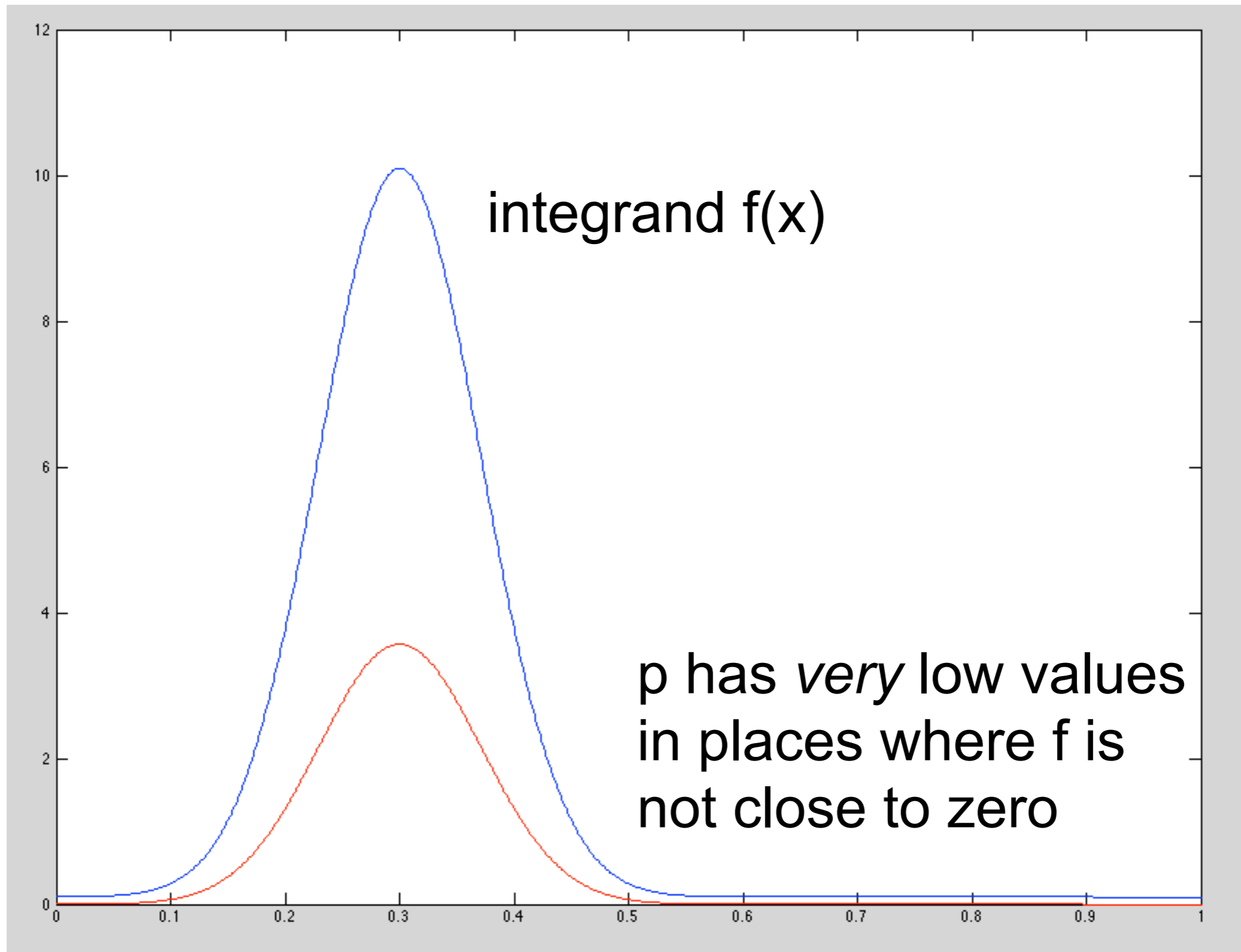
# Ok, how do you do it?



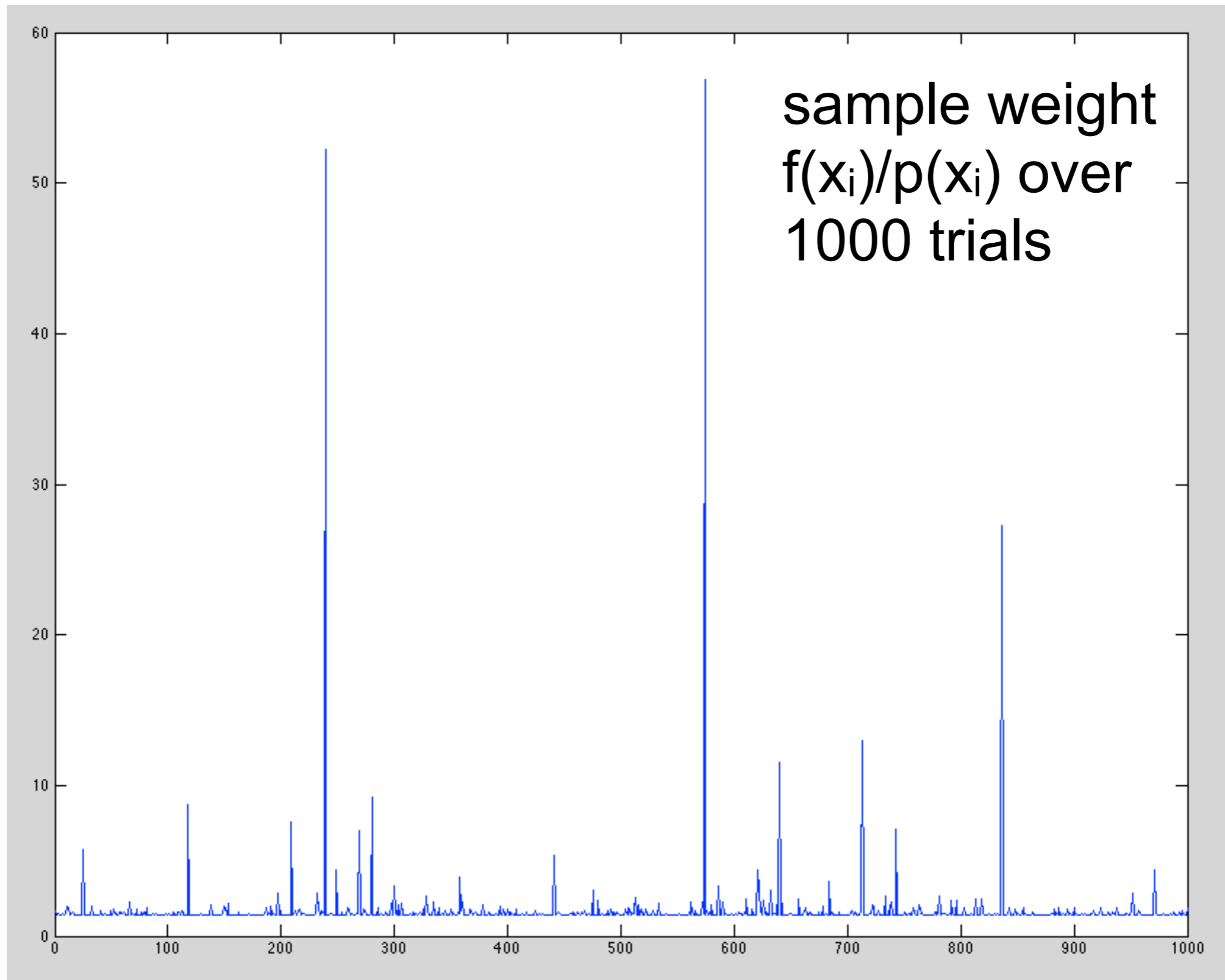
# Why is the Red Gaussian bad for IS?



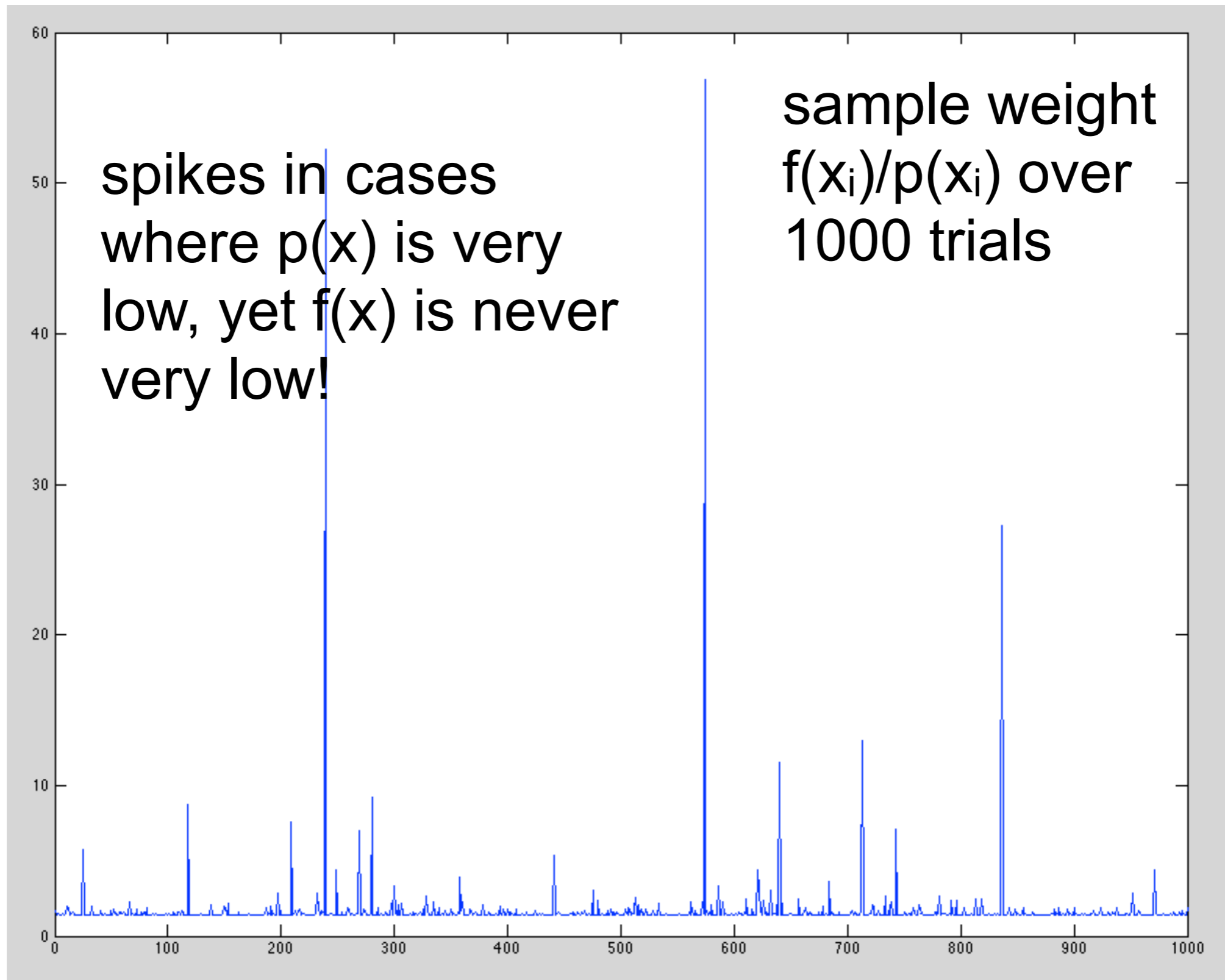
# Why the Red Gaussian is *bad* for IS



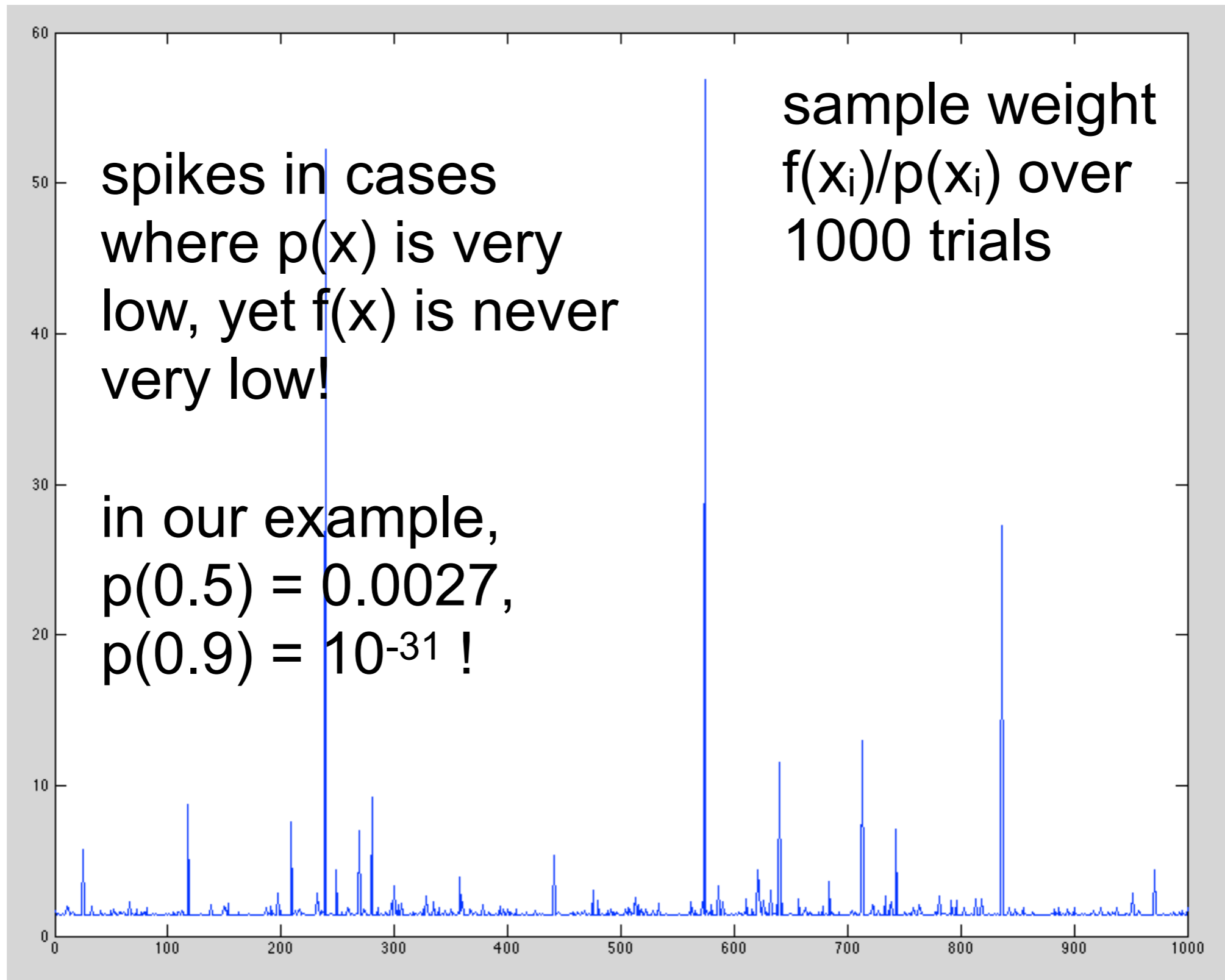
# Why This Matters



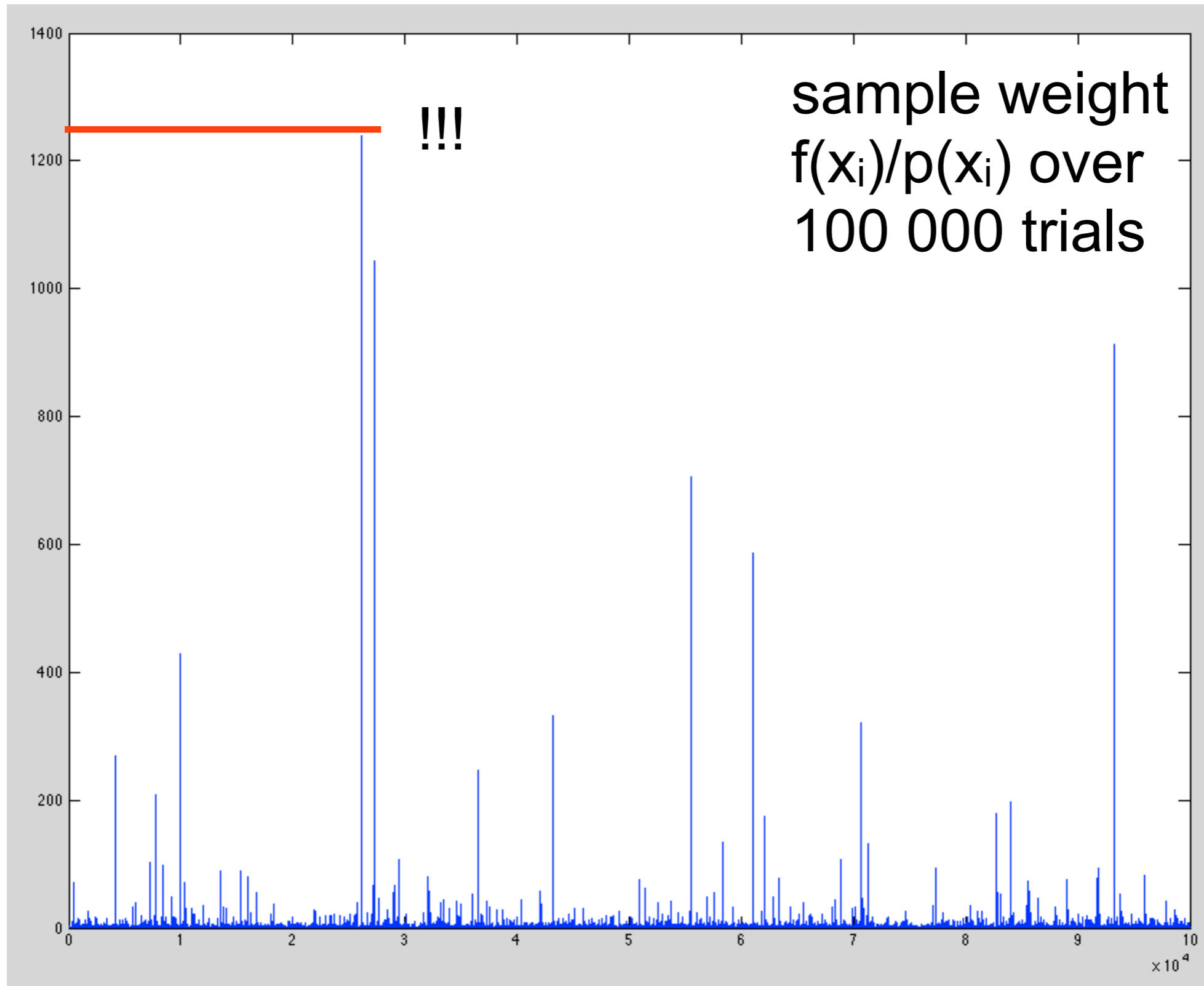
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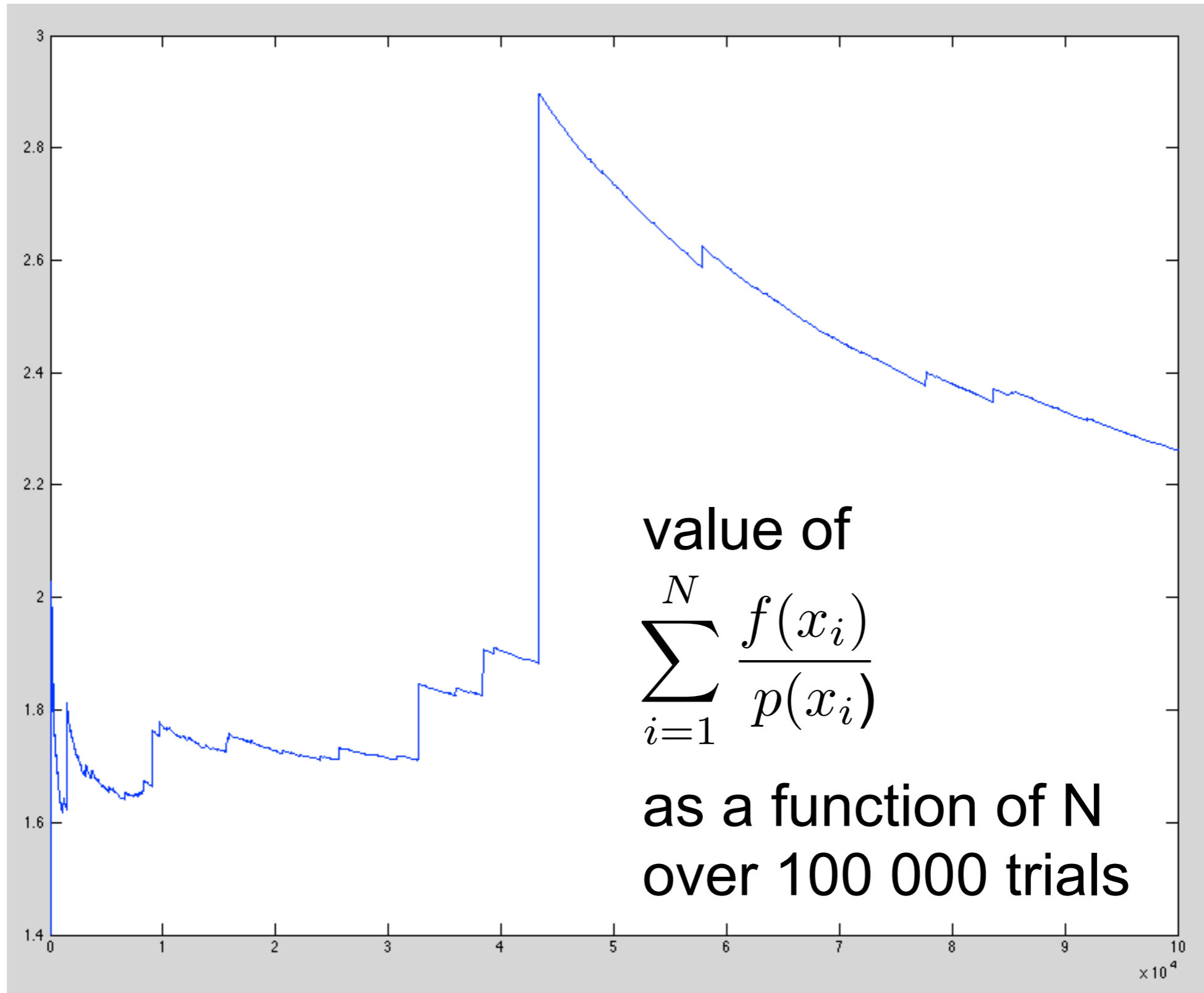
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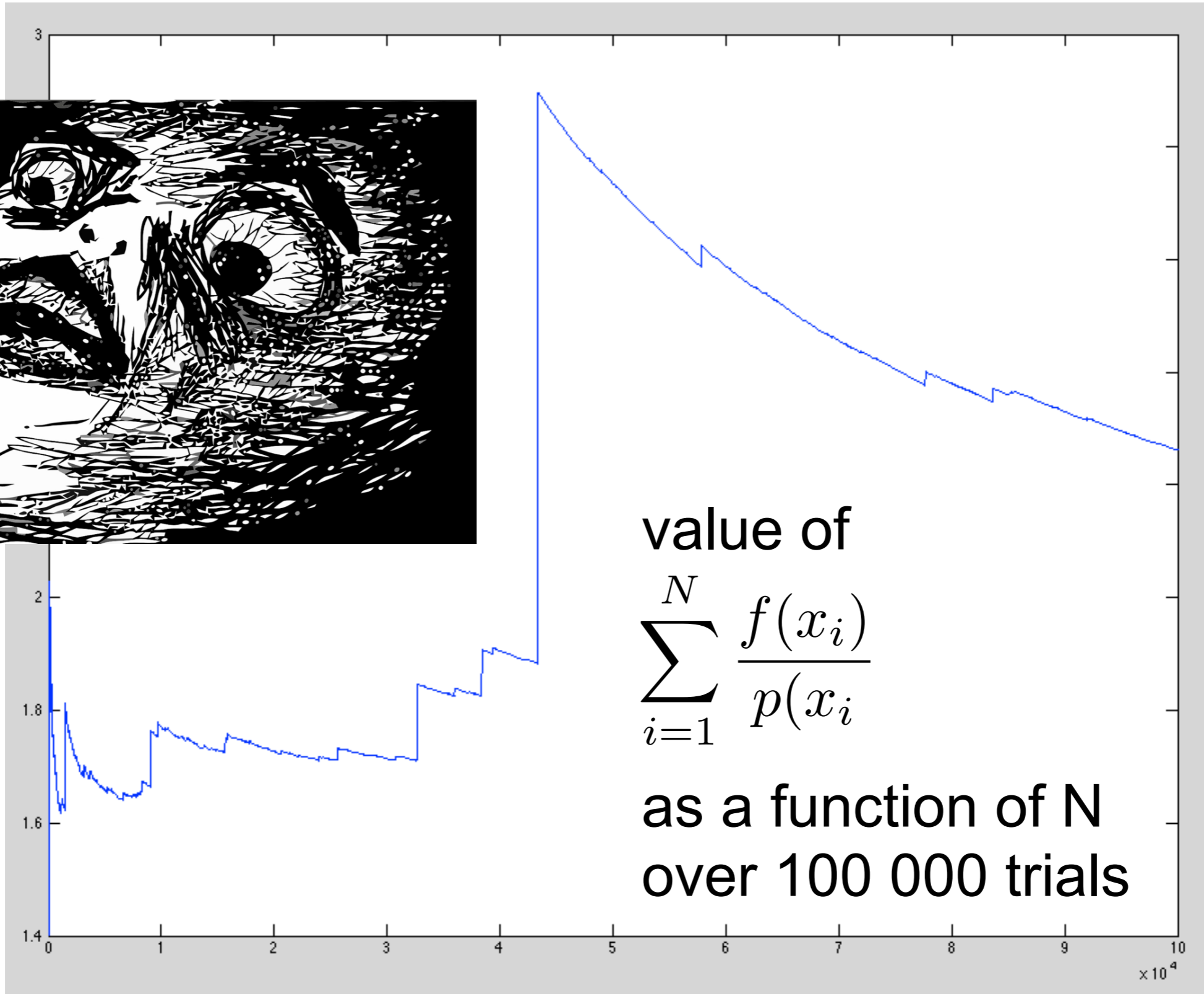
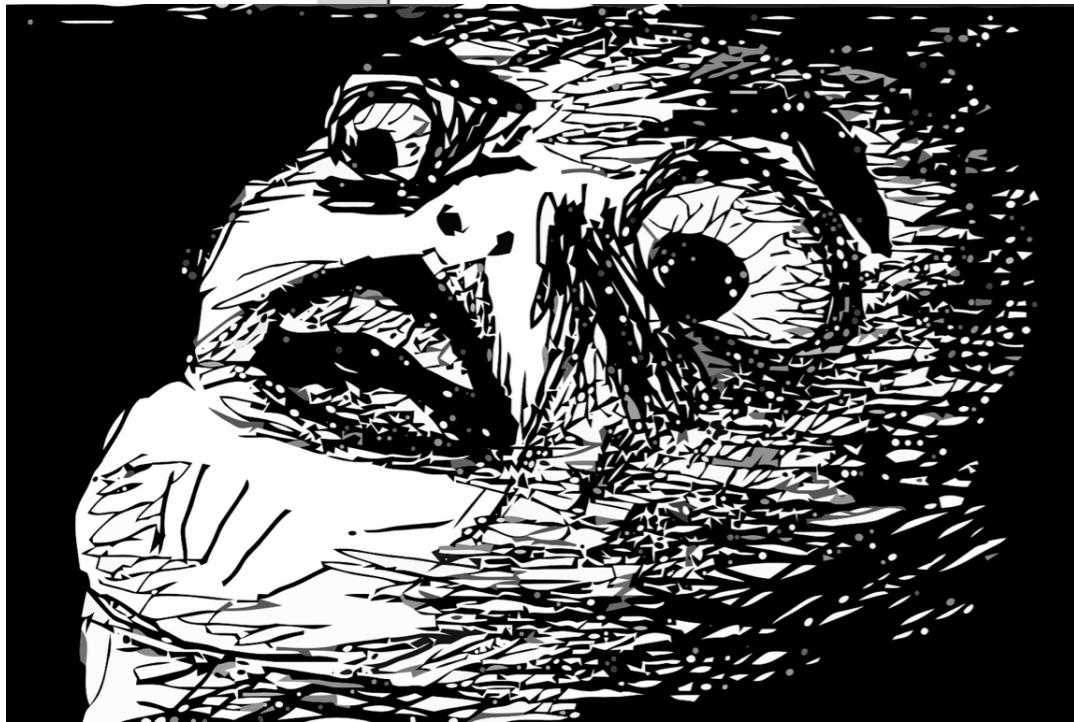
# Spikes get worse with higher N



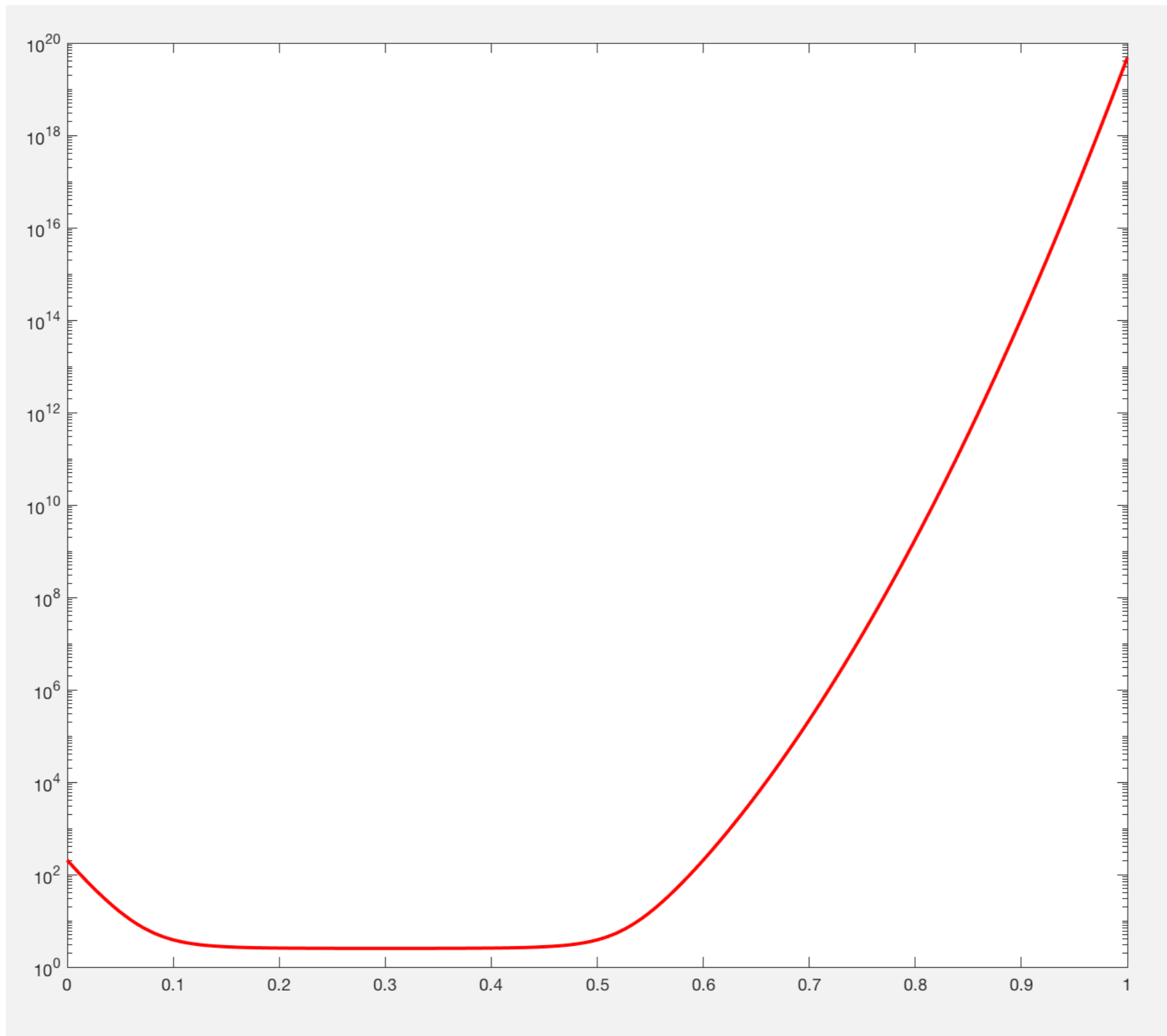
# Effect of Spikes on Integral Estimate



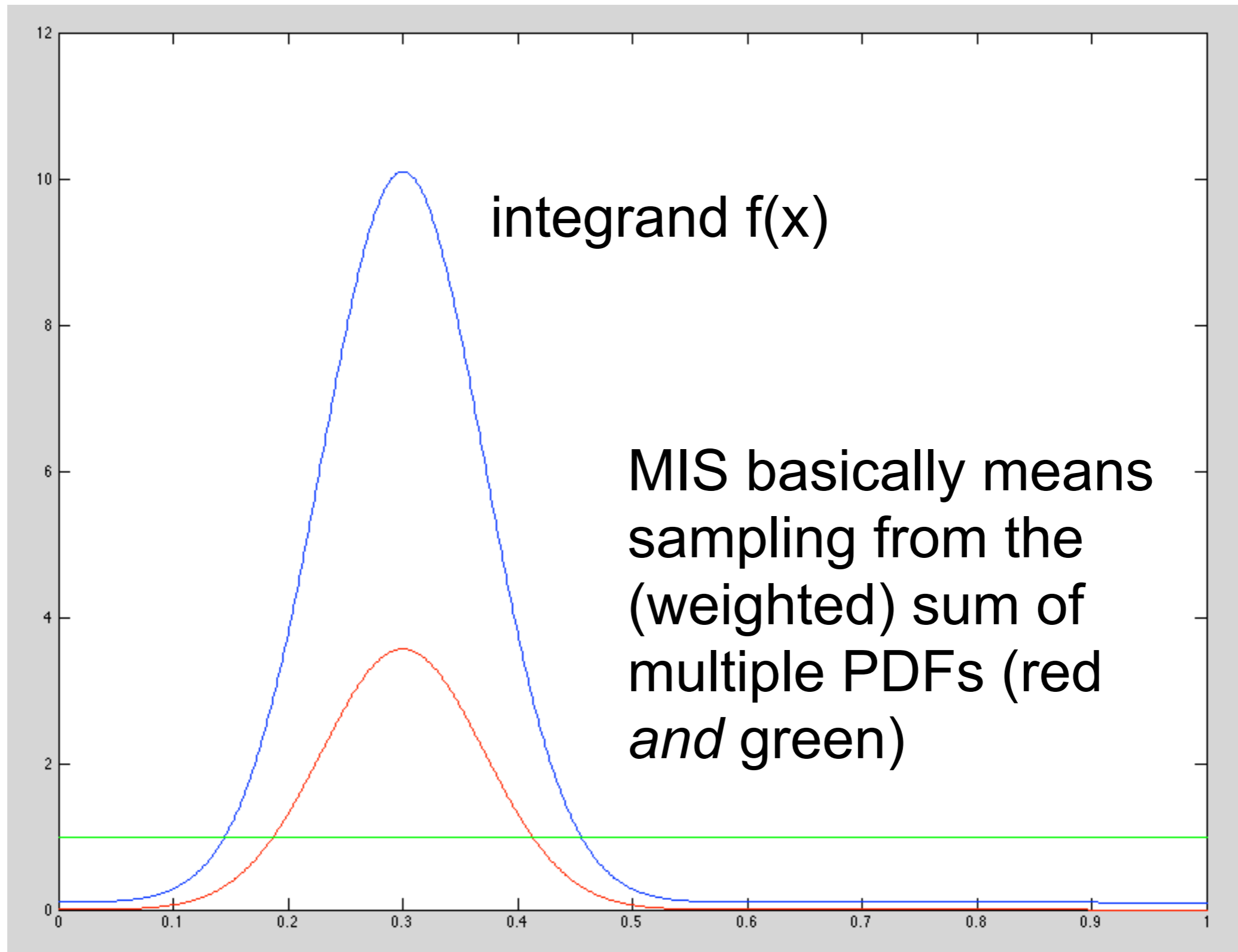
# Effect of Spikes on Integral Estimate



# Graph of $f/p$ (note log scale in y!)



# Better: Let's mix in a constant PDF



# Basic MIS Recipe

- You have  $M$  sampling distributions.
- For each sample  $i$ 
  - Pick one distribution at random, let's say it's the  $j$ th one
    - You can't do much better than equal chances, i.e. using probability  $p(j) = 1/M$  for all  $j$  (Veach 1995, Sec. 5.2) (I assume this below.)
  - Draw a sample  $x_i$  from the  $j$ th distribution
  - Compute
$$W_i = \frac{f(x_i)}{\sum_{j=1}^M p(j)p_j(x_i)}$$
  - Take the average of the  $W_i$
  - Done!

# What's Going On?

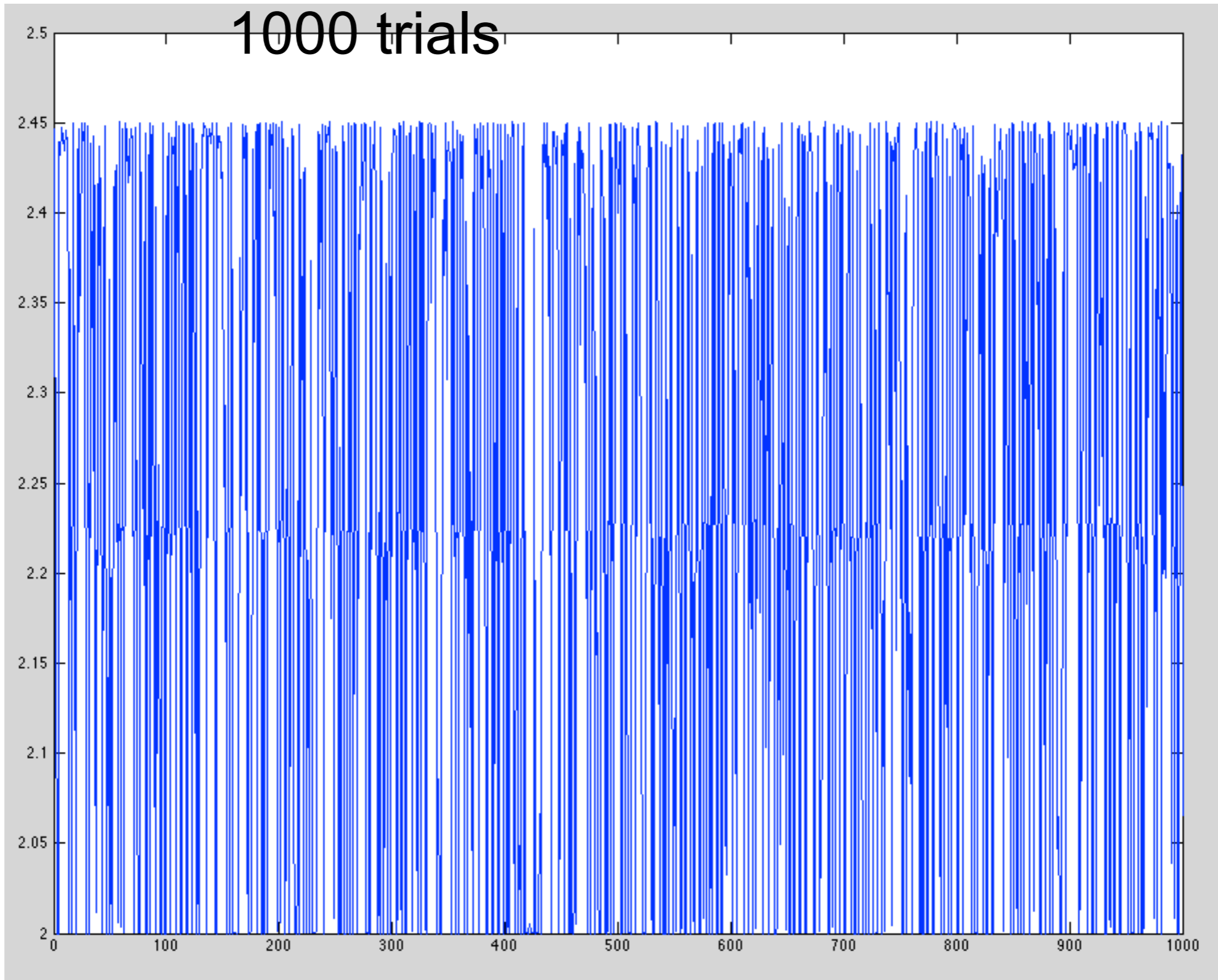
- The above process generates samples with the joint distribution

$$\bar{p}(x) = \sum_{j=1}^M p(j)p_j(x)$$

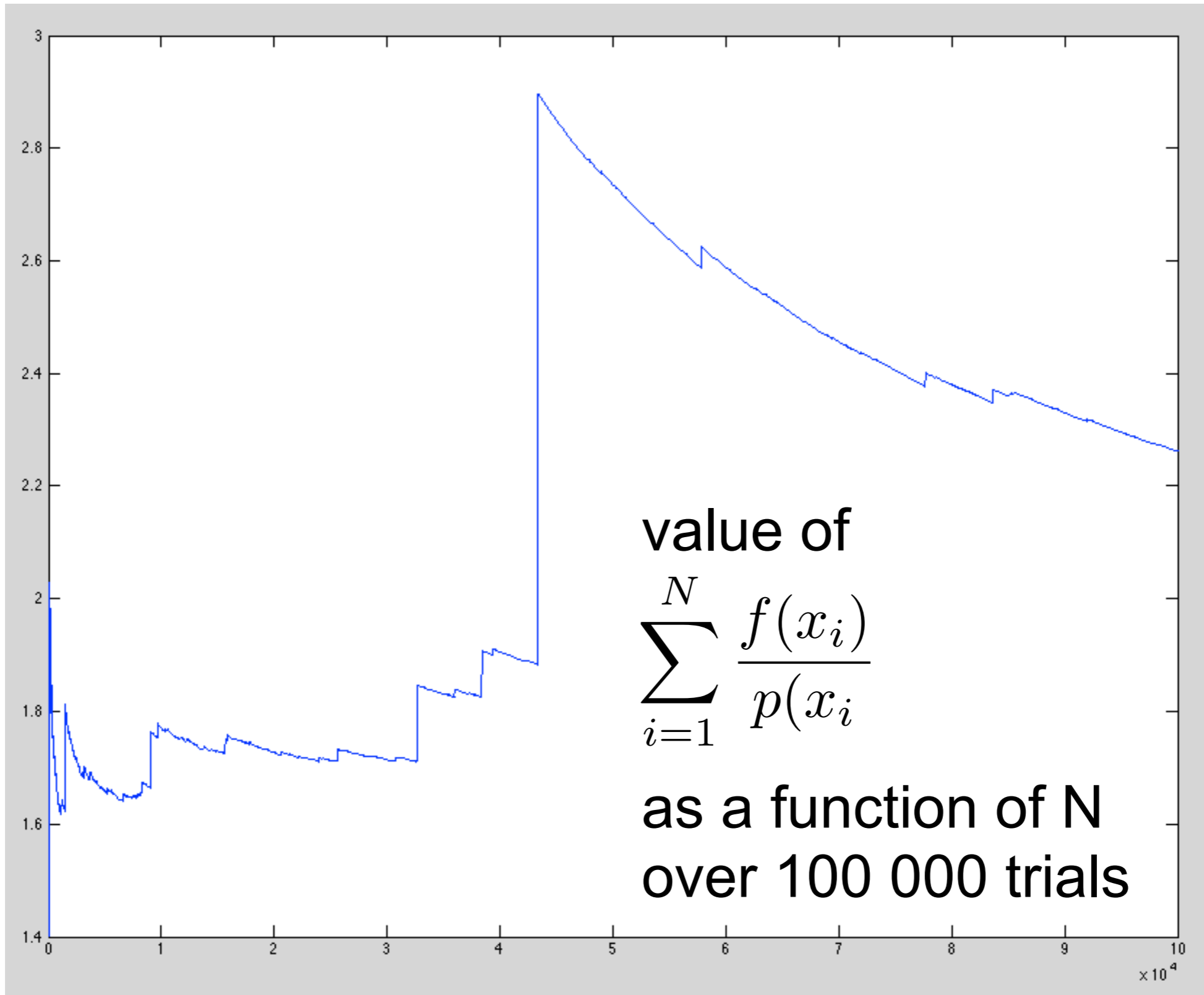
- Hence, we're just computing f/p with this new PDF.
  - Note that the  $p(j)$ 's are a discrete distribution, their sum must be 1!
- *This is an unbiased estimate, just like regular MC.*

# Ha!

sample weight  
 $f(x_i)/p(x_i)$  over  
1000 trials

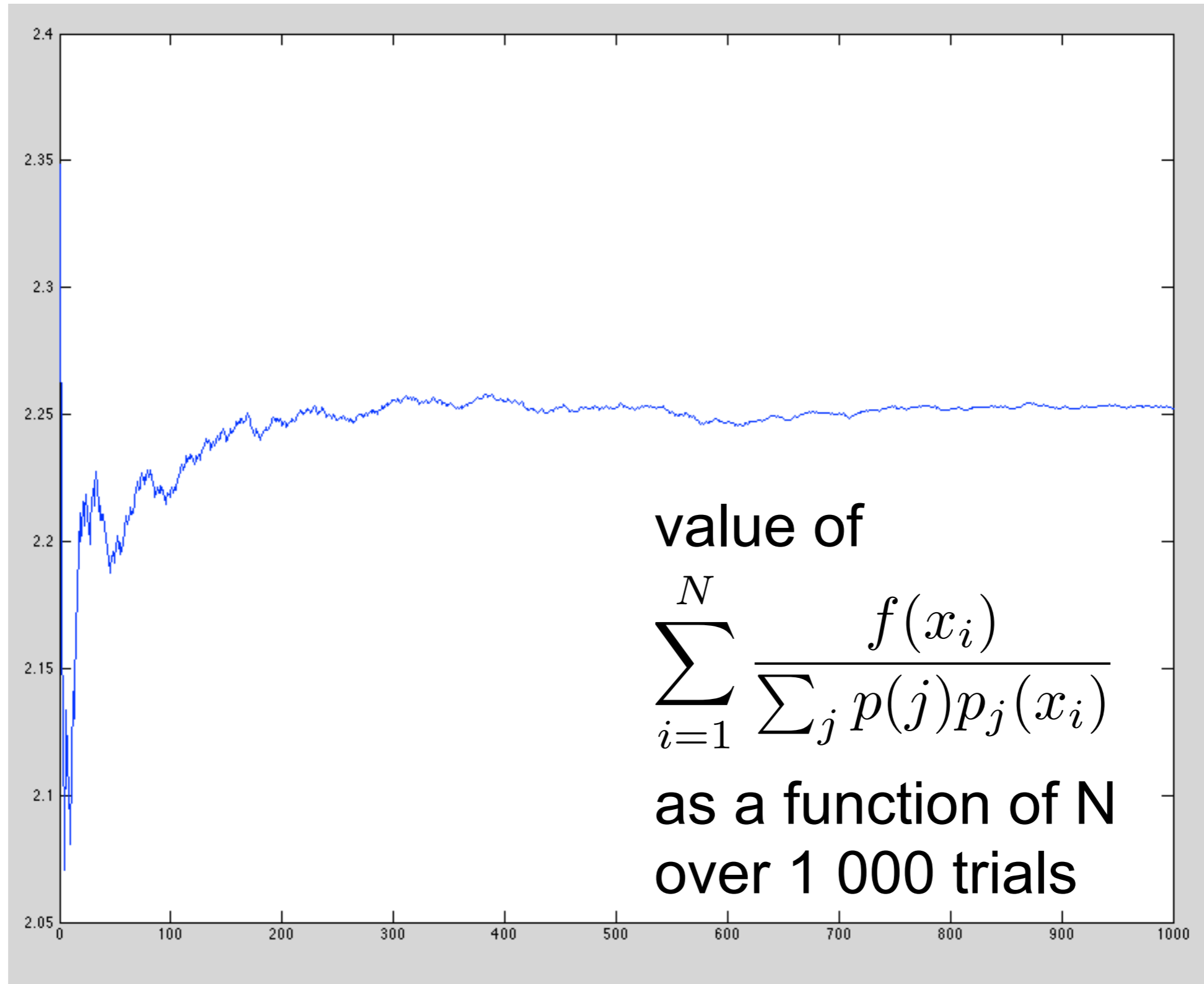


# Integral Estimate, No MIS, 100k samples



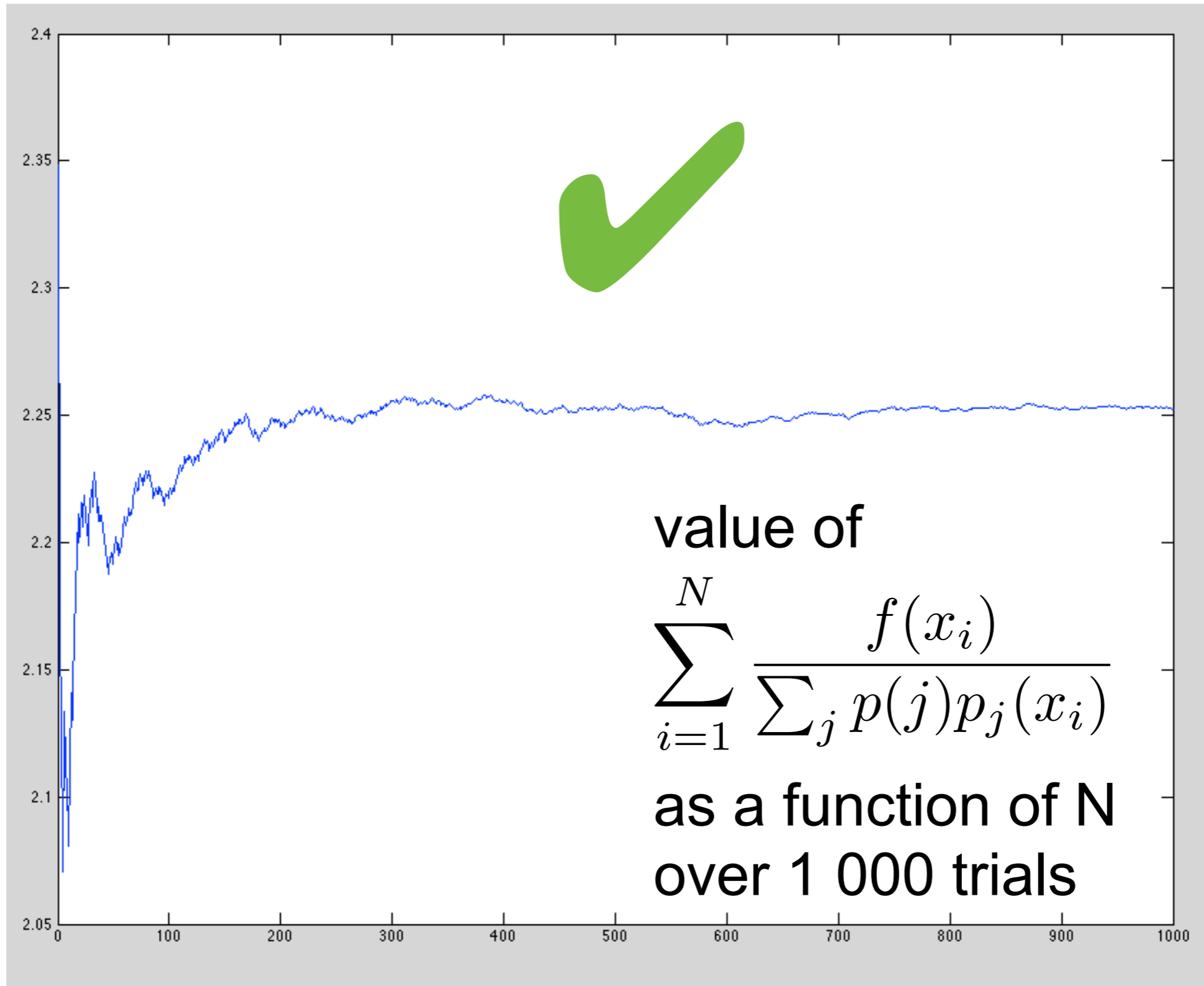
# Integral Estimate, MIS, 1k samples

(100x fewer than previous terrible non-MIS result)



# Integral Estimate, MIS, 1k samples

(100x fewer than previous terrible non-MIS result)



# Bells And Whistles

- This is the basic intuition and approach.
- Veach's 1995 paper contains a long treatment on how to choose the relative weighting between the PDFs and more general ways of constructing  $\bar{p}(x)$  based on the individual distributions.
- However, we won't go into this. This process is really general and applies wherever MC can be applied.

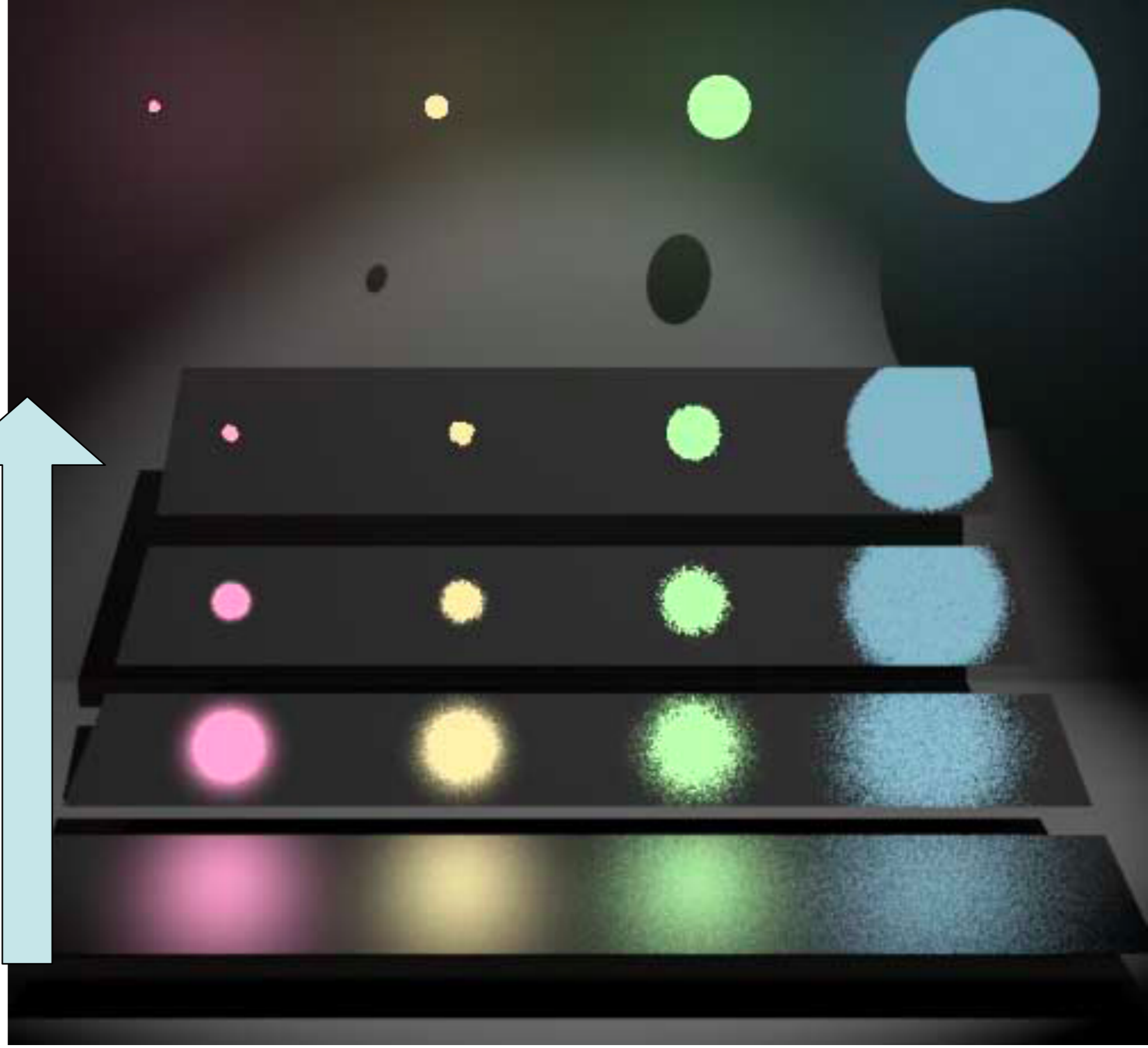
# Example: Use in a Path Tracer

- Apart from the direct eye ray, our basic path tracer only accounts for light through shadow rays
  - If the extension ray, which is sampled from the BRDF, hits a light source, we set its contribution to zero.
  - Is this the best we can do?
- Indeed, we can repurpose the extension ray for another purpose: we'll try to make the light connection by both light sampling and BRDF sampling.
  - However we deterministically use both samplers, no random picking.

# Multiple Importance Sampling

**MIS = Sample both ways and optimally combine the samples**

increasing gloss



# Questions?

