CS-C3100 Computer Graphics

Bézier Curves and Splines

3.1 Introduction to Splines

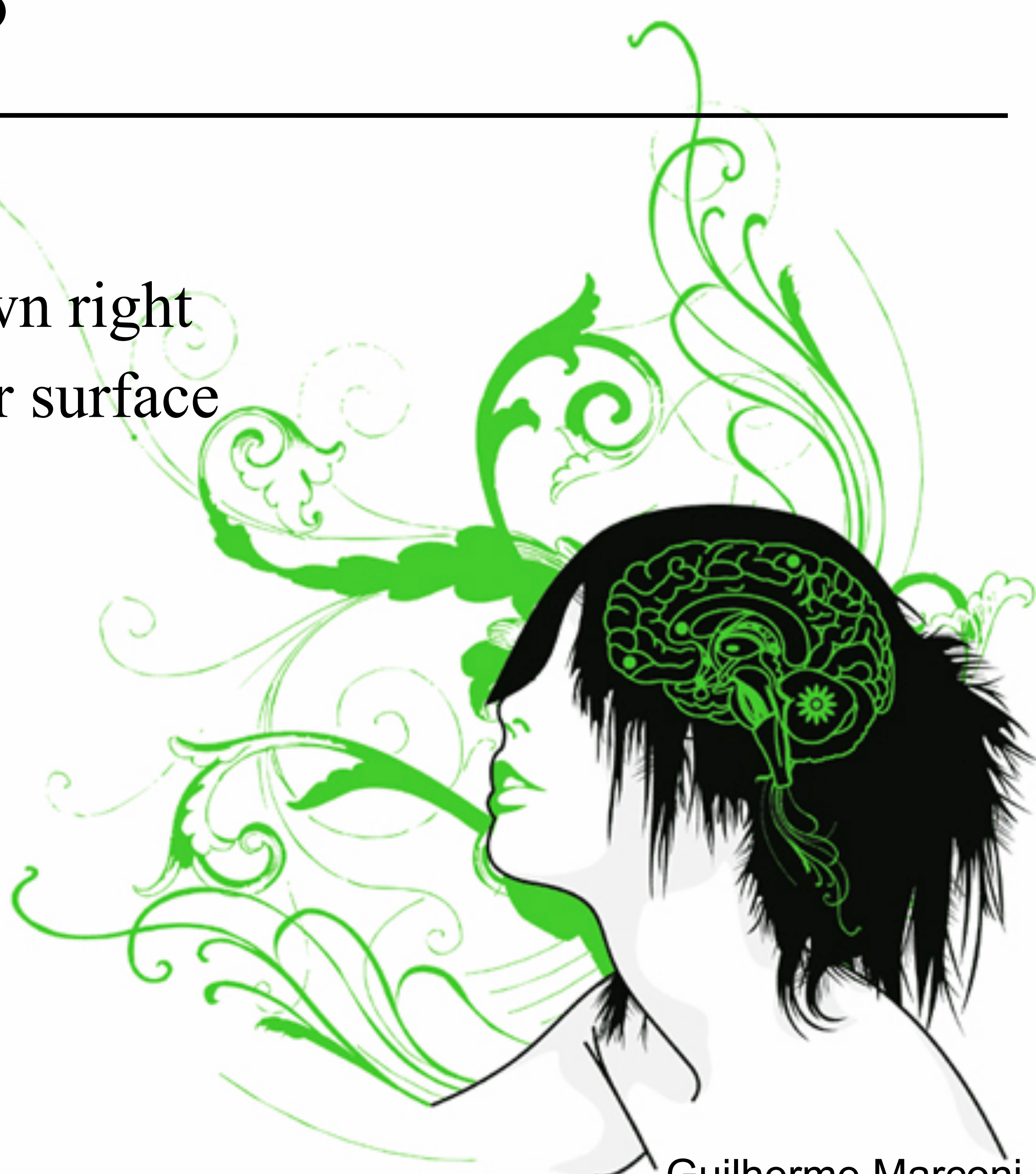
Majority of slides from Frédo Durand

In This Video

- Representing 2D curves (3D: simple extension)
- Splines: piecewise polynomial curves
  - Defined by a sequence of *control points* and a mathematical rule that turns them into a smooth curve
- In particular: cubic Bézier curves

Why Curves?

- Curves in 2D
  - Useful in their own right
  - Provides basis for surface modeling

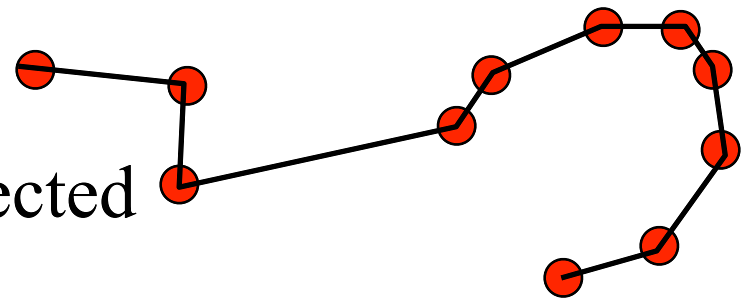


Guilherme Marconi

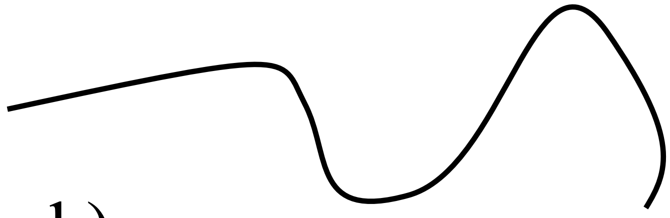
Modeling Curves in 2D

- How?

Modeling Curves in 2D



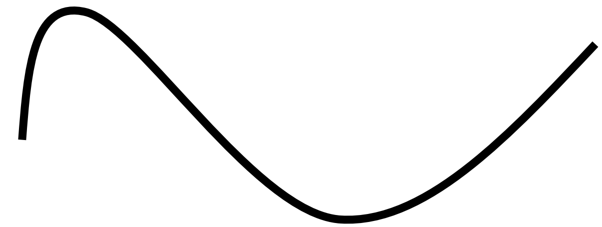
- Polylines
  - Sequence of vertices connected by straight line segments
  - Useful, but not for smooth curves
  - This is the representation that usually gets drawn in the end (smooth curves converted into these)



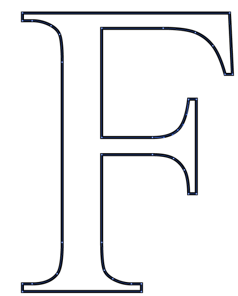
- **Smooth curves**
  - How do we specify them?
  - A little harder (but not too much)

Splines

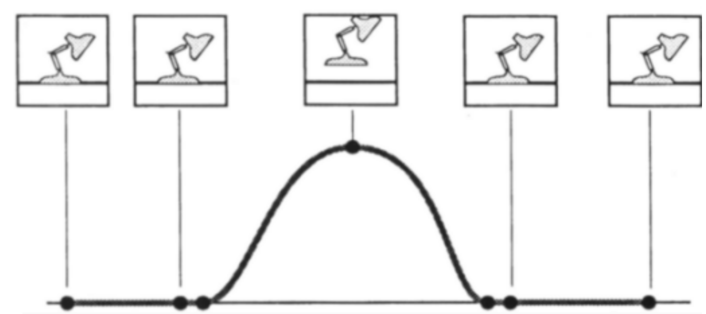
- A type of smooth curve in the plane or in 3D



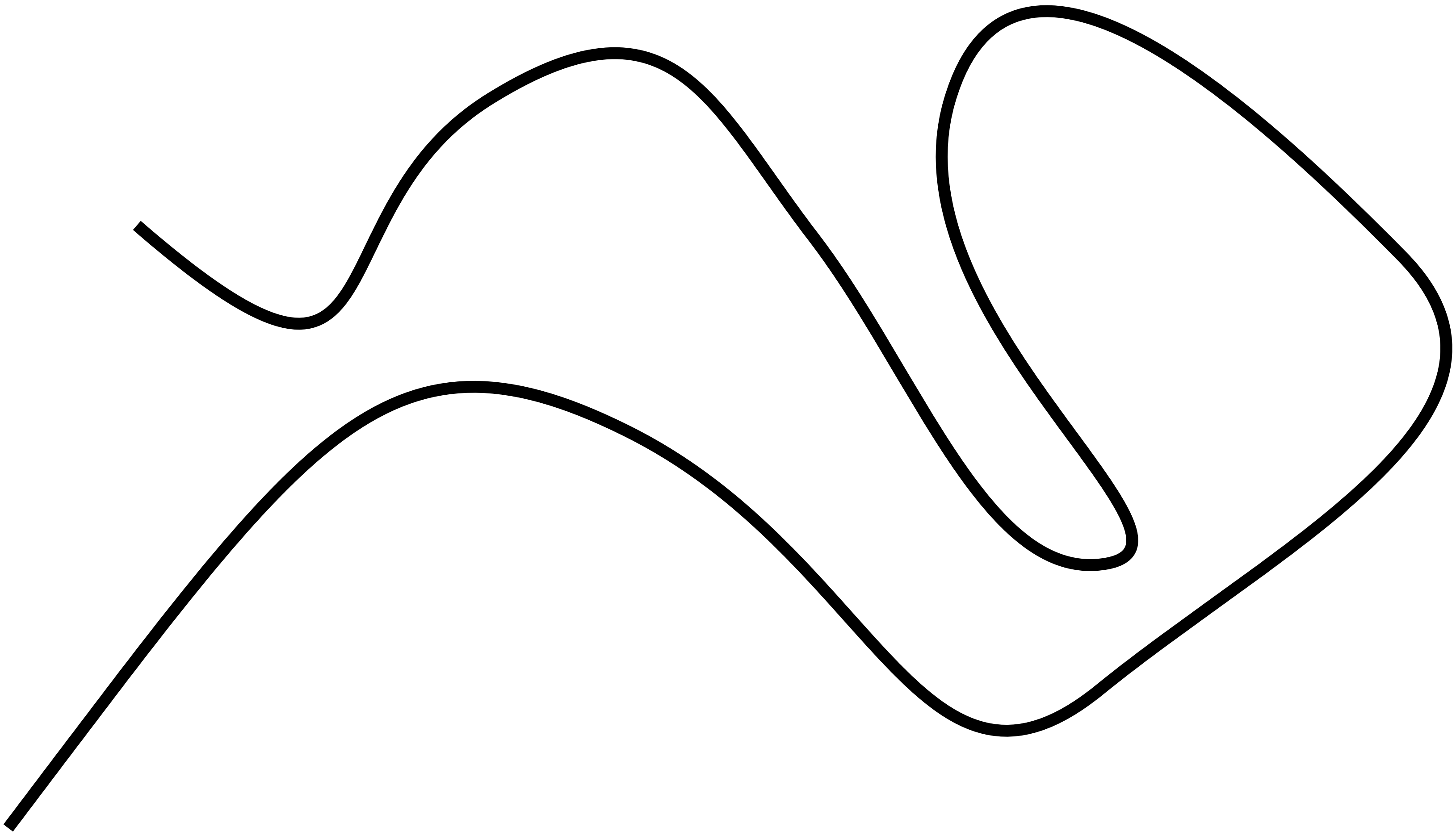
- Many uses
  - 2D Illustration (e.g. Adobe Illustrator)
  - Fonts
  - 3D Modeling
  - Animation: Trajectories



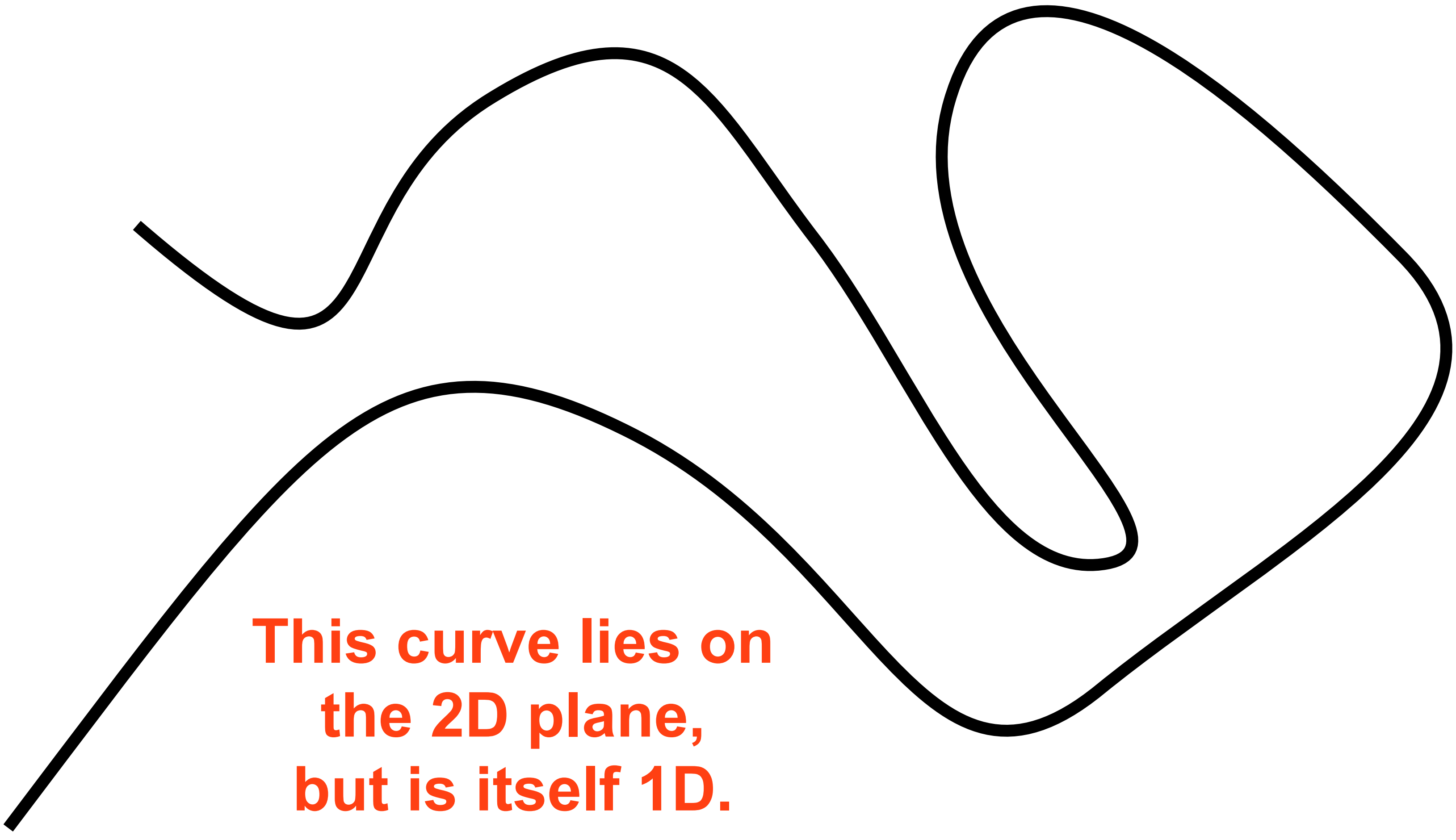
- In general: Interpolation and approximation



How Many Dimensions?



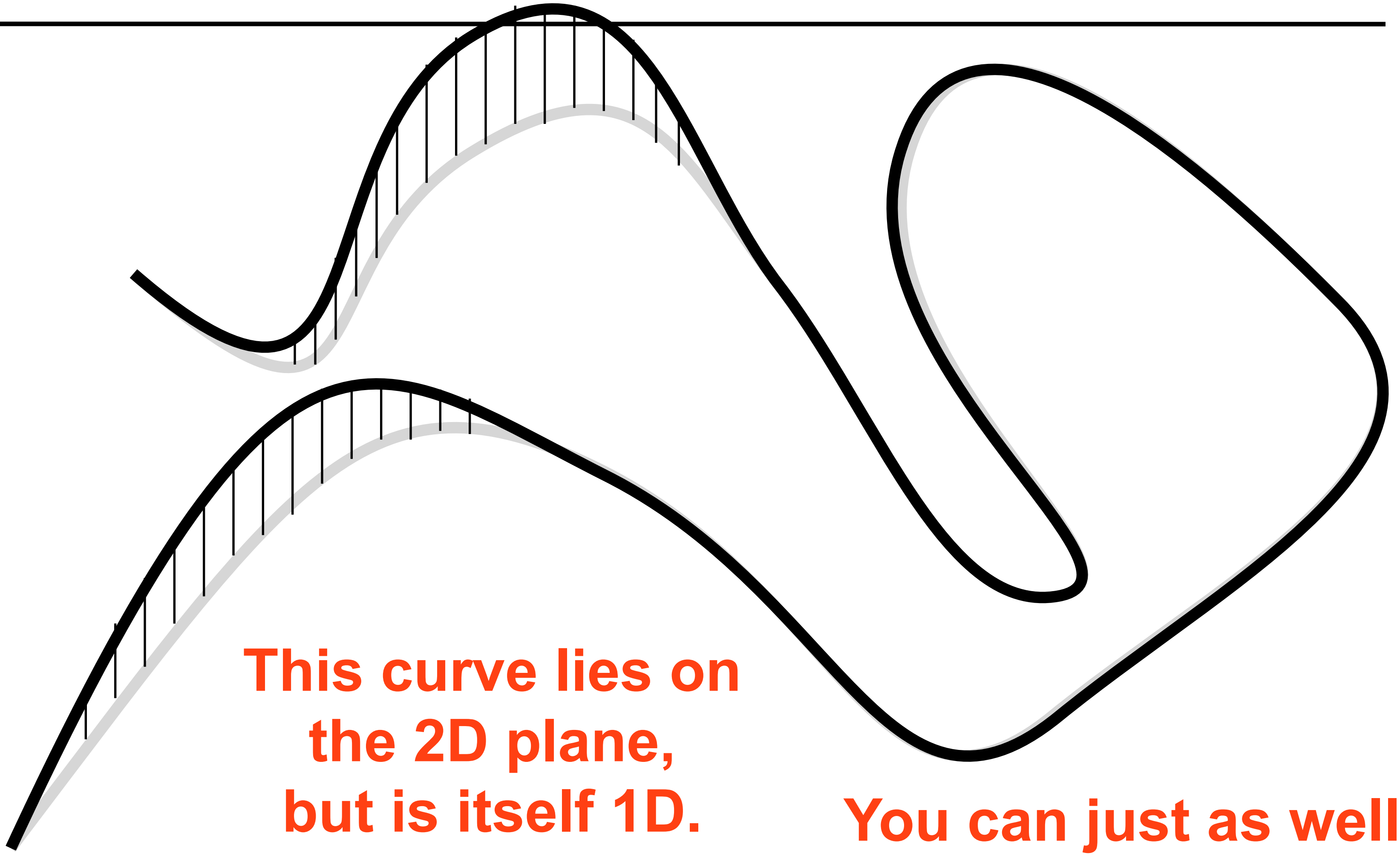
How Many Dimensions?



This curve lies on the 2D plane, but is itself 1D.

= you need one number to identify any point on the curve

How Many Dimensions?



This curve lies on the 2D plane, but is itself 1D.

You can just as well define 1D curves in 3D space.

Two Definitions of a Curve

1. A continuous 1D point set on the plane or space
2. A mapping from an interval S onto the plane
   - That is, P(t) is the point of the curve at parameter t

$$P : \mathbb{R} \ni S \mapsto \mathbb{R}^2, \quad P(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

- Big differences
  - It's easy to generate points on the curve from the 2nd
  - The second definition can describe trajectories, the speed at which we move on the curve

Example of the First Definition?

- **A continuous 1D point set on the plane or space**
- A mapping from an interval S onto the plane
  - That is, P(t) is the point of the curve at parameter t

$$P : \mathbb{R} \ni S \mapsto \mathbb{R}^2, \quad P(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

- An "algebraic curve", e.g. intersection of

$$x^2 + y^2 + z^2 = 1 \quad \text{and} \quad z = 0$$

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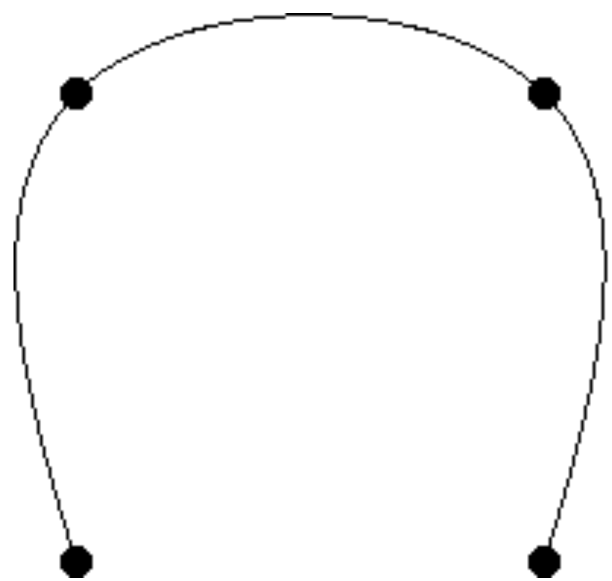
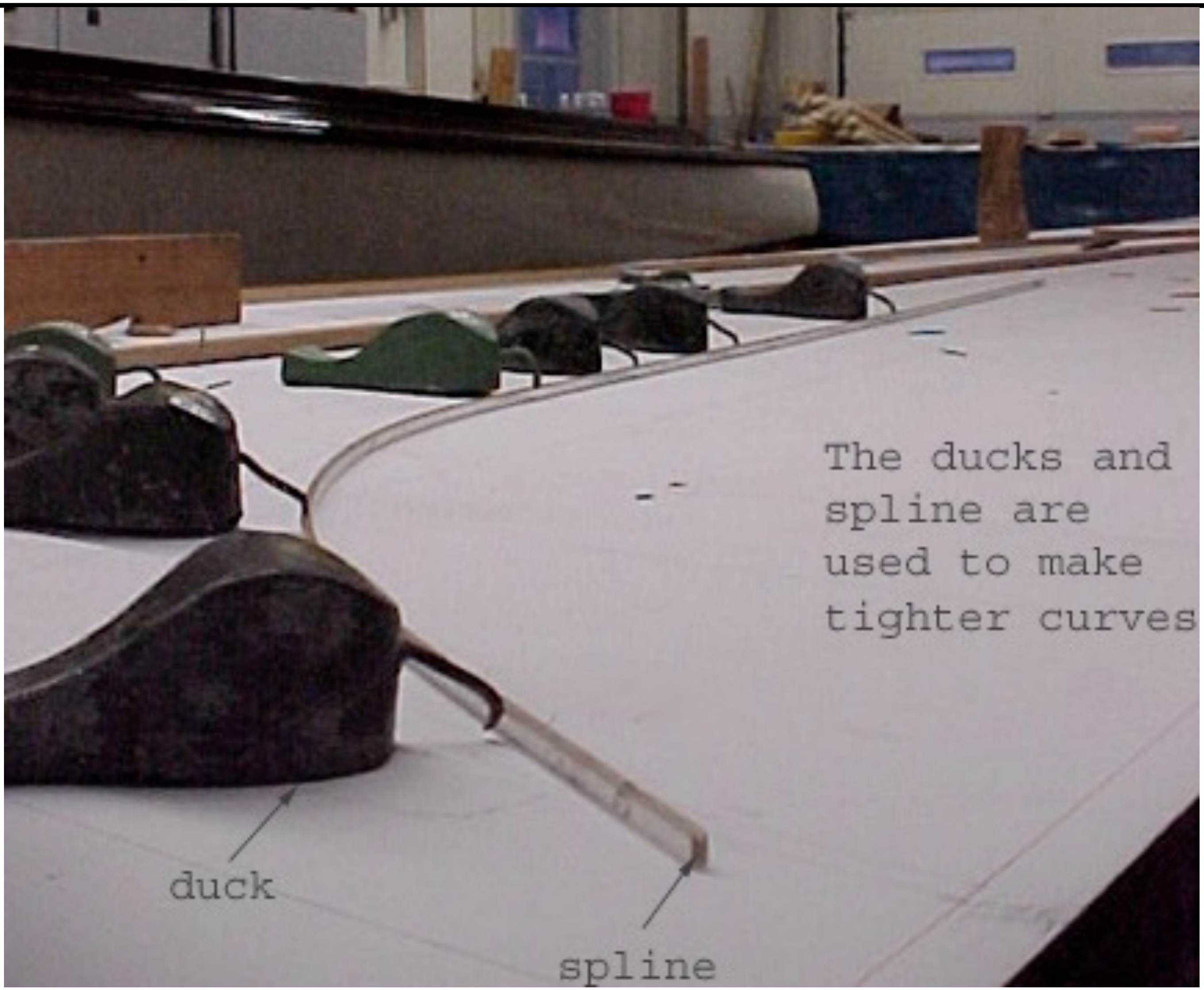
It's a circle on the xy plane :)

General Principle for Modeling

- User specifies "**control points**"
- We'll interpolate the control points by a smooth curve
  - The curve is completely determined by the control points.
  - I.e., we need an unambiguous rule that gives us the curve points given the control points.



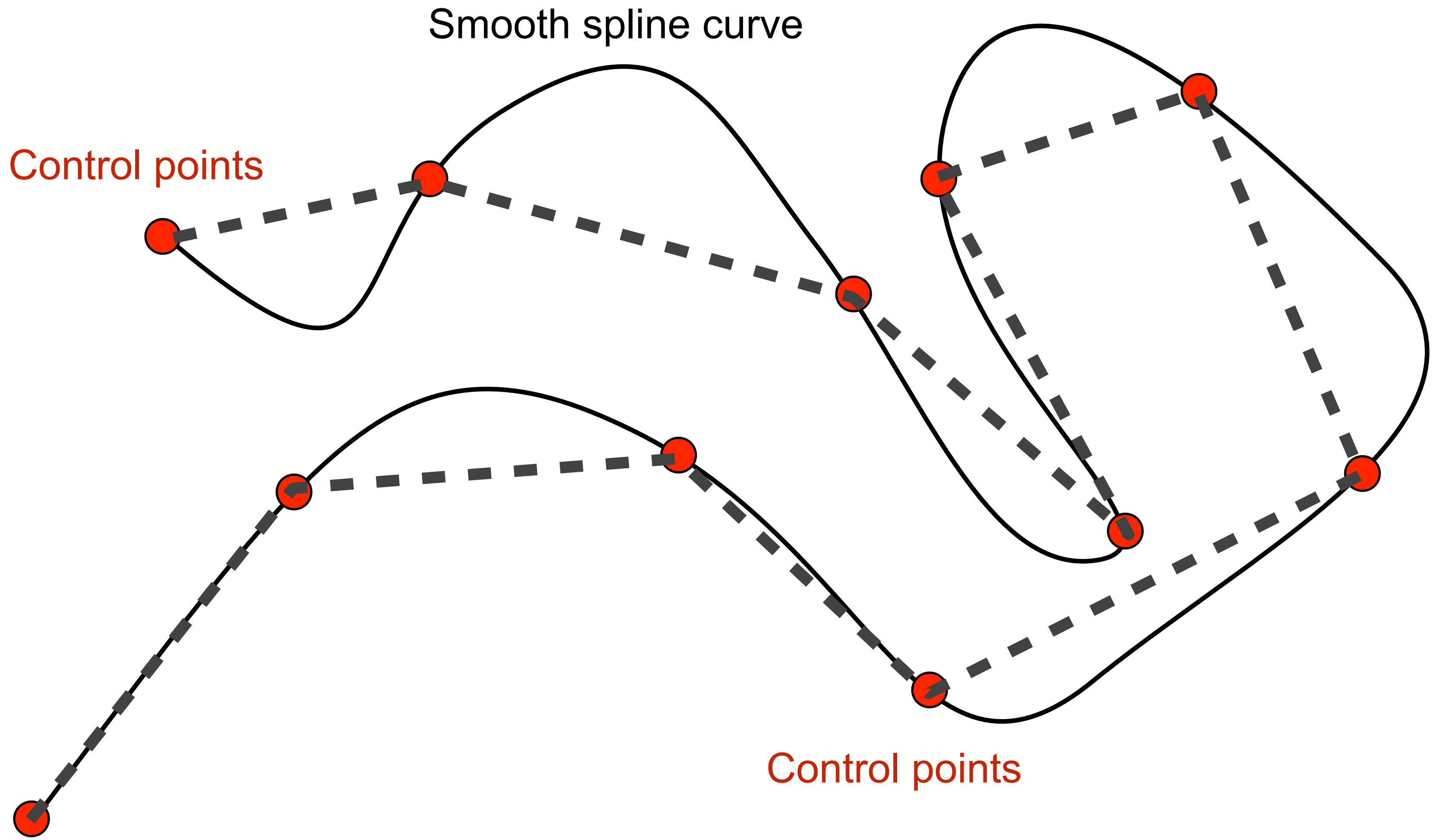
Physical Splines



www.abm.org

See http://en.wikipedia.org/wiki/Flat_spline

Splines in graphics = mathematical rule for turning sequence of points into smooth curve

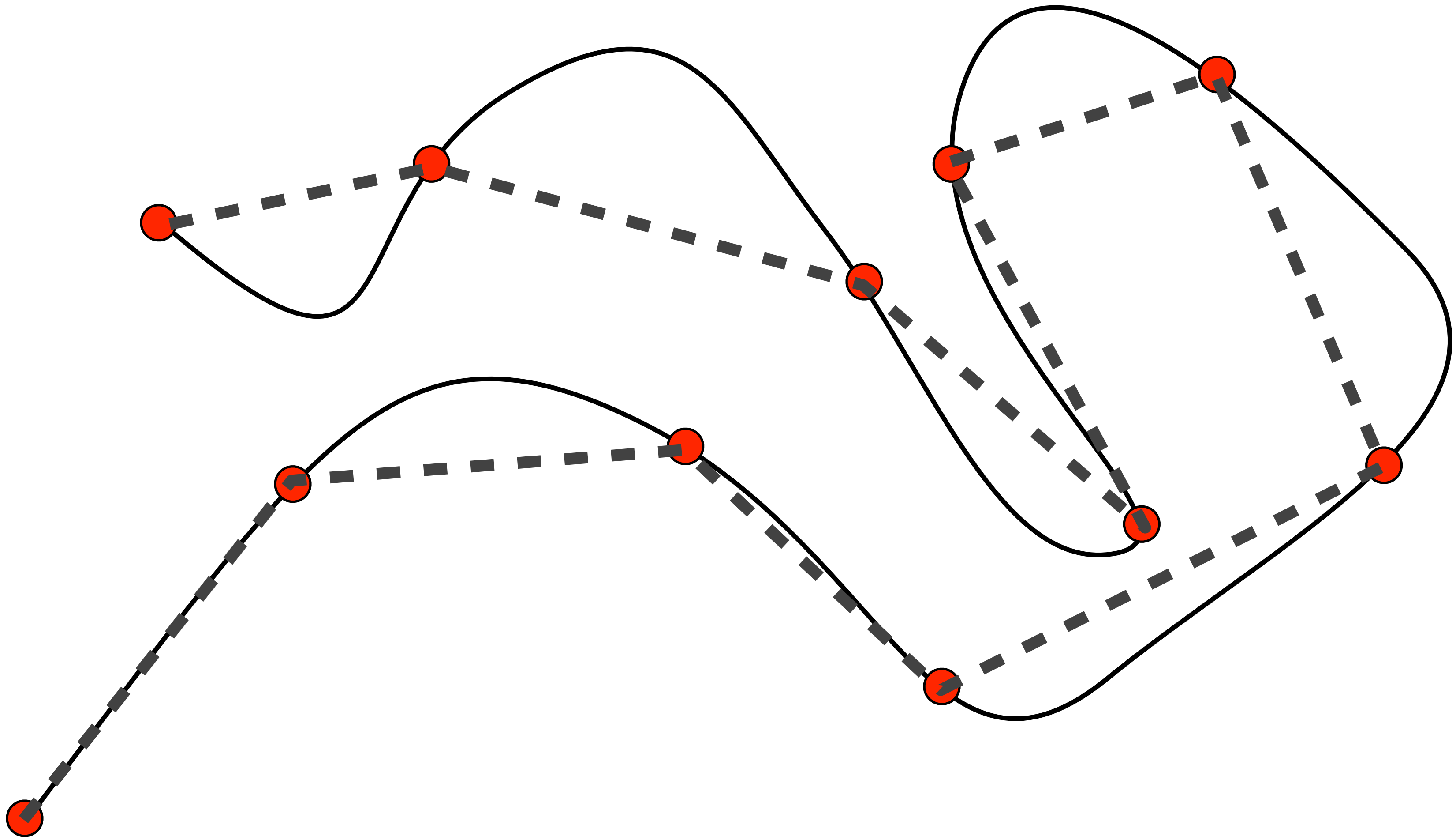


Splines

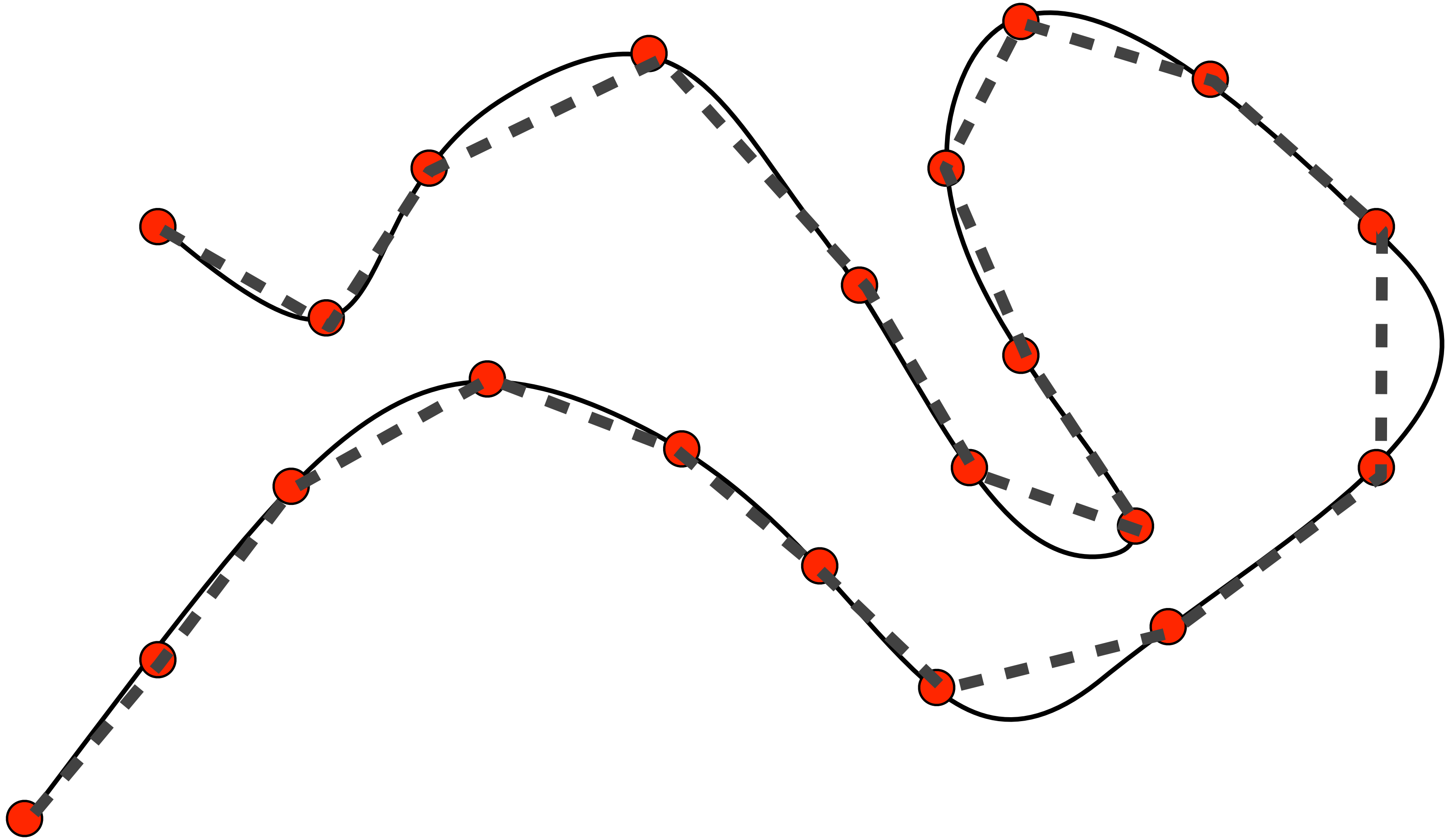
- Specified by a few control points
  - Good for UI
  - Good for storage

- Results in a smooth parametric curve P(t)
  - Just means that we specify $x(t)$ and $y(t)$
  - In practice **low-order polynomials chained together**
  - Convenient for animation, where t is time
  - Convenient for *tessellation* because **we can discretize *t* and approximate the curve with a polyline**

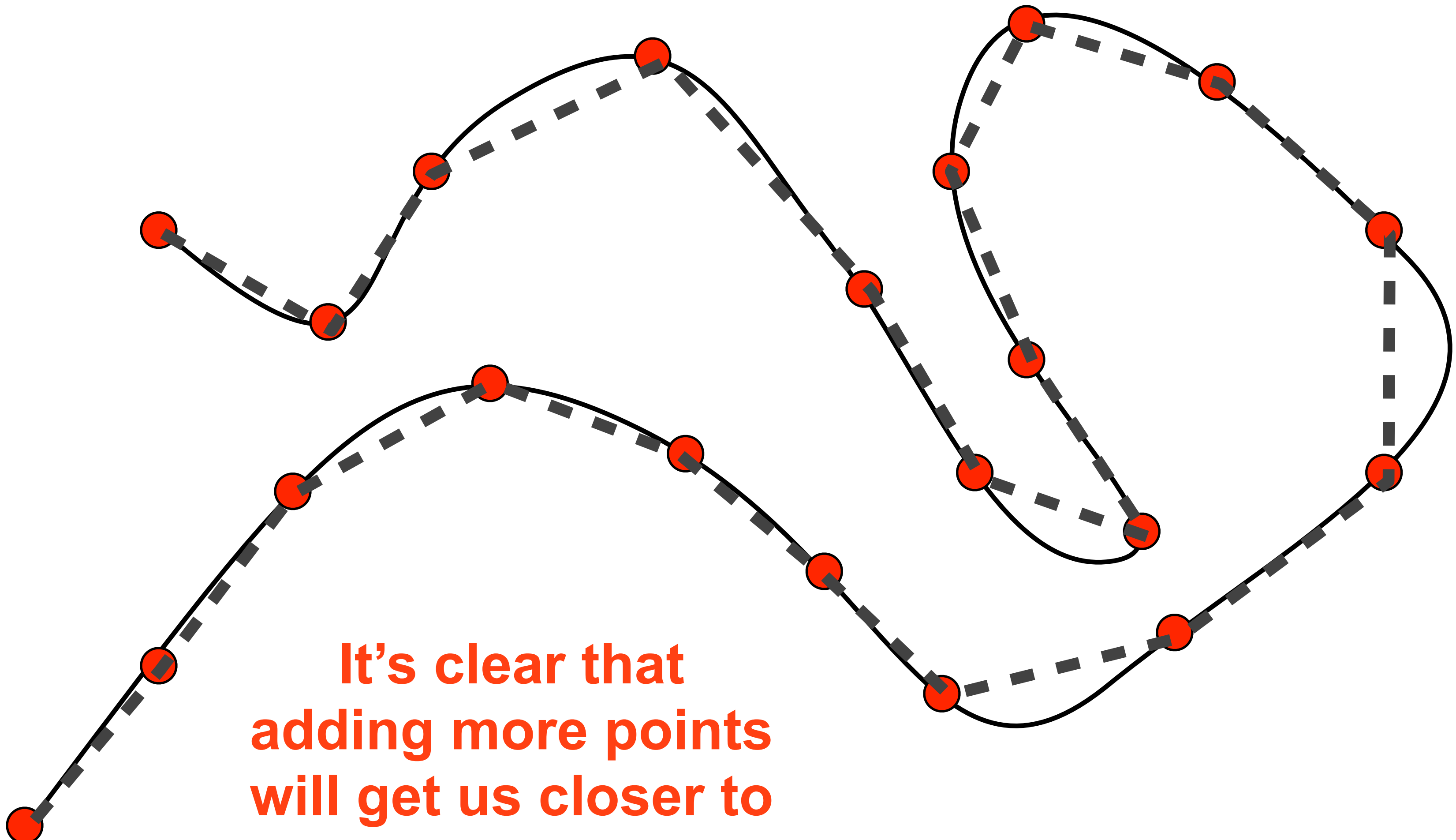
Tessellation



Tessellation



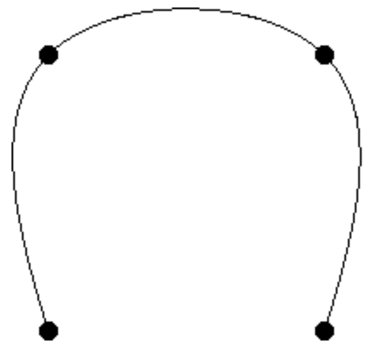
Tessellation



It's clear that adding more points will get us closer to the curve.

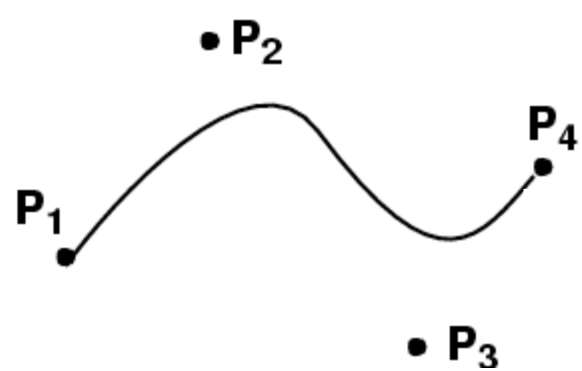
Interpolation vs. Approximation

- Interpolation
  - Goes through all specified points
  - Sounds more logical



Interpolation

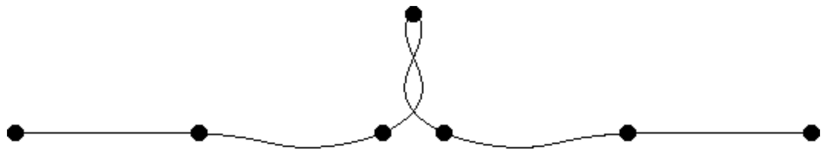
- Approximation
  - Does not go through all points



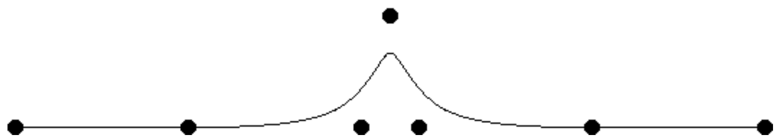
Approximation

Interpolation vs. Approximation

- Interpolation
  - Goes through all specified points
  - Sounds more logical
  - But can be more unstable, “ringing”
- Approximation
  - Does not go through all points
  - Turns out to be convenient
- In practice, we’ll do something in between.



Interpolation



Approximation


Next up:
The Cubic Bézier Curve