CS-C3100 Computer Graphics

Bézier Curves and Splines

3.4 Splitting cubic Bézier curves with the De Casteljau construction
In These Slides

• Splitting cubic Bézier curves in two: the De Casteljau construction
Higher-Order Bézier Curves

- > 4 control points
- Bernstein Polynomials as the basis functions
  - For polynomial of order $n$, the $i^{th}$ basis function is
    \[ B_i^n(t) = \frac{n!}{i!(n-i)!} t^i (1 - t)^{n-i} \]
- Every control point affects the entire curve
  - Not simply a local effect
  - More difficult to control for modeling
- You will not need this in this class
Higher-Order Bézier Curves

• > 4 control points
• Bernstein Polynomials as basis functions
  – For polynomial of order \( n \), \( i^{th} \) basis function is
    \[
    B_i^n(t) = \binom{n}{i} t^i (1 - t)^{n-i}
    \]
• Every control point affects the entire curve
  – Not simply a local effect
  – More difficult to control for modeling
• You will not need this in this class
Can we split a Bezier curve into two in the middle, using two new Bézier curves?
- Would be useful for adding detail, as a single cubic doesn’t get you very far, and higher-order curves are nasty.
Subdivision of a Bezier curve

- Can we split a Bezier curve into two in the middle, using two new Bézier curves?
  - The resulting curves are again a cubic (Why?)
Subdivision of a Bezier curve

• Can we split a Bezier curve into two in the middle, using two new Bézier curves?
  – The resulting curves are again a cubic
    (Why? A cubic in $t$ is also a cubic in $2t$)
  • (Why? $a_0 (2t)^3 = 8a_0 t^3$, etc.)
Subdivision of a Bezier curve

- Can we split a Bezier curve into two in the middle, using two new Bézier curves?
  - The resulting curves are again a cubic (Why? A cubic in $t$ is also a cubic in $2t$)
  - Hence it must be representable using the Bernstein basis. So yes, we can!
“De Casteljau Construction”
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
- Construct the two segments joining them
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
- Construct the two segments joining them
- Take the middle of those two new segments
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
- Construct the two segments joining them
- Take the middle of those two new segments
- Join them
“De Casteljau Construction”

- Take the middle point of each of the 3 segments
- Construct the two segments joining them
- Take the middle of those two new segments
- Join them
- Take the middle point P’’’
The two new curves are defined by
- $P_1$, $P'_1$, $P''_1$, and $P'''$
- $P'''$, $P''_2$, $P'_3$, and $P_4$

Together they exactly replicate the original curve!
- Originally 4 control points, now 7 (more control)
Sanity Check

• Do we get the middle point?
• \( B_1(t) = (1-t)^3 \)
• \( B_2(t) = 3t(1-t)^2 \)
• \( B_3(t) = 3t^2(1-t) \)
• \( B_4(t) = t^3 \)

\[
\begin{align*}
P'_1 &= 0.5(P_1 + P_2) \\
P'_2 &= 0.5(P_2 + P_3) \\
P'_3 &= 0.5(P_3 + P_4) \\
P''_1 &= 0.5(P'_1 + P'_2) \\
P''_2 &= 0.5(P'_2 + P'_3) \\
P''' &= 0.5(P''_1 + P''_2) \\
&= 0.5 \left( 0.5(P'_1 + P'_2) + 0.5(P'_2 + P'_3) \right) \\
&= 0.5 \left( 0.5 \left[ 0.5(P_1 + P_2) + 0.5(P_2 + P_3) \right] + 0.5 \left[ 0.5(P_2 + P_3) + 0.5(P_3 + P_4) \right] \right) \\
&= \frac{1}{8}P_1 + \frac{3}{8}P_2 + \frac{3}{8}P_3 + \frac{1}{8}P_4
\end{align*}
\]
De Casteljau Construction

- Actually works to construct a point at any $t$, not just 0.5
- Just subdivide the segments with ratio $(1-t)$, $t$ (not in the middle)
De Casteljau Construction

- Actually works to construct a point at any \( t \), not just 0.5
- Just subdivide the segments with ratio \( (1-t) \), \( t \) (not in the middle)
De Casteljau Construction

• Actually works to construct a point at any $t$, not just 0.5
• Just subdivide the segments with ratio $(1-t)$, $t$ (not in the middle)
De Casteljau Construction

- Actually works to construct a point at any $t$, not just 0.5
- Just subdivide the segments with ratio $(1-t)$, $t$ (not in the middle)