Deterministic subgraph detection in broadcast CONGEST

Janne H. Korhonen · Aalto University
Joel Rybicki · University of Helsinki
1. Introduction
Introduction:

CONGEST model

- CONGEST model
  - $n$ nodes, connected by communication links
  - unique identifiers, synchronous communication
  - unlimited local computation
  - message size $O(\log n)$ bits/round
  - time measure: number of rounds
Introduction:

CONGEST model

- CONGEST model
  - $n$ nodes, connected by communication links
  - unique identifiers, synchronous communication
  - unlimited local computation
  - message size $O(\log n)$ bits/round
  - time measure: number of rounds

- Upper bounds: broadcast CONGEST
- Lower bounds: unicast CONGEST
Introduction:

**Subgraph detection**

- H-subgraph detection problem
  - given a fixed pattern graph $H$ on $k$ nodes
  - does the network $G$ contain $H$ as a subgraph?

- triangle detection, cycle detection, clique detection, …
Introduction:

Subgraph detection

• Detection:
  • if node belongs to a copy of $H$, output one copy of $H$

• Listing/enumeration:
  • all copies of $H$ are a part of some node’s output
Introduction:

**Subgraph detection**

- $H$ has constant size $k$
  - In LOCAL: $O(1)$ for any $H$ trivially
  - In CONGEST: trivial upper bound $O(n^2)$
Introduction:

Prior work

• Upper bounds
  • triangle finding in $\tilde{O}(n^{2/3})$ rounds [Izumi & Le Gall, PODC 2017]
  • triangle enumeration in $\tilde{O}(n^{3/4})$ rounds [Izumi & Le Gall, PODC 2017]
  • 4-cycle finding in $O(n^{1/2})$ rounds [Drucker, Kuhn, Ostmann, PODC 2014]
  • clique enumeration in $O(n)$ rounds (trivial)

• Lower bounds
  • $k$-cycles ($k$ even) $\tilde{\Omega}(n^{2/k})$ rounds [Drucker, Kuhn, Ostmann, PODC 2014]
  • $k$-cycles ($k$ odd, $k \geq 5$) $\tilde{\Omega}(n)$ rounds [Drucker, Kuhn, Ostmann, PODC 2014]
  • triangle enumeration $\tilde{\Omega}(n^{1/3})$ rounds [Izumi & Le Gall, PODC 2017]
Prior work, DISC 2017


Appearing together as *Three notes on distributed property testing*, DISC 2017.

- tree detection in $O(1)$ rounds
Our Results: Overview
Results 1:

Finding Trees and Cycles

- **Upper bounds**
  - $k$-trees in $O(1)$ rounds*
  - $k$-cycles in $O(n)$ rounds
  - $k$-pseudotrees (tree + 1 edge) in $O(n)$ rounds

- **Lower bounds**
  - $k$-cycles ($k$ even) require $\Omega(n^{1/2}/\log n)$ rounds
Results 1:

Finding Trees and Cycles

• Upper bounds
  • $k$-trees in $O(k2^k)$ rounds*
  • $k$-cycles in $O(k2^kn)$ rounds
  • $k$-pseudotrees (tree + 1 edge) in $O(k2^kn)$ rounds

• Lower bounds
  • $k$-cycles ($k$ even) require $\Omega(n^{1/2}/\log n)$ rounds
Results 1:

Finding Trees and Cycles

• Some tight results…
  • trees in $O(1)$ rounds
  • odd cycles are $\tilde{\Theta}(n)$

• …and some not tight
  • gap for even cycles between $O(n)$ and $\tilde{\Omega}(n^{1/2})$
Results 2:

**Enumeration in sparse graphs**

- does it help if the input graph $G$ is sparse?

- notion of sparseness: bounded degeneracy
  - input graph $G$ with degeneracy $d$
  - degeneracy $\approx$ arboricity
Results 2:

**Enumeration in sparse graphs**

- **Upper bounds**
  - $k$-cliques and 4-cycles in $O(d + \log n)$ rounds
  - 5-cycles in $O(d^2 + \log n)$ rounds

- **Lower bounds**
  - finding 4-cycles and 5-cycles requires $\tilde{\Omega}(d)$ rounds
  - bounded degeneracy does not help with 6-cycles
    - need $\tilde{\Omega}(n^{1/2})$ rounds on graphs with degeneracy 2
Our Results:
Finding Trees and Cycles
Technical tool:

Representative families

• Well-known algorithmic technique
  • used in centralised fixed-parameter algorithms for subgraph detection
  • running times of type $2^{O(k)} \cdot \text{poly}(n)$
  • compare with other FPT techniques: colour-coding, polynomial sieving,…

• Pierre Fraigniaud, Pedro Montealegre, Dennis Olivetti, Ivan Rapaport, and Ioan Todinca.
explicit construction of all partial subtrees

+ “filtering” with representative families
\( O(k2^k) \cdot n = O(k2^k n) \)

\( O(k2^k) \cdot n = O(k2^k n) \)
\[ \Omega(n^{1/2}/\log n) \]
very standard communication complexity reduction
4.

Our Results:
Enumeration in sparse graphs
$O(d + \log n)$

$O(d^2 + \log n)$
Preliminaries:

Degeneracy

• The following are equivalent:
  • graph $G$ has degeneracy $d$
  • graph $G$ has acyclic orientation with out-degree $d$
Preliminaries:

**Degeneracy**

- The following are equivalent:
  - graph \( G \) has degeneracy \( d \)
  - graph \( G \) has acyclic orientation with out-degree \( d \)

- acyclic orientation with out-degree \( O(d) \) can be found in \( O(\log n) \) rounds [Barenboim & Elkin 2010]
Basic idea: all nodes broadcast their outgoing edges $(O(d))$ rounds
Basic idea: all nodes broadcast their outgoing edges ($O(d)$ rounds)
Basic idea: all nodes broadcast their outgoing edges ($O(d)$ rounds)
Basic idea: all nodes broadcast their outgoing edges \((O(d))\) rounds)
Basic idea: all nodes broadcast their outgoing edges \((O(d))\) rounds

cliques: the sink will see all edges
Basic idea: all nodes broadcast their outgoing edges \((O(d))\) rounds

cliques: the sink will see all edges
Basic idea: all nodes broadcast their outgoing edges ($O(d)$ rounds)

4-cycles: some node will see all edges (3 cases to consider)
Basic idea: all nodes broadcast their outgoing edges ($O(d)$ rounds)
Basic idea: all nodes broadcast their outgoing edges ($O(d)$ rounds)
Basic idea: all nodes broadcast their outgoing edges \((O(d))\) rounds
Basic idea: all nodes broadcast their outgoing edges ($O(d)$ rounds)
Basic idea: all nodes broadcast their outgoing edges $(O(d))$ rounds

4-cycles: some node will see all edges (3 cases to consider)
Basic idea: all nodes broadcast their outgoing edges ($O(d)$ rounds)

5-cycles: broadcast outgoing 2-paths ($O(d^2)$ rounds)
$\Omega(d/\log n)$

no degeneracy upper bound
5. Conclusions
Conclusions:

General upper/lower bounds?

• General question: given arbitrary $H$, what is the complexity of detecting $H$?
  • general upper bound $O(n)$?
  • connection to tree-width: trees 1, cycles 2, …?

• Special cases:
  • triangles: ???
  • even cycles: gap between $O(n)$ and $\Omega(n^{1/2})$
Conclusions:

General upper/lower bounds?

• Graphs requiring $\Omega(n^{2-\varepsilon})$ rounds for any $\varepsilon > 0$
  • diameter 3 [Fischer, Gonen & Oshman 2017]
  • tree-width 2 [our work]

$\Omega(n^{2-1/2})$  $\Omega(n^{2-1/3})$  $\Omega(n^{2-1/4})$  …
Conclusions:

General upper/lower bounds?

• Graphs requiring $\Omega(n^{2-\varepsilon})$ rounds for any $\varepsilon > 0$
  • diameter 3 [Fischer, Gonen & Oshman 2017]
  • tree-width 2 [our work]

• Corresponding upper bound?
  • lower bound $\Omega(n^2/polylog n)$ does not seem possible with standard techniques
  • conjecture: for any $H$, some $O(n^{2-\varepsilon})$ upper bound
Thanks! Questions?