Brief Announcement:
Deterministic MST Sparsification in the Congested Clique

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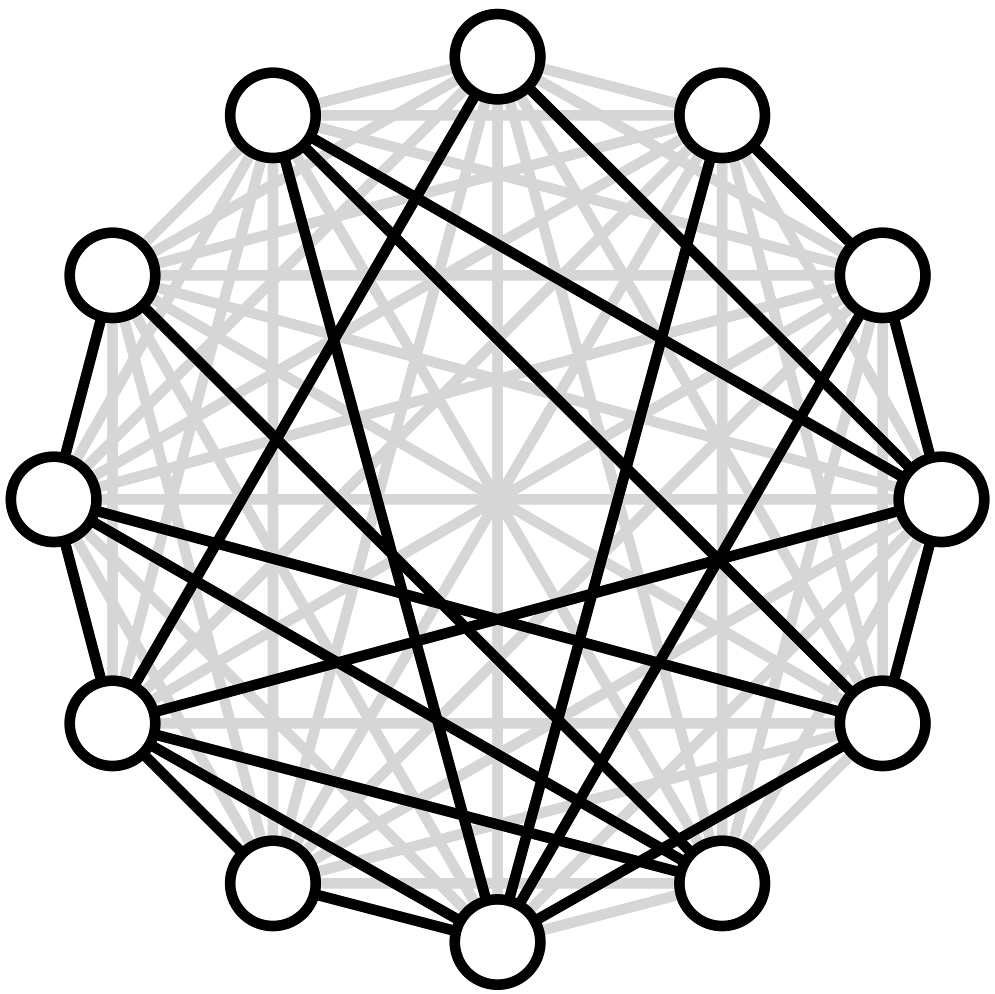
Introduction 1:

Congested Clique Model

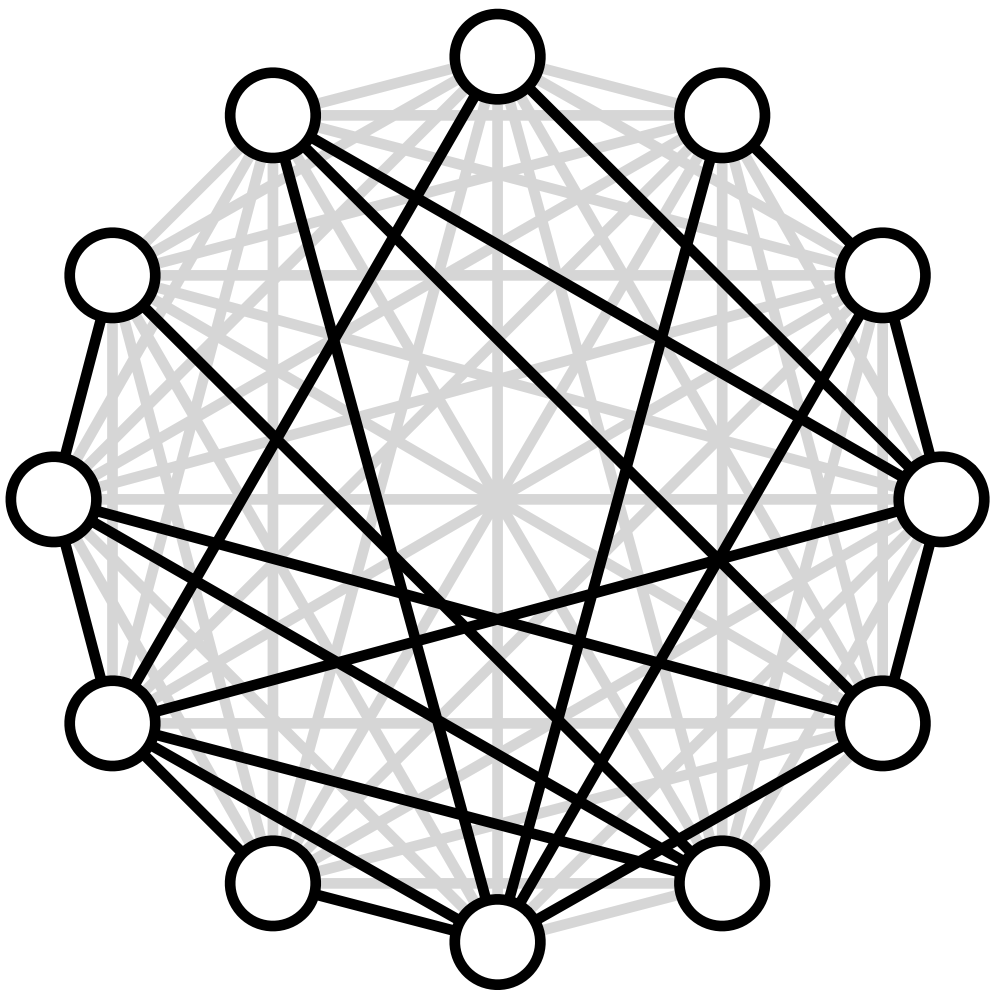


- specialisation of **CONGEST**
- communication graph = **clique** on n nodes
- input graph = **arbitrary graph** on n nodes
  - **local input:** incident edges

- synchronous, error-free
- $O(\log n)$ bandwidth/edge/round
- unlimited local computation
- we are interested in **round complexity**

Introduction 2:

MST in the Congested Clique



- **undirected** graph, poly(n) weights
- find a **minimum spanning tree**

O(log log n)	Det.	2005	Lotker, Patt-Shamir, Pavlov, Peleg
O(log log log n)	Rand.	2015	Hegeman, Pandurangan, Pemmaraju, Sardeshmukh, Scquizzato
O(log* n)	Rand.	2016	Ghaffari, Parter

Introduction 3:

MST Sparsification

- Randomised MST based on fast **connectivity** algorithms
- Solving **MST** via connectivity:
  - reduce MST to MST on sparse graphs
  - reduce sparse MST to many connectivity instances
  - solve connectivity instances in parallel

Lemma (Karger, Klein and Tarjan 1995). There is a randomised reduction from MST to **two** instances of MST on graphs with $O(n^{3/2})$ edges.

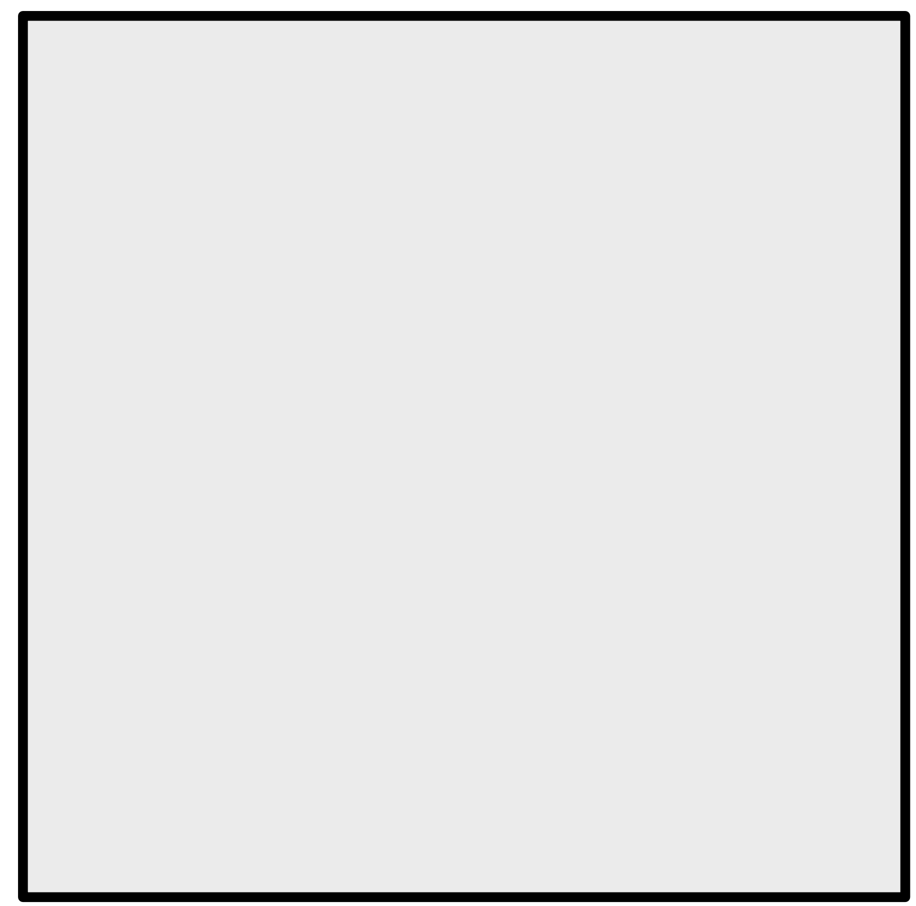
Main Result

Theorem. There is a $O(k)$ round deterministic congested clique algorithm on that sparsifies the input graph to $O(n^{1+1/2^k})$ edges and does not remove any edge of the minimum spanning tree.

- $O(n^{1+\varepsilon})$ edges in constant rounds for any constant $\varepsilon > 0$
  - very sparse instances already the worst case for MST
- gives MST algorithm for $k = O(\log \log n)$

Proof Sketch:

Block-sparsification

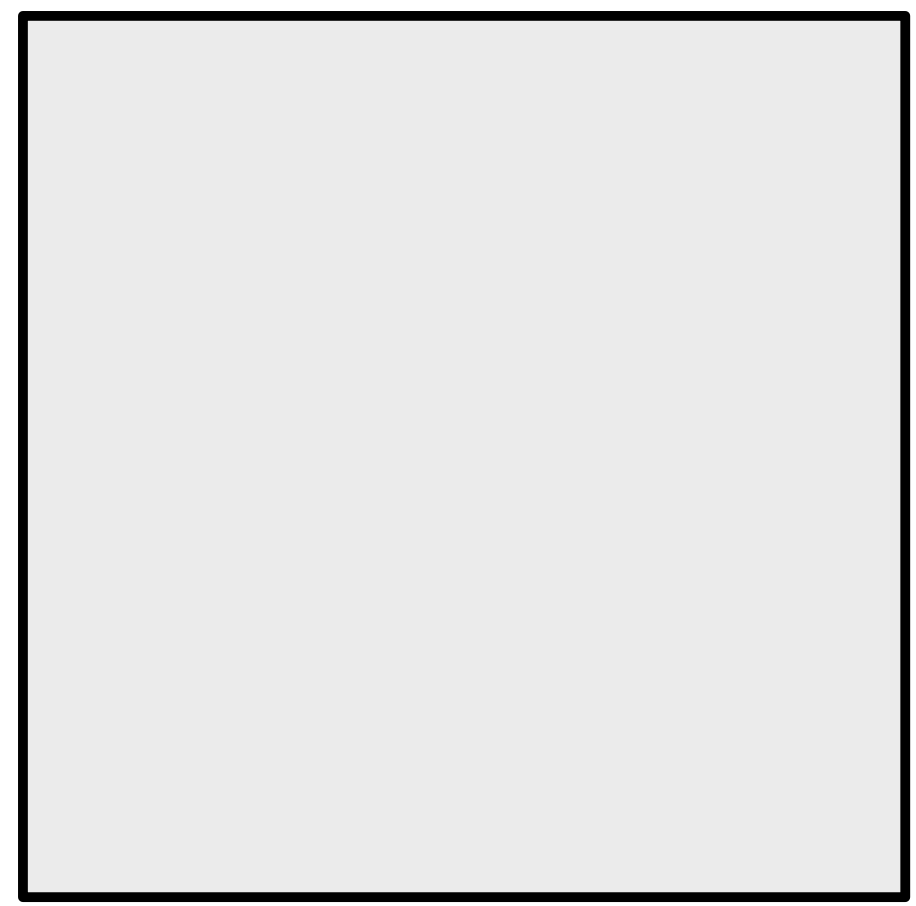


weighted adjacency matrix A

Proof Sketch:

Block-sparsification

1. Partition the adjacency matrix to n blocks of size $n^{1/2} \times n^{1/2}$



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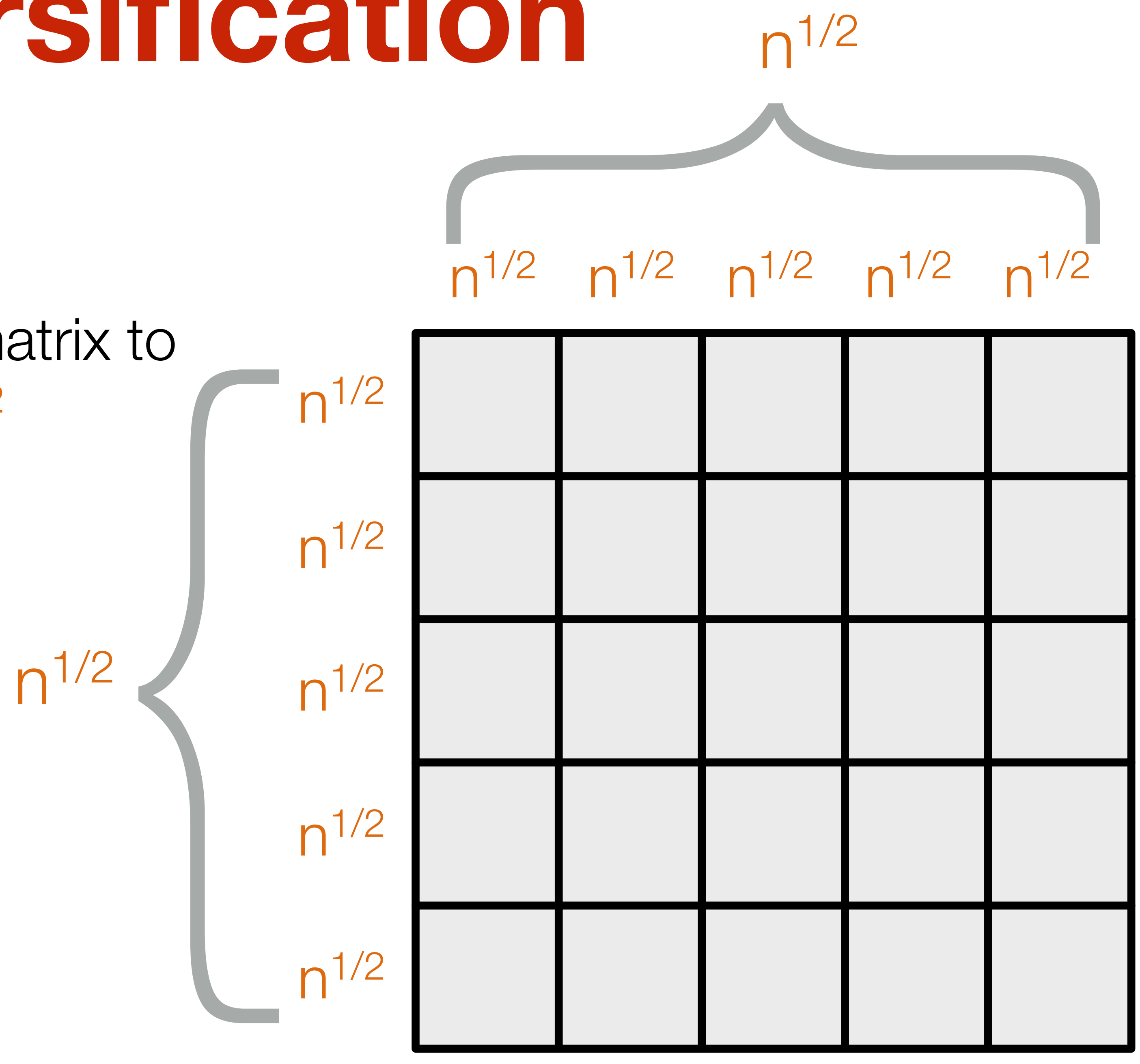

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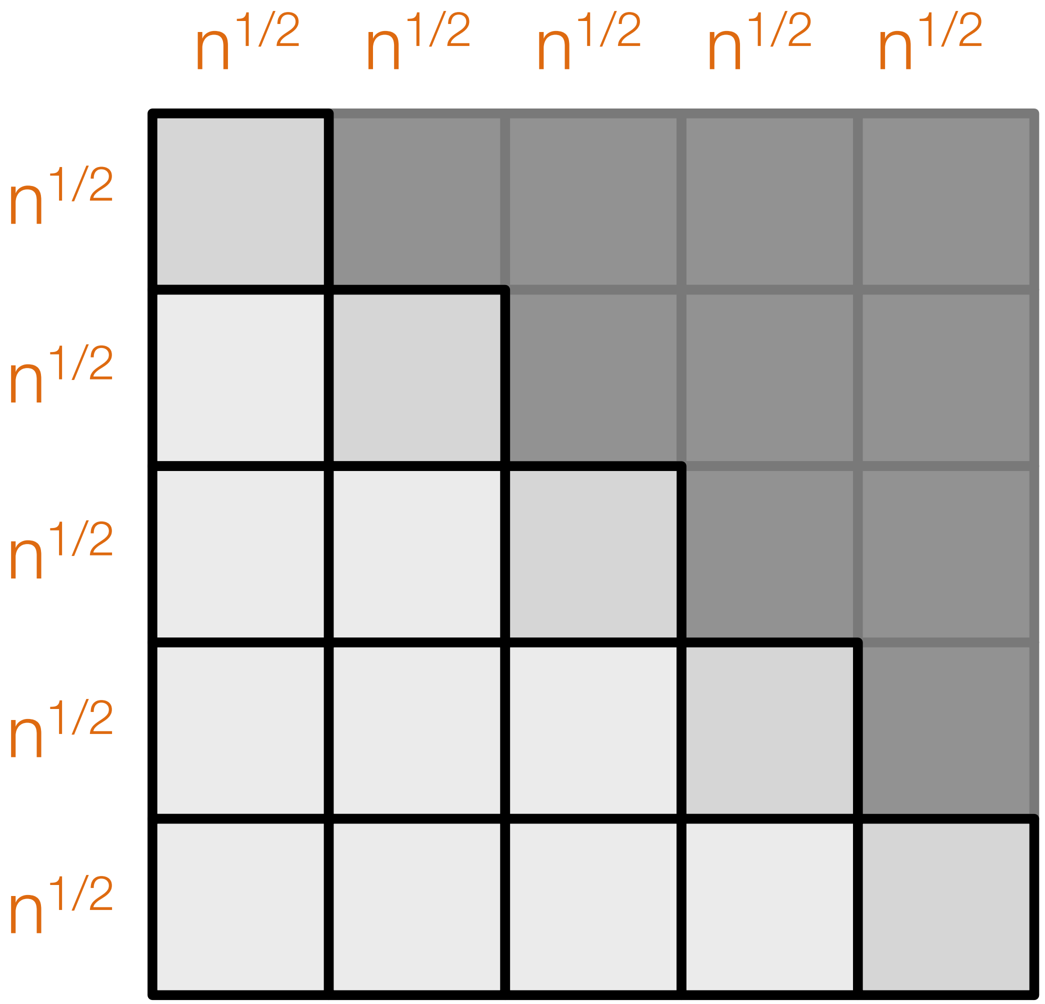
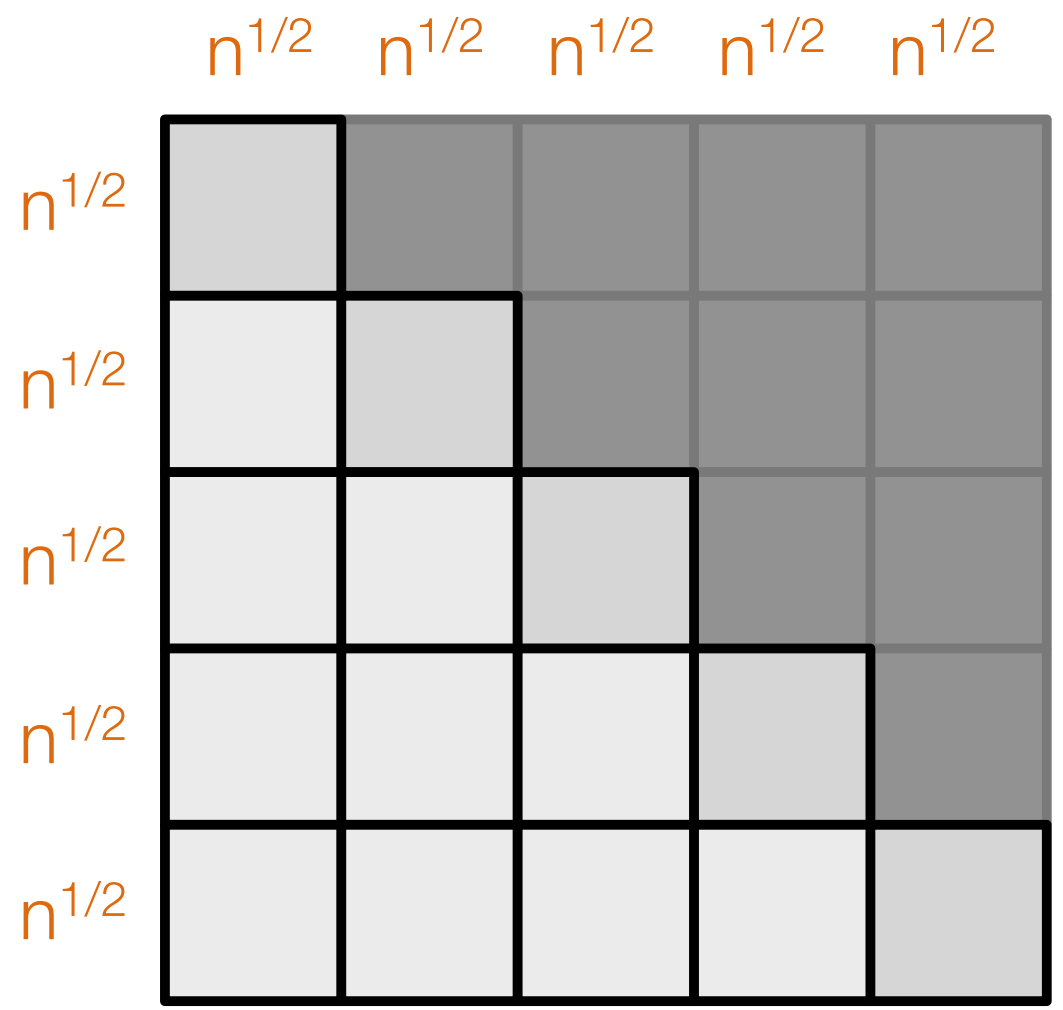


Proof Sketch:

Block-sparsification

1. Partition the adjacency matrix to n blocks of size $n^{1/2}$ x $n^{1/2}$
2. Each node learns a single block [Lenzen 2013]



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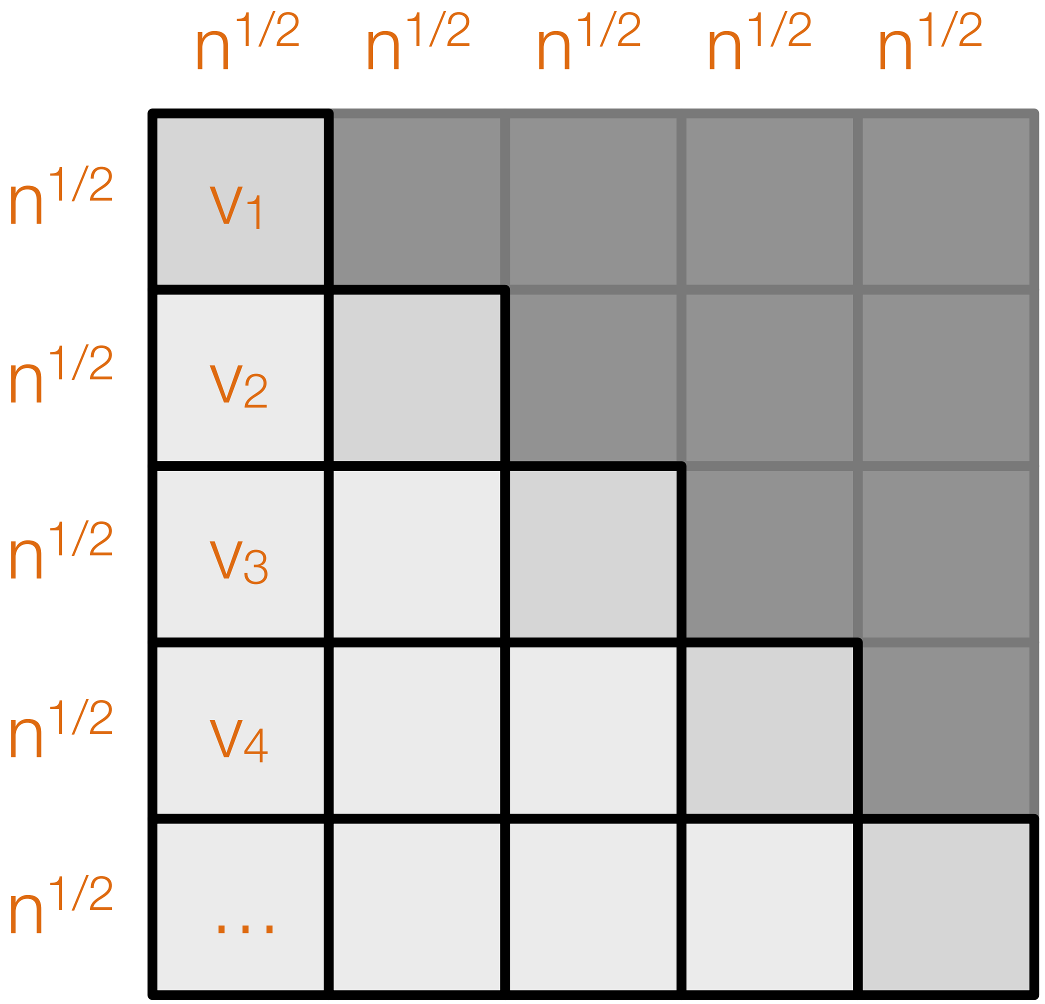

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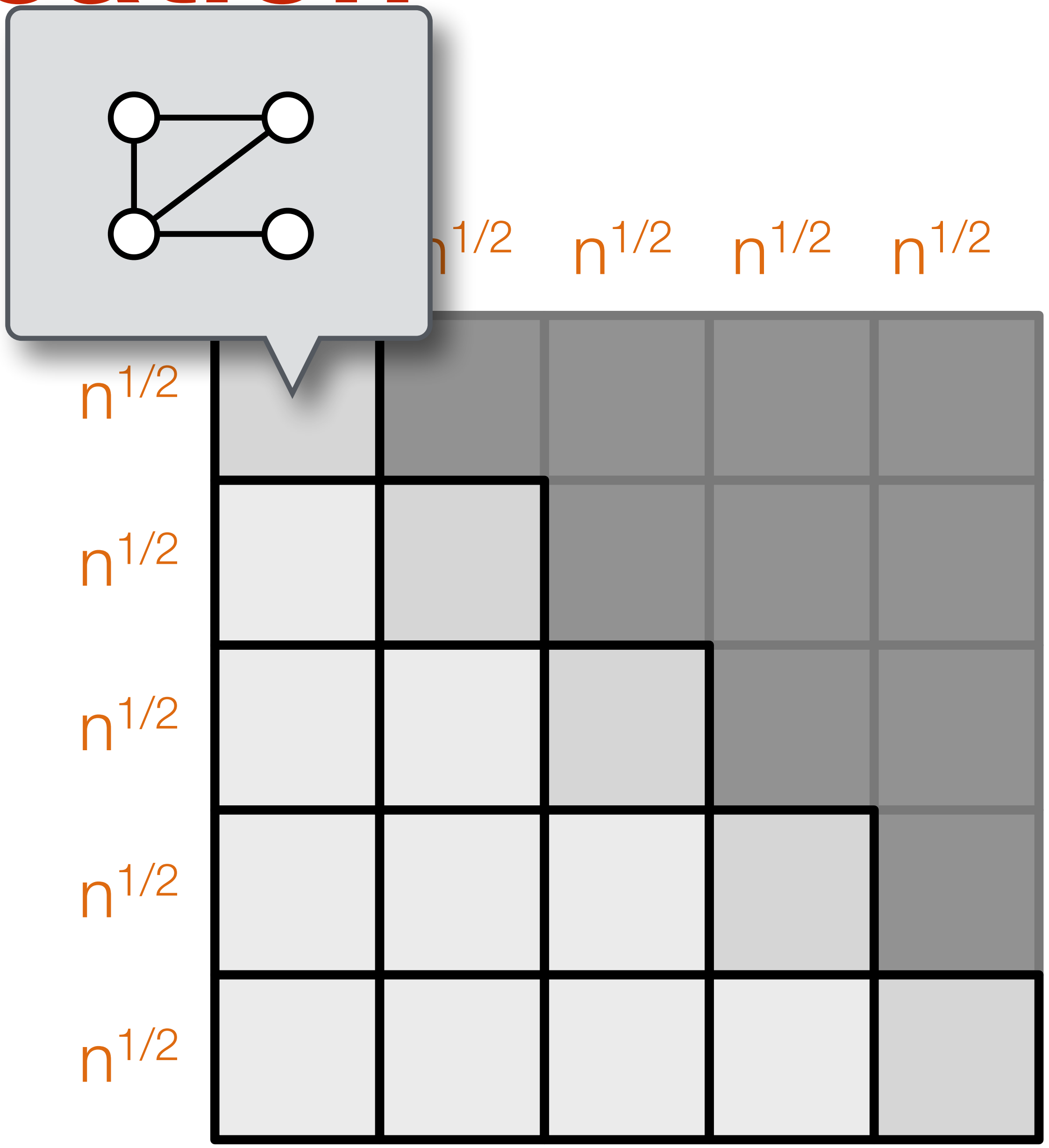

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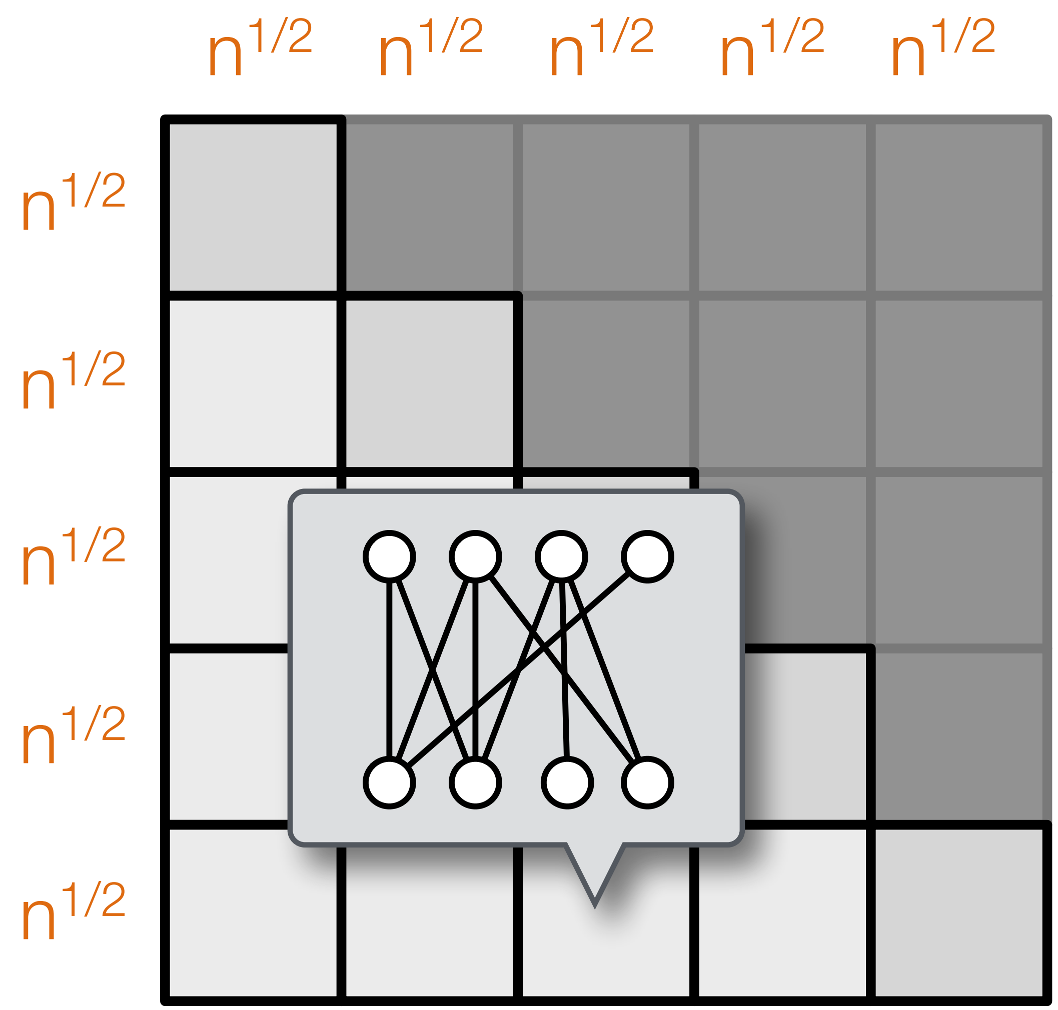

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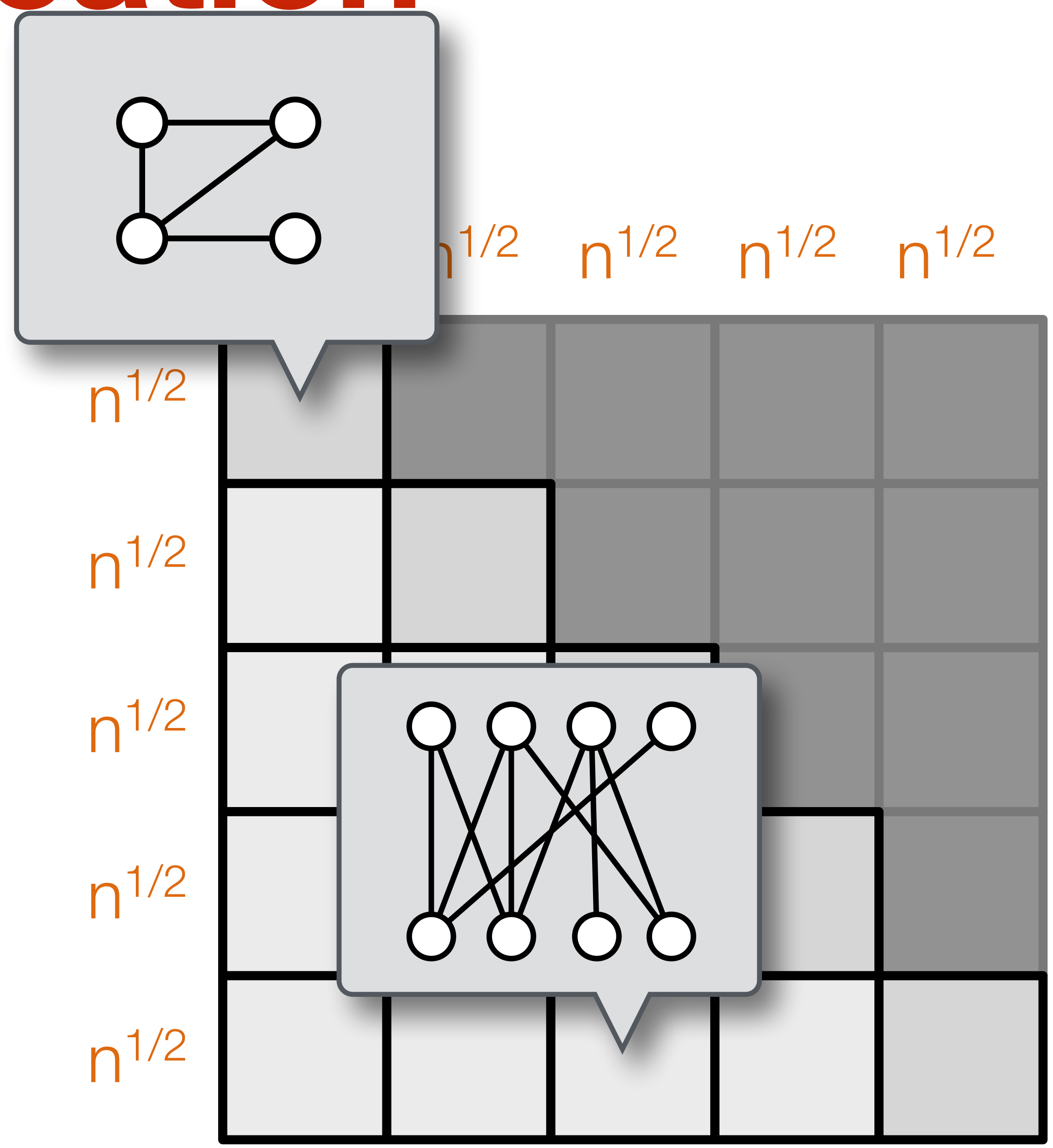
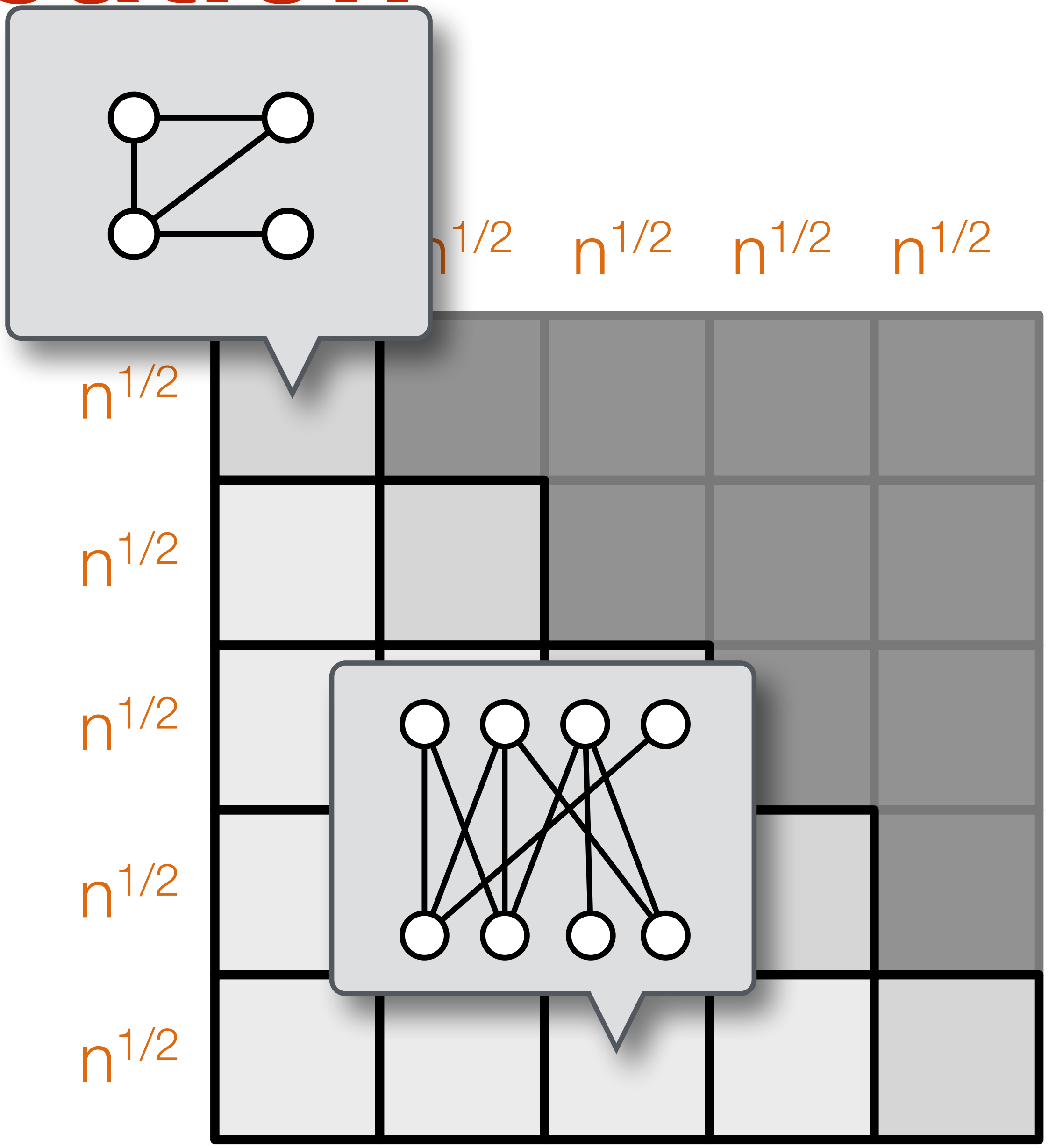


Proof Sketch:

Block-sparsification

1. Partition the adjacency matrix to n blocks of size $n^{1/2}$ x $n^{1/2}$
2. Each node learns a single block [Lenzen 2013]
3. locally find minimum spanning forest to the subgraph given by the block
   - subgraph has $2n^{1/2}$ nodes
   - each MSF has $2n^{1/2}$ edges
   - total $O(n^{3/2})$ edges



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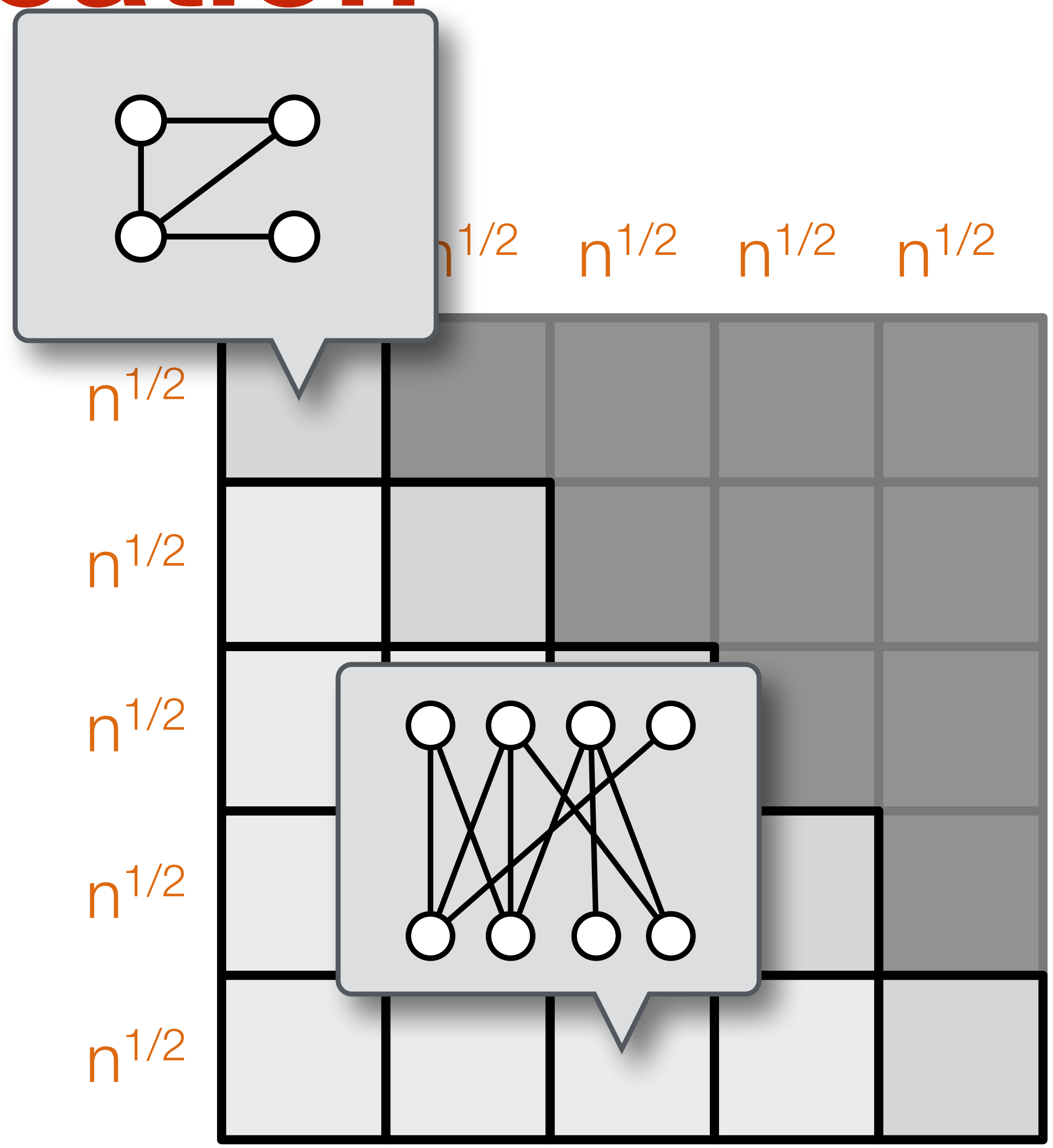
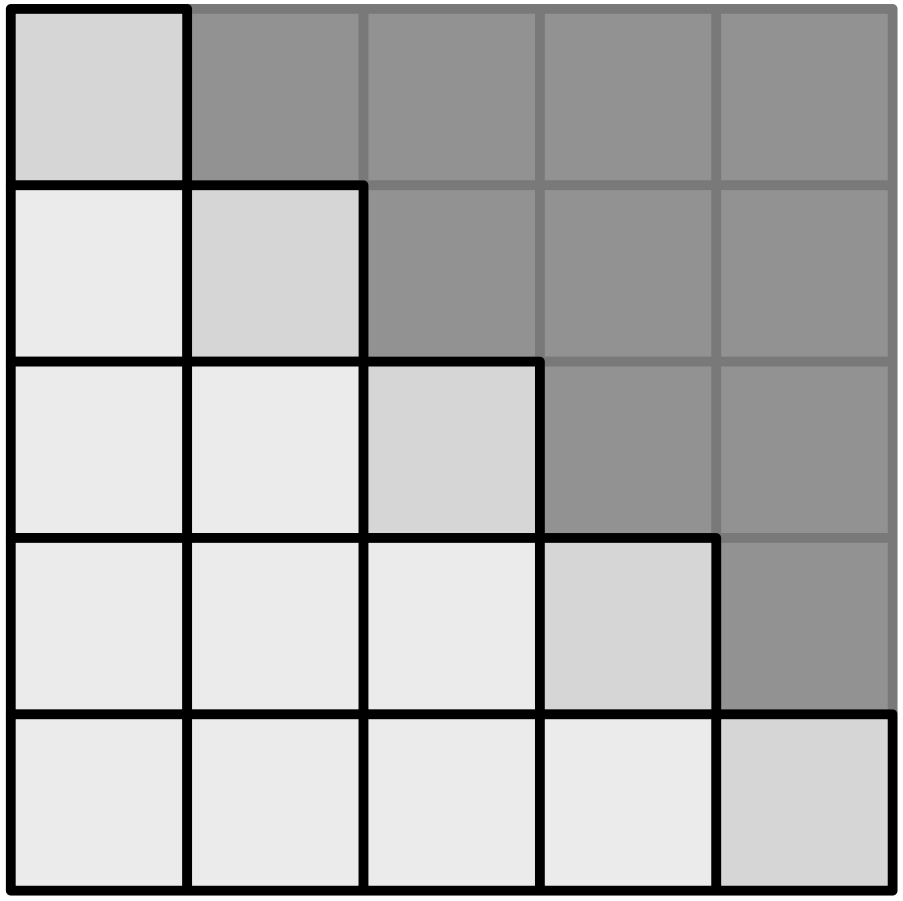


(repeat with larger blocks to get better sparsity)



- **Other applications** of block-sparsification?
  - need **sparse** representations of partial solutions
  - approximate APSP, build spanners in blocks?

Thanks!
Questions?