AFFINE REGISTRATION WITH MULTI-SCALE AUTOCONVOLUTION

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ABSTRACT

In this paper we propose a novel method for the recovery of affine transformation parameters between two images. Registration is achieved without separate feature extraction by directly utilizing the intensity distribution of the images. The method can also be used for matching point sets under affine transformations. Our approach is based on the same probabilistic interpretation of the image function as the recently introduced Multi-Scale Autoconvolution (MSA) transform. Here we describe how the framework may be used in image registration and present two variants of the method for practical implementation. The proposed method is experimented with binary and grayscale images and compared with other non-feature-based registration methods. The experiments show that the new method can efficiently align images of isolated objects and is relatively robust.

1. INTRODUCTION

There are basically two main approaches to image registration: feature-based and featureless solutions [7]. In the feature-based solutions the aim is to first extract some salient features from both images and then find correspondence between these features. Thereafter the feature correspondences are used to recover the geometric transformation that registers the images. The problem with this approach is that it is not always possible to find a sufficient number of features which can be localized accurately from both images and matched reliably between them.

In the featureless approach the registration is achieved by directly utilizing the intensity information of the images. For example, in the Fourier-Mellin transform the log-polar mapping of the spectral magnitude makes it possible to efficiently match images under similarity transformations [2]. Unfortunately, there are quite a few direct registration methods for the more general class of affine transformations. The affine invariant spectral signatures [1] could be used but the computational cost of this method is quite high and it is primarily designed for object recognition purposes. The cross-weighted moments [6] and affine moment descriptors [4] are two possible approaches that have been proposed for affine registration. However, due to its computational complexity the cross-weighted moment method is somewhat cumbersome for large images.

In this paper, we propose a novel approach for the affine registration problem. The approach is based on the MSA transform [3], which is briefly described in Section 2. In Section 3, we derive our registration method by utilizing the framework behind the MSA. We also present a variant of the method that can be used to match affine transformed point sets without knowing the point correspondences. The implementational issues are covered in Section 4. The results are presented and discussed in Sections 5 and 6.

2. MULTI-SCALE AUTOCONVOLUTION

Let $f : \mathbb{R}^2 \to \mathbb{R}$ with $f \geq 0$ be an image intensity function in $L^1(\mathbb{R}^2) \cap L^2(\mathbb{R}^2)$. Then $p(\mathbf{x}) = f(\mathbf{x})/\|f\|_{L^1}$ is a probability density function, and we may take $\mathbf{X}_0, \mathbf{X}_1$, and $\mathbf{X}_2$ to be independent and identically distributed random variables with the probability density function p. For $\alpha, \beta \in \mathbb{R}$, we define a random variable

$$\mathbf{U}_{\alpha,\beta} = \alpha\mathbf{X}_1 + \beta\mathbf{X}_2 + \gamma\mathbf{X}_0, \tag{1}$$

where also the notation $\gamma = 1 - \alpha - \beta$ is introduced. It can be shown that the probability density function of $\mathbf{U}_{\alpha,\beta}$ is the double convolution

$$p_{\mathbf{U}_{\alpha,\beta}}(\mathbf{u}) = (p_\alpha * p_\beta * p_\gamma)(\mathbf{u}), \tag{2}$$

where $p_a(\mathbf{x}) = \frac{1}{a^2}p(\frac{\mathbf{x}}{a})$ if $a \neq 0$, and $p_a(\mathbf{x}) = \delta(\mathbf{x})$ if $a = 0$ (Dirac delta).

The MSA transform of f is defined as the expectation value of $f(\mathbf{U}_{\alpha,\beta})$, i.e.,

$$\begin{aligned} F(\alpha, \beta) &= E[f(\mathbf{U}_{\alpha,\beta})] \\ &= \int_{\mathbb{R}^2} f(\mathbf{u})(p_\alpha * p_\beta * p_\gamma)(\mathbf{u})d\mathbf{u}. \end{aligned} \tag{3}$$

It is essential for the efficient implementation of MSA that instead of computing the double convolution (2) and the integral (3) one may compute the transform in frequency domain. Indeed, using the Plancherel formula, $\int_{\mathbb{R}^2} f\bar{g} =$

$\int_{\mathbb{R}^2} \hat{f}\bar{\hat{g}}$, and noting that Fourier transform takes convolutions into products, we obtain from (3) that

$$F(\alpha,\beta) = \int_{\mathbb{R}^2} \hat{f}(-\boldsymbol{\xi})\hat{p}_\alpha(\boldsymbol{\xi})\hat{p}_\beta(\boldsymbol{\xi})\hat{p}_\gamma(\boldsymbol{\xi})d\boldsymbol{\xi}. \tag{4}$$

Since $\hat{f}_a(\boldsymbol{\omega}) = \hat{f}(a\boldsymbol{\omega})$ when $f_a(\mathbf{x}) = \frac{1}{a^2}f(\frac{\mathbf{x}}{a})$, the MSA transform of f may be written also in the form

$$F(\alpha,\beta) = \frac{1}{\hat{f}(\mathbf{0})^3}\int_{\mathbb{R}^2} \hat{f}(-\boldsymbol{\xi})\hat{f}(\alpha\boldsymbol{\xi})\hat{f}(\beta\boldsymbol{\xi})\hat{f}(\gamma\boldsymbol{\xi})d\boldsymbol{\xi}, \tag{5}$$

which holds for all α, β.

The MSA transform values are invariant to affine transformations of the image and can be used as affine invariant features in object recognition and classification [3, 5].

3. AFFINE REGISTRATION

3.1. MSA descriptors

In image registration, we would need such descriptors that allow to recover the geometric transformation between the images, instead of being transformation invariant. Therefore we slightly modify the approach above and define the *MSA descriptors* of f in $\mathbb{R}^2$ as follows.

Definition 1 *Let $f \geq 0$ be a function in $L^1(\mathbb{R}^2) \cap L^2(\mathbb{R}^2)$ with a compact support, and let $p(\mathbf{x}) = f(\mathbf{x})/\|f\|_{L^1}$ be the corresponding probability density function. Take $\mathbf{X}_0, \mathbf{X}_1$, and $\mathbf{X}_2$ to be independent and identically distributed random variables in $\mathbb{R}^2$ with the probability density function p. For $\alpha, \beta \in \mathbb{R}$ set $\gamma = 1 - \alpha - \beta$ and define the random variable $\mathbf{U}_{\alpha,\beta}$ by* (1). *The MSA descriptor $\mathbf{H}(\alpha,\beta)$ of f is defined as*

$$\mathbf{H}(\alpha,\beta) = E[\mathbf{U}_{\alpha,\beta} f(\mathbf{U}_{\alpha,\beta})] \tag{6}$$

By explicitly writing out the expectation values in (6) we get

$$\begin{aligned}\mathbf{H}(\alpha,\beta) &= \int_{\mathbb{R}^2} \mathbf{u} f(\mathbf{u})(p_\alpha * p_\beta * p_\gamma)(\mathbf{u})d\mathbf{u} \\ &= \int_{\mathbb{R}^2} \hat{\mathbf{h}}(-\boldsymbol{\xi})\hat{p}_\alpha(\boldsymbol{\xi})\hat{p}_\beta(\boldsymbol{\xi})\hat{p}_\gamma(\boldsymbol{\xi})d\boldsymbol{\xi} \\ &= \frac{1}{\hat{f}(\mathbf{0})^3}\int_{\mathbb{R}^2} \hat{\mathbf{h}}(-\boldsymbol{\xi})\hat{f}(\alpha\boldsymbol{\xi})\hat{f}(\beta\boldsymbol{\xi})\hat{f}(\gamma\boldsymbol{\xi})d\boldsymbol{\xi},\end{aligned} \tag{7}$$

where $\hat{\mathbf{h}}(\boldsymbol{\xi})$ is the Fourier transform of $\mathbf{x}f(\mathbf{x})$.

The MSA descriptors have the following important property.

Property 1 *Let $\mathcal{A}(\mathbf{x}) = \mathbf{T}\mathbf{x} + \mathbf{t}$ be an affine transformation, where $\mathbf{T}$ is a nonsingular matrix. Let f be an image intensity function and f' the $\mathcal{A}$ transformed version of f, i.e., $f'(\mathbf{x}) = f(\mathcal{A}^{-1}(\mathbf{x}))$. Then the MSA descriptor $\mathbf{H}'(\alpha,\beta)$ of f' is obtained from the corresponding descriptor of f by*

$$\mathbf{H}'(\alpha,\beta) = \mathbf{T}\mathbf{H}(\alpha,\beta) + \mathbf{t}F(\alpha,\beta), \tag{8}$$

where $F(\alpha,\beta)$ is the MSA transform of f.

Proof: From (7) we get

$$\mathbf{H}'(\alpha,\beta) = \frac{1}{\hat{f}'(\mathbf{0})^3}\int_{\mathbb{R}^2} \hat{\mathbf{h}}'(-\boldsymbol{\xi})\hat{f}'(\alpha\boldsymbol{\xi})\hat{f}'(\beta\boldsymbol{\xi})\hat{f}'(\gamma\boldsymbol{\xi})d\boldsymbol{\xi}, \tag{9}$$

where

$$\begin{aligned}\hat{\mathbf{h}}'(\boldsymbol{\xi}) &= \int_{\mathbb{R}^2} e^{-j2\pi\boldsymbol{\xi}\cdot\mathbf{x}}\mathbf{x}f'(\mathbf{x})d\mathbf{x} \\ &= \int_{\mathbb{R}^2} e^{-j2\pi\boldsymbol{\xi}\cdot(\mathbf{T}\mathbf{y}+\mathbf{t})}(\mathbf{T}\mathbf{y}+\mathbf{t})f(\mathbf{y})|\det\mathbf{T}|d\mathbf{y} \\ &= e^{-j2\pi\boldsymbol{\xi}\cdot\mathbf{t}}|\det\mathbf{T}|\left(\mathbf{T}\hat{\mathbf{h}}(\mathbf{T}^\top\boldsymbol{\xi}) + \mathbf{t}\hat{f}(\mathbf{T}^\top\boldsymbol{\xi})\right).\end{aligned} \tag{10}$$

For the Fourier transform of f' it holds

$$\hat{f}'(\boldsymbol{\xi}) = e^{-j2\pi\boldsymbol{\xi}\cdot\mathbf{t}}|\det\mathbf{T}|\hat{f}(\mathbf{T}^\top\boldsymbol{\xi}). \tag{11}$$

By substituting (10) and (11) into (9), noting that $\alpha + \beta + \gamma = 1$ and changing variables, one sees that (8) holds. □

Hence, by computing $\mathbf{H}(\alpha_i,\beta_i)$, $\mathbf{H}'(\alpha_i,\beta_i)$ and $F(\alpha_i,\beta_i)$ for at least three different pairs (α_i,β_i) one obtains a set of linear equations from which $\mathbf{T}$ and $\mathbf{t}$ may be solved. If more than three points are used, a linear least-squares solution of

$$\min_{\mathbf{T},\mathbf{t}} \sum_i \|\mathbf{H}'(\alpha_i,\beta_i) - (\mathbf{T}\mathbf{H}(\alpha_i,\beta_i) + \mathbf{t}F(\alpha_i,\beta_i))\|^2 \tag{12}$$

can be computed.

In order to avoid singular configurations when choosing the suitable set of points (α_i,β_i) one should take into account the symmetries of $\mathbf{H}(\alpha,\beta)$.

Property 2 *The descriptor values $\mathbf{H}(\alpha,\beta)$ have symmetry over three lines: $\alpha - \beta = 0$, $\alpha + 2\beta = 1$ and $2\alpha + \beta = 1$.*

Proof: The symmetries are the same as those of $F(\alpha,\beta)$ [3] and they follow from the symmetries of $p_{\mathbf{U}_{\alpha,\beta}}$. □

Remark 1 *Notice that for the invariance property*

$$E[g'(\mathbf{U}'_{\alpha,\beta})] = E[g(\mathbf{U}_{\alpha,\beta})]$$

and the transformation property

$$E[\mathbf{U}'_{\alpha,\beta}g'(\mathbf{U}'_{\alpha,\beta})] = \mathbf{T}E[\mathbf{U}_{\alpha,\beta}g(\mathbf{U}_{\alpha,\beta})] + \mathbf{t}E[g(\mathbf{U}_{\alpha,\beta})]$$

it is not required that the function g is equal to f which is used to define the random variable $\mathbf{U}_{\alpha,\beta}$. It is sufficient that $g' = g \circ \mathcal{A}^{-1}$, where the affine transformation $\mathcal{A}$ is the same as in $f' = f \circ \mathcal{A}^{-1}$.

Remark 2 *Moreover, if $E[g(\mathbf{U}_{\alpha,\beta})] \neq 0$, we may write the transformation property in the form*

$$\frac{E[\mathbf{U}'_{\alpha,\beta}g'(\mathbf{U}'_{\alpha,\beta})]}{E[g'(\mathbf{U}'_{\alpha,\beta})]} = \mathbf{T}\frac{E[\mathbf{U}_{\alpha,\beta}g(\mathbf{U}_{\alpha,\beta})]}{E[g(\mathbf{U}_{\alpha,\beta})]} + \mathbf{t}. \tag{13}$$

Here we may replace g' with some scalar multiplied version $sg'(\mathbf{x}) = sg(\mathcal{A}^{-1}(\mathbf{x}))$, $s \neq 0$, so that (13) *still holds.*

3.2. Matching point sets

Assume that we have a set of two-dimensional points, $\mathbf{x}_i$, and an affine transformed version of it, $\mathbf{x}'_i = \mathcal{A}(\mathbf{x}_i)$, and we would like to solve the transformation between the point patterns without knowing the point correspondences.

We consider that the points $\mathbf{x}_i$ are random samples of a random variable $\mathbf{X}$ with some probability density p. Then the points $\mathbf{x}'_i$ are samples of $\mathbf{X}'$ which has density $p' = (p \circ \mathcal{A}^{-1})/\|p \circ \mathcal{A}^{-1}\|_{L^1}$. We denote the mean and covariance of $\mathbf{X}$ by $\boldsymbol{\mu}$ and $\mathbf{C}$, and those of $\mathbf{X}'$ by $\boldsymbol{\mu}'$ and $\mathbf{C}'$. For $\alpha, \beta \in \mathbb{R}$ we may use p and p' to define random variables $\mathbf{U}_{\alpha,\beta}$ and $\mathbf{U}'_{\alpha,\beta}$ as in Definition 1. By defining

$$g(\mathbf{x}) = N(\boldsymbol{\mu}, \mathbf{C}), \quad g'(\mathbf{x}) = N(\boldsymbol{\mu}', \mathbf{C}'), \tag{14}$$

where $N(\boldsymbol{\mu}, \mathbf{C})$ is the Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\mathbf{C}$, we have $g'(\mathbf{x}) = g(\mathcal{A}^{-1}(\mathbf{x}))/|\det \mathbf{T}|$. Therefore, in principle, we could use (13) to solve $\mathbf{T}$ and $\mathbf{t}$.

In practice, we do not know the functions p and p', which implies that we do not know the probability density functions of $\mathbf{U}_{\alpha,\beta}$ and $\mathbf{U}'_{\alpha,\beta}$. Neither do we know the means and covariances of $\mathbf{X}$ and $\mathbf{X}'$. However, we may estimate these as sample means and sample covariances from the point sets $\{\mathbf{x}_i\}$ and $\{\mathbf{x}'_i\}$. We may also compute the expectation values in (13) as sample means. Namely, each point $(\mathbf{x}_{i_1}, \mathbf{x}_{i_2}, \mathbf{x}_{i_3})$ of the Cartesian product set $\{\mathbf{x}_i\} \times \{\mathbf{x}_i\} \times \{\mathbf{x}_i\}$ defines a sample of $\mathbf{U}_{\alpha,\beta}$ by $\mathbf{u} = \alpha\mathbf{x}_{i_1} + \beta\mathbf{x}_{i_2} + (1 - \alpha - \beta)\mathbf{x}_{i_3}$. The samples of $\mathbf{U}'_{\alpha,\beta}$ are obtained respectively.

Thus, by using at least three (α, β)-pairs we get from (13) a set of equations from which $\mathbf{T}$ and $\mathbf{t}$ may be solved. In the noiseless case the affine transformation is recovered exactly, up to a numeric round off error, but the above probabilistic framework provides a justification of the method also when there is noise in the point coordinates or some points in the other set have no counterparts in the other.

Remark 3 *The approach presented here for point pattern matching can be applied also in image registration. The pixels of an image are considered as 2D points and the grayscale values are used as weight factors when computing the sample means.*

4. IMPLEMENTATION

4.1. MSA descriptors

The implementation of the MSA descriptors is based on the Fast Fourier Transform (FFT) [3]. By discretizing the second integral in (7) we get

$$H_k(\alpha, \beta) = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1 N_2} \mathcal{H}_k(\mathbf{v}_i)\mathcal{P}_\alpha(\mathbf{v}_i)\mathcal{P}_\beta(\mathbf{v}_i)\mathcal{P}_\gamma(\mathbf{v}_i), \tag{15}$$

where $k = \{1, 2\}$ and $\mathcal{H}_k$ is the discrete Fourier transform (DFT) of $x_k f(\mathbf{x})$ and each $\mathcal{P}_a$ is the DFT of the corresponding discrete function p_a. If the size of the image f is $M_1 \times M_2$, we must choose the DFT lengths so that $N_i \geq (|\alpha| + |\beta| + |\gamma|)M_i - 2$ in order to avoid the wrap-around error. To avoid large DFT lengths α and β should be reasonably small numbers.

To compute (15) we need scaled versions of the original image. We do the interpolation and decimation in such a way that the probability mass of each image region is approximately preserved in the scaling.

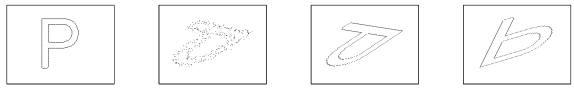
The computational complexity of the MSA descriptors for an $N \times N$ image is $O(N^2 \log N)$ since the complexity of the FFT is $O(N^2 \log N)$ and the summations and multiplications in (15) are only $O(N^2)$ operations.

4.2. Point-based method

Implementation of the point-based matching method of Section 3.2 is straightforward. However, in order to increase efficiency we choose the values α, β so that $1 - \alpha - \beta = 0$. Then the number of samples of $\mathbf{U}_{\alpha,\beta}$ may be reduced from n^3 to n^2, where n is the number of points in the set $\{\mathbf{x}_i\}$. Since an $N \times N$ image contains N^2 pixels the computational complexity of the method is $O(N^4)$ which is the same as the complexity of the cross-weighted moment method [6].

5. EXPERIMENTS

5.1. Point pattern



(a) (b) (c) (d)

Fig. 1: Pattern matching: original (a), transformed and noise added (b), recovered transformation with MSAP (c), and with MD (d).

Table 1: The average values of the matching error ϵ among 1000 estimated transformations at six different levels of noise.

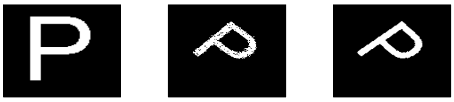
λ	0	0.02	0.04	0.06	0.08	0.10
MD	0.00	0.01	0.07	0.14	0.24	0.31
MSAP	0.00	0.04	0.08	0.12	0.16	0.22
CW	0.00	0.05	0.10	0.16	0.23	0.30

First we experimented our method with the point pattern shown in Fig. 1(a). The points were transformed with random affine transformations and isotropic Gaussian noise was added to the coordinates before matching, cf. Fig. 1(b). The random transformation matrices were chosen according to

$$\mathbf{T} = \begin{pmatrix} \cos(\omega) & -\sin(\omega) \\ \sin(\omega) & \cos(\omega) \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & d \end{pmatrix} \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix},$$

where $\omega, \phi \in [0, 2\pi]$ and $d \in [0.3, 1]$ are uniformly distributed random variables. The standard deviation σ of the Gaussian noise was chosen to be proportional to the standard deviation of the x-coordinates of the original data points, i.e., $\sigma = \lambda\sigma_x$, where values $\lambda \in [0, 0.1]$ were used.

Patterns were matched with three different methods: affine moment descriptors (MD) [4], point-based MSA method (MSAP) and cross-weighted moments (CW) [6]. In MSAP we used three (α, β)-pairs: $(0, 1)$, $(\frac{1}{3}, \frac{2}{3})$ and $(\frac{1}{2}, \frac{1}{2})$.



(a) (b) (c)

Fig. 2: Image registration: original (a), transformed and noise added ($P=0.08$) (b), recovered transformation ($\epsilon=0.13$) (c)

Table 2: Results for the binary image with different levels of noise.

P	0	0.02	0.04	0.06	0.08	0.10
MD	0.47	0.62	0.67	0.74	0.74	0.79
CW	0.11	0.17	0.21	0.28	0.29	0.34
MSAP	0.07	0.13	0.16	0.19	0.22	0.27
MSAP*	0.04	0.07	0.08	0.10	0.13	0.15
MSAD	0.05	0.09	0.13	0.15	0.16	0.20

For each estimated transformation matrix $\hat{\mathbf{T}}$ we evaluated the distance ϵ to the true matrix $\mathbf{T}$ by defining the points $\mathbf{p}_1 = (1,0)^\top$ and $\mathbf{p}_2 = (0,1)^\top$ and computing

$$\epsilon = \frac{1}{2}\sum_{i=1}^{2} \frac{\|(\mathbf{T}-\hat{\mathbf{T}})\mathbf{p}_i\|}{\|\mathbf{T}\mathbf{p}_i\|}. \qquad (16)$$

This is the measure we used to assess the matching result.

In Table 1 we have tabulated the average values of (16) among 1000 estimated transformations at different levels of noise. The results show that the new method seems to be most tolerant to noise. Although the moment descriptor method often gives a good result it sometimes badly fails, as in Fig. 1(d). The point-based MSA method and the cross-weighted moment method seemed to behave more steadily.

5.2. Binary image

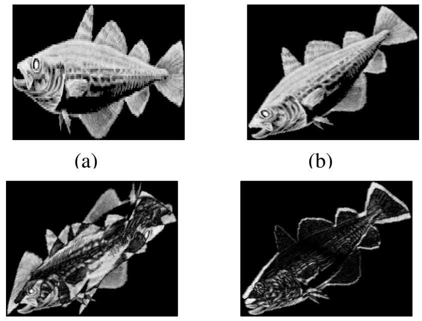
The second experiment was quite similar to the first but now we considered the binary image shown in Fig. 2(a). The random transformations were obtained as above. The noise added to the transformed images was uniformly distributed binary noise with the noise level P indicating the probability of a single pixel to change its value. After adding the noise we removed the separated noise pixels from the background, cf. Fig. 2(b).

We did 500 random affine transformations and the average errors ϵ at different noise levels are shown in Table 2. The method MSAP* is the point-based MSA method but with three additional (α,β)-pairs, and MSAD is the frequency implementation of the MSA method with the following (α,β)-pairs: $(0,0)$, $(\frac{1}{3},\frac{1}{3})$, $(\frac{1}{2},\frac{1}{2})$, $(1,1)$ and $(1,\frac{1}{4})$.

We can see that the moment descriptor method works badly with this data. It works fine for some transformations but it fails so often that the average error is quite high. All the MSA-based methods work reasonably well.

5.3. Grayscale image

We did experiments also with grayscale images. Since the methods MSAP and CW are computationally heavy and not very convenient for large images we used only the methods MD and MSAD to register the grayscale image in Fig. 3(a) with its affine transformed and noisy version, Fig. 3(b). The noise was Gaussian with standard deviation of 1% of the maximum intensity and was added also to the black background. As the difference images illustrate only the MSA method succeeds in registration.



(a) (b) (c) (d)

Fig. 3: (a) Original image, (b) transformed and noise added, (c) difference image with MD, (d) difference image with MSAD

6. CONCLUSIONS

We have proposed a novel method for affine registration of images and point patterns. The method is based on the MSA descriptors which were defined by utilizing the probabilistic interpretation of the image function analogously to the MSA transform [3]. It was shown that the MSA descriptors have the important transform property that allows the recovery of affine transformation between two images without separate feature extraction. For the registration of digital images we proposed an efficient frequency space implementation which is similar to the discrete MSA transform. The experiments showed that the new method performs robustly when compared to other similar methods. Nevertheless, there are still open questions for future research. One such is the optimal choice of (α,β)-pairs.

7. REFERENCES

[1] J. Ben-Arie and Z. Wang. Pictorial recognition of objects employing affine invariance in the frequency domain. *IEEE Trans. Pattern Anal. Mach. Intell.*, 20(6):604–618, 1998.

[2] Q. Chen, M. Defrise, and F. Deconinck. Symmetric phase-only matched filtering of fourier-mellin transforms for image registration and recognition. *IEEE Trans. Pattern Anal. Mach. Intell.*, 16(12):1156–1168, 1994.

[3] J. Heikkilä. Multi-scale autoconvolution for affine invariant pattern recognition. In *Proc. ICPR*, pages 119–122, 2002.

[4] J. Heikkilä. Pattern matching with affine moment descriptors. *Pattern Recognit.*, 37(9):1825–1834, 2004.

[5] E. Rahtu and J. Heikkilä. Object classification with multi-scale autoconvolution. In *Proc. ICPR*, pages 37–40, 2004.

[6] Z. Yang and F. Cohen. Cross-weighted moments and affine invariants for image registration and matching. *IEEE Trans. Pattern Anal. Mach. Intell.*, 21(8):804–814, 1999.

[7] B. Zitová and J. Flusser. Image registration methods: a survey. *Image Vis. Comput.*, 21:977–1000, 2003.