Deterministic Local Algorithms, Unique Identifiers, and Fractional Graph Colouring

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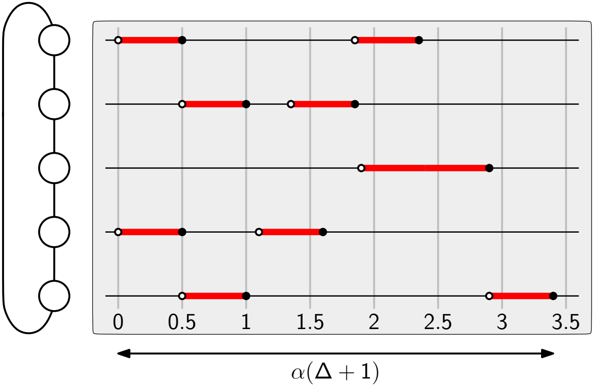
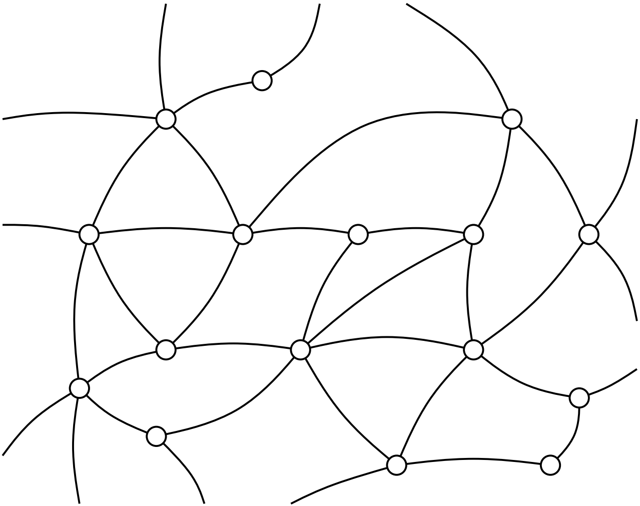
SIROCCO 2012
30 June 2012

Our Result



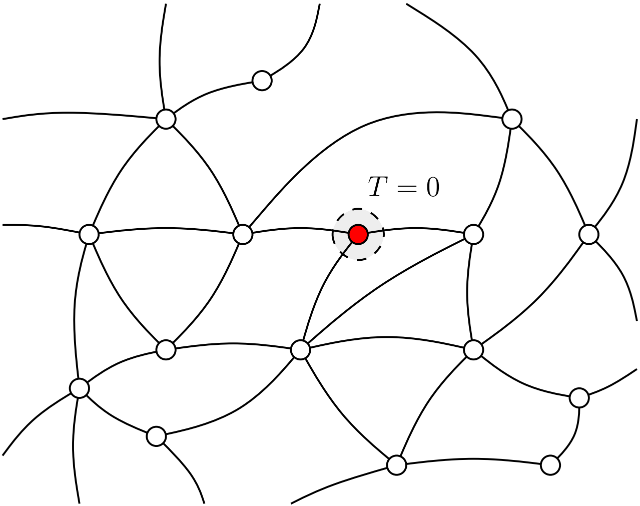
There is a deterministic distributed algorithm that runs in 1 communication round that, for any $\alpha > 1$, finds a fractional graph colouring of length at most $\alpha(\Delta + 1)$

Model of Computation: LOCAL



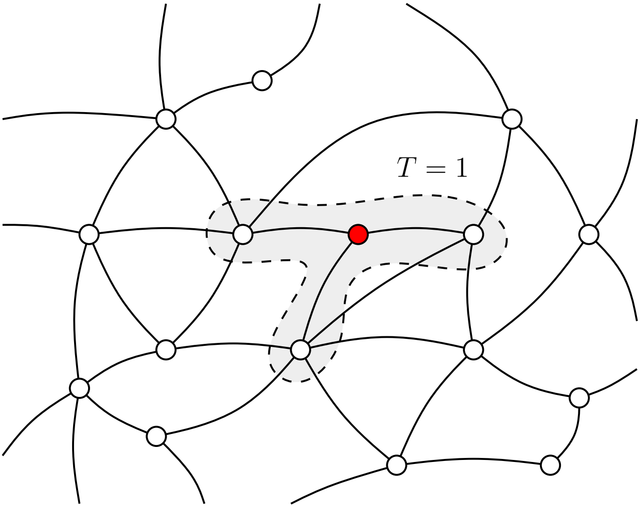
- Communication graph

Model of Computation: LOCAL



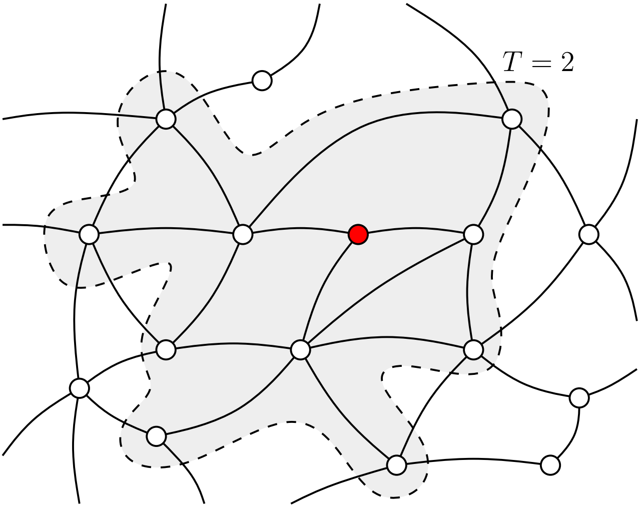
- Synchronous communication

Model of Computation: LOCAL



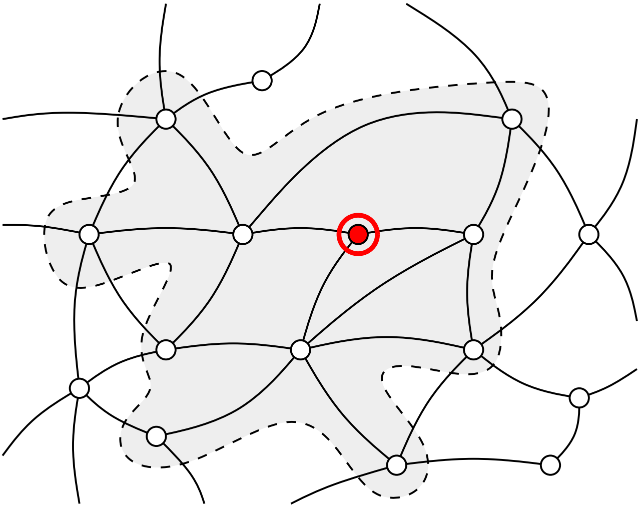
- Synchronous communication

Model of Computation: LOCAL



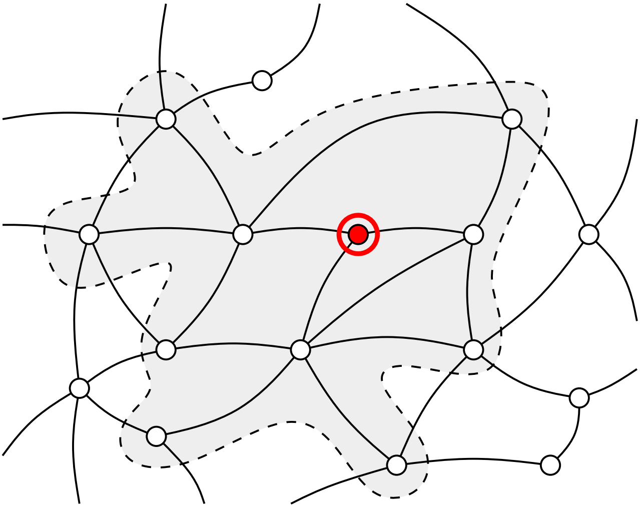
- In T rounds gather radius-T neighbourhood

Model of Computation: LOCAL



- Constant-time algorithms

Model of Computation: LOCAL

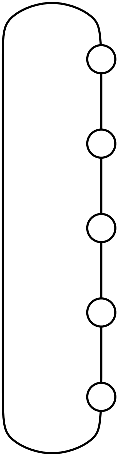


- Each node maps neighbourhood to output

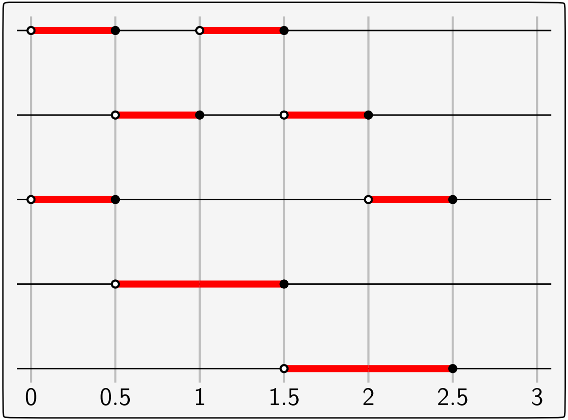
Fractional Graph Colouring

Fractional Graph Colouring

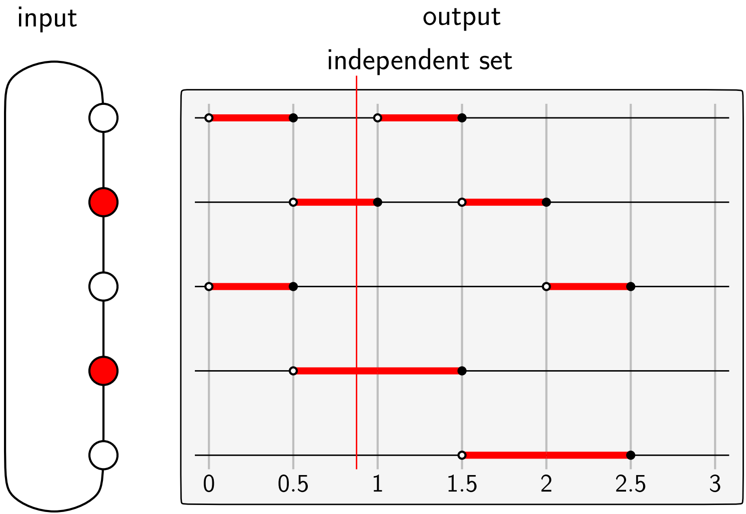
input



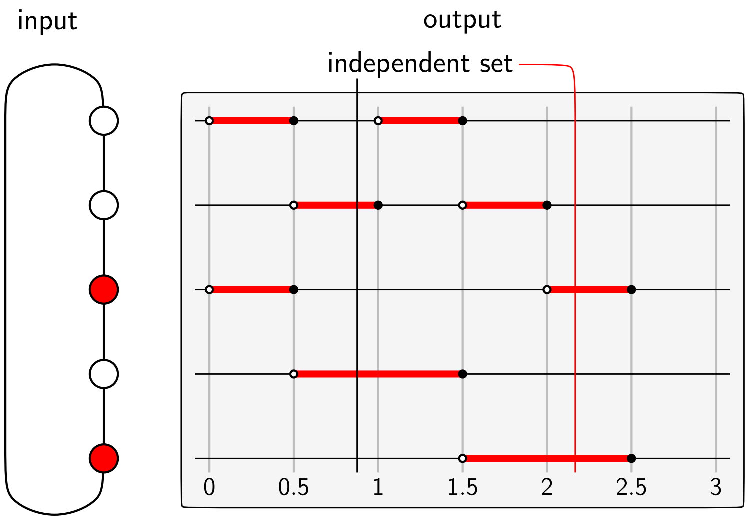
output



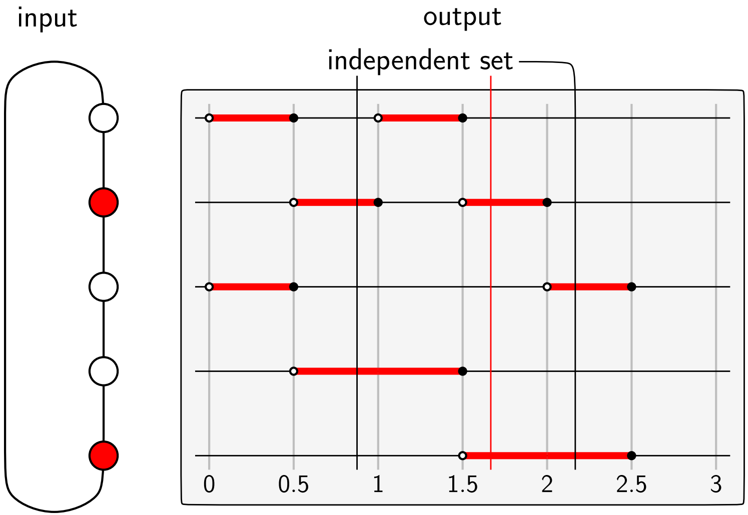
Fractional Graph Colouring



Fractional Graph Colouring



Fractional Graph Colouring



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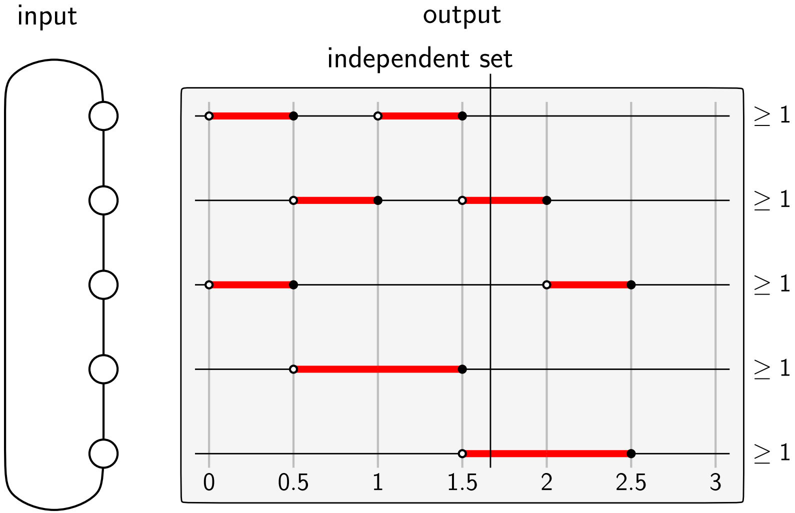

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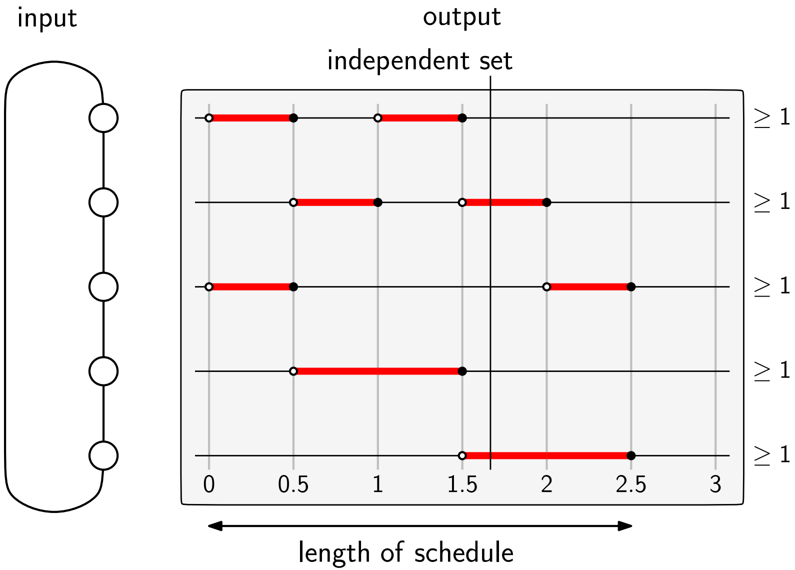

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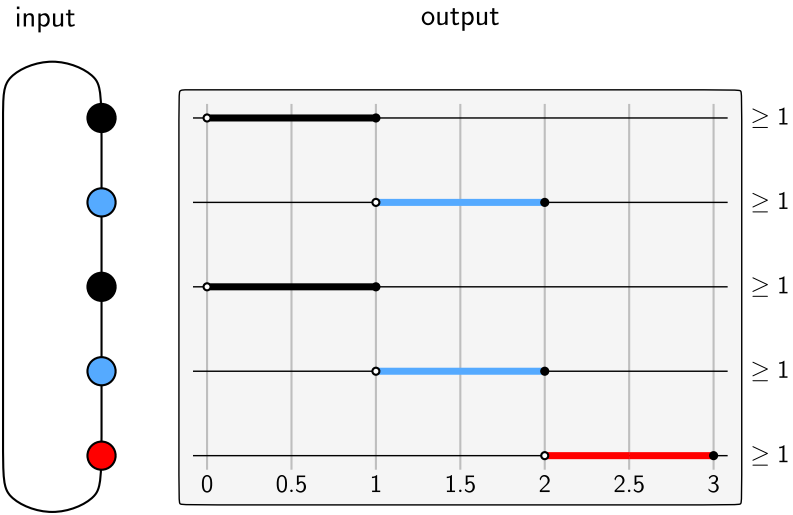
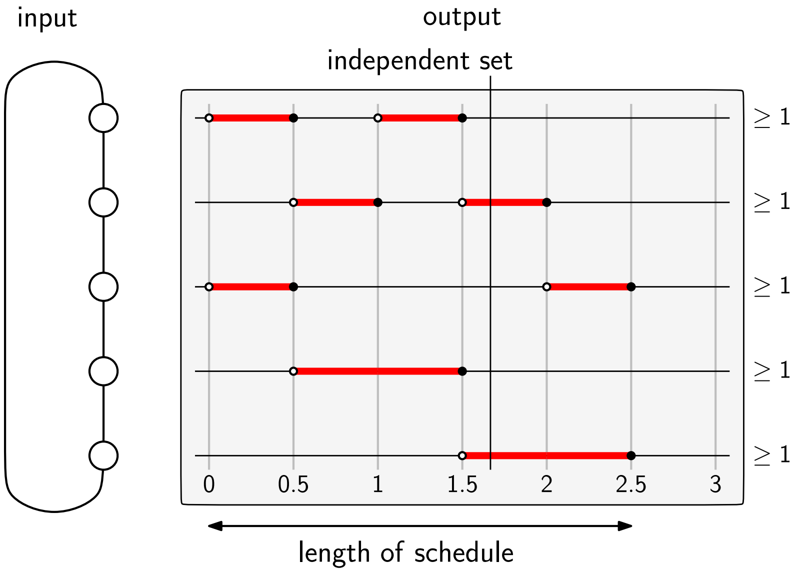
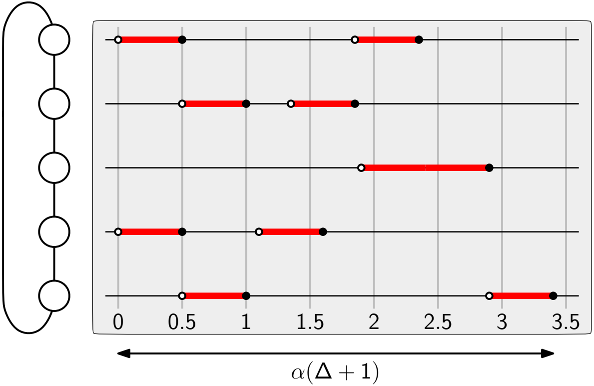


Fractional Graph Colouring

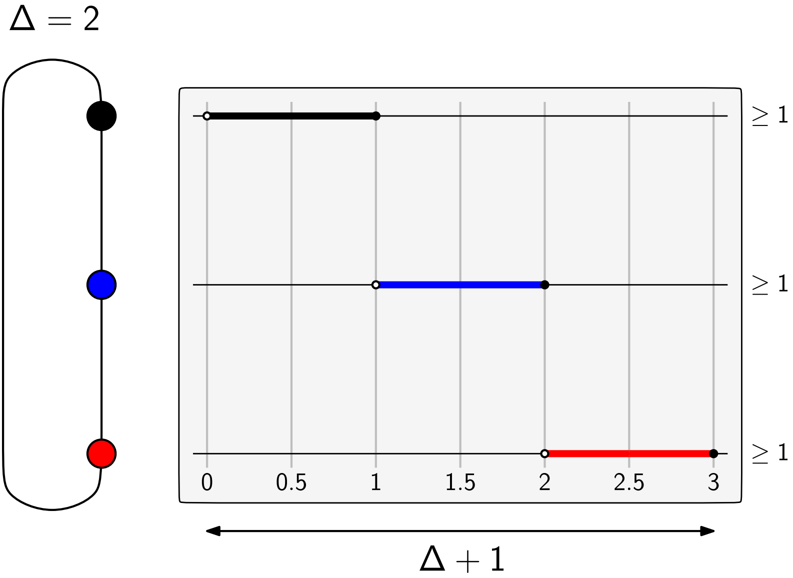


Our Result Again



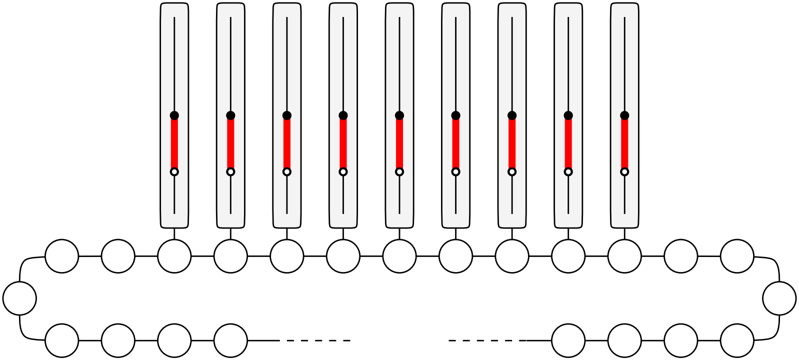
There is a deterministic distributed algorithm that runs in 1 communication round that, for any $\alpha > 1$, finds a fractional graph colouring of length at most $\alpha(\Delta + 1)$

Lower Bound



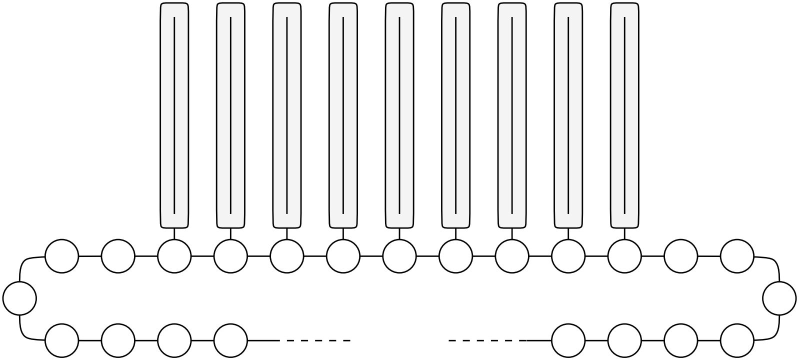
Finding a Fractional Graph Colouring

Finding a Fractional Graph Colouring



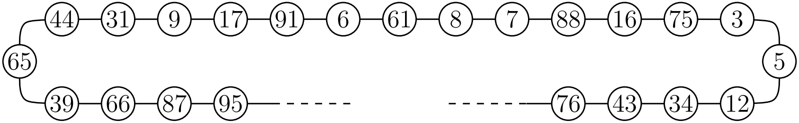
- Impossible to break symmetry in an anonymous cycle

Finding a Fractional Graph Colouring



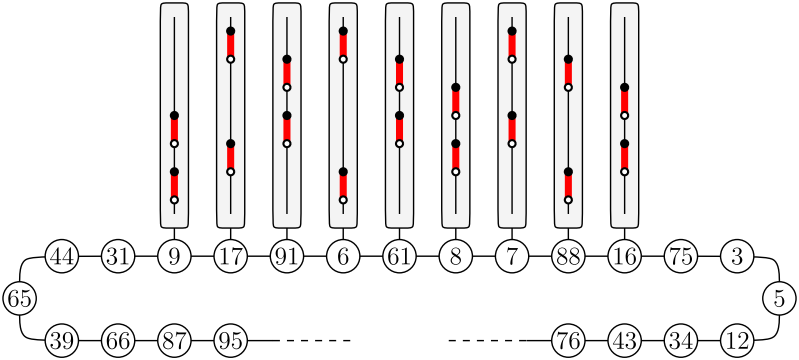
- Nodes must produce an empty schedule

Finding a Fractional Graph Colouring



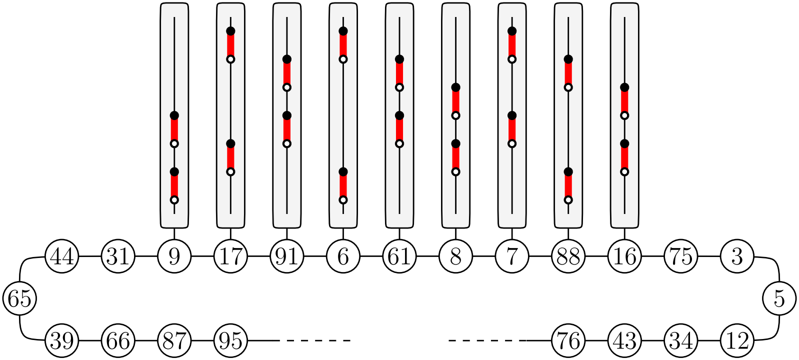
- Standard assumption: numeric identifiers

Finding a Fractional Graph Colouring



- Standard assumption: numeric identifiers

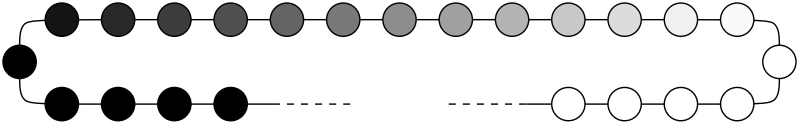
Finding a Fractional Graph Colouring

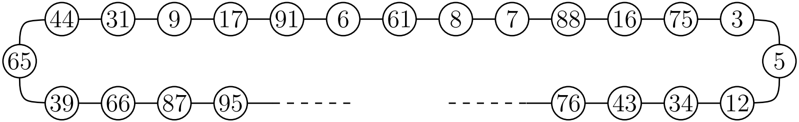


- Standard assumption: numeric identifiers
- FGC is the first example where numeric identifiers give a constant-time algorithm

Why Numeric Identifiers Do Not Help?

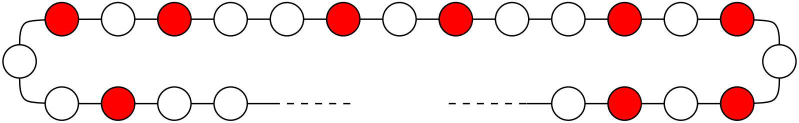
Numeric Identifiers Not Needed





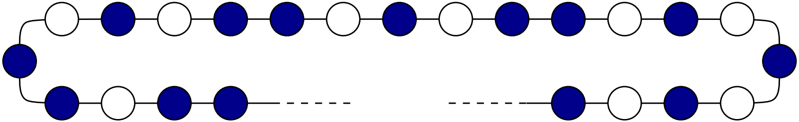
- Naor & Stockmeyer (1995) studied when numeric identifiers are necessary
- LCL-problems

LCL-problems



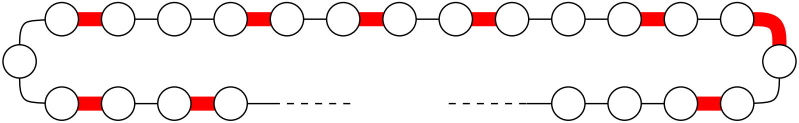
- Maximal Independent Set

LCL-problems



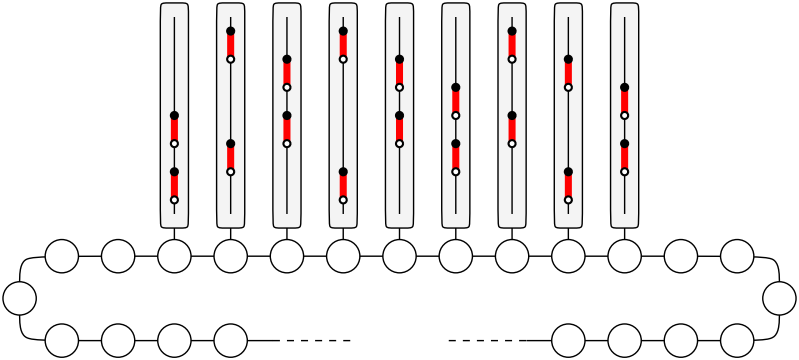
- Vertex Cover

LCL-problems



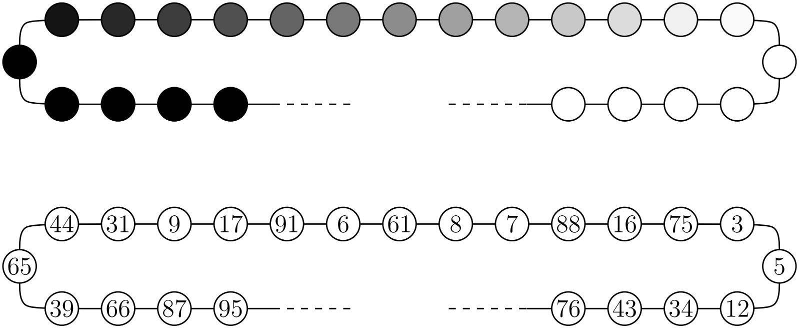
- Maximal Matching

LCL-problems



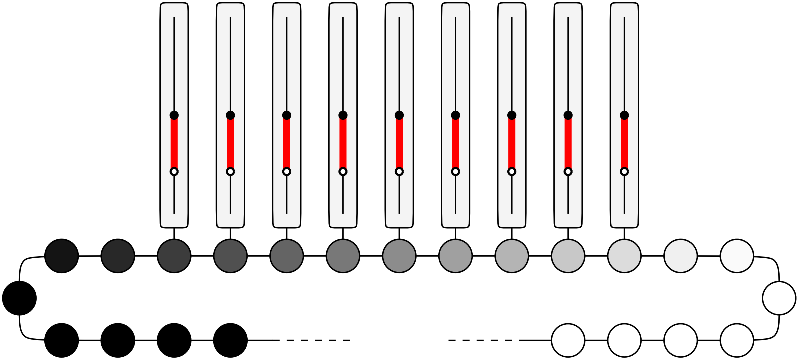
- Fractional Graph Colouring

Numeric Identifiers Not Needed



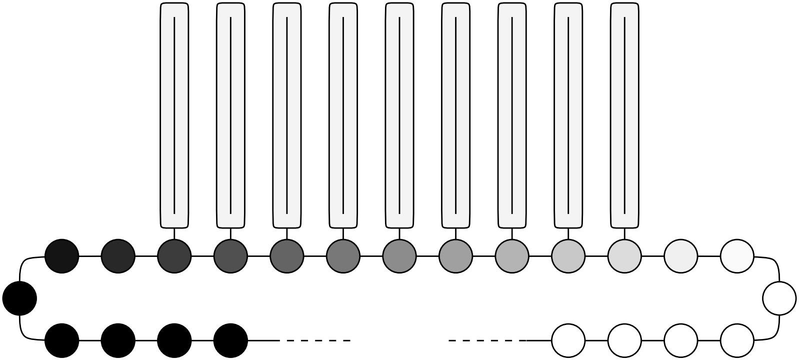
- Naor & Stockmeyer (1995): In LCL-problems numeric identifiers not necessary
  - Technicality: applies if output bounded

No FGC with Comparisons



- Identifiers arranged in an ascending order

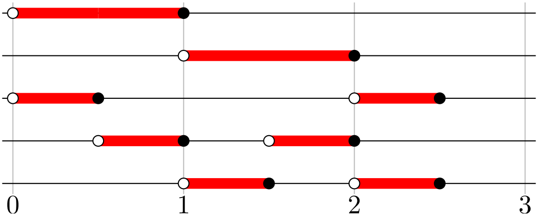
No FGC with Comparisons



- Some nodes must produce an empty schedule

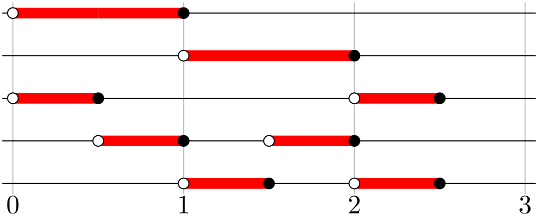
Why Numeric Identifiers Help with FGC?

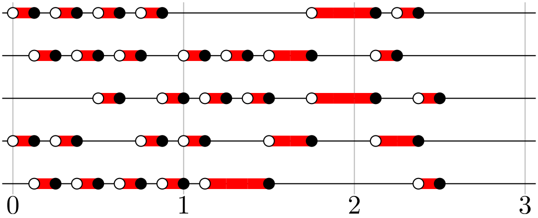
Non-constant output



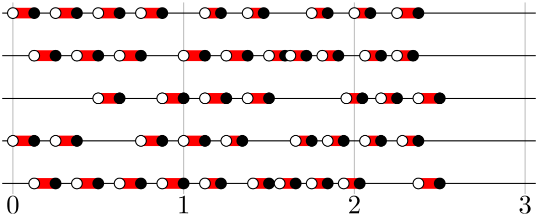
- In FGC natural encoding of solution not bounded in size

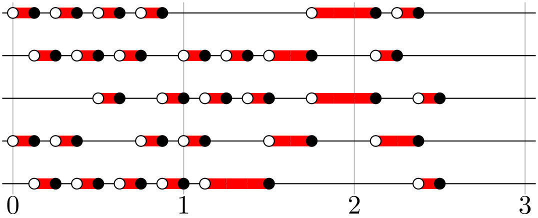
Non-constant output





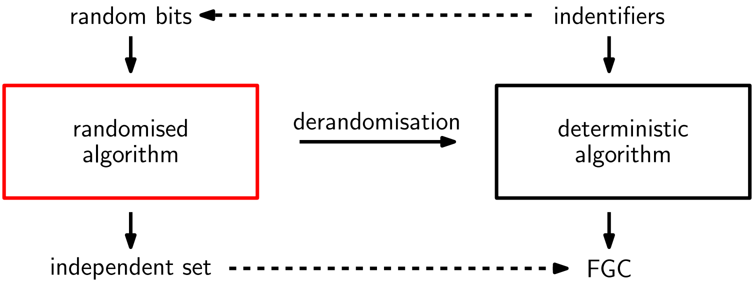
Non-constant output





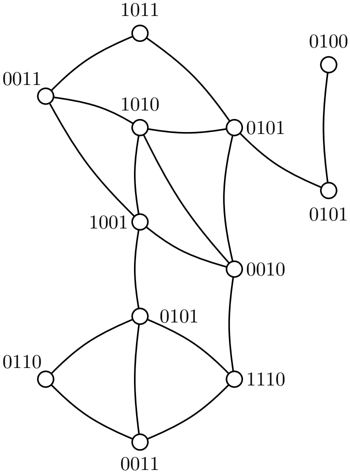
The Algorithm

Algorithm Design Idea



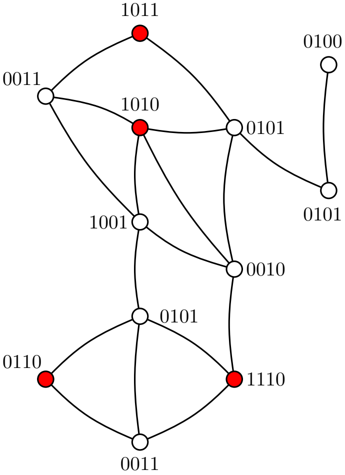
- Use a randomised independent set algorithm as a black box
- Iterate over possible random bit strings for the black box to get a deterministic algorithm

A Randomised Algorithm



- A randomised algorithm for the independent set problem
- Each node gets a random bit string

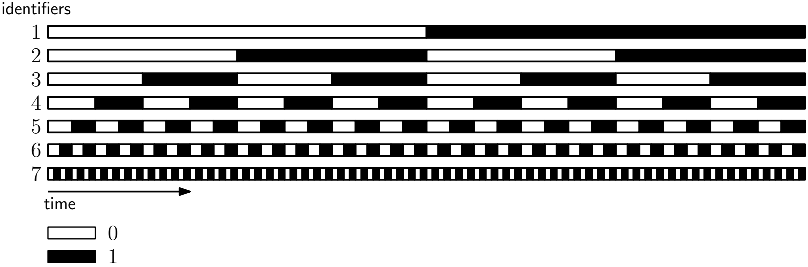
A Randomised Algorithm



- Local maxima join the independent set
- Guarantee: each node v joins with probability at least

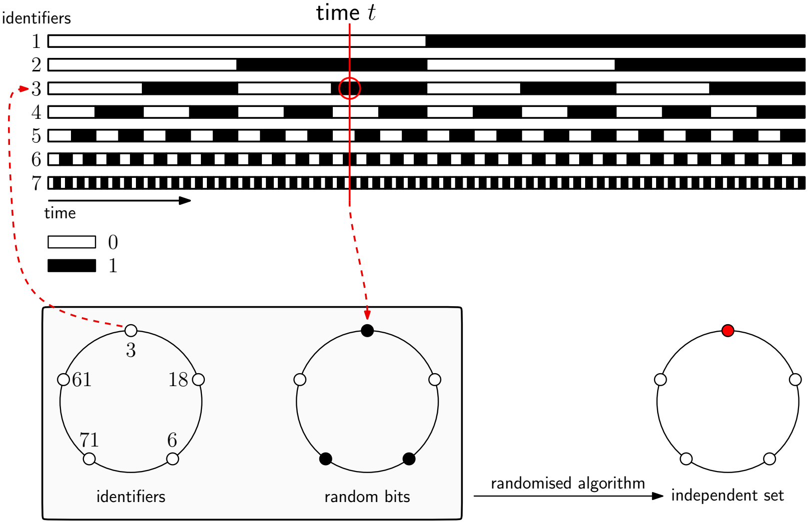
$$\frac{1-\varepsilon}{\deg(v)+1}$$

Deterministic Algorithm (Oversimplified)



- Simulate the random algorithm by iterating over all combinations of inputs
- Encoding of the output grows with size of the network
- By Naor & Stockmeyer, dependence on n is necessary

Deterministic Algorithm (Oversimplified)



Tradeoffs

Granularity Tradeoff

granularity	unbounded
length	$O(1)$
running time	$O(1)$

- Any two can be kept constant in bounded degree graphs
- Constant running time and length of schedule
  - This work
  - granularity of schedule grows with size of the network

Running Time Tradeoff

granularity	$O(1)$
length	$O(1)$
running time	$\log^* n$

- Any two can be kept constant in bounded degree graphs
- Constant length of schedule and granularity
  - find a $(\Delta + 1)$-colouring in $O(\log^* n)$ rounds

Length of Schedule Tradeoff

granularity	$O(1)$
length	$\text{poly}(n)$
running time	$O(1)$

- Any two can be kept constant in bounded degree graphs
- Constant running time and granularity
  - node of colour $c(v)$ is active during time interval

    $$\big(c(v) - 1, c(v)\big]$$

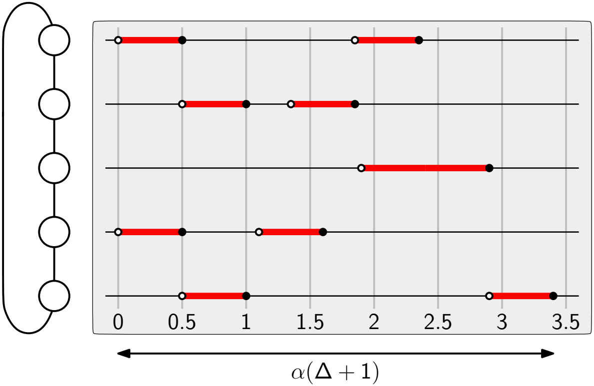
  - length of schedule $\text{poly}(n)$

Time-Length-Granularity Tradeoff—Summary

granularity	$O(1)$
length	$O(1)$
running time	$O(1)$

- Impossible to have constant running time, length and granularity at the same time
  - Naor & Stockmeyer

Our Result



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