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Origin of non-Gaussian velocity distributions in steady-state sedimentation

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Abstract. – We study the settling dynamics of non-Brownian particles during steady-state sedimentation under realistic conditions by computer simulations. We show that the velocity fluctuations parallel to gravity change systematically with the particle volume fraction Φ. For dilute systems, their distribution function has a non-Gaussian form, with an extended tail in the direction of the downward velocity, while for dense systems the situation is the opposite. In the intermediate regime at $\Phi \approx 0.05$ the fluctuations are Gaussian. We show that these distributions can be understood from the dependence of the settling velocity and its fluctuations on the local density around each particle.

The sedimentation of non-colloidal particles is a common phenomenon in nature on which many important technological processes are based, *e.g.* in paper and pulp industry. It is also an interesting example of non-equilibrium dynamics whose details are still inadequately understood in the case of a finite volume fraction Φ of the particles [1]. Under appropriate boundary conditions, such as in fluidized beds, a sedimenting system driven by gravity can reach a steady-state velocity distribution $P(v)$ [2]. In such experiments the average velocity of the particles $\langle v_\parallel \rangle$ is zero while the fluid flows upward with velocity v_f which depends on Φ. In the horizontal direction perpendicular to gravity, $P(v_\perp)$ is centered around $\langle v_\perp \rangle = 0$ for obvious symmetry reasons.

While the behavior of $v_\mathrm{f}(\Phi)$ has been the subject of intense study [1], there are far fewer studies of the velocity distribution function itself. In the most simple-minded approximation, one would expect the particle velocities v to be uncorrelated leading to a Gaussian distribution for $P(\delta v)$ as in the case of ordinary Brownian motion. Interestingly enough, this is not the case. Ichiki and Hayakawa [4] observed in their model simulations of a 2D fluidized bed that at $\Phi = 0.327$ $P(v_\parallel)$ was asymmetric. Both of its branches could be fitted separately to a Gaussian distribution, but the upward branch was more extended.

In the subsequent experiments of Rouyer *et al.* [5], a suspension of spherical particles was studied experimentally by using a quasi-2D fluidized bed. They considered the case of high Peclét and low Reynolds numbers with $\Phi \in [0.08, 0.76]$ and confirmed that $P(v_\parallel)$ was asymmetric. The downward branch was near Gaussian with

$$P\big(v_\parallel/\sigma(v_\parallel)\big) \propto \exp\Big[-\beta\big(|v_\parallel|/\sigma(v_\parallel)\big)^\xi\Big], \tag{1}$$

where $\xi = 2$. The velocities have been normalized by the average fluctuation $\sigma(v_\parallel) = \sqrt{\langle v_\parallel^2\rangle}$ from the Gaussian part, and β is a constant. The upward branch was, however, a stretched exponential with the value of ξ decreasing from about 1.8 to 1 when the volume fraction increased from 0.12 to 0.70.

To explain their results, Rouyer *et al.* suggested that the particles can be considered to be "slow" or "fast": In the denser areas the particles form a kind of clusters, where the motion of single particles is Brownian-like. The motion of the "fast particles" in the more dilute streams between the "clusters" is more correlated and they are typically moving fast upwards. They also studied $P(v_\perp)$ and found it to be non-Gaussian. They pointed out that the correlated feature of the motion of the fast particles should also stretch the tails of the horizontal velocity distribution.

Most recently, Miguel and Pastor-Satorras [6] performed computer simulations for a 2D system (with an additional velocity-dependent friction term in order to mimic a quasi-2D experimental setup) using the Oseen tensor method. For $\Phi = 0.01$, they found again that $P(v_\parallel)$ was asymmetric, but this time more stretched in the *downward* direction. They also adopted an argument based on fast and slow particles: The "fast particles" are those in downward streams and the "slow" ones are those caught into the swirls between the streams. Furthermore, they studied the autocorrelation function of the particle velocities and found that after an initial rapid decay region there was a region of slower decay. These two different decaying regions were connected to the two different types of particles.

Although at first sight appealing, the idea of separate fast and slow particles is not supported by the recent experiments of Lei *et al.* [7]. They measured particle density fluctuations during sedimentation and showed that, in all cases, the density fluctuations were *smaller* than those in a uniformly random configuration. Thus the origin of the non-Gaussian velocity distributions requires a more thorough study.

In this work our aim is to study the velocity distributions of sedimenting non-Brownian particles over a wide range of volume fractions. This is achieved by a simulation method which accurately describes a sedimenting two-phase system under realistic physical conditions [8]. First, we verify that $P(v_\parallel)$ is indeed non-Gaussian for small and large values of Φ, but it changes shape so that the extended tail at small $\Phi \approx 0.005$ is in the direction of downward velocities, while for $\Phi \approx 0.3$ the situation is the opposite. This means that at intermediate, semi-dilute values of $\Phi \approx 0.05$–0.1, $P(v_\parallel)$ is essentially Gaussian. We show that these results can be explained by the dependence of the settling velocity and its fluctuations on the local density around each particle. We demonstrate that, while the particles with the highest local density settle fastest, the overall velocity fluctuations are largest in the semi-dilute case here. Using this idea, we show how the dependence of these fluctuations as the *local density* lead to the observed distribution functions.

We employ an immersed boundary type of simulation method described in detail in ref. [8]. The fluid is treated in continuum by using a finite-difference method on a regular grid to solve the Navier-Stokes equation. To properly include the hydrodynamic interactions, the boundary conditions between the fluid phase and the solid particles are taken into account by adding a fictitious force density to the equation of motion of the fluid so that in the interior of the particles the fluid moves like a rigid object. This force is derived by tracking explicitly the

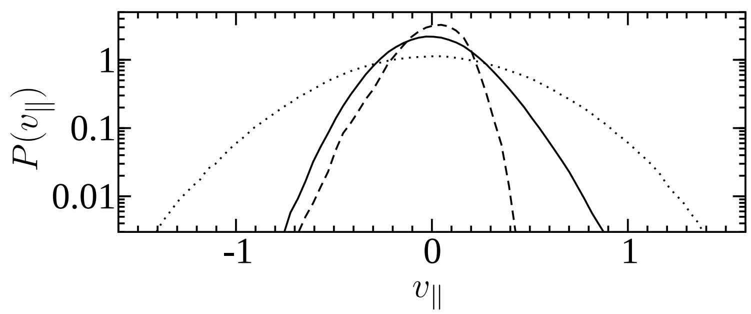

Fig. 1 – The vertical velocity distributions $P(v_{\|})$ for three different volume fractions: $\Phi = 0.005$ (dashed line), 0.05 (dotted line) and 0.30 (solid line).

motion of the solid particles and, whenever the motion of the fluid and the particle templates differ in certain predefined points, a restoring force is added. The hydrodynamic interaction between the particles is thus generated by the dynamics of the fluid. In addition to this, a direct elastic collision potential is used between the particles to prevent overlap. It is required, since once two particles are separated by less than one lattice spacing, the hydrodynamic interaction is modelled inadequately. Thus the collision potential mimics the short-range repulsion due to lubrication forces between the particles. The method is suitable for modelling non-Brownian suspensions up to particle Reynolds numbers of $\mathrm{Re} = rV_0/\eta \approx 10$, where η is the viscosity of the fluid, r the radius of the particle and V_0 is the terminal velocity of a single settling particle. The method has been tested for a variety of different cases in refs. [8–10].

The data shown here are for a suspension of monodisperse spherical particles whose density is 2.5 times the fluid density. The system sizes used in this work are $32 \times 32 \times 64$ in units of the radius of the particles, where the larger dimension is in the direction of gravity. Periodic boundary conditions in all directions were used to obtain the steady state which was determined by the average settling velocity and its fluctuations. We fixed the fluid viscosity so that the particle Reynolds number $\mathrm{Re} \approx 0.5$. In order to mimic the fluidized bed experiments, the particle velocities have been measured in the frame where $\langle v_{\|} \rangle = 0$.

To verify the present system, we computed v_f and the velocity fluctuations $\sigma(v_{\|})$ and $\sigma(v_\perp)$. We find v_f to follow very closely the expected Richardson-Zaki relation $V_0(1-\Phi)^{4.5}$ up to $\Phi = 0.3$ [11]. For the velocity fluctuations scaled by v_f we find scaling as Φ^m with $m \approx 0.50$ for $\sigma(v_{\|})$ and 0.55 for $\sigma(v_\perp)$. In sedimentation experiments it has been found that m varies from 1/3 to 1/2 [3, 12].

The actual velocity distributions $P(v_{\|})$ from simulations are shown in fig. 1. The remarkable feature in these data is the systematic change of P with Φ. For the smallest $\Phi = 0.005$ (dashed line), the distribution is non-Gaussian with a longer tail in the direction of downward velocities. At an intermediate value of $\Phi = 0.05$ (dotted line), the distribution is symmetric and closely Gaussian. Finally, for a system with $\Phi = 0.30$ the distribution is skewed to the opposite direction, *i.e.* it has a longer tail in the direction of upward velocities (solid line). These distributions are in good agreement with the fluidized bed experiments of Rouyer *et al.* [5] and with the previous simulations [4, 6]. We have also computed the horizontal distributions $P(v_\perp)$ and found that they are symmetric, but leptokurtic (see below). These data will be published elsewhere [13].

To demonstrate the systematic variation of P with Φ we have first computed the skewness of the distributions, defined by $\gamma_1 = \mu_3/\mu_2^{3/2}$, where $\mu_n(v) = \langle v^n \rangle$. For symmetric distributions

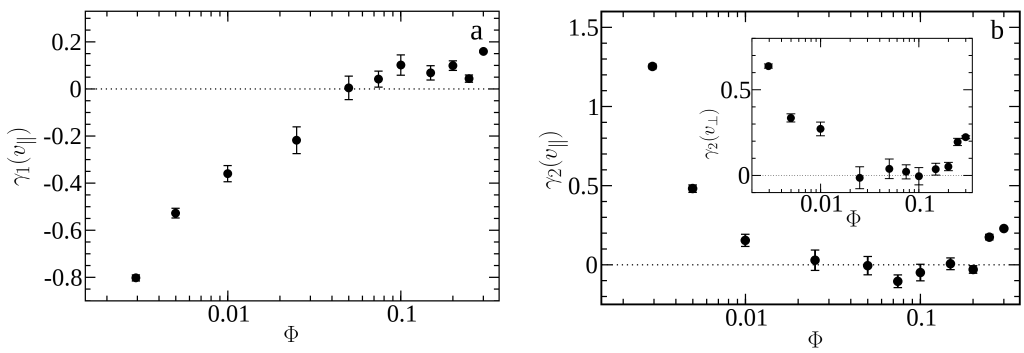

Fig. 2 – (a) The skewness γ_1 of $P(v_\parallel)$. (b) The corresponding kurtosis $\gamma_2(v_\parallel)$. The kurtosis of the horizontal velocities $\gamma_2(v_\perp)$ is shown in the inset.

such as for $P(v_\perp)$, $\gamma_1 = 0$. In fig. 2(a) we show $\gamma_1(v_\parallel)$ up to $\Phi = 0.3$. The systematic change from negative to positive values of γ_1 is evident, and in accordance with fig. 1 the distribution is symmetric at $\Phi \approx 0.05$.

The second quantity we consider here is the kurtosis which describes the weight of the tails of the distribution. We define it as $\gamma_2 = \mu_4/\mu_2^2 - 3$ so that it is zero for a Gaussian distribution. In fig. 2(b) we show $\gamma_2(v_\parallel)$. We can see that the tails are most extended for small volume fractions, while again around $\Phi \approx 0.05$, $\gamma_2 \approx 0$. For larger Φ, the tails become extended again. For completeness, in the inset of fig. 2(b) we also show $\gamma_2(v_\perp)$ for the horizontal distribution. It can be seen that its tails are also extended and correlate with the changes in $P(v_\parallel)$. This demonstrates that the vertical velocity fluctuations indeed influence the horizontal ones.

For the non-Gaussian cases found here it is possible to try to fit to the stretched exponential function of eq. (1) as in ref. [5]. However, we find that in the regime $\Phi < 0.01$, $P(v_\parallel)$'s do not fit well to this form. For the large volume fractions $\Phi > 0.1$, the ξ's fitted to the upward branch of the distribution decrease from 2.0 to 1.7 while Φ increases from 0.1 to 0.3. For the same range of Φ, Rouyer *et al.* measured ξ to decrease from about 1.75 to 1.5. For the downward branch they found that $\xi \approx 2$ independent of Φ. Here we obtain $\xi \approx 2.2$ with no clear Φ-dependence, either. Thus, we can conclude that our results are in quantitative agreement with the experiments, but the non-Gaussian distributions in a dilute system are not well described by stretched exponentials.

Next we will discuss the physical reasons behind the vertical distributions. To quantify the role of density inhomogeneities in determining $P(v_\parallel)$, we introduce here the concept of a *local volume fraction* ϕ. It is defined as the number of particles within a certain volume V_p around a test particle, multiplied by the ratio of the one-particle volume and the volume of the region. The choice of the shape and size of V_p is somewhat arbitrary. Based on the symmetry of the system, a spheroidal region, with a possibly different radius in the direction parallel to the gravity, is a natural choice. The size of the region has here been chosen such that ϕ correlates as much as possible with the vertical velocity. This can be achieved by maximizing the square of the normalized cross correlation between ϕ and $v_\parallel$, defined by

$$c^2_{v_\parallel,\phi} = \frac{\big(\langle v_\parallel \phi\rangle - \langle v_\parallel\rangle\langle\phi\rangle\big)^2}{\sigma^2\big(v_\parallel\big)\sigma^2(\phi)} \,. \tag{2}$$

Such a procedure can be motivated by the assumption that there is a characteristic spatial size

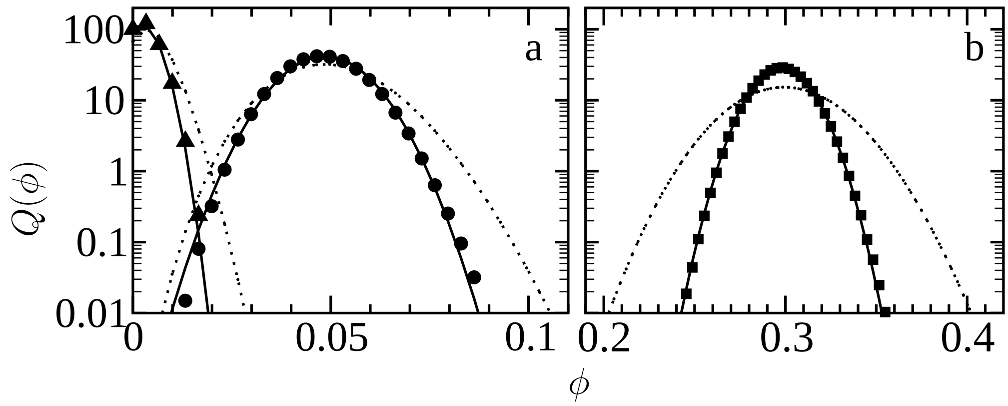

Fig. 3 – The distributions of the local volume fraction from the simulations with $\Phi = 0.005$, 0.05 (triangles and circles in a) and 0.30 (squares in b). Gaussian fits are shown with solid lines. The corresponding distributions for uniformly random configurations are shown with dotted lines.

of velocity fluctuations that can be recognized by its ability to explain the particle velocity fluctuations. Such a characteristic size has been notified in the sedimentation experiments by Segrè *et al.* [14, 15]. Numerically, we found that $c^2_{v_\parallel,\phi}$ can be maximized to a good degree of approximation by choosing a spheroidal region of size $11 \times 11 \times 20$ for all values of Φ studied here. The values for $c^2_{v_\parallel\phi}$ vary from 0.20 for $\Phi = 0.005$ to 0.25 for $\Phi = 0.30$. It is reasonable to assume that the shape and the size of the chosen region reflect the vorticity structure of the fluid. The fact that such a region is more elongated in the direction of gravity is in good agreement with the experimental results of Segrè *et al.* [14].

The distributions of the local volume fraction ϕ are shown in fig. 3 for different values of Φ. As expected, $Q(\phi)$'s are Gaussian and their maxima coincide with the total volume fraction. However, the distributions $Q(\phi)$ are much narrower than those corresponding to uniformly random configurations which are shown with dotted lines. This result is in good agreement with the experiments of Lei *et al.* [7] on the particle number fluctuations during sedimentation. These distributions suggest that the particles form one continuous phase rather than a separation into "cluster" and "intermediate" particles. In order to further study this assumed homogeneity, we have also calculated the distribution of the smallest distances between particles and found no indication of two phases.

The key point in understanding the role of the local density is the observation that regions of larger ϕ tend to sediment faster than regions with smaller ϕ. An intuitive reason for this behavior is that an area of large local density behaves like a blob of heavier fluid obtaining a downward velocity relative to the surrounding fluid [15, 16]. This is demonstrated in fig. 4 where we show the dependence of the average vertical velocity of particles with fixed ϕ $\langle v_\parallel\rangle_\phi$ on the local volume fraction ϕ for several different total volume fractions Φ. Based on these data, we can write the total particle velocity as $v_\parallel = \langle v_\parallel\rangle_\phi + \delta v_\parallel$, where $\langle v_\parallel\rangle_\phi \approx a(\Phi - \phi)$ describes the part of the velocity that is determined by ϕ, and $\delta v_\parallel$ is the residual fluctuation induced by all the other factors, *e.g.* details of the particle configuration (here a is a positive Φ-dependent coefficient). To quantify this, we define the remaining *residual velocity fluctuation* (RVF) as $\sigma_\phi(v_\parallel) = \sqrt{\langle \delta v_\parallel{}^2\rangle_\phi} = \sqrt{\langle v_\parallel{}^2\rangle_\phi - \langle v_\parallel\rangle^2_\phi}$. These data for different values of Φ are shown in fig. 5. The fluctuations are smallest for small and large volume fractions, and have a maximum around $\phi \approx 0.08$.

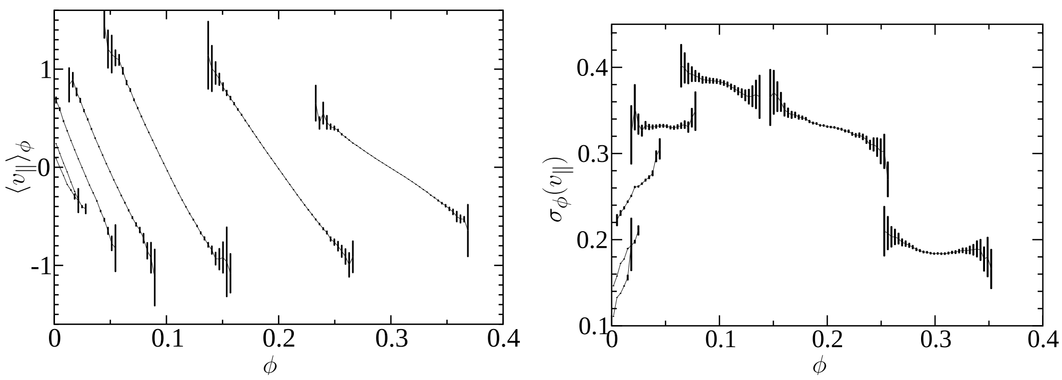

Fig. 4 Fig. 5

Fig. 4 – The average vertical velocity of the particles as a function of ϕ with fixed total volume fractions (from left to right): $\Phi = 0.005$, 0.01, 0.02, 0.05, 0.10, 0.20, and 0.30.

Fig. 5 – The RVFs in the direction parallel to gravity as a function of ϕ. From left to right the data are for fixed values of $\Phi = 0.005$, 0.01, 0.02, 0.05, 0.10, 0.20, and 0.30.

The behavior of the distributions $P(v_{\|})$ can now be explained by the ϕ-dependence of the RVFs. Namely, the whole function $P(v_{\|})$ can be obtained by summing up the velocities of all the particles with different ϕ's. Since the $Q(\phi)$'s are Gaussian, and $\langle v_{\|}\rangle_\phi$ depends linearly on ϕ, $P(v_{\|})$ would be Gaussian, too, if the RVFs did not depend on ϕ. This is, however, the case only at $\Phi \approx 0.05$, *i.e.* in a semi-dilute suspension. For dilute suspensions we can see from fig. 5 that the RVFs increase strongly with ϕ, which means that the fluctuations in the downward velocity part of $P(v_{\|})$ are enhanced, in accordance with our data. For the opposite case of suspensions denser than $\Phi \approx 0.05$, the RVFs decrease with increasing ϕ, and thus there are enhanced fluctuations in the opposite (upward) direction. This explains the systematic change in the sign of the skewness of $P(v_{\|})$ as a function of Φ.

Thus, although there is no need to conceptually separate "fast" and "slow" particles with qualitatively different behavior, in the dilute case the fastest particles (as in ref. [6]) are those with the highest local concentration ϕ around them, and their fluctuations are largest leading to the non-Gaussian tail in $P(v_{\|})$. On the other hand, for dense systems the largest fluctuations are caused by the particles with the lowest ϕ (Rouyer *et al.* call these particles fast ones [5]). This produces a non-Gaussian tail on the other side of $P(v_{\|})$.

We note that these tails undermine the use of the concept of "temperature" in sedimentation even though it is often used in the context of granular media [17]. Our results also show that the concept of "gravitational temperature" introduced recently by Segrè *et al.* [15] can only be used to explain the part of $P(v_{\|})$ coming from the dependence of $v_{\|}$ on local density ϕ.

Finally, we can also reinterpret the two regions of decay in the particle velocity autocorrelation function observed by Miguel and Pastor-Satorras [6]. Instead of "fast" and "slow" particles, our data indicate that the fast initial decay corresponds to the decay in $\delta v_{\|}$ and the slow asymptotic decay is due to the decay in the autocorrelation of ϕ.

To summarize, we have shown here that for steady-state sedimentation of non-Brownian particles, the velocity fluctuations have a non-Gaussian character for dilute and dense suspensions. Most remarkably, the skewness of the distribution $P(v_{\|})$ changes as a function of Φ: for small Φ there is more weight at the large (negative) downward velocities, while the situation is opposite for $\Phi > 0.1$. These results can be explained by considering the dependence of the vertical particle velocity and its fluctuations on the local particle concentration ϕ.

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