Handedness in plasmonics: electrical engineer's perspective

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Abstract In this article, the concepts of handedness and negative material parameters are analyzed at a general and qualitative level. Three different usages of handedness in metamaterials and electromagnetics are distinguished: left-handedness as characterization of double-negative materials, handedness of the polarization of a plane wave, and chirality in the structure of matter. The symmetry of the treatment between left and right is discussed from the point of view of the three uses of the handedness. It is essential to distinguish the helicity of the spatial shape of the field vector as opposed to the temporal behavior of the field at a given position in space. Negative refraction and backward-wave characteristics are discussed in the case when structural chirality of the medium splits the wavenumbers of the eigenwaves. Finally, negative refraction is connected with anisotropic and bi-anisotropic materials.

1 Introduction

Metamaterials form such a wide and uncharted range of materials that a coherent presentation of the possible examples of media that fall in this class is beyond reach [1]. With powerful new technologies that allow processing of materials at nanoscales, many types of artificial materials can be fabricated which are endowed with engineered properties that could only be dreamed of in the past [2]. In addition to man-made materials, also many naturally existing media display strange and emergent properties [3]. This fact contributes further to the fact that it is difficult to draw lines that would separate metamaterials from other, evenly interesting substances.

Typical to an emergent research field like metamaterials is also its interdisciplinary character that transcends previously respected boundaries between research fields. We see people from different traditions and backgrounds approaching and attacking metamaterials problems, solving those, and creating new ideas.

These researchers come from electrical engineering, electromagnetics, solid state physics, microwave and antenna engineering, optoelectronics, classical optics, materials science, semiconductor engineering, nanoscience, etc.

The diversity of backgrounds and paradigms is a source of fruitful cross-fertilization of ideas, and potential for joint research networks where different strengths and capacities can be successfully combined. But the challenge is to find the internal cohesion for such joint efforts. Within different domains of science and engineering, the traditions of doing research and analyzing problems may differ. Even the formalism and terminology of quantities of the same physical phenomenon may vary when researchers from different backgrounds approach the problem. For example in discussions about magnetism and reciprocity, or plasmonics and surface waves, concepts may cause initial difficulties in understanding the language of a colleague.

The present article attempts to cast some light into one area of metamaterials studies where misunderstandings and ambiguous concepts are being used. What does it mean to say that something is "left-handed" or "right-handed"? How does the important phenomenon of negative refraction or backward wave propagation connect with handedness? How symmetric is the distinction between left and right? Various ways of looking at handedness, helicity, chirality, order, and symmetry are discussed. The metamaterials under study are also analyzed from the point of view of bi-anisotropic classification where the number of degrees of freedom in characterizing the medium becomes many times larger.

2 Close-reading of the term "handedness"

Handedness is a term that is very much used in the metamaterials literature. Let us start by discussing the various meanings of handedness within the electromagnetics discipline. It is essential to define and analyze these meanings because very often in even scientific discussions they are used parallel and grave misunderstandings may result from a wrong association of this term.

2.1 The three meanings of handedness in electromagnetics

Let us separate three different meanings of handedness in electromagnetics and materials [4]:

1. Metamaterials as "left-handed" media
2. Handedness of the circularly (or elliptically) polarized wave
3. Chirality as geometrical structure of matter

2.1.1 Metamaterials as left-handed media

The use of the label "left-handed" materials for a certain class of metamaterials has its rationale from the handedness of the vector triplet (**E**, **H**, **k**) of a linearly polarized wave propagating in such media; these vectors here refer to the electric field, magnetic field, and wave vector. Such left-handedness is the situation if both the dielectric permittivity and magnetic permeability are both negative.

With time-harmonic convention exp(jωt), Maxwell curl equations read for plane-wave functional dependence exp(-j**k**·**r**) as

$$\mathbf{k} \times \mathbf{E} = \omega\, \mathbf{B} = \omega\mu\mathbf{H}, \qquad \mathbf{H} \times \mathbf{k} = \omega\, \mathbf{D} = \omega\varepsilon\mathbf{E} \tag{1}$$

in homogeneous, isotropic, source-free background with permittivity ε and permeability μ. For ordinary media (which have positive permittivity and positive permeability), the (**E**, **H**, **k**) triplet is right-handed; however in case of both $\varepsilon < 0$ and $\mu < 0$, Eq. (1) makes it left-handed.[1] Likewise, the time-dependent Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ is parallel to **k** in the first case and antiparallel in the latter one, as shown in Fig. 1.[2]

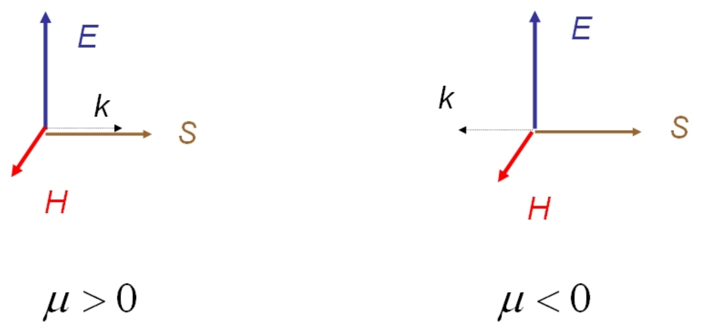

Fig. 1 For "ordinary" isotropic media (left), the triplet (**E**, **H**, **k**) is right-handed and the wavevector **k** points parallel to the Poynting vector **S**, whereas for negative parameters (right), the wavevector changes direction and the triplet becomes left-handed (and **k** and **S** become anti-parallel).

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[1] Note, however, that even if the handedness of (**E**, **H**, **k**) depends on the sign of ε and μ, the triplets (**E**, **B**, **k**) and (**D**, **H**, **k**) are both right-handed, irrespective of the signs of the material parameters.

[2] Faraday's law $\mathbf{k} \times \mathbf{E} = \omega\, \mathbf{B} = \omega\mu\mathbf{H}$ seems to lead to the conclusion that it is only the sign of μ that determines the right/left-handedness of the triplet (**E**, **H**, **k**). However, it is important to keep in mind that in order for the waves to propagate, a negative μ has to be complemented with a negative ε, due to the wave number dependence on the material parameters $k = \omega(\mu\varepsilon)^{1/2}$.

Such type of metamaterials obey many other names in addition to “left-handed media” [5]: double-negative materials, negative-index materials, negative-phase-velocity materials, backward-wave media, and—due to the theoretical prediction of their existence in the 1960’s [6]—Veselago media.

2.1.2 Handedness of a circularly polarized wave

In the language of electrical engineers, especially antenna engineers, the term handedness appears in connection with polarization of the electromagnetic wave or a radio wave. Polarization then refers to the direction and behavior of the electric field vector which for a circular or elliptical polarization has a character of helicity, or handedness. The wave is propagating in a certain direction and (in isotropic media) the electric field is transversal ($\mathbf{k}\cdot\mathbf{E} = 0$). In the transversal plane, the temporal oscillations of the field vector follow an ellipse or circle (in the case of linear polarization, the ellipse shrinks to a line).

When looking along the wave propagation direction, the wave may rotate in two directions. According to the Federal Standard 1037C, the polarization is defined right-handed if the temporal rotation is clockwise when looking from the transmitter (in the propagation direction), and left-handed if the rotation is counterclockwise. Hence, the wave depicted in Fig. 2 would be right-handed circularly polarized.

It is important to bear in mind that this handedness definition is not universal. For example, astronomers [7] are always looking towards the source (transmitter), and hence into the opposite direction of the wave propagation. Then also clockwise and counterclockwise senses swap as compared to the engineering point of view, and likewise the definition of right- and left-handedness is just the opposite. Figure 2, however, shows also that if we focus on the *spatial* behavior of the electric field instead of the time dependence, the field vector in space at a certain moment draws a left-handed spiral. The handedness of a fixed object remains the same even if it is turned or rotated. Both the antenna engineer behind the transmitter and the astronomer observing the emitted signal in front of the source agree that the spatial spiral is left-handed. (This fact would favor the astronomy definition of handedness by removing some arbitrariness in the handedness of the polarization.)[3]

[3] The talk about handedness is very human-centered. One may ask whether there might be more “objective” ways to define this difficult concept; for example, by defining in cold mathematical terms a positive or negative attribute to a structure with a given helicity. It has been pointed out that the emphasis on right-hand rules and memorizations in engineering education (which is very common) does not take into account the natural capabilities of all students, especially handicapped: how does an invalid who has lost both hands benefit from such rules of thumb?

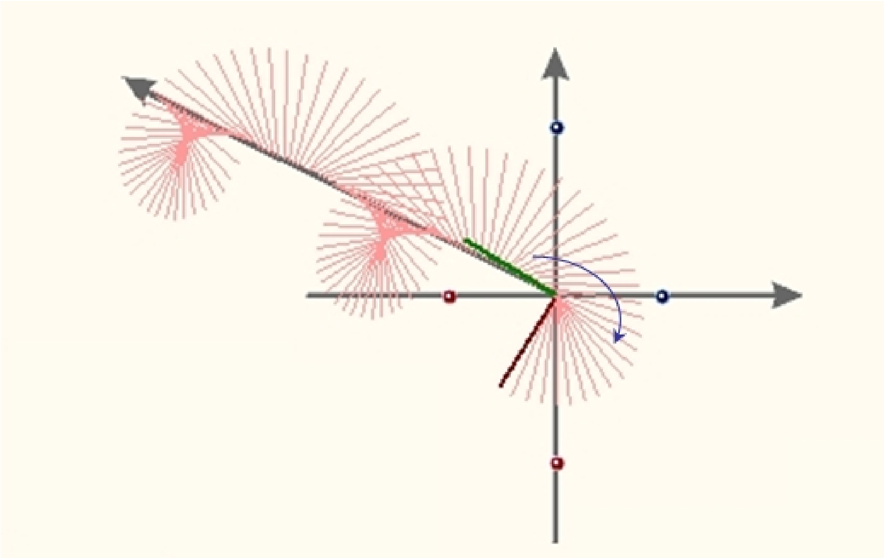

Fig. 2 A circularly polarized wave has a sense of rotation and hence can be identified with a definition of handedness. In this figure the curved arrow shows the temporal rotation of the field vector in the plane of the two axes and the phase propagation direction is into the paper, shown with slightly oblique perspective. According to the Federal Standard [8] definition, this wave is right-handed circularly polarized. Note, however, that the curved spatial structure is a left-handed spiral. For an interactive applet showing the real temporal behavior of this polarization, see [9].

2.1.3 Chirality as geometrical structure of matter

Handedness is also an everyday concept affiliated with material objects, like cork-screws, ice hockey clubs, scissors, and construction tools. The mirror image of a right-handed object is otherwise the same as the original but it is left-handed. A non-handed object remains the same within this mirror-image operation, because such an object, after imaging, can be brought into congruence with the original by simple translations and rotations.

A handed object is called "chiral"[4], and if molecules or other small elements with a particular handedness form a macroscopically homogeneous medium with net handedness, such a composite can be called chiral medium. Chiral media possess so-called *optical activity,* meaning that the polarization is affected. The effect of the handed microstructure is that the polarization plane of a linearly polarized electromagnetic wave is rotated along the propagation path. The connection of chiral microstructure to macroscopic optical rotatory power was discovered by

[4] From the word in Greek language for "hand."

Louis Pasteur in 1840's. Although analogous to the Faraday rotation in magneto-plasma, the chirality-induced optical activity is a reciprocal effect, whereas the classical Faraday effect is non-reciprocal and anisotropic due to the biasing magnetic field [10].

The mirror image operation is also called parity transformation (all spatial axes are reversed when parity is changed), and it is a fundamental property of physics that parity symmetry is broken in subatomic interactions [11]. And also on several different scales and levels of nature, parity is not balanced. From amino acids through bacteria, winding plants, right-handed human beings to spiral galaxies, one of the handednesses dominates over the other [12].

Obviously chirality in structural objects and continuous media is a very basic manifestation of handedness. Since it also causes observable and particular electromagnetic effects (optical activity), one has to pay particular attention in analyzing phenomena where other meanings of handedness are relevant.

2.2 Equal or discriminative treatment between left and right

Symmetry is an essential concept in analyzing handedness. Symmetry would also call for equality between the corresponding right- and left-handed objects; at least it is very compatible with the idea of equal status of two entities that are the same in all other respects except that they are each other's mirror images.

How do the three ways of looking at handedness, introduced in the previous section, differ as far as this aspect of left–right classification is concerned?

We can immediately note that very obviously the circular-polarization-based handedness property is fully symmetric. In other words, both senses of polarization are as easy to generate, and certainly one is as useful as the other. Even if the conventions differ and definitions regarding which wave is right-handed were not the same over scientific disciplines, at least it is only a question of phase shift in dipole antennas with which handedness the radiated wave is polarized.

On the other hand, the use of handedness with the reference to Veselago-type metamaterials is the totally opposite in this respect. There the "natural" state of affairs is the ordinary world of "right-handed" materials, as in Fig. 1: permittivity and permeability are positive as in ordinary materials. Only metamaterials, which by certain definitions are such media that do not exist naturally [3,5], can display material parameters which both are simultaneously negative. As "left-handed" materials they belong certainly to another class of media which cannot by any means be treated as parallel status as "right-handed" ones.

What can we say about the third possibility of looking at handedness: the structural chirality? Very tempting would be to observe that the parity operation (mirror-imaging the object) does not change chemical or physical properties, capacity to be of use for something, or value of the original. The whole world and universe would probably not notice any difference if it were suddenly mirror-imaged at all levels.

On the other hand, it is clear that when parity is broken at a certain scale and entities of a given handedness dominate, such balance is self-preserving and positive feedback mechanisms perpetuate such a state. When all DNA molecules twist with the right-handed sense,[5] there is no chance of the opposite handedness to survive. However, the parity balance is turned around on the level of amino acids: only left-handed ones exist in natural proteins [12].

So, even if by coincidence of evolution and chance parity is broken on various scales in the natural world, it is nevertheless broken to both directions, left and right, and one might expect equal treatment with respect to this left–right variation on the global level. However, the human preference of right over left can be seen in the taxonomy of animal species. As Fig. 3 illustrates, the sea snail of genus *Busycon,* displaying left-handed chirality in its structure, has been given as its species identification the label *perversum*. The origin of such a derogatory name is most probably due the abnormality of left-handedness: such whelk molluscs are predominantly dextral, right-handed.

Fig. 3 The marine mollusk *Busycon perversum* (lightning whelk) is left-handed in the sense of a three-dimensional screw (on the right-hand side of the figure). The strangeness of the handedness (most seashells show rotation along the right-handed sense) is most probably the reason behind the discriminative label (perverse!) for this species. (Image of the whelk taken from Wikipedia Commons.)

[5] The left-handed version of such nucleotides can be prepared in the laboratory; but in nature, only right-handed isomers exist.

3 Chirality and negative material parameters

Isotropic Veselago materials with simultaneously negative permittivity and permeability cause waves to behave in interesting ways. However, if structural handedness is added to such materials, the parameters that account for the chirality also affect the wave propagation problem. Chirality brings in magnetoelectric coupling and birefringence. But it also leads to the imperative that the phenomenon of negative refraction needs to be generalized.

3.1 Bi-isotropy and eigenwaves

As is well known [10], net chirality in the continuum matter brings forth magnetoelectric coupling. In other words, electric field excitation causes magnetic polarization in the medium and vice versa. Hence the material response relations have to be rewritten from their dielectric–magnetic form.

3.1.1 Bi-isotropic constitutive relations

On the level on constitutive relations, the magnetoelectric cross-coupling means that in addition to the permittivity ε and permeability μ, a parameter measuring the strength of chirality κ has to appear:

$$\mathbf{D} = \varepsilon\mathbf{E} + (\chi - \mathrm{j}\kappa)\mathbf{H}, \quad \mathbf{B} = (\chi + \mathrm{j}\kappa)\mathbf{E} + \mu\mathbf{H} \tag{2}$$

with the electric (**E**) and magnetic (**H**) field strengths, and electric (**D**) and magnetic (**B**) flux densities.

In isotropic media, the material parameters are equivalent to scalars. Here now, to emphasize the magnetoelectric coupling, the relations are termed *bi-isotropic*, and due to the fact that there are two exciting fields and two response flux densities, in the most general case four material parameters are needed.

The missing fourth parameter is already included in Eq. (2): it is the so-called Tellegen parameter χ, which is a measure of non-reciprocal magnetoeletric coupling. This mechanism is another type of coupling that can be achieved by artificial (or natural) coupling of permanent electric and magnetic moments [13,14]. Note the appearance of the imaginary unit j in the relations (2). It shows the 90-degree phase shift of the response due to chirality effect (charge separation in a spiral creates a circulating current which is proportional to the time derivative of charge density), as compared to the in-phase response of the non-reciprocal magnetoelectric (Tellegen) effect.

Media with nonzero non-reciprocity parameter χ are sometimes called Tellegen media, whereas in case of chiral materials (nonzero κ) one often speaks about Pasteur media.

3.1.2 Effect of magnetoelectric coupling on wave propagation

Unlike to the ordinary isotropic materials and double-negative Veselago media, bi-isotropic materials are birefringent: the two eigenwaves propagating in bi-isotropic media have different propagation factors:

$$k_{\pm} = \omega\left(\sqrt{\mu\varepsilon - \chi^2} \pm \kappa\right) \tag{3}$$

Both magnetoelectric parameters χ and κ affect the phase of the wave.[6]

The effect of the chirality parameter is to split the refractive index. This leads to a need to take a closer look at the backward-wave properties of media with negative permittivity and permeability. Figure 4 shows on the left panel the division of isotropic media into four classes depending on whether ε and/or μ are positive or negative [16], and a propagating wave requires the parameters be of the same sign. Double-negative media support backward waves.

However, when the chirality parameter is allowed to be nonzero, the situation is characterized by the right panel in Fig. 4 (there, for simplicity, the Tellegen parameter χ is assumed to be zero). The permittivity and permeability are assumed to be of the same sign, and the square root of their product is assumed to have the same sign as ε and μ. Depending on the magnitude of the chirality parameter, both eigenwaves can be forward (the wave vector amplitudes both positive), both backward (both wave vector amplitude negative), or one of the waves forward, one backward. A corollary is that in order to create negative-index materials (backward-wave media), it is not a necessary condition to have double-negative material. It suffices that the chirality parameter exceeds the magnitude of square root of $\mu\varepsilon$.

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[6] It turns out that the losslessness condition [15] leads to the requirement that all the four material parameters in Eq. (2) be real-valued. (Of course, when dispersion is taken into consideration, losses and the imaginary parts for these parameters need be accounted for, and we need to deal with four medium parameters.) It is therefore wrong to interpret in the constitutive relations (2) the magnetoelectric coefficients $\chi + j\kappa$ and $\chi - j\kappa$ as complex conjugates of each other in the sense that κ would be the imaginary part of the whole coefficient. (The complex conjugate of $\chi + j\kappa$ is $\chi^* - j\kappa^*$.)

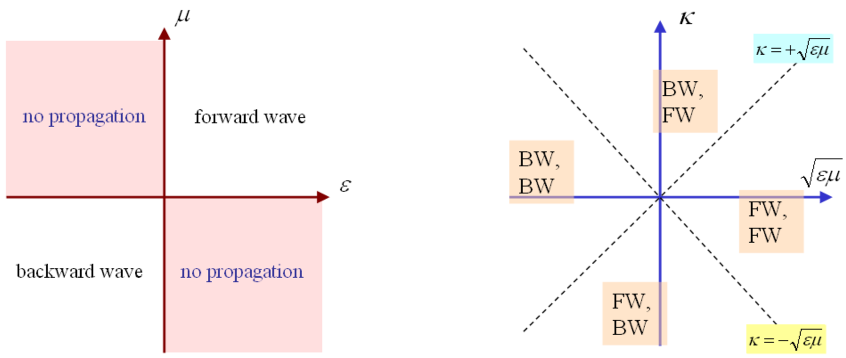

Fig. 4 The effect of chirality on the backward-wave characteristics of bi-isotropic media. Left side: in isotropic media, waves propagate only if the permittivity and permeability are of the same sign, and positive parameters lead to forward waves, negative to backward ones. Right side: when ε and μ have the same sign, a sufficiently large chirality parameter κ creates a situation where one of the eigenwaves is backward, while the other is forward. (The sign of the square root of εμ is assumed the same as that of ε and μ in this figure.)

Hence sufficiently small (absolute) values of ε and μ lead to interesting possibilities in connection with chiral properties. The extreme case in this respect is the so-called *chiral nihility* [17] where $\mu\varepsilon = 0$ but the medium still possesses a nonzero chirality parameter. The nonzero value of κ splits the eigenwaves such that they have equal but opposite wavenumbers.

Also even without the chirality effect, in the plain isotropic domain, the regime of very small values for the permittivity and permeability are of interest in metamaterials research. Such media, also called as zero-index media (ZIM),[7] would be extremely useful in very-high frequency applications of novel materials, such as for example optical circuits. To transfer the machinery electronics into optical wavelengths and to make use of the theories and powerful results of circuit theory in the nanoworld would create a new paradigm, *metactronics,* as has been visioned by Nader Engheta [19].

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[7] It is worth noting that although media with "very small" permittivity *and* permeability lead to a material with very small index of refraction and eventually ZIM, the reverse is not necessarily true. Even using non-magnetic materials (for which the permeability is the same as that in free space), one can approximate ZIM if only the permittivity is sufficiently small. Such media have been called epsilon-near-zero materials (ENZ). Such materials are being studied due to their potential applications, for example, in directive emission [18] and squeezing light in optical nanocircuits.

3.1.3 Non-reciprocity and wave propagation

The effect of the Tellegen parameter χ on the forward–backward characteristics is different from that of the chirality parameter κ. When the magnitude of χ exceeds that of the square root of εμ, the waves change character: they attain an imaginary part and become attenuating. (However, in such case there may still be phase variation: the real part of the wave vector is given by κ, as Eq. (3) shows). Nevertheless, also in the case when the Tellegen parameter of the medium is nonzero, backward waves are possible.

The interrelation of the domains of backward-wave media and the four subclasses of bi-isotropic materials is illustrated in Fig. 5.

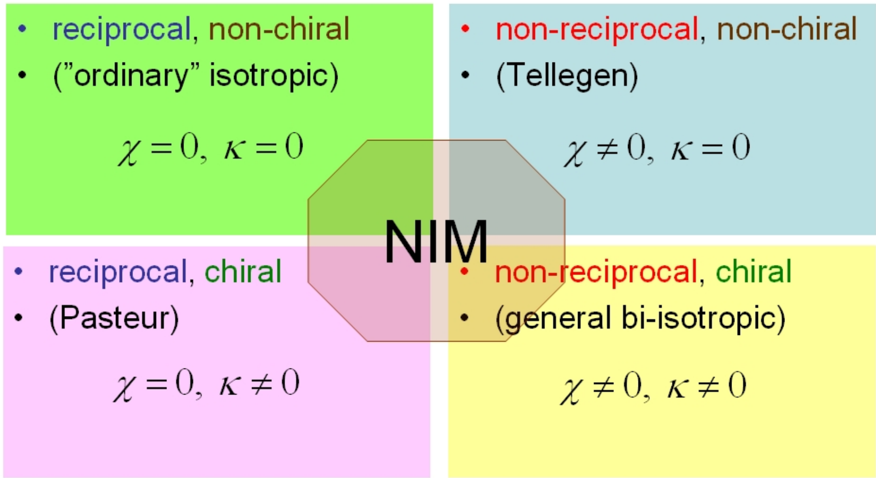

Fig. 5 Negative-index media (NIM, here referring to media in which a plane eigenwave is allowed to display backward propagation, in other words negative phase velocity) are allowed in all subfields of bi-isotropic materials: chiral (handed) and non-chiral, reciprocal and non-reciprocal.

3.2 Optical activity and polarization rotation

Pasteur's discovery (connection of microstructural chirality with macroscopically observable polarization rotation) was qualitative. To present a formula for calculating the angle of rotation as function of the chiral activity requires solution of Maxwell's equations with the constitutive relations (2). In light of the theme of the present article, handedness, it is especially interesting to find out which is the sense of helicity (right- or left-handed) for a given chirality parameter (positive or negative).

The eigenwaves in homogeneous chiral media are the two circularly polarized (right- and left-handed) waves [10]. Let us assume that the waves are propagating into the direction of the positive *z* axis. Then their dependence upon the propagation distance is exp(-j$k_\pm$z), with $k_\pm$ being the propagation constant of the right-handed (+) and left-handed (-) eigenwave.

The electric field vectors rotate in the *xy* plane (unit vectors $\mathbf{u}_x$ and $\mathbf{u}_y$), and as complex vectors they can be written as

$$\begin{cases} \text{RCP:} \quad \mathbf{u}_x - j\mathbf{u}_y; \quad \exp(-jk_+z), \quad k_+ = \omega(\sqrt{\mu\varepsilon} + \kappa) \\ \text{LCP:} \quad \mathbf{u}_x + j\mathbf{u}_y; \quad \exp(-jk_-z), \quad k_- = \omega(\sqrt{\mu\varepsilon} - \kappa) \end{cases} \tag{4}$$

If the wave is linearly *x* polarized at *z* = 0, it is the sum of RCP and LCP, both of equal amplitudes. For positive κ, the wave number of RCP is larger than that of LCP. Hence its phase changes faster. The result is that at a distance into positive *z*, the vector direction of the electric field is

$$\begin{aligned} &(\mathbf{u}_x - j\mathbf{u}_y)\exp(-jk_+z) + (\mathbf{u}_x - j\mathbf{u}_y)\exp(-jk_-z) \\ &\quad = 2\left[\mathbf{u}_x\cos(\omega\kappa z) - \mathbf{u}_y\sin(\omega\kappa z)\right]\exp(-j\omega\sqrt{\mu\varepsilon}\,z) \end{aligned} \tag{5}$$

from which it can be seen that at position *z* = 0, the field vector is *x*-polarized as assumed.

In particular, Eq. (5) shows that at any fixed position *z*, the electric field is linearly polarized (the vector multiplying the phase exponential is real). The plane of polarization of the wave in the transversal *xy*-plane, however, depends on the position *z*. As the field penetrates into the chiral medium and *z* increases, the field polarization starts to attain a negative *y* component. This means that the polarization rotation is counterclockwise (for positive κ) when one is looking along the propagation direction.

This analysis can be illustrated by the situation in Fig. 6. There the sign of κ is assumed positive in the chiral material, and the plane of polarization of the propagating wave rotates counterclockwise.[8]

[8] One has to bear in mind that over a larger spectral range, the chirality parameter is dispersive: its magnitude varies as function of frequency. For resonating particles, the rotatory dispersion can be so strong that it even changes sign [20]. This leads to a paradoxical situation: a sample of material which is, say, dextral (i.e., it has right-handed microstructure), can have the power of rotating the wave polarization in either right- or left-handed sense, depending on the frequency of the radiation.

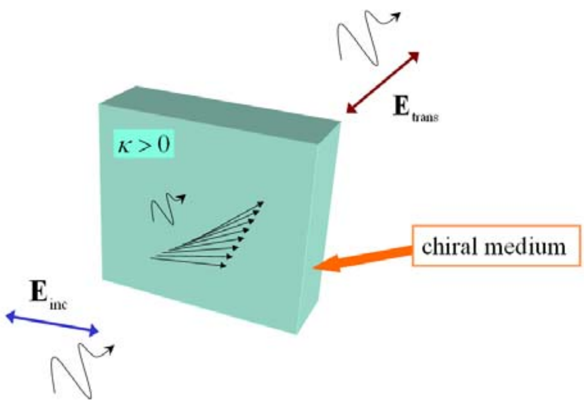

Fig. 6 The rotation of the polarization plane of a linearly polarized wave as it enters a chiral medium and propagates through it. Here the chirality parameter κ is assumed to be *positive* (for negative κ, the rotation would run in the opposite sense). The important conclusion of the figure is that the rotation is *counterclockwise* as one looks into the propagation direction. (Of course there also is a reflection, caused by the possible impedance mismatch at the boundary; it is omitted here because the chirality parameter does not affect the reflection coefficient for normal incidence.) Note that here the constitutive relations defined in (2) are used (the non-reciprocity parameter χ does not affect the rotation). Also, the time-harmonic convention is exp(jωt); evidently the convention exp(-iωt) would chance the sense of rotation.[9]

There are two "wavelengths" that can be distinguished in wave propagation in chiral medium, evident from the field dependence in Eq. (5). As Fig. 6 shows, the polarization rotates, and after a certain distance λ_{pol}, the field polarization aligns with its original direction. This could be termed the "polarization wavelength." According to Eq. (5) this happens when $\omega\kappa\lambda_{pol} = 2\pi$.

The other wavelength is the ordinary wavelength λ_{ph} separating spatial places where the phase of the wave has increased by 2π. Eq. (5) gives the condition for this: $\omega(\mu\varepsilon)^{1/2}\,\lambda_{ph} = 2\pi$.

The ratio between these two wavelengths is

[9] A comparison of the field rotations of Fig. 6 and Fig. 2 needs a word of caution because a fixed-in-time image may lead to wrong associations. Here (Fig. 6) the fields shown are linearly polarized; hence, in time they always keep the same vector direction shown in the picture but oscillate sinusoidally. The situation of the circularly polarized wave of Fig. 2 the situation is different: there the picture shows a snapshot (at a certain time instant) of the field vector direction simultaneously at different places. With time, the helical spiral representing the field moves forward so that at any fixed point in space, the field vector draws a clockwise circle like RCP.

$$\frac{\lambda_{\mathrm{ph}}}{\lambda_{\mathrm{pol}}} = \frac{\kappa}{\sqrt{\mu\varepsilon}} \tag{6}$$

which relation reveals an interesting connection to the backward-wave characteristics in chiral media. As shown in Fig. 4, if the chirality parameter κ exceeds the value of the square root of the product εμ, one of the eigenwaves is backward. According to Eq. 6, this very same limiting condition corresponds to the case of the two wavelengths (polarization, λ_{pol}, and phase, λ_{ph}) being equal.

3.3 Rotation of reflection from Tellegen medium

The other magnetoelectric effect, the non-reciprocal Tellegen coupling, has a complementary effect on wave propagation. Instead of affecting, in a rotatory manner, the plane of the transmitted wave, it has an effect on the reflection. As is known, reflection coefficient is determined by the wave impedance. A Tellegen medium is bi-impedant whereas the chiral medium is bi-refringent.

For an ordinary isotropic dielectric–magnetic medium, there are two Fresnel reflection coefficients, one for the parallel and one for the perpendicular polarization. Since a magnetoelectric medium is more complex than an isotropic medium, the eigenpolarizations are no longer these simple linear solutions. The reflection problem needs to be described by a reflection matrix.

Let us study the reflection problem from a half space of Tellegen material with material parameters ε, μ, and non-reciprocity parameter χ. For simplicity, let the incident wave fall from free space (wave impedance η_0) with normal incidence on the planar boundary, according to Fig. 7. Then [10] the co-polarized and cross-polarized reflection coefficients can be written as functions of the parameters of the Tellegen half space:

$$\begin{gathered} R_{\mathrm{co}} = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2 + 2\eta\eta_0\cos\theta}, \quad R_{\mathrm{cross}} = \frac{-2\eta\eta_0\sin\theta}{\eta^2 + \eta_0^2 + 2\eta\eta_0\cos\theta} \\ \text{with} \quad \sin\theta = \frac{\chi}{\sqrt{\mu\varepsilon}}, \quad \cos\theta = \sqrt{1 - \frac{\chi^2}{\mu\varepsilon}}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} \end{gathered} \tag{7}$$

Here the Tellegen character of the medium is measured by a slightly more convenient parameter θ which has a straightforward connection with χ. The special case of reflection problem from an isotropic, reciprocal half space (for which the Tellegen parameter χ vanishes: $\sin\theta = 0$, $\cos\theta = 1$) is easily seen to follow from

Eq. (7): no cross-polarized reflection, and the co-polarized reflection coefficient becomes the well known relation $(\eta - \eta_0) / (\eta + \eta_0)$.

Very interesting in the result (7) is the direction of the cross-polarization. When a linearly polarized wave is reflected from Tellegen medium, the reflected field is also linearly polarized but it is pointing to a different direction. The rotation angle is affected by both the impedance contrast of the medium and vacuum, and particularly the Tellegen parameter. For a “high-impedance” surface ($\eta > \eta_0$), the reflected wave is rotated counterclockwise for positive values of χ, and clockwise for negative values.[10] Due to the fact that the co-polarized reflection coefficient keeps the sign of $\eta - \eta_0$, the rotation is inverted for “low-impedance” surfaces ($\eta < \eta_0$). In that case there is also a 180° phase shift in the co-polarized reflection but nevertheless the polarization plane of the reflected field is rotated clockwise (for positive χ).

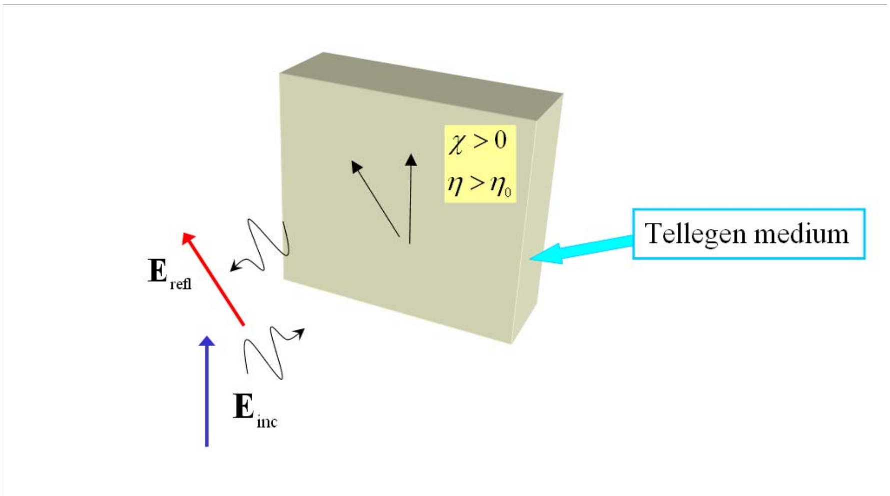

Fig. 7 The rotation of the polarization plane of the reflected field when a linearly polarized wave hits a planar boundary between free space and Tellegen medium. The Tellegen parameter χ is assumed positive and its impedance η is assumed higher than that of free space. In this case the reflection is counterclockwise as shown. Changing either of the conditions causes the rotation be clockwise. (In the general case, there also exists a transmitted wave that penetrates the Tellegen medium; it is, however, omitted here because the most interesting effect of χ is on reflection.)

[10] From the rotatory power in reflection it can be inferred that Tellegen medium is non-reciprocal: one cannot change the incident and reflected fields in Fig. 7 because a field approaching the surface with the orientation of the reflected field would suffer a further rotation into the *same* direction as the rotation in the figure, and the rotation would not be unwound.

3.4 Generalization to bi-anisotropy

When the assumption of isotropy (or bi-isotropy) is relaxed, very many more dimensions are needed to characterize media. Anisotropy means that the direction of the field force of excitation affects the amplitude of the response and such a situation may appear when the microstructure of the medium does not possess a spherical or cubic symmetry, it is, for instance, composed of oriented needle-like elements. For anisotropic media, in the constitutive relations, permittivity and permeability have to be described by dyadics or second-rank tensors, instead of scalars like in Eq. (2). In three-dimensional space, a dyadic can be expanded as a 3×3 matrix, and hence the most general anisotropic permittivity has 9 degrees of freedom, likewise the anisotropic permeability.

When anisotropy and magnetoelectric coupling are both allowed, all four parameters ε, μ, χ, and κ become dyadics (in the following denoted by a bar under the symbol), and one can say that to span the full material space requires 36 dimensions [21], cf. Fig. 8.

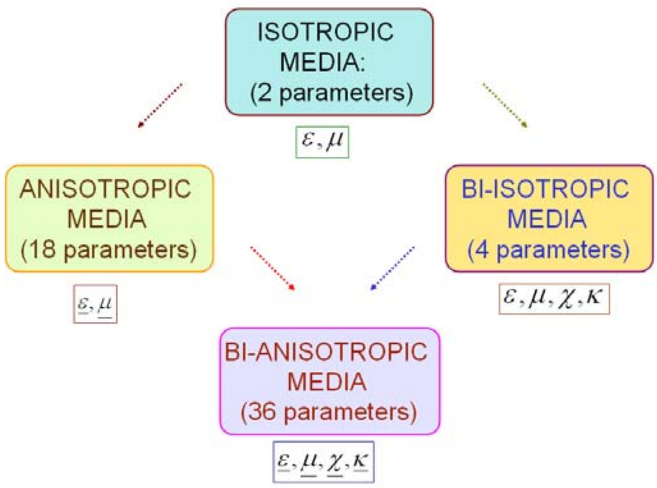

Fig. 8 Bi-anisotropic media are very general linear media, and the four material dyadics comprise together 36 parameters responsible for the magnetoelectric behavior. Anisotropic, bi-isotropic, and isotropic media can be considered as subclasses of bi-anisotropic materials with 18, 4, and 2 degrees of freedom, respectively.

On the level of equations, the constitutive relations can be collected into a six-vector presentation where electric and magnetic vector quantities (field strengths **E** and **H** on one hand, and the flux densities **D** and **B** on the other) are joined together in the following manner:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \underline{\varepsilon} & \underline{\chi}^{\mathrm{T}} - \mathrm{j}\underline{\kappa}^{\mathrm{T}} \\ \underline{\chi} + \mathrm{j}\underline{\kappa} & \underline{\mu} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = C \cdot \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \tag{8}$$

and here the four dyadics are generalizations of the scalar bi-isotropic material parameters. The equation also defines the material matrix C. Note the transpose operation of the magnetoelectric dyadics in the upper right position of the material matrix. It is with this definition that the reciprocal (κ) and non-reciprocal (χ) parts of the magnetoelectric effects are kept separate.

How, then, does the discussion in the previous subsection concerning the backward-wave characteristics of media apply to the bi-anisotropic environment? An effective tool for answering this question turns out [22] to be the material matrix C in Eq. (4). For lossless media, the matrix C is Hermitian (it is equal to its conjugate transpose) [15]. For ordinary double-positive isotropic medium, the eigenvalues of the C matrix are all positive, for double-negative media they are all negative. Indeed, for general bi-anisotropic materials, the character of definiteness of matric C determines the forward–backward character of its eigenwaves. If C is positive definite (all of its six eigenvalues are positive; note that the eigenvalues of a Hermitian matrix are real), all waves that propagate in it are forward waves. In the case of negative definite matrix C, the waves are all backward. However, if the matrix is non-definite, there are several possibilities: some of the waves may be forward, some backward, or it may even happen that some or all waves are evanescent, they do not propagate.

4 Conclusion

Within a multidisciplinary research field such as metamaterials, the terminology and labeling of phenomena and quantities under discussion need be carefully analyzed. Many aspects on parity, symmetry, and handedness were discussed in the present article. The division of semantics of handedness—with respect to electromagnetics—into three categories (negative-index and plasmonic media as left-handed materials, polarization of circularly/elliptically polarized wave, and geometrical parity-breaking structure of matter) helps in understanding and categorizing complex effects involving metamaterials. Especially the question of treatment between left and right (whether on equal basis or as the left-handed phenomenon as anomaly) was shown to be essential. In a general frame of electromagnetics of bi-anisotropic media, the three aspects of handedness can be clearly seen to occupy distinct places in the characterization of macroscopic effects of metamaterials.

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