

COMBINING ERROR-CORRECTION CODING AND CUTOFF RATE MAXIMIZATION BASED PRECODING

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ABSTRACT

A new linear block precoding technique is combined with error-correction coding for wireless orthogonal frequency-division multiplexing (OFDM) transmissions through frequency-selective fading channels. Our precoder is based on the maximization of the mean cutoff rate and requires only the knowledge of the average relative channel multipath gains and delays at the transmitter. Convolutional coding is used as an outer channel coding (CC). Two decoding schemes are developed for the proposed joint channel-coded and linearly-precoded OFDM communication system. The first decoding scheme consists of the cascade of maximum likelihood (ML) symbol detector and Viterbi decoder. The ML detector decodes the symbols encoded by the linear precoder, and the Viterbi decoder decodes the convolutional channel codes. The second decoding scheme is based on the iterative (turbo) decoding approach, which exploits the soft information and enables to further improve the decoding performance. Simulation results with convolutional codes for UMTS and HIPERLAN/2 channels show an improved performance of the proposed joint coding-precoding technique as compared to entirely precoding-based, entirely CC-based, and other known joint coding-precoding techniques. Among the two proposed decoding approaches, the turbo decoder provides better performance at the price of higher computational complexity and larger memory consumption.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising multiuser communication scheme which enables

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to mitigate multiple-access interference (MAI) by means of providing each user with a non-intersecting fraction of subcarriers [1]. Due to the inverse fast Fourier transform (IFFT) at the transmitter and the fast Fourier transform (FFT) at the receiver, the frequency selective fading channel is converted by OFDM into parallel flat fading channels [2]. This greatly simplifies the equalizer design at the receiver. However, a well known disadvantage of the OFDM scheme is that, at each subcarrier, the channel may be subject to a deep fading. This makes a reliable detection of the information-bearing symbols at this particular subcarrier very difficult and, as a result, the overall performance of the system may degrade substantially.

A popular recent approach to improve the performance of OFDM systems in fading environments is to use linear block precoding at the transmitter [3]. In a recent paper [4], the linear precoder based on the channel cutoff rate maximization has been proposed, and its improved performance as compared to other known precoders has been demonstrated. However, to mitigate fading and noise effects, practical wireless systems often employ some form of outer channel coding (CC).

In this paper, we extend the results of [4] for the case when a linear precoder is combined with an outer CC. We propose two decoding schemes for the joint channel-coded and linearly-precoded OFDM communication system. The first one consists of the cascade of the ML symbol detector and Viterbi decoder. The ML detector is designed to detect the symbols encoded by the linear precoder, while the Viterbi decoder is used to decode the outer (convolutional) CC. The second decoding scheme is based on the iterative (turbo) decoding approach, which enables to further improve the decoding performance. The turbo decoding scheme makes use of the soft extrinsic information between the channel codes and the linear precoded symbols. It should also be mentioned here that a related joint coding-precoding scheme has been recently proposed in [5]. How-

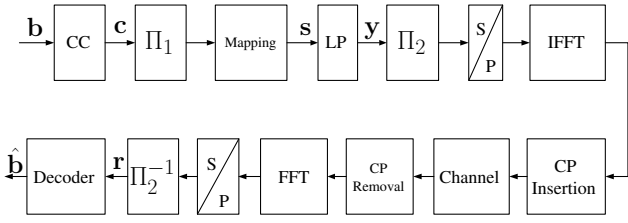


Fig. 1. System block diagram.

ever, the precoder design in [5] is different from the proposed design. An important advantage of our precoder is that only the knowledge of the average relative path gains and delays of the multipath channel is required at the transmitter, whereas most of other existing precoders require to know the full channel state information (CSI) at the transmitter [3], [17], [18]. The knowledge of average relative path gains and delays can be obtained in practical communication systems through a low-rate feedback.

2. SYSTEM MODEL

For the sake of simplicity, we consider the single-user block transmission system with N subcarriers. The extension to the multi-user case can be done straightforwardly by allocating different nonintersecting groups of subcarriers to different users.

The OFDM communication system discussed in this paper is shown in Fig. 1. It operates in the following way. A stream of information bits is first encoded by error-correction CC. The coded bits are interleaved by the interleaver Π_1 . The output bits of Π_1 are mapped to constellation symbols. After constellation mapping, successive blocks of N symbols

$$\mathbf{s}(t) = [s(tN), \dots, s(tN + N - 1)]^T \quad (1)$$

are linearly precoded (LP) by a matrix \mathbf{T} of the size $N \times N$, where t is the block index. As a result, a precoded block of symbols

$$\mathbf{y}(t) = \mathbf{T}\mathbf{s}(t) \quad (2)$$

is obtained. This block of symbols is passed through the second interleaver Π_2 . The output of Π_2 is passed to the OFDM modulator, which includes serial-to-parallel conversion, IFFT, and cyclic prefix (CP) insertion. The resulting output is serialized for transmission.

The frequency selective wireless channel between the transmitter and the user is characterized by the path gains h_l ($l = 1, \dots, L$) and the delays τ_l ($l = 1, \dots, L$), where all path gains are assumed to be independent (but not necessarily identically distributed) zero-mean complex Gaussian random variables.

At the receiver, after OFDM demodulation and deinterleaving Π_2^{-1} , we have the following relationship:

$$\begin{aligned} \mathbf{r}(t) &= \sqrt{E_s} \mathbf{D} \mathbf{y}(t) + \mathbf{v}(t) \\ &= \sqrt{E_s} \mathbf{D} \mathbf{T} \mathbf{s}(t) + \mathbf{v}(t) \end{aligned} \quad (3)$$

where

$$\mathbf{r}(t) = [r(tN), \dots, r(tN + N - 1)]^T \quad (4)$$

is the $N \times 1$ vector of the received symbols after removing CP, the FFT operation, and deinterleaving; E_s is the transmitted symbol power;

$$\mathbf{y}(t) = [y(tN), \dots, y(tN + N - 1)]^T \quad (5)$$

is the $N \times 1$ vector of the transmitted symbols without CP and interleaving;

$$\mathbf{v}(t) = [v(tN), \dots, v(tN + N - 1)]^T \quad (6)$$

is the $N \times 1$ vector of white complex Gaussian noise with the covariance matrix

$$\mathbb{E}\{\mathbf{v}(t)\mathbf{v}^H(t)\} = \sigma_v^2 \mathbf{I}_N \quad (7)$$

$\mathbb{E}\{\cdot\}$ is the statistical expectation; \mathbf{I}_N is the $N \times N$ identity matrix; σ_v^2 is the noise power; \mathbf{D} is the $N \times N$ diagonal matrix whose (n, n) th element is given by

$$[\mathbf{D}]_{n,n} = \frac{1}{\sqrt{N}} \sum_{l=1}^L h_l \exp\left(-\frac{j2\pi n\tau_l}{NT}\right) \quad (8)$$

T is the sampling interval; and $j = \sqrt{-1}$.

3. PROPOSED LINEAR PRECODER

The channel cutoff rate R_0 is a lower bound on the Shannon channel capacity, beyond which the sequential decoding becomes impractical [6], [7]. It also specifies an upper bound on the error rate of the optimal ML decoder. R_0 has been frequently used as a practical coding limit because it can be calculated in a much simpler way as compared to the channel capacity [7]. Therefore, the cutoff rate appears to be a proper criterion for the design of linear block precoders. Note that it has been previously used as a performance metric for OFDM systems [8], [9], and as a design criterion for transmitter optimization in multiple-input multiple-output (MIMO) channels [10].

Hereafter, we assume that a discrete constellation is used at the transmitter, and the full CSI is available at the receiver. Moreover, it is assumed that the ML technique is used to detect the symbols $\mathbf{s}(t)$ from the received data $\mathbf{r}(t)$.

The conditional probability density function of the received data can be written as

$$f(\mathbf{r}|\mathbf{s}^{(i)}, \mathbf{T}, \mathbf{D}) = \frac{1}{(\pi\sigma_v^2)^N} \exp\left(-\frac{\|\mathbf{r} - \sqrt{E_s}\mathbf{D}\mathbf{T}\mathbf{s}^{(i)}\|^2}{\sigma_v^2}\right) \quad (9)$$

where $\mathbf{s}^{(i)}$ is the i th member of the transmit vector constellation, $\|\cdot\|$ denotes the Frobenius matrix norm or the Euclidean vector norm, and the dependence on t is omitted in the interest of notational brevity. To further simplify the notation, let us denote $f(\mathbf{r}|\mathbf{s}^{(i)}, \mathbf{T}, \mathbf{D})$ as $f(i)$. Then the mean cutoff rate can be calculated as [6, p. 361]

$$\begin{aligned} R_0 &= -\log E_{\mathbf{D}} \left\{ \int_{\mathbf{r}} \left[\frac{1}{M^N} \sum_{i=1}^{M^N} \sqrt{f(i)} \right]^2 d\mathbf{r} \right\} \\ &= -\log \left[\frac{1}{M^N} + \frac{1}{M^{2N}} \sum_{i=1}^{M^N} \sum_{\substack{l=1 \\ l \neq i}}^{M^N} E_{\mathbf{D}} \left\{ \int_{\mathbf{r}} \sqrt{f(i)f(l)} d\mathbf{r} \right\} \right] \end{aligned} \quad (10)$$

where M is the constellation size.

Inserting (9) into (10) we obtain the following expression for the mean cutoff rate

$$\begin{aligned} R_0 &= -\log \left[\frac{1}{M^N} + \frac{1}{M^{2N}} \right. \\ &\quad \left. \cdot \sum_{i=1}^{M^N} \sum_{\substack{l=1 \\ l \neq i}}^{M^N} E_{\mathbf{D}} \left\{ \exp\left(-\frac{E_s \|\mathbf{D}\mathbf{T}(\mathbf{s}^{(i)} - \mathbf{s}^{(l)})\|^2}{4\sigma_v^2}\right) \right\} \right]. \end{aligned} \quad (11)$$

Using the results of [11], the expectation of exponential quadratic form in (11) can be written as

$$\begin{aligned} &E_{\mathbf{D}} \left\{ \exp\left(-\frac{E_s \|\mathbf{D}\mathbf{T}(\mathbf{s}^{(i)} - \mathbf{s}^{(l)})\|^2}{4\sigma_v^2}\right) \right\} \\ &= \prod_{k=1}^{\text{r}\{\mathbf{E}_{i,l}\}} \left(1 + \frac{E_s}{4\sigma_v^2} \lambda_k \right)^{-1} \end{aligned} \quad (12)$$

where

$$\mathbf{E}_{i,l} \triangleq E_{\mathbf{D}} \{ \mathbf{D}\mathbf{T}\mathbf{e}_{i,l}\mathbf{e}_{i,l}^H \mathbf{T}^H \mathbf{D}^H \} \quad (13)$$

$$\mathbf{e}_{i,l} \triangleq \mathbf{s}^{(i)} - \mathbf{s}^{(l)} \quad (14)$$

λ_k is the k th non-zero eigenvalue of the matrix $\mathbf{E}_{i,l}$, and $\text{r}\{\cdot\}$ denotes the matrix rank.

Substituting (12) in (11), after straightforward manipulations we obtain

$$\begin{aligned} R_0 &= -\log \left[\frac{1}{M^N} \right. \\ &\quad \left. + \frac{1}{M^{2N}} \sum_{i=1}^{M^N} \sum_{\substack{l=1 \\ l \neq i}}^{M^N} \prod_{k=1}^{\text{r}\{\mathbf{E}_{i,l}\}} \left(1 + \frac{E_s}{4\sigma_v^2} \lambda_k \right)^{-1} \right]. \end{aligned} \quad (15)$$

Interestingly, the expression (15) for the mean cutoff rate is closely related to the expression for the Chernoff bound on the pairwise error probability (PEP). In particular, the second term under the logarithm in (15) can be seen as an average of the Chernoff bounds on PEP for all distinct pairs of symbols $(\mathbf{s}^{(i)}, \mathbf{s}^{(l)})$, $i, l = 1, \dots, M^N$, $i \neq l$. In other words, the maximization of mean cutoff rate is equivalent to the minimization of averaged PEP. This observation provides further motivation and justification to choose the mean cutoff rate as a criterion for precoder design.

To compute the matrix $\mathbf{E}_{i,l}$ explicitly, let us introduce the vector

$$\mathbf{d} \triangleq [[\mathbf{D}]_{1,1}, \dots, [\mathbf{D}]_{N,N}]^T. \quad (16)$$

Then, $\mathbf{E}_{i,l}$ can be rewritten as

$$\mathbf{E}_{i,l} = \mathbf{R}_{\mathbf{d}} \odot (\mathbf{T}\mathbf{e}_{i,l}\mathbf{e}_{i,l}^H \mathbf{T}^H) \quad (17)$$

where \odot stands for the Schur-Hadamard matrix product and

$$\mathbf{R}_{\mathbf{d}} \triangleq E_{\mathbf{d}} \{ \mathbf{d}\mathbf{d}^H \}. \quad (18)$$

The (n, k) th entry of $\mathbf{R}_{\mathbf{d}}$ is given by

$$[\mathbf{R}_{\mathbf{d}}]_{n,k} = \frac{1}{N} \sum_{l=1}^L P_l \exp\left(-\frac{j2\pi(n-k)\tau_l}{NT}\right) \quad (19)$$

where P_l is the average power of the l th path relative to the first path.

Our task now is to design the precoding matrix \mathbf{T} which maximizes R_0 in (15) subject to the power constraint

$$\|\mathbf{T}\| = \sqrt{N}. \quad (20)$$

Towards this end, we can use either algebraic number-theoretic techniques or computer search over compact parameterizations of unitary matrices [12]. In this paper, we obtain \mathbf{T} through computer search over the unitary¹ parameterization that uses Givens rotation matrices. Details of this technique can be found, for example, in [12]. Provided that each user occupies a moderate number of subcarriers (not more than 3 per user), and since the precoding matrices can be designed for each user independently, we can conclude that the total number of real optimization parameters for one particular user is $N(N-1) \leq 6$. If the number of optimization parameters is small, full search is computationally feasible. Therefore, the design of our precoder becomes practically feasible as well.

It can be seen from (15)-(19) that for the design of our linear precoder, only the knowledge of the average relative path gains and delays of the multipath channel is required at

¹Note that unitary precoders have the merit that they do not alter the Euclidian distance between the entries of any block of information-bearing symbols.

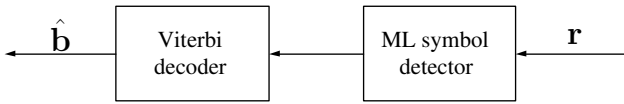


Fig. 2. Block diagram of concatenated ML symbol detector and Viterbi decoder.

the transmitter. Although the channel state variations can be very fast due to small-scale fading, the average path gains and delays vary typically much slower. Therefore, a low-rate feedback can be used to convey this information to the transmitter.

4. DECODING SCHEMES

In this section, we design the decoder for the joint channel-coded and linearly-precoded OFDM communication system. From the system block diagram in Fig. 1 we can observe that the channel encoder, the interleaver Π_1 , and the linear precoder together represent a serial concatenated encoder [13]. In such an encoder, the linear precoder and the channel encoder can be seen as the inner encoder and the outer encoder, respectively. Therefore, the standard decoding schemes for serial concatenated codes can be used to decode the joint channel-coded and linearly-precoded symbols.

4.1. Hard Decision Decoding Scheme

The block diagram of the first scheme, which is referred to as the hard decision decoding scheme (HDDS), is shown in Fig. 2. HDDS consists of the ML symbol detector and Viterbi decoder. In this scheme, the ML symbol detector is used to detect the symbols encoded by the linear precoder, while the Viterbi decoder is applied to decode the convolutional CCs. Note that such decoding scheme is suboptimal because the hard decision is used at the output of the ML symbol detector. However, the computational complexity of this decoder is relatively low, provided that each user occupies only a moderate number of subcarriers.

4.2. Soft Decision Decoding Scheme

The second decoding scheme we refer to as the soft decision decoding scheme (SDDS). The block diagram of SDDS is shown in Fig. 3. It is based on the iterative (turbo) decoding technique. The main components of SDDS are two maximum a posteriori (MAP) soft-input soft-output (SISO) modules. Each SISO module is a four-port device which receives bits of soft information and outputs the updated soft information calculated by the MAP algorithm [13]. The updated soft information is exchanged between two SISO modules in an iterative way. Normally the soft information

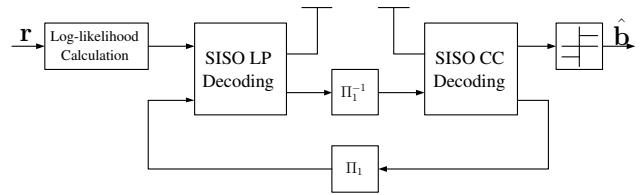


Fig. 3. Block diagram of iterative (turbo) decoder.

is represented through the log-likelihood ratio. Compared to the hard information, the soft information not only contains the result of a decision, but also reflects the reliability of this decision [13].

The merit of the iterative (turbo) decoding scheme is that during each iteration, the so-called extrinsic information (which is the soft information passed from one SISO module to another) increases the reliability of the decision. Therefore, after a finite number of iterations, the reliability of the decision will be high enough, and the iteration process will be terminated. Then, the final decision can be made by passing the value of the likelihood ratio of each bit through a threshold detector. Detailed discussion about the turbo principle is beyond the scope of this paper, and we refer readers to [5] and [13] for more information.

Note that SDDS can greatly improve the performance of the joint channel-coded and linearly-precoded OFDM system. However, compared to HDDS, SDDS has higher computational complexity, because the computations involved in the MAP algorithm are at least 4 times of that involved in the Viterbi algorithm [5]. Moreover, SDDS requires more memory as compared to HDDS. Therefore, HDDS and SDDS offer different tradeoffs in terms of performance, system requirement and hardware/software complexity.

5. SIMULATIONS

In this section, we investigate the performance of the combined channel coding and the proposed linear precoding scheme for both multipath indoor and outdoor channel environments. As an example of a multipath Rayleigh fading outdoor channel, we choose the ETSI “Vehicular A” channel environment, which has been defined for the evaluation of UMTS radio interface proposals [14]. The multipath time delays and variances of the multipath gains of the “Vehicular A” channel are shown in Table 1. Correspondingly, as an example of a multipath Rayleigh fading indoor channel we choose the HIPERLAN/2 “Model A” channel, which represents a typical office environment [15]. The multipath time delays and variances of the multipath gains of the HIPERLAN/2 “Model A” channel are shown in Table 2. The Doppler frequencies for these two channels are set to be equal to 100 Hz and 50 Hz, respectively.

Table 1. Characteristics of the ETSI “Vehicular A” channel environment.

Tap	Time Delays (T)	Average Power (dB)
1	0	0
2	1.55	-1
3	3.55	-9
4	5.45	-10
5	8.65	-15
6	12.55	-20

Table 2. Characteristics of the HIPERLAN/2 “Model A” channel environment.

Tap	Time Delays (T)	Average Power (dB)
1	0	0
2	0.2	-0.9
3	0.4	-1.7
4	0.6	-2.6
5	0.8	-3.5
6	1	-4.3
7	1.2	-5.2
8	1.4	-6.1
9	1.6	-6.9
10	1.8	-7.8
11	2.2	-4.7
12	2.8	-7.3
13	3.4	-9.9
14	4	-12.5
15	4.8	-13.7
16	5.8	-18.0
17	6.8	-22.4
18	7.8	-26.7

Throughout the simulations, a multi-user block transmission system with 64 subcarriers is assumed. All subcarriers are allocated among the users and interleaved such that the subcarriers assigned to the same user are as less correlated to each other as possible. Each user is provided with $N = 3$ subcarriers. The sequence of information bits is encoded by the convolutional code and then BPSK modulated. The interleaver Π_1 is chosen to be random with the size corresponding to 256 OFDM symbols. For the optimization of the precoding matrix \mathbf{T} , we carry out 10^5 Monte Carlo trials, and pick up the parameters which maximize R_0 in (15).

5.1. Example 1

In the first example, six different precoding techniques are compared: the approach where no precoding is used, the minimum mean square error for zero-forcing equalization

(MMSE-ZF) precoder of [3], the minimum bit error rate for ZF equalization (MBER-ZF) precoder of [17], the MBER for MMSE equalization (MBER-MMSE) precoder of [18], the Vandermonde precoder of [16], and the proposed precoder.

Figs. 4 and 5 display the channel mean cutoff rate of different precoding schemes versus the signal-to-noise ratio (SNR) for the ETSI “Vehicular A” channel environment and the HIPERLAN/2 “Model A” channel environment, respectively. It can be seen from these two figures that, as expected, the proposed linear precoder has the highest mean cutoff rate among all the techniques tested.

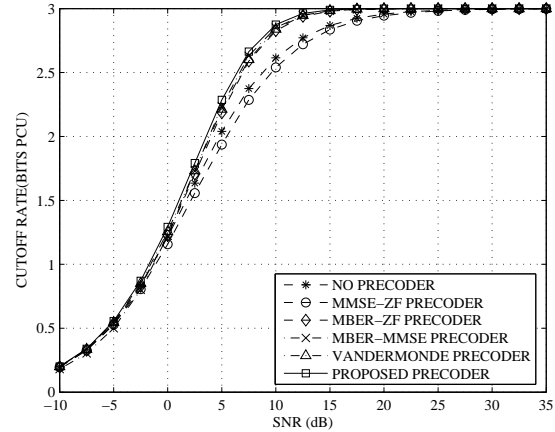


Fig. 4. Cutoff rate versus SNR. The ETSI “Vehicular A” channel environment.

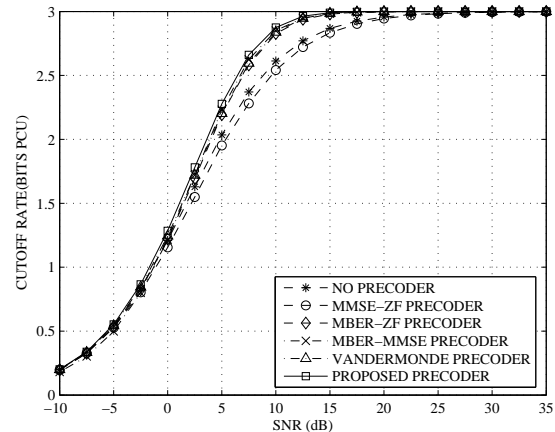


Fig. 5. Cutoff rate versus SNR. The HIPERLAN/2 “Model A” channel environment.

Figs. 6 and 7 compare the symbol-error-rate (SER) performances of the same techniques with different symbol

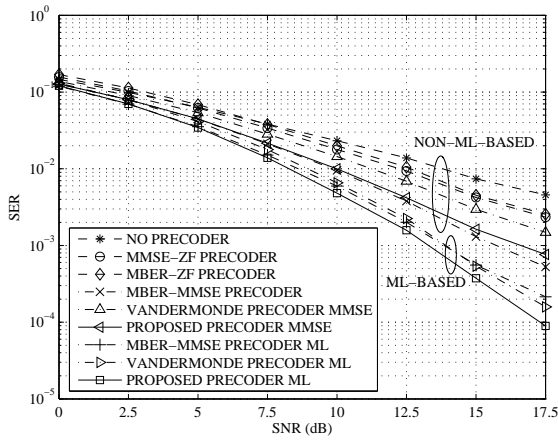


Fig. 6. SER versus SNR. First example with the ETSI “Vehicular A” channel environment.

detectors for the ETSI “Vehicular A” channel environment and the HIPERLAN/2 “Model A” channel environment, respectively. In these two figures, the performances of the proposed, MBER-MMSE, and Vandermonde precoders are displayed both in the cases when the ML and MMSE symbol detectors are used. Additionally, the SER performances of the MMSE-ZF and MBER-ZF precoders are displayed along with the SER of the standard approach where no precoding is used. All results are averaged over 1000 simulation runs.

We can observe that our linear precoder substantially outperforms all other techniques tested in terms of SER for both the ETSI “Vehicular A” and HIPERLAN/2 “Model A” channel environments. Interestingly, this conclusion is true when the ML-based as well as non-ML (MMSE) receivers are used, with the only exception for the MBER-MMSE precoder. In particular, the performance of the latter precoder is comparable to the performance of our precoder used with the MMSE receiver. This fact demonstrates that although the mean cutoff rate based precoder has been proposed for the ML-based symbol detector, it also provides a good performance when applied with the simpler MMSE symbol detector.

5.2. Example 2

In the second example, we compare the BER performance of OFDM systems with combined coding-precoding, with CC only, with precoding only, and without CC and precoding. The proposed precoder and the Vandermonde precoder are used in this simulation example. The Vandermonde precoder is chosen for comparison to the proposed precoder as one that shows the best performance among known precoders tested in the previous example.

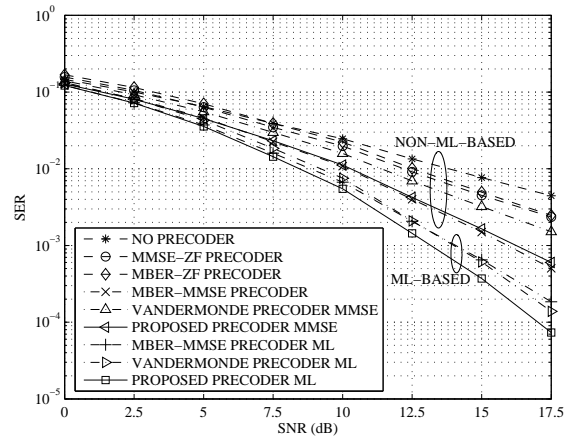


Fig. 7. SER versus SNR. First example with the HIPERLAN/2 “Model A” channel environment.

First, we study the system performance when HDDS is used at the receiver. For this simulation, the rate 1/2 convolutional code in the HIPERLAN/2 standard [15] with generator (133, 171) is used. Figs. 8 and 9 show the BER performance of different OFDM configurations for the ETSI “Vehicular A” and the HIPERLAN/2 “Model A” channel environments, respectively. From these two figures, it can be seen that OFDM system with combined coding-precoding shows the best performance among the techniques tested. Moreover, the scheme where the cutoff rate maximization based precoder is used outperforms the scheme based on the Vandermonde precoder of [16].

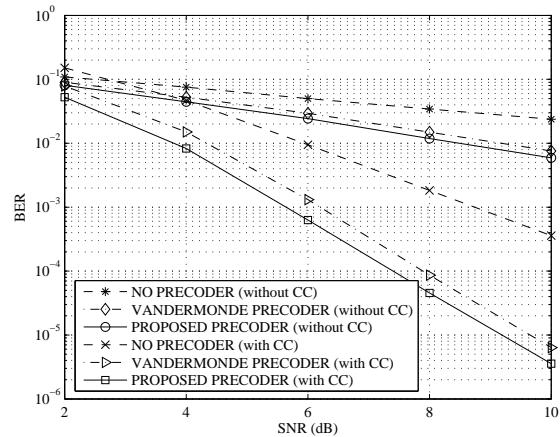


Fig. 8. BER versus SNR. Second example with the ETSI “Vehicular A” channel environment; HDDS.

Second, we investigate the system performance when SDDS is applied at the receiver. For this simulation the rate 1/2 systematic convolutional code with generator (5, 7) is

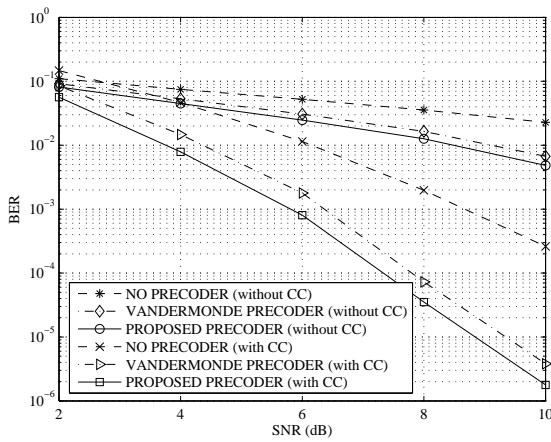


Fig. 9. BER versus SNR. Second example with the HIPER-LAN/2 “Model A” channel environment; HDDS.

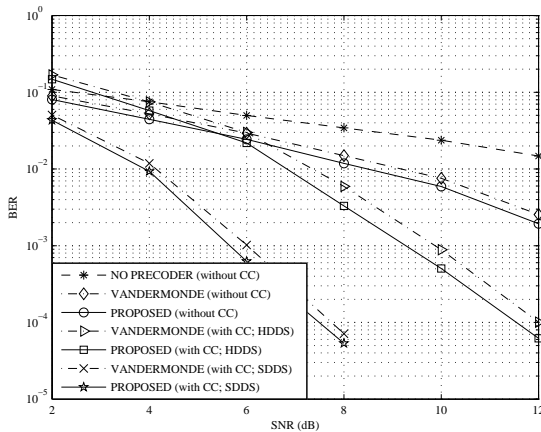


Fig. 10. BER versus SNR. Second example with the ETSI “Vehicular A” channel environment; SDDS.

used. Fig. 10 shows the system performance for the ETSI “Vehicular A” channel environment. 2 iterations are carried out before the final decision. From Fig. 10, we observe that SDDS substantially enhances the overall system performance. However, this performance improvement is at the price of higher decoding complexity and larger memory consumption. We can also see from Fig. 10 that the scheme where the cutoff rate maximization based precoder is used outperforms the scheme based on the Vandermonde precoder of [16].

6. CONCLUSIONS

A new linear block precoding technique is combined with error-correction coding for block OFDM transmissions. The

proposed precoder is based on the maximization of the channel mean cutoff rate and requires only the knowledge of the average relative channel multipath gains and delays at the transmitter. Two decoding schemes are studied for the joint channel-coded and linearly-precoded OFDM communication system. The first one is the cascade of the ML symbol detector and Viterbi decoder, while the second one is based on the turbo decoding approach. Simulation results show substantial performance improvements achieved by the proposed precoding technique as compared to entirely precoding-based, entirely CC-based, and earlier joint coding-precoding techniques. Among the two decoding approaches, the turbo decoder offers drastic performance improvements at the price of higher computational complexity and larger memory consumption. The proposed precoding technique can be readily extended to the MIMO case.

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