From Robust Adaptive Beamformers to Robust Multi-User MIMO Receivers

A. Gershman, Y. Rong, S. Shahbazpanahi, and S. Vorobyov

Communication Systems Group

Darmstadt University of Technology

Darmstadt, Germany

October 2005

Outline

- Introduction
- Overview of Robust MV Beamforming Based on Worst-Case Designs
- Array Processing-Type MIMO Model
- Robust Linear Minimum Variance MIMO Receivers
- Numerical Examples
- Conclusions

Introduction

Robust MV beamforming:

- earlier approaches include fixed diagonal loading [Abramovich'81], [Carlson'88]; adaptive diagonal loading [Cox Zeskind Owen'87]; eigenspace-based beamforming [Feldman Griffiths'91], and other techniques.
- More recent approaches are based on *worst-case designs* [Vorobyov Gershman Luo'01], [Lorenz Boyd'01], or can be interpreted by means of such designs [Li Stoica Wang'02].

Our goal: to extend the worst-case MV beamformer designs to multi-user MIMO space-time receivers to make them robust against channel state information (CSI) errors.

Traditional and Robust MV Beamforming

Classical (non-robust) MV beamforming:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_s = 1 \quad \rightarrow \quad \mathbf{w}_{\mathrm{MV}} = (\mathbf{a}_s^H \mathbf{R}^{-1} \mathbf{a}_s)^{-1} \mathbf{R}^{-1} \mathbf{a}_s$$

Robust worst-case MV beamforming:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{s.t.} \quad |\mathbf{w}^H (\mathbf{a}_s + \boldsymbol{\delta})| \geq 1 \;\; \forall \; \|\boldsymbol{\delta}\| \leq \varepsilon \tag{1}$$

The latter problem can be transformed to

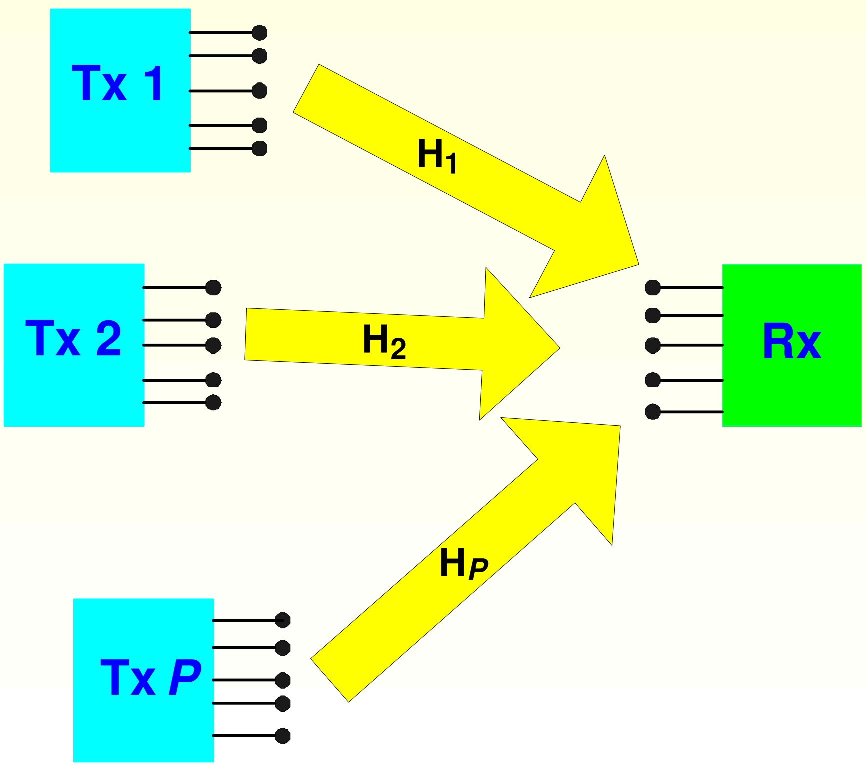
$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_s \geq \varepsilon \|\mathbf{w}\| + 1 \tag{2}$$

Problem (2) is *convex* and can be solved with the complexity $O(N^3)$ by second-order cone programming (SOCP) algorithms [Vorobyov Gershman Luo'01] or Newton-type algorithms [Lorenz Boyd'01], [Li Stoica Wang'02]

Array Processing-Type MIMO Model

Assumptions:

- uplink multi-user MIMO; block flat fading channel;
- P transmitters (users); each user has N antennas; the receiver has M antennas



Array Processing-Type MIMO Model Cont'd

Signal model:

$$\mathbf{Y}_{(T\times M)} = \sum_{p=1}^{P} \mathbf{X}_{p\,(T\times N)} \mathbf{H}_{(N\times M)} + \mathbf{V}_{(T\times M)}$$

Let the transmitted symbols of the pth user be given by the vector $\mathbf{s}_p \triangleq [s_{p,1} \ \cdots \ s_{p,K}]^T$ and let an orthogonal space-time code (OSTBC) [Tarokh Jafarkhani Calderbank'98], [Alamouti'98] be used:

$$\mathbf{X}_p = \mathbf{X}(\mathbf{s}_p), \quad \mathbf{X}^H(\mathbf{s}_p)\mathbf{X}(\mathbf{s}_p) = \|\mathbf{s}_p\|^2 \mathbf{I}_N$$

Array Processing-Type MIMO Model Cont'd

The matrix $\mathbf{X}(\mathbf{s}_p)$ can be written as

$$\mathbf{X}(\mathbf{s}_p) = \sum_{k=1}^{K} \left(\mathbf{C}_k \mathrm{Re}\{s_{p,k}\} + \mathbf{D}_k \mathrm{Im}\{s_{p,k}\} \right) \qquad (3)$$

where $\mathbf{C}_k \triangleq \mathbf{X}(\mathbf{e}_k)$ and $\mathbf{D}_k \triangleq \mathbf{X}(j\mathbf{e}_k)$.

Using (3) yields the following *array processing-type* model:

$$\underline{\mathbf{Y}}_{(2MT\times 1)} = \sum_{p=1}^{P} \mathbf{A}_{p\,(2MT\times 2K)} \underline{\mathbf{s}}_{p\,(2K\times 1)} + \underline{\mathbf{V}}_{(2MT\times 1)}$$

$$\begin{aligned} \mathbf{A}_p = \mathbf{A}(\mathbf{H}_p) &\triangleq [\underline{\mathbf{C}_1\mathbf{H}_p} \cdots \underline{\mathbf{C}_K\mathbf{H}_p}\ \underline{\mathbf{D}_1\mathbf{H}_p} \cdots \underline{\mathbf{D}_K\mathbf{H}_p}] \\ &\triangleq [\mathbf{a}_{p,1}\ \cdots\ \mathbf{a}_{p,2K}] \end{aligned}$$

where $\underline{\mathbf{P}} \triangleq [\mathrm{vec}\{\mathrm{Re}(\mathbf{P})\}^T\, \mathrm{vec}\{\mathrm{Im}(\mathbf{P})\}^T]^T$ for any matrix $\mathbf{P}$.

Linear Receivers

The ML decoder may be prohibitively expensive in the multi-user case. Simpler suboptimal linear receivers can be used:

$$\underline{\hat{\mathbf{s}}_1} = \mathbf{W}^T \underline{\mathbf{Y}}$$

where w.l.g. user # 1 is assumed to be the user-of-interest (UOI) and $\mathbf{W}$ is the $2MT \times 2K$ matrix of receiver coefficients.

Example: the matched filter (MF) receiver

$$\underline{\hat{\mathbf{s}}_1} = \frac{1}{\|\mathbf{H}_1\|_F^2} \mathbf{A}_1^T \underline{\mathbf{Y}}$$

- optimal in the ML sense in the single user case,
- far away from the optimality in the multi-user case.

Linear Receivers Cont'd

The MV receiver was used in [Shahbazpanahi et al'05]:

$$\min_{\mathbf{w}_k} \mathbf{w}_k^T \mathbf{R} \mathbf{w}_k \quad \text{s.t.} \quad \mathbf{a}_{1,k}^T \mathbf{w}_k = 1 \;\rightarrow\; \mathbf{w}_{\mathrm{MV},k} = (\mathbf{a}_{1,k}^H \mathbf{R}^{-1} \mathbf{a}_{1,k})^{-1} \mathbf{R}^{-1} \mathbf{a}_{1,k}$$

where $\mathbf{R} \triangleq \mathrm{E}\{\underline{\mathbf{Y}}\,\underline{\mathbf{Y}}^T\}$. Then, $\mathbf{W}_{\mathrm{MV}} = [\mathbf{w}_{\mathrm{MV},1} \; \cdots \; \mathbf{w}_{\mathrm{MV},2K}]$

Self-interference zero-forcing by additional constraints:

$$\mathbf{a}_{1,l}^T \mathbf{w}_k = 0 \quad \text{for all} \quad l \neq k$$

Generalized MV (GMV) receiver [Shahbazpanahi et al'05]:

$$\min_{\mathbf{W}} \mathrm{tr}\{\mathbf{W}^T \mathbf{R} \mathbf{W}\} \;\text{s.t.}\; \mathbf{A}_1^T \mathbf{W} = \mathbf{I}_{2K} \;\rightarrow\; \mathbf{W}_{\mathrm{GMV}} = \mathbf{R}^{-1} \mathbf{A}_1 (\mathbf{A}_1^T \mathbf{R}^{-1} \mathbf{A}_1)^{-1}$$

How to add robustness against CSI errors at the receive side?

Robust MV Receivers

Let

$\mathbf{H}_1$	true channel matrix of the UOI
$\hat{\mathbf{H}}_1$	presumed channel matrix of the UOI
$\mathbf{\Delta} \triangleq \mathbf{H}_1 - \hat{\mathbf{H}}_1$	CSI error

The worst-case robust modification of $\mathbf{W}_{\mathrm{MV}}$:

$$\min_{\mathbf{w}_k} \mathbf{w}_k^T \mathbf{R} \mathbf{w}_k \quad \text{s.t.} \quad \mathbf{w}_k^T \mathbf{a}_k(\hat{\mathbf{H}}_1 + \mathbf{\Delta}) \geq 1 \quad \forall \quad \|\mathbf{\Delta}\|_F \leq \varepsilon \tag{4}$$

where $\mathbf{a}_k(\cdot)$ is the kth column of $\mathbf{A}(\cdot)$.

Robust MV Receivers Cont'd

Lemma: For any OSTBC,

$$\|\mathbf{\Delta}\|_F = \|\boldsymbol{\delta}_k\| \quad \text{for all} \quad k = 1, \ldots, 2K$$

where $\boldsymbol{\delta}_k \triangleq \mathbf{a}_k(\mathbf{H}_1) - \mathbf{a}_k(\hat{\mathbf{H}}_1)$.

Using this lemma, we can prove that problem (4) takes the following form [Rong Shahbazpanahi Gershman'05]:

$$\min_{\mathbf{w}_k} \mathbf{w}_k^T \mathbf{R} \mathbf{w}_k \quad \text{s.t.} \quad \mathbf{w}_k^T (\mathbf{a}_k(\hat{\mathbf{H}}_1) + \boldsymbol{\delta}_k) \geq 1 \quad \forall \quad \|\boldsymbol{\delta}\| \leq \varepsilon \qquad (5)$$

which is *mathematically identical* to the robust MV beamforming problem in (1)! Hence, (5) can be directly solved using the algorithms developed in [Vorobyov Gershman Luo'01], [Lorenz Boyd'01], and [Li Stoica Wang'02].

Robust MV Receivers Cont'd

- A robust formulation has also been developed for the GMV receiver: more complicated formulation, but still convertible to the SOCP form [Rong Shahbazpanahi Gershman'05].

- As the worst-case designs may be overly conservative, *chance programming* designs with probabilistic constraints have been proposed in [Rong Vorobyov Gershman'05]:

$$\min_{\mathbf{w}_k} \mathbf{w}_k^T \mathbf{R} \mathbf{w}_k \quad \text{s.t.} \quad \Pr\{\mathbf{w}_k^T(\mathbf{a}_k(\hat{\mathbf{H}}_1) + \boldsymbol{\delta}_k) \geq 1\} > p_o$$

  where $1 - p_o$ can be viewed as the *outage probability*. More flexibility, but still convertible to convex optimization problems. More work on this idea is required.

Numerical Examples

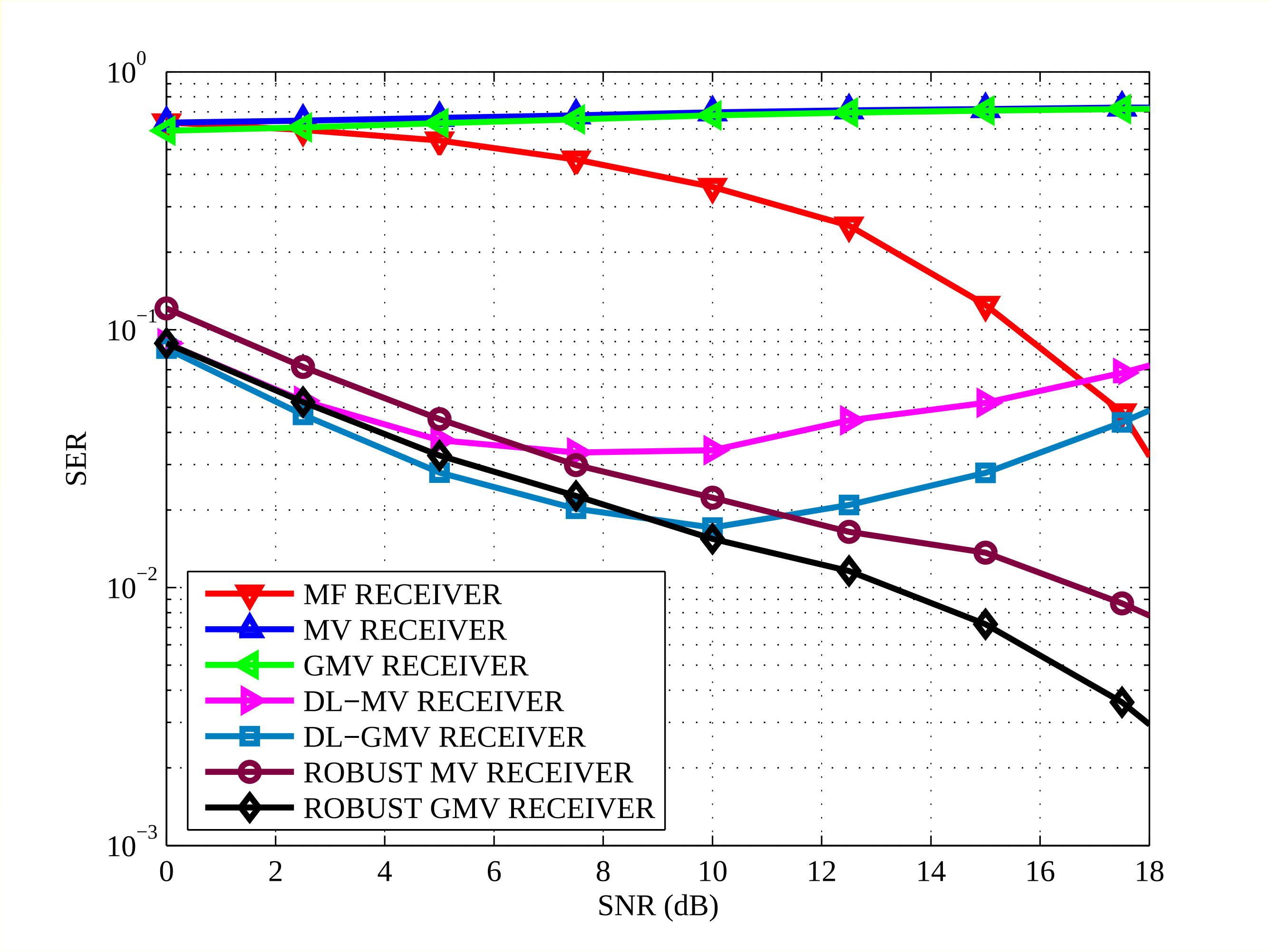
Example # 1:

- $P = 2$ users of $N = 2$ antennas each; $M = 8$ receive antennas, Alamouti's code ($T = 2$, $K = 2$), QPSK symbols, $\mathrm{INR} = 20$ dB.
- UOI channel matrix is drawn from a zero-mean Gaussian unit-variance distribution.
- UOI channel matrix is known up to CSI errors drawn from a zero-mean Gaussian distribution with variance $\sigma_e^2 = 0.1$

Example # 2: the same as Example # 1 except that $P = 3$ users of $N = 3$ antennas each and Tarokh's $3/4$-rate OSTBC ($T = 4$, $K = 3$) are assumed.

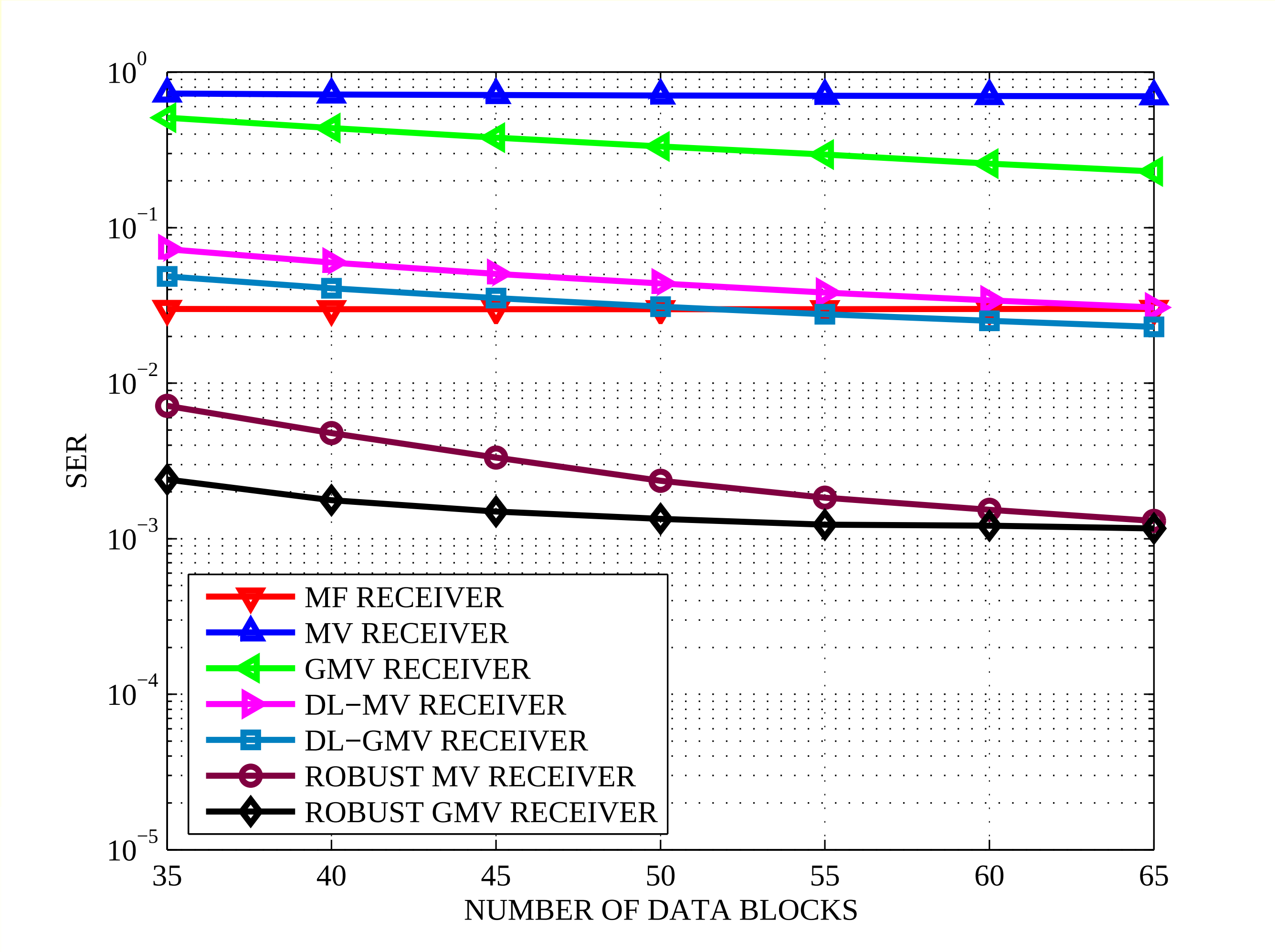
Numerical Examples Cont'd

Example # 1: Alamouti's OSTBC, $J = 35$.



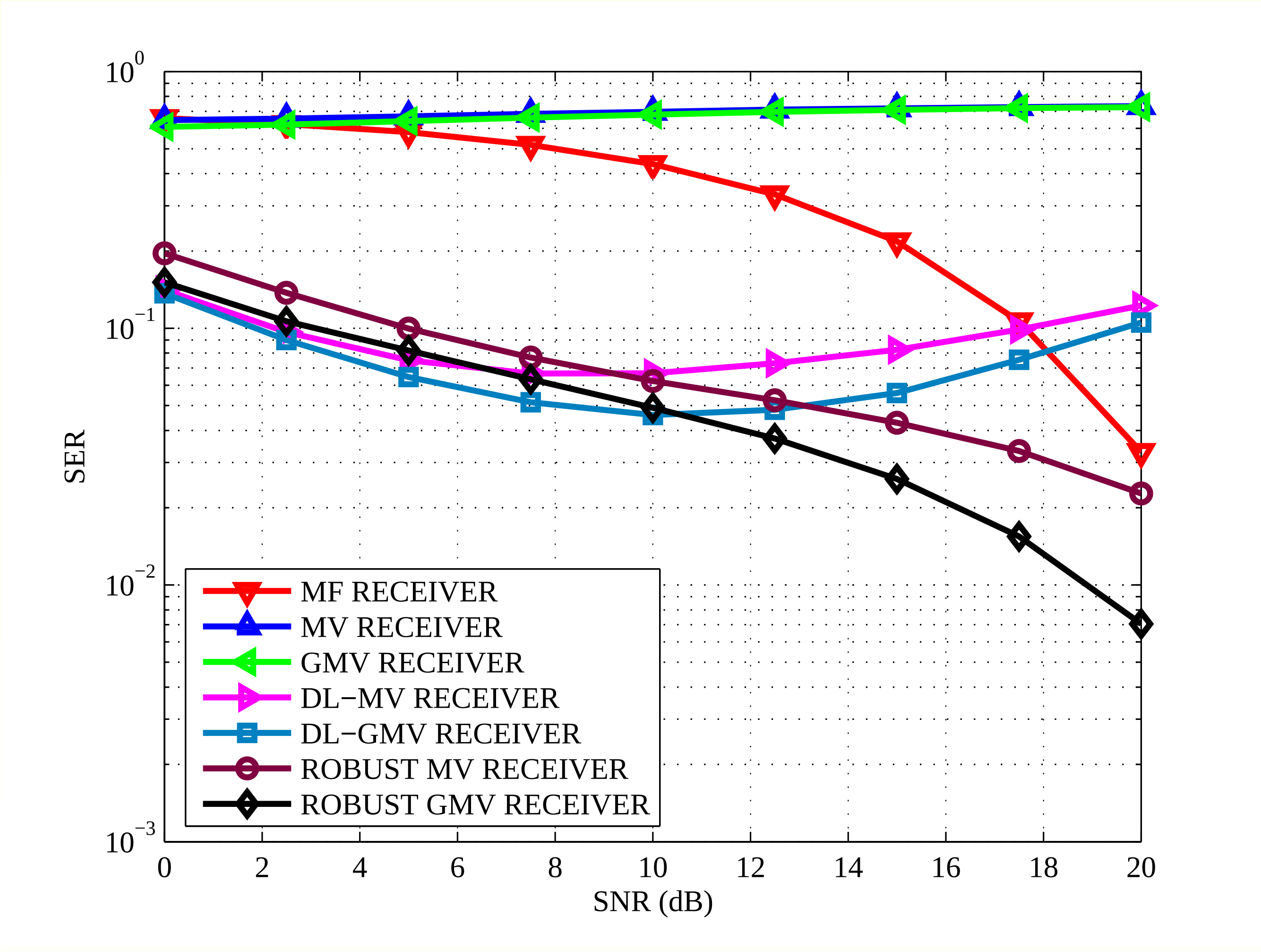
Numerical Examples Cont'd

Example # 1: Alamouti's OSTBC, $\mathrm{SNR} = 18$ dB.



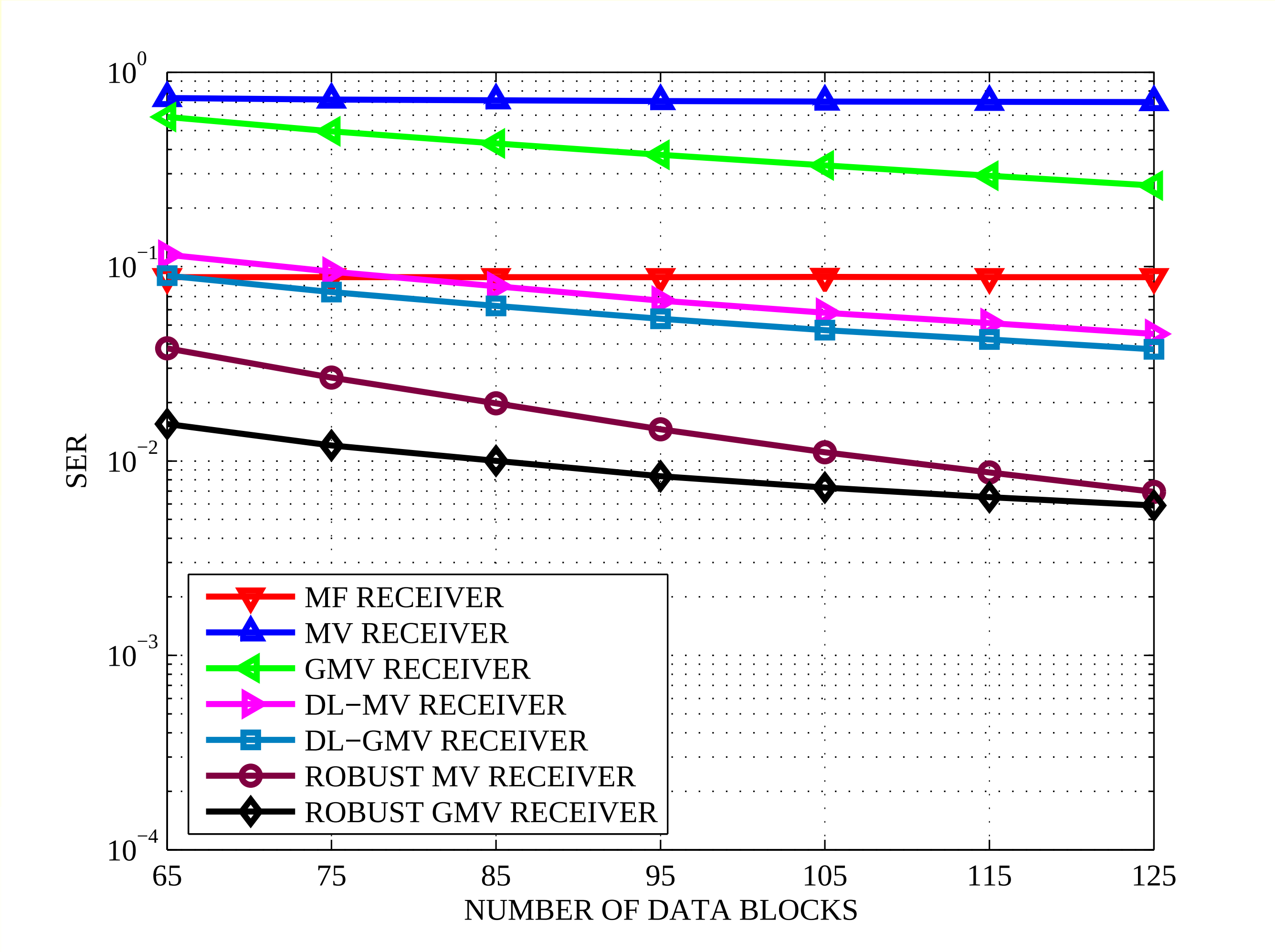
Numerical Examples Cont'd

Example # 2: Tarokh's OSTBC, $J = 70$.



Numerical Examples Cont'd

Example # 2: Tarokh's OSTBC, $\mathrm{SNR} = 18$ dB.



Conclusions

- Worst-case robust beamformer designs have been extended to multi-user receivers for orthogonally space-time block coded MIMO systems.
- Further extensions: non-orthogonal high-rate space-time codes, outage probability based formulations, computationally efficient on-line algorithms, combining linear and non-linear receivers.