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University of Alberta

Joint Bandwidth and Power Allocation in Wireless Communication Networks

by

Xiaowen Gong

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of **Master of Science**.

Department of Electrical and Computer Engineering

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University of Alberta

Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Joint Bandwidth and Power Allocation in Wireless Communication Networks** submitted by Xiaowen Gong in partial fulfillment of the requirements for the degree of **Master of Science**.

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Chintha Tellambura and Sergiy A. Vorobyov (Co-supervisors)

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Abstract

Wireless communication networks have recently attracted significant research attention. As a critical issue for improving network performance, efficient and intelligent resource allocation strategies have been intensively studied.

This thesis consists of two studies on resource allocation strategy, specifically, joint bandwidth and power allocation strategy, for wireless communication networks where both bandwidth and power are constrained resources. In the first study, joint bandwidth and power allocation strategy is proposed for wireless multi-user networks without relaying and with decode-and-forward relaying based on three system-wide objectives. It is shown that the formulated resource allocation problems are convex and, thus, the optimal solutions can be obtained efficiently using convex optimization techniques. Admission control based on the joint bandwidth and power allocation strategy is further considered. A greedy search algorithm is developed for solving the admission control problem efficiently, and the optimality conditions of the greedy search algorithm are derived and shown to be mild. In the second study, joint bandwidth and power allocation strategy is presented for maximizing the sum ergodic capacity of secondary users under fading channels in cognitive radio networks. Optimal bandwidth allocation is derived in closed-form for any given power allocation. The structure of optimal power allocation under each combination of four types of power constraints is derived. Using these structures, efficient algorithms are developed for finding the optimal power allocations. In summary, this thesis has proposed, analyzed and solved joint bandwidth and power allocation problems in wireless communication networks.

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My deepest gratitude goes to my parents for their endless love and support throughout my life. I owe them more than anyone else and they are the most important part of my life.

Contents

List of Figures

Chapter 1

Introduction

Recently, wireless communication networks have attracted a lot of research efforts. The main interest concerns developing efficient and intelligent resource allocation strategies. This thesis focuses on the resource allocation in wireless communication networks.

1.1 Motivation

Wireless communication networks serve as essential means to carry out data communications among multiple users. One of the critical issues in wireless communication networks is the efficient allocation of available radio resources in order to improve network performance. Future wireless networks, such as cellular and ad hoc networks, are expected to provide users with reliable data transmissions at high rates. Thus, it is a challenging task to achieve a system-wide goal for the network, while users' individual Quality of Service (QoS) requirements also need to be satisfied. Intelligent resource allocation schemes should capture the tradeoff between user-centric constraints and a particular network-centric objective. Moreover, there exists a conflict between the increasing demand for wireless services and the availability of radio resources, including both bandwidth and power resources. The overly crowded spectrum allocation charts given by Federal Communications Commission (FCC) indicates that the radio spectrum available for emerging wireless applications is scarce. Transmission power is also a constrained resource at wireless devices due to their finite battery energy and hardware constraints. Therefore, there is a strong motivation for the research on effective resource management and distribution that can make the best use of limited radio resources. Furthermore, efficient resource utilization is desirable for exploiting the dynamics and diversity nature of wireless multi-user networks. Since conventional fixed resource allocation schemes are designed regardless of the time-varying characteristic of wireless channel conditions, they certainly can not achieve high efficiency. On the contrary, a dynamic resource allocation scheme can take full advantage of the channel diversity among users by distributing resources adaptively according to their available channel state information (CSI) and, thus, enhance the performance substantially.

Numerous works have been conducted on the resource allocation of wireless communication networks. Power allocation strategies have been a research focus for both energy efficiency and interference management. Power control techniques for interference-limited networks, such as cellular networks, have been studied intensively in the literature (see, for example, [1]- [6]), and aimed at achieving optimal network performance while guaranteeing a target signal-to-interference-plus-noise ratio (SINR) for each user. On the other hand, joint bandwidth and power allocation strategy has received much less attention [10]- [12]. In fact, the joint allocation of bandwidth and power is especially critical in practical wireless networks, where both the available transmission power of individual nodes and the total available bandwidth for all users are limited. Due to the limited resources in wireless networks, there are situations where not all users can be satisfied with their QoS requirements and, therefore, admission control should be carried out to determine which users can be admitted into the network. Power allocation with admission control have been investigated for interference-limited networks [1], [3], [13], [14], where a removal approach has been proposed for removing users until the remaining users in the network are feasible.

1.2 Contribution

Motivated by the need to improve the efficiency of the conventional disjoint allocation strategies for bandwidth and power resources, this thesis aims at studying the fundamental performance limits of joint bandwidth and power allocation strategy for wireless communication networks where both bandwidth and power are constrained resources. In particular, the joint bandwidth and power allocation strategy is studied for two setups.

In the first setup, joint bandwidth and power allocation strategy is proposed for wireless multi-user networks without relaying and with decode-and-forward relaying based on (i) sum capacity maximization; (ii) worst user capacity maximization; and (iii) total network power minimization. The formulated resource allocation problems are shown to be convex and, thus, can be solved efficiently using convex optimization techniques. Due to limited resources, the network may not be able to support all users with their QoS requirements. Therefore, admission control based on the joint bandwidth and power allocation strategy is further considered, which aims at maximizing the number of users that can be admitted into the network. Since finding the optimal solution to the admission control problem has high complexity, a suboptimal greedy search algorithm is developed for solving it efficiently. The optimal conditions of the greedy search algorithm are derived and shown to be mild.

In the second setup, joint bandwidth and power allocation strategy is presented for maximizing the sum ergodic capacity of secondary users (SUs) under fading channels in cognitive radio networks. Optimal bandwidth allocation is derived in closed-form for any given power allocation. The structures of the optimal power allocation under all possible combinations of four types of power constraints are derived, which indicate the possible numbers of users that transmit at nonzero power but below their corresponding peak power, and show that other users do not transmit or transmit at their corresponding peak power. Efficient algorithms are developed based on these structures for finding the optimal power allocations. The solutions and algorithms obtained in both works achieve significant performance improvements compared to conventional methods, which is verified by numerical results provided in simulations.

1.3 Mathematical Background

In this section, we briefly introduce convex optimization preliminaries, which serve as the main mathematical tool for studying the resource allocation problems in this thesis.

The modeling, design, and optimization of wireless communication networks rely on optimization theory, which has found a wide range of applications in wireless communications and networking. Convexity and non-convexity is the great watershed in optimization theory. It is recognized that non-convex optimization problems are computationally difficult to solve and, thus, have received limited attention [5]- [7]. Convex optimization, however, is much more appealing [15]- [18], since many problems can be identified or formulated as convex optimization problems and their optimal solutions can be computed reliably and efficiently through established techniques such as interior point methods [8], even if they involve nonlinear objectives and constraints. Apart from computational efficiency, convex optimization also offers theoretical advantages by giving insightful interpretations for optimal solutions such as, for example, Lagrange duality. The availability of software packages for solving convex optimization problems, such as [9], further enhances the popularity of convex optimization.

1.3.1 Convex functions

Consider a function $f : \mathcal{R}^n \rightarrow \mathcal{R}$ defined on a convex set $\mathcal{D}$. We say f is a convex function if for any two $\mathbf{x}, \mathbf{y} \in \mathcal{D}$, the following inequality holds for any $\alpha \in [0, 1]$

$$f(\alpha\mathbf{x} + (1-\alpha)\mathbf{y}) \leq \alpha f(\mathbf{x}) + (1-\alpha)f(\mathbf{y}). \tag{1.1}$$

A geometric interpretation of (1.1) is that the plot of f along the linear interval from $\mathbf{x}$ to $\mathbf{y}$ is below the line segment connecting $(\mathbf{x}, f(\mathbf{x}))$ and $(\mathbf{y}, f(\mathbf{y}))$. We say f is concave if $-f$ is convex.

Suppose f is first-order differentiable. The first-order convexity condition states that f is convex if and only if the following inequality holds for any two $\mathbf{x}, \mathbf{y} \in \mathcal{D}$

$$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}). \tag{1.2}$$

Suppose f is second-order differentiable, i.e., the second-order derivative exists. The second-order convexity condition states that f is convex if and only if the second-order derivative is positive semidefinite, i.e.,

$$\nabla^2 f(\mathbf{x}) \succeq 0. \tag{1.3}$$

Some basic convex functions include linear functions, exponential and logarithmic functions, and power functions. There are some operations that preserve the convexity of convex functions, including addition, nonnegative scaling, and pointwise maximum.

1.3.2 Convex optimization problems

Mathematically, an optimization problem can be written in the following standard form

$$\min_{\mathbf{x}\in\mathcal{D}} \quad f_0(\mathbf{x}) \tag{1.4a}$$

$$\text{subject to (s.t.)} \quad f_i(\mathbf{x}) \leq 0, \quad i = 1, \cdots, m \tag{1.4b}$$

$$h_i(\mathbf{x}) = 0, \quad i = 1, \cdots, l \tag{1.4c}$$

where f_0 denotes the objective function, f_i denotes the i-th inequality constraint function, h_i denotes the i-th equality constraint function, and $\mathcal{D}$ denotes the domain of the optimization problem.

A point $\mathbf{x} \in \mathcal{D}$ is feasible if it satisfies the constraints (1.4b) and (1.4c). The problem (1.4a)-(1.4c) is feasible if there exists at least one feasible point, and is infeasible if there does not exist any feasible point. A feasible point is optimal, denoted by $\mathbf{x}^*$, if $f_0(\mathbf{x}) \geq f_0(\mathbf{x}^*)$ holds for any feasible point $\mathbf{x}$. The optimal value of the problem (1.4a)-(1.4c), denoted by v^*, is defined as the value of the objective function at the optimal point, i.e., $v^* = f_0(\mathbf{x}^*)$.

The problem (1.4a)-(1.4c) is a convex optimization problem if the objective function and inequality constraint functions are convex, and the equality constraint functions are linear.

1.3.3 Lagrange duality and Karush-Kuhn-Tucker conditions

The Langrangian $L : \mathcal{R}^n \times \mathcal{R}^m \times \mathcal{R}^l \to \mathcal{R}$ associated with the problem (1.4a)-(1.4c) is defined as

$$L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{l} \mu_i h_i(\mathbf{x}) \tag{1.5}$$

where $\boldsymbol{\lambda} \triangleq [\lambda_1 \cdots \lambda_m]$, $\boldsymbol{\mu} \triangleq [\mu_1 \cdots \mu_l]$, λ_i is the Lagrange multiplier associated with the ith inequality constraint, and μ_i is the Lagrange multiplier associated with the ith equality constraint. The Lagrange dual function is defined as

$$g(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \inf_{\mathbf{x}\in\mathcal{D}} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \inf_{\mathbf{x}\in\mathcal{D}} \left(f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{l} \mu_i h_i(\mathbf{x}) \right). \tag{1.6}$$

It can be seen that the optimal value v^* of the problem (1.4a)-(1.4c) is lower bounded by the dual function, i.e., $g(\boldsymbol{\lambda}, \boldsymbol{\mu}) \leq v^*$, for any $\boldsymbol{\lambda} \succeq 0$ and any $\boldsymbol{\mu}$. The tightest lower bound for v^* can be obtained by solving the dual problem defined as follows

$$\max_{\boldsymbol{\lambda},\boldsymbol{\mu}} \quad g(\boldsymbol{\lambda},\boldsymbol{\mu}) \tag{1.7a}$$

$$\text{s.t.} \quad \boldsymbol{\lambda} \succeq 0. \tag{1.7b}$$

Note that the dual problem (1.7a)-(1.7b) is always convex regardless of the convexity of the original problem (1.4a)-(1.4c). The original problem (1.4a)-(1.4c) is also called the primal problem in this context.

Let d^* denote the optimal value of the dual problem (1.7a)-(1.7b). Then $v^* - d^*$ is defined as the optimal duality gap between the primal problem (1.4a)-(1.4c) and the dual problem (1.7a)-(1.7b). If the primal problem (1.4a)-(1.4c) is convex, the optimal duality gap is zero, i.e., $v^* - d^* = 0$, and we say that strong duality holds. Using the property of strong duality, if the primal problem (1.4a)-(1.4c) is convex, it can be solved equivalently by solving the dual problem (1.7a)-(1.7b).

Suppose the objective function f_0 and the inequality functions f_i, $i = 1, \cdots, m$ are differentiable. The optimal solution of the primal problem (1.4a)-(1.4c) and the dual problem (1.7a)-(1.7b), denoted by $\mathbf{x}^*$ and $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$, respectively, satisfy the following Karush-Kuhn-Tucker (KKT) conditions

$$f_i(\mathbf{x}^*) \leq 0, \quad i = 1, \cdots, m \tag{1.8a}$$

$$h_i(\mathbf{x}^*) = 0, \quad i = 1, \cdots, l \tag{1.8b}$$

$$\lambda_i^* \geq 0, \quad i = 1, \cdots, m \tag{1.8c}$$

$$\lambda_i^* f_i(\mathbf{x}^*) = 0, \; i = 1, \cdots, m \tag{1.8d}$$

$$\nabla f_0(\mathbf{x}^*) + \sum_{i=1}^{m} \lambda_i^* \nabla f_i(\mathbf{x}^*) + \sum_{i=1}^{l} \mu_i^* \nabla h_i(\mathbf{x}^*) = 0. \tag{1.8e}$$

In general, the KKT conditions are only necessary conditions for the optimal solutions $\mathbf{x}^*$ and $(\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*)$. However, if the primal problem (1.4a)-(1.4c) is convex, the KKT conditions are both necessary and sufficient conditions for the optimal solutions and, therefore, solving for the KKT conditions is equivalent to solving the primal problem (1.4a)-(1.4c).

1.3.4 Solving convex optimization problems

Although intensive study has been done on analyzing the properties of various classes of optimization problems and developing algorithms for solving them, optimization problems are generally computationally difficult to solve, even if the objective and constraint functions are smooth. The efficiency of computing the optimal solutions of general optimization problems depends on different factors, including the particular forms and structures of the objective and constraint functions, and the numbers of the variables and constraints. However, there exist few classes of optimization problems that can be reliably and efficiently solved by effective algorithms, even if the problems involve a large number of variables and constraints. Convex optimization problems can serve as an example of such problems.

Analytical solutions of convex optimization problems can be obtained, if possible, using Lagrange duality or the KKT conditions. However, general analytical formulas for the optimal solutions are not available and, therefore, effective methods like interior-point methods should be used. Interior-point methods can solve a convex optimization problem in an almost constant number of iterations regardless of the structure of the problem. In practice, a convex optimization problem with hundreds or even thousands of variables and constraints can be solved efficiently on a desktop computer in a few tens of seconds. Therefore, once we can recognize and formulate a research problem as a convex optimization problem, we can claim that we have found a method to solve this research problem.

1.4 Thesis Outline

This thesis studies resource allocation, specifically, bandwidth and power allocation in wireless communication networks. The outline of each chapter is given below.

Chapter 1 provides the motivation, contribution, and outline of the thesis, and introduces basic convex optimization theory.

Chapter 2 presents joint bandwidth and power allocation strategy for wireless multiuser networks without relaying and with decode-and-forward relaying by taking into account three network performance measures, i.e., the sum capacity, the worst capacity, and the total network power consumption. The admission control problem based on the joint bandwidth and power allocation strategy is further considered. A greedy search algorithm is developed to solve the admission control problem efficiently. The complexity and optimality conditions of the greedy search algorithm are investigated.

Chapter 3 proposes joint bandwidth and power allocation strategy for the sum ergodic capacity maximization of SUs under fading channels in cognitive radio networks. Optimal bandwidth allocation is derived first in terms of any given power allocation. Then optimal power allocation is obtained subject to each combination of four types of power constraints.

Chapter 4 summarizes the results of the thesis and proposes future work directions.

Chapter 2

Joint Bandwidth and Power Allocation with Admission Control in Wireless Multi-User Networks With and Without Relaying

Equal allocation of bandwidth and/or power may not be efficient for wireless multi-user networks with limited bandwidth and power resources. Joint bandwidth and power allocation strategies for wireless multi-user networks with and without relaying are proposed in this chapter for (i) the maximization of the sum capacity of all users; (ii) the maximization of the worst user capacity; and (iii) the minimization of the total power consumption of all users subject to rate requirements. It is shown that the proposed allocation problems are convex and, therefore, can be solved efficiently. Moreover, the admission control based joint bandwidth and power allocation is considered. A suboptimal greedy search algorithm is developed to solve the admission control problem efficiently. The conditions under which the greedy search is optimal are derived and shown to be mild. The performance improvements offered by the proposed joint bandwidth and power allocation are demonstrated by simulations. The advantages of the suboptimal greedy search algorithm for admission control are also shown.

The rest of this chapter is organized as follows. Section 2.1 gives the overview of the related literature and summarizes the contributions. System models of multi-user networks without relaying and with decode-and-forward relaying are given in Section 2.2. In Section 2.3, joint bandwidth and power allocation problems for the three aforementioned objectives are formulated and solved for both types of networks with and without relaying. Admission control problem based on joint bandwidth and power allocation is formulated in Section 2.4, where the greedy search algorithm is also developed and investigated for both types of systems with and without relaying. Numerical results are reported in Section 2.5, followed by concluding remarks in Section 2.6.

2.1 Introduction

It has been shown that the efficiency of wireless communications can be improved by using relays [19]- [20]. In a relay-assisted communication system, the data transmitted from a source is forwarded via relaying to the corresponding destination. Since relay-assisted communication has significant advantages such as extended coverage and enhanced communication quality, relay networks are considered promising candidates for future wireless networks. One critical issue in relay networks is the efficient allocation of available radio resources to enhance the performance of relaying. Therefore, numerous works have been done on the resource allocation for relay networks (see, for example, [21]- [33]). Note that [22]- [30] as well as most of the existing works consider a single user, i.e., a single source-destination pair, while only a few works have studied resource allocation for multi-user relay networks. Power allocation aiming at optimizing the sum capacity of multiple users for four different relay transmission strategies has been studied in [31], while an AF based strategy in which multiple sources share multiple relays using power control has been developed in [32], [33].

In practical wireless networks where both the available transmission power of individual nodes and the total available bandwidth of the network are limited, joint bandwidth and power allocation should be considered [10]– [12]. It is worth noting that most of the works mentioned above on the resource allocation for relay networks have assumed equal and fixed bandwidth allocation for the one-hop links from a source to a destination. In fact, it is inefficient to allocate the bandwidth equally when the total available bandwidth is limited. Therefore, joint bandwidth and power allocation is important for both networks with and without relaying.

Various performance metrics for resource allocation in multi-user networks have been considered. System throughput maximization and the worst user throughput maximization are studied using convex optimization in [15]. Sum capacity maximization is taken as an objective for power allocation in [31], while max-min SNR, power minimization, and throughput maximization are used as power allocation criteria in [32].

In some applications, certain minimum transmission rates must be guaranteed for the users in order to satisfy their quality-of-service (QoS) requirements. For instance, in real-time voice and video applications, a minimum rate should be guaranteed for each user to satisfy the delay constraints of the services. However, when the rate requirements can not be supported for all users, admission control is adopted to decide which users to be admitted into the network. The admission control in wireless networks typically aims at maximizing the number of admitted users and has been recently considered in several works. A single-stage reformulation approach for a two-stage joint resource allocation and admission control problem is proposed in [34], [35], while another approach is based on user removals [1], [3], [13], [14], [36]. To the best of our knowledge, admission control based on joint bandwidth and power allocation has never been considered.

In this chapter[1], the problem of joint bandwidth and power allocation for wireless multi-user networks with and without relaying is considered, which is especially efficient for the networks with both limited bandwidth and limited power. The joint bandwidth and power allocation are proposed to (i) maximize the sum capacity of all users; (ii) maximize the capacity of the worst user; (iii) minimize the total power consumption of all users. The corresponding joint bandwidth and power allocation problems can be formulated as optimization problems that are shown to be convex. Therefore, these problems can be solved efficiently by using convex optimization techniques. The joint bandwidth and power allocation together with admission control is further considered, and a greedy search algorithm is developed in order to reduce the computational complexity of solving the admission control problem. The optimality conditions of the greedy search are derived and shown to be mild.

[1]This work has been presented in [37], [38] and [39].

2.2 System Model

Without Relaying

Consider a wireless network, which consists of M source nodes S_i, $i \in \mathcal{M} = \{1, 2, \cdots, M\}$, and K destination nodes D_i, $i \in \mathcal{K} = \{1, 2, \cdots, K\}$, as shown in Fig. 2.1. The network serves N users U_i, $i \in \mathcal{N} = \{1, 2, \cdots, N\}$, where each user represents a one-hop link from a source to a destination. The set of users which are served by S_i is denoted by $\mathcal{N}_{S_i}$.

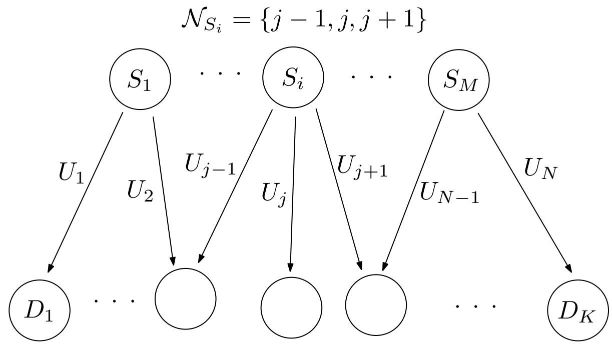


Fig. 2.1. Wireless multi-user network without relaying.

A spectrum of total bandwidth W is available for the transmission from the sources. This spectrum can be divided into distinct and nonoverlapping channels of unequal bandwidths, so that the sources share the available spectrum through frequency division and, therefore, do not interfere with each other.

Let P_i^{S} and W_i^{S} denote the allocated transmit power and channel bandwidth of the source to serve U_i. Then the received SNR at the destination of U_i is

$$\gamma_i^{\mathrm{D}} = \frac{P_i^{\mathrm{S}} h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}} N_0} \tag{2.1}$$

where h_i^{SD} denotes the channel gain of the source–destination link of U_i and $W_i^{\mathrm{S}} N_0$ stands for the power of additive white Gaussian noise (AWGN) over the bandwidth W_i^{S}. The channel gain h_i^{SD} results from such effects as path loss, shadowing, and fading. Due to the fact that the power spectral density (PSD) of AWGN is constant over all frequencies with the constant value denoted by N_0, the noise power in the channel is linearly increasing with the channel bandwidth. It can be seen from (2.1) that a channel with larger bandwidth introduces higher noise power and, thus, reduces the SNR.

Channel capacity gives an upper bound on the achievable rate of a link. Given γ_i^{D}, the source–destination link capacity of U_i is

$$C_i^{\mathrm{SD}} = W_i^{\mathrm{S}} \log(1+\gamma_i^{\mathrm{D}}) = W_i^{\mathrm{S}} \log\left(1+\frac{P_i^{\mathrm{S}} h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}} N_0}\right). \tag{2.2}$$

It can be seen that W_i^{S} characterizes channel bandwidth, and $\log(1+\gamma_i^{\mathrm{R}})$ characterizes spectral efficiency and, thus, C_i^{SD} characterizes data rate over the source–destination link in bits per second. Moreover, for fixed W_i^{S}, C_i^{SD} is a concave increasing function of P_i^{S}. It can be also shown that C_i^{SD} is a concave increasing function of W_i^{S} for fixed P_i^{S}, although γ_i^{D} is a linear decreasing function of W_i^{S}. Indeed, it can be proved that C_i^{SD} is a concave function of P_i^{S} and W_i^{S} jointly [11], [12].

With Relaying

Consider L relay nodes R_i, $i \in \mathcal{L} = \{1, 2, \cdots, L\}$ added to the network described in the previous subsection and used to forward the data from the sources to the destinations, as shown in Fig. 2.2. Then each user represents a two-hop link from a source to a destination via relaying. To reduce the implementation complexity at the destinations, single relay assignment is adopted so that each user has one designated relay. Then the set of users served by R_i is denoted by $\mathcal{N}_{R_i}$. The relays work in a half-duplex manner due to the practical limitation that they can not transmit and receive at the same time. A two-phase decode-and-forward (DF) protocol is assumed, i.e., the relays receive and decode the transmitted data from the sources in the first phase, and re-encode and forward the data to the destinations in the second phase. The sources and relays share the total available spectrum in the first and second phases, respectively. It is assumed that the direct links between the sources and the destinations are blocked and, thus, are not available. Note that although the two-hop relay model is considered in the paper, the results are applicable for multi-hop relay models as well.

Let P_i^{R} and W_i^{R} denote the allocated transmit power and channel bandwidth of the relay to serve U_i. The two-hop source–destination link capacity of U_i is given by

$$C_i^{\mathrm{SD}} = \min\{C_i^{\mathrm{SR}}, C_i^{\mathrm{RD}}\} = \left\{W_i^{\mathrm{S}} \log\left(1+\frac{P_i^{\mathrm{S}} h_i^{\mathrm{SR}}}{W_i^{\mathrm{S}} N_0}\right), W_i^{\mathrm{R}} \log\left(1+\frac{P_i^{\mathrm{R}} h_i^{\mathrm{RD}}}{W_i^{\mathrm{R}} N_0}\right)\right\} \tag{2.3}$$

where C_i^{SR} and C_i^{RD} are the one-hop source–relay and relay–destination link capacities of U_i, respectively, and h_i^{SR} and h_i^{RD} denote the corresponding channel gains.

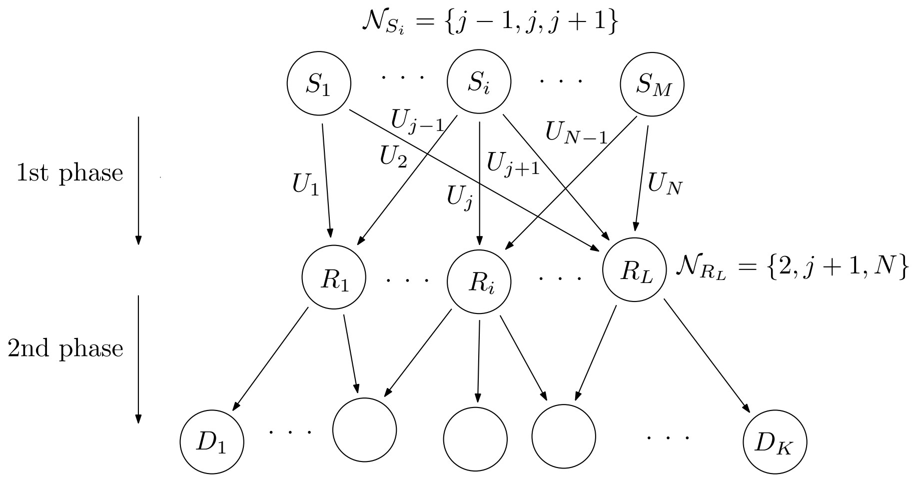


Fig. 2.2. Wireless multi-user network with relaying.

It can be seen from (2.3) that if equal bandwidth is allocated to W_i^{S} and W_i^{R}, C_i^{SR} and C_i^{RD} can be unequal due to the power limits on P_i^{S} and P_i^{R}. Then the source–destination link capacity C_i^{SD} is constrained by the minimum of C_i^{SR} and C_i^{RD}. Note that since all users share the total bandwidth of the spectrum, equal bandwidth allocation for all one-hop links can be inefficient. Therefore, the joint allocation of bandwidth and power is necessary.

2.3 Joint Bandwidth and Power Allocation

Different objectives can be considered while jointly allocating bandwidth and power in wireless multi-user networks. The widely used objectives for network optimization are (i) the sum capacity maximization; (ii) the worst user capacity maximization; and (iii) the total network power minimization. In this section, the problems of joint bandwidth and power allocation are formulated for the aforementioned objectives for both considered systems with and without relaying. It is shown that all these problems are convex and, therefore, can be efficiently solved using standard convex optimization methods.

2.3.1 Sum capacity maximization

In the applications without delay constraints, a high data rate from any user in the network is preferable. Thus, it is desirable to allocate the resources to maximize the overall network performance, e.g., the sum capacity of all users.

Without Relaying

In this case, the joint bandwidth and power allocation problem aiming at maximizing the sum capacity of all users can be mathematically formulated as

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}} \sum_{i\in\mathcal{N}} C_i^{\mathrm{SD}} \tag{2.4a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}_{S_j}} P_i^{\mathrm{S}} \le P_{S_j},\ j\in\mathcal{M} \tag{2.4b}$$

$$\sum_{i\in\mathcal{N}} W_i^{\mathrm{S}} \le W. \tag{2.4c}$$

The nonnegativity constraints on the optimization variables $\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}$ are natural and, thus, omitted throughout the paper for brevity. In the problem (2.4a)–(2.4c), the constraint (2.4b) stands that the total power at S_j is limited by P_{S_j}, while the constraint (2.4c) indicates that the total bandwidth of the channels allocated to the sources is also limited by W.

Note that since C_i^{SD} is a jointly concave function of P_i^{S} and W_i^{S}, the objective function (2.4a) is convex. The constraints (2.4b) and (2.4c) are linear and, thus, convex. Therefore, the problem (2.4a)–(2.4c) itself is convex. Using the convexity, the closed-form optimal solution of the problem (2.4a)–(2.4c) can be found as it is shown below. It is worth noting that the optimal solution demonstrates that for a set of users served by one source, the sum capacity maximization based allocation strategy allocates all the power of each source only to one user, that is, the user with the highest channel gain. Therefore, it results in highly unbalanced resource allocation among the users. The following proposition describes the result formally.

Proposition 2.1: *The optimal solution of the problem* (2.4a)–(2.4c)*, denoted by* $\{P_i^{\mathrm{S}*}, W_i^{\mathrm{S}*} | i\in\mathcal{N}\}$*, is* $P_i^{\mathrm{S}*} = P_i^{\mathrm{S}\star}$, $W_i^{\mathrm{S}*} = W h_i^{\mathrm{SD}} P_i^{\mathrm{S}\star} / \sum_{j\in\mathcal{I}} h_j^{\mathrm{SD}} P_j^{\mathrm{S}\star}$, $\forall i \in \mathcal{I}$*, and* $P_i^{\mathrm{S}*} = W_i^{\mathrm{S}*} = 0$, $\forall i \notin \mathcal{I}$*, where* $P_i^{\mathrm{S}\star}$ *is the total power of the source serving* U_i*, i.e.,* $P_i^{\mathrm{S}\star} = P_{S_k}$ *for* $i \in \mathcal{N}_{S_k}$*, and* $\mathcal{I} = \{i \,|\, i = \arg\max_{j\in\mathcal{N}_{S_k}} h_j^{\mathrm{SD}}, k\in\mathcal{M}\}$.

Proof: We first give the following lemma.

Lemma 2.1: *The optimal solution of the problem*

$$\max_{\{p_i, w_i\}} \quad \sum_{i \in \mathcal{N}} w_i \log\left(1 + \frac{h_i p_i}{w_i}\right) \tag{2.5a}$$

$$\text{s.t.} \sum_{i \in \mathcal{N}} p_i \leq p \tag{2.5b}$$

$$\sum_{i \in \mathcal{N}} w_i \leq w \tag{2.5c}$$

which is denoted by $\{p_i^* | i \in \mathcal{N}\}$, *is* $p_k^* = p$, $w_k^* = w$, *and* $p_i^* = w_i^* = 0$, $\forall i \neq k$, *where* $k = \arg\max_{i \in \mathcal{N}} h_i$.

Proof of Lemma 2.1: Consider if $\mathcal{N} = \{1, 2\}$. Then the problem (2.5a)–(2.5c) is equivalent to

$$\max_{p_1 \leq p,\, w_1 \leq w} \quad g(w, p) = w \log\left(1 + \frac{h_1 p_1}{w_1}\right) + (w - w_1) \log\left(1 + \frac{h_2(p - p_1)}{w - w_1}\right). \tag{2.6}$$

Assume without loss of generality that $h_1 > h_2$. Consider if the constraints $0 \leq p_1 \leq p$ and $0 \leq w_1 \leq w$ are inactive at optimality. Since the problem (2.6) is convex, using the Karush-Kuhn-Tucker (KKT) conditions, we have

$$\log\left(1 + \frac{h_1 p_1^*}{w_1^*}\right) - \frac{h_1 p_1^*}{w_1^* + h_1 p_1^*} - \log\left(1 + \frac{h_2(p - p_1^*)}{w - w_1^*}\right) + \frac{h_2(p - p_1^*)}{w - w_1^* + h_2(p - p_1^*)}$$
$$= y\left(\frac{h_1 p_1^*}{w_1^*}\right) - y\left(\frac{h_2(p - p_1^*)}{w - w_1^*}\right) = 0 \tag{2.7a}$$

$$\frac{h_1 w_1^*}{w_1^* + h_1 p_1^*} - \frac{h_2(w - w_1^*)}{w - w_1^* + h_2(p - p_1^*)} = 0. \tag{2.7b}$$

where $y(x) \triangleq \log(1 + x) - x/(1 + x)$. Since $y(x)$ is monotonically increasing, it can be seen from (2.7a) that

$$\frac{h_1 p_1^*}{w_1^*} = \frac{h_2(p - p_1^*)}{w - w_1^*}. \tag{2.8}$$

Combining (2.7b) and (2.8), we obtain $h_1 = h_2$, which contradicts the condition $h_1 > h_2$. Therefore, at least one of the constraints $0 \leq p_1 \leq p$ and $0 \leq w_1 \leq w$ is active at optimality. Then it can be shown that $p_1^* = p$ and $w_1^* = w$. Note that this is also the optimal solution if $h_1 = h_2$ is assumed. Furthermore, this conclusion can be directly extended to the case of $N > 2$ by induction. This completes the proof. □

Now we are ready to show Proposition 2.1. It can be seen from Lemma 2.1 that $P_i^{\mathrm{S}*} = P_i^{\mathrm{S}\star}$, $\forall i \in \mathcal{I}$, and $P_i^{\mathrm{S}*} = 0$, $\forall i \notin \mathcal{I}$. Then the problem (2.4a)–(2.4c) is equivalent to

$$\max_{\{W_i^{\mathrm{S}}\}} \sum_{i\in\mathcal{I}} W_i^{\mathrm{S}} \log\left(1 + \frac{P_i^{\mathrm{S}\star} h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}} N_0}\right) \tag{2.9a}$$

$$\text{s.t.} \sum_{i\in\mathcal{I}} W_i^{\mathrm{S}} \leq W. \tag{2.9b}$$

Since the problem (2.9a)–(2.9b) is convex , using the KKT conditions, we have

$$\log\left(1 + \frac{P_i^{\mathrm{S}\star} h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}*} N_0}\right) - \frac{P_i^{\mathrm{S}\star} h_i^{\mathrm{SD}}}{W_i^{\mathrm{S}*} N_0 + P_i^{\mathrm{S}\star} h_i^{\mathrm{SD}}} - \lambda^* = y\left(\frac{P_i^{\mathrm{S}\star} h_i^{\mathrm{SR}}}{W_i^{\mathrm{S}*} N_0}\right) - \lambda^* = 0,\ i \in \mathcal{I} \tag{2.10a}$$

$$W - \sum_{i\in\mathcal{I}} W_i^{\mathrm{S}*} = 0 \tag{2.10b}$$

where λ^* denotes the optimal Lagrange multiplier, and $y(x) \triangleq \log(1+x) - x/(1+x)$. Since $y(x)$ is monotonically increasing, it follows from (2.10a) that

$$\frac{P_i^{\mathrm{S}\star} h_i^{\mathrm{SR}}}{W_i^{\mathrm{S}*} N_0} = \frac{P_j^{\mathrm{S}\star} h_j^{\mathrm{SR}}}{W_j^{\mathrm{S}*} N_0}, \quad \forall i, j \in \mathcal{I}_1,\ i \neq j. \tag{2.11}$$

Solving the system of equations (2.10b) and (2.11), we obtain $W_i^{\mathrm{S}*} = W h_i^{\mathrm{SD}} P_i^{\mathrm{S}\star} / \sum_{j\in\mathcal{I}} h_j^{\mathrm{SD}} P_j^{\mathrm{S}\star}$, $i \in \mathcal{I}$. This completes the proof. □

With Relaying

The sum capacity maximization based joint bandwidth and power allocation problem for the network with DF relaying is given by

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}} \sum_{i\in\mathcal{N}} C_i^{\mathrm{SD}} \tag{2.12a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}_{S_j}} P_i^{\mathrm{S}} \leq P_{S_j},\ j \in \mathcal{M} \tag{2.12b}$$

$$\sum_{i\in\mathcal{N}_{R_j}} P_i^{\mathrm{R}} \leq P_{R_j},\ j \in \mathcal{L} \tag{2.12c}$$

$$\sum_{i\in\mathcal{N}} W_i^{\mathrm{S}} \leq W \tag{2.12d}$$

$$\sum_{i\in\mathcal{N}} W_i^{\mathrm{R}} \leq W. \tag{2.12e}$$

Introducing new variables $\{T_i | i \in \mathcal{N}\}$, the problem (2.12a)–(2.12e) can be equivalently rewritten as

$$\min_{\{P_i^{\rm S}, W_i^{\rm S}, P_i^{\rm R}, W_i^{\rm R}, T_i\}} \quad -\sum_{i\in\mathcal{N}} T_i \tag{2.13a}$$
$$\text{s.t. } T_i - C_i^{\rm SR} \leq 0, \ i \in \mathcal{N} \tag{2.13b}$$
$$T_i - C_i^{\rm RD} \leq 0, \ i \in \mathcal{N} \tag{2.13c}$$
$$\text{the constraints (2.12b)–(2.12e).} \tag{2.13d}$$

Note that the constraints (2.13b) and (2.13c) are convex since $C_i^{\rm SR}$ and $C_i^{\rm RD}$ are jointly concave functions of $P_i^{\rm S}$, $W_i^{\rm S}$ and $P_i^{\rm R}$, $W_i^{\rm R}$, respectively. The constraints (2.13d) are linear and, thus, convex. Therefore, the problem (2.13a)–(2.13d) itself is convex. It can be seen that the closed-form optimal solution of the problem (2.13a)–(2.13d) can not be obtained due to the coupling of the constraints (2.13b) and (2.13c). However, the convexity of the problem (2.13a)–(2.13d) allows to use standard numerical convex optimization algorithms for solving the problem efficiently [8].

Intuitively, the sum capacity maximization based allocation for the network with DF relaying should not result in as unbalanced resource allocation as that for the network without relaying. It is because the channel gains in both transmission phases for the networks with relaying affect the achievable capacity of each user. Below we give the conditions under which the sum capacity maximization based resource allocation strategy for the network with relaying does not allocate any resources to some users. In particular, if two users are served by the same source and the same relay, and one user has lower channel gains than the other user in both transmission phases, then no resource is allocated to the former user. The result can be formally stated in terms of the following proposition.

Proposition 2.2: *If* $h_i^{\rm SR} \geq h_j^{\rm SR}$ *and* $h_i^{\rm RD} \geq h_j^{\rm RD}$ *where* $\{i,j\} \subseteq \mathcal{N}_{S_k}$ *and* $\{i,j\} \subseteq \mathcal{N}_{R_l}$, *then* $P_j^{\rm S*} = W_j^{\rm S*} = P_j^{\rm R*} = W_j^{\rm R*} = 0$.

Proof: It can be seen that

$$C_i^{\rm SD} + C_j^{\rm SD} = \min\{C_i^{\rm SR}, C_i^{\rm RD}\} + \min\{C_j^{\rm SR}, C_j^{\rm RD}\} \leq \min\{C_i^{\rm SR} + C_j^{\rm SR}, C_i^{\rm RD} + C_j^{\rm RD}\} \tag{2.14}$$

When $P_j^{\rm S} = W_j^{\rm S} = P_j^{\rm R} = W_j^{\rm R} = 0$, it follows from Lemma 2.1 that the maximum value of the right hand side of (2.14) is achieved and equals to $C_i^{\rm SD}$ and, on the other hand, the left hand side of (2.14) also equals to C_i^{SD}. Therefore, the maximum value of $C_i^{\mathrm{SD}} + C_j^{\mathrm{SD}}$ is achieved when $P_j^{\mathrm{S}} = W_j^{\mathrm{S}} = P_j^{\mathrm{R}} = W_j^{\mathrm{R}} = 0$. This completes the proof. □

2.3.2 Worst user capacity maximization

Fairness among users is also an important issue for resource allocation. If the fairness issue is considered, the achievable rate of the worst user is commonly used as the network performance measure. In this case, the joint bandwidth and power allocation problem for the network without relaying can be mathematically formulated as

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}} \quad \min_{i \in \mathcal{N}} C_i^{\mathrm{SD}} \tag{2.15a}$$

$$\text{s.t. the constraints (2.4b)–(2.4c) .} \tag{2.15b}$$

Similar, for the networks with relaying, the joint bandwidth and power allocation problem can be formulated as

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}} \quad \min_{i \in \mathcal{N}} C_i^{\mathrm{SD}} \tag{2.16a}$$

$$\text{s.t. the constraints (2.12b)–(2.12e) .} \tag{2.16b}$$

Introducing a variable T, the problem (2.16a)–(2.16b) can be equivalently written as

$$\min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}, T\}} \quad -T \tag{2.17a}$$

$$\text{s.t. } T - C_i^{\mathrm{SR}} \leq 0, \ i \in \mathcal{N} \tag{2.17b}$$

$$T - C_i^{\mathrm{RD}} \leq 0, \ i \in \mathcal{N} \tag{2.17c}$$

$$\text{the constraints (2.12b)–(2.12e).} \tag{2.17d}$$

Similar to the sum capacity maximization based allocation problems, it can be shown that the problems (2.15a)–(2.15b) and (2.17a)–(2.17d) are convex. Therefore, the optimal solutions can be efficiently obtained using standard convex optimization methods.

The next proposition indicates that the worst user capacity maximization based allocation leads to absolute fairness among users, just the opposite to the sum capacity maximization based allocation. The proof is intuitive from the fact that the total bandwidth is shared by all users, and is omitted for brevity.

Proposition 2.3: *In the problems* (2.15a)–(2.15b) *and* (2.16a)–(2.16b)*, the capacities of all users are equal at optimality.*

Proof: Consider the problem (2.15a)–(2.15b). Assume that the capacity of one user is larger than the minimum capacity among the capacities of other users at optimality. Then we can always take an arbitrary small amount of bandwidth allocated to this user and reallocate it to the user(s) with the minimum capacity such that the minimum capacity of all users is increased. This contradicts the optimality assumption. Thus, the capacities of all users are equal at optimality in the problem (2.15a)–(2.15b). Similarly, it can be shown that all users achieve the same capacity at optimality in the problem (2.16a)–(2.16b). This completes the proof. □

2.3.3 Total network power minimization

Another widely considered design objective is the minimization of the total power consumption of all users. This minimization is performed under the constraint that the rate requirements of all users are satisfied. The corresponding joint bandwidth and power allocation problem for the network without relaying can be written as

$$\min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}} \sum_{i \in \mathcal{N}} P_i^{\mathrm{S}} \tag{2.18a}$$

$$\text{s.t. } c_i - C_i^{\mathrm{SD}} \leq 0, \; i \in \mathcal{N} \tag{2.18b}$$

$$\text{the constraints (2.4b)–(2.4c)} \tag{2.18c}$$

where c_i is the minimum acceptable capacity for U_i, while the same problem for the network with relaying is

$$\min_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}} \sum_{i \in \mathcal{N}} (P_i^{\mathrm{S}} + P_i^{\mathrm{R}}) \tag{2.19a}$$

$$\text{s.t. } c_i - C_i^{\mathrm{SR}} \leq 0, \; i \in \mathcal{N} \tag{2.19b}$$

$$c_i - C_i^{\mathrm{RD}} \leq 0, \; i \in \mathcal{N} \tag{2.19c}$$

$$\text{the constraints (2.12b)–(2.12e)} \tag{2.19d}$$

where the constraints (2.19b) and (2.19c) indicate that the one-hop link capacities of U_i should be no less than the given capacity threshold. Similar to the sum capacity maximization and worst user capacity maximization based allocation problems, the problems

(2.18a)–(2.18c) and (2.19a)–(2.19d) are convex and, thus, can be solved efficiently as mentioned before.

2.4 Admission Control Based on Joint Bandwidth and Power Allocation

In the multi-user networks under consideration, admission control is required if a certain minimum capacity must be guaranteed for each user. Thus, we next consider admission control problem for both systems with and without relaying.

Without Relaying

The objective of admission control is to maximize the number of users whose capacity requirements can be satisfied subject to the bandwidth and power constraints of the network. The admission control problem based on joint bandwidth and power allocation in the network without relaying can be mathematically expressed as

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{2.20a}$$

$$\text{s.t. } c_i - C_i^{\mathrm{SD}} \leq 0, \ i \in \mathcal{I} \tag{2.20b}$$

$$\text{the constraints } (2.4b)\text{–}(2.4c) \tag{2.20c}$$

where $|\mathcal{I}|$ stands for the cardinality of $\mathcal{I}$.

Note that the problem (2.20a)–(2.20c) can be solved using exhaustive search among all possible subsets of users. However, the computational complexity of the exhaustive search can be very high since the number of possible subsets of users is exponentially increasing with the number of users, which is not acceptable for practical implementation. Therefore, we develop a suboptimal greedy search algorithm that significantly reduces the complexity of solving the admission control problem (2.20a)–(2.20c).

2.4.1 Greedy search algorithm

Given that all power constraints and capacity requirements are satisfied, the minimum total bandwidth required to support a set of users $\mathcal{I}$ can be defined as $G(\mathcal{I})$, where

$$G(\mathcal{I}) \triangleq \min_{\{P_i^{\rm S}, W_i^{\rm S}\}} \sum_{i\in\mathcal{I}} W_i^{\rm S} \tag{2.21a}$$

$$\text{s.t. } c_i - C_i^{\rm SD} \leq 0,\ i \in \mathcal{I} \tag{2.21b}$$

$$\text{the constraint (2.4b).} \tag{2.21c}$$

The following proposition provides a necessary and sufficient condition for the admissibility of a set of users.

Proposition 2.4: *A set of users $\mathcal{I}$ is admissible if and only if $G(\mathcal{I}) \leq W$.*

Proof: It is equivalent to show that there exists a feasible point $\{P_i^{\rm S}, W_i^{\rm S} | i \in \mathcal{I}\}$ of the problem (2.20a)–(2.20c) if and only if $G(\mathcal{I}) \leq W$. If $\{P_i^{\rm S}, W_i^{\rm S} | i \in \mathcal{I}\}$ is a feasible point of the problem (2.20a)–(2.20c), then since it is also a feasible point of the problem (2.21a)–(2.21c), we have $G(\mathcal{I}) \leq \sum_{i\in\mathcal{I}} W_i^{\rm S} \leq W$. If we have $G(\mathcal{I}) \leq W$, then the optimal solution of the problem (2.21a)–(2.21c) for $\mathcal{I}$, denoted by $\{P_i^{\rm S*}, W_i^{\rm S*} | i \in \mathcal{I}\}$, is a feasible point of the problem (2.20a)–(2.20c) since $\sum_{i\in\mathcal{I}} W_i^{\rm S*} = G(\mathcal{I}) \leq W$. This completes the proof. □

Proposition 2.4 is instrumental in establishing our greedy search algorithm, which removes users one by one until the remaining users are admissible. The 'worst' user, i.e., the user whose removal reduces the total bandwidth requirement to the maximum extent, is removed at each greedy search iteration. In other words, the removal of the 'worst' user results in the minimum total bandwidth requirement of the remaining users.[2] Thus, the removal criterion can be stated as

$$n(t) \triangleq \arg\max_{n\in\mathcal{N}(t-1)} \left(G(\mathcal{N}(t-1)) - G(\mathcal{N}(t-1) \setminus \{n\})\right) = \arg\min_{n\in\mathcal{N}(t-1)} G(\mathcal{N}(t-1) \setminus \{n\}) \tag{2.22}$$

where $n(t)$ denotes the user removed at the t-th greedy search iteration, $\mathcal{N}(t) \triangleq \mathcal{N}(t-1) \setminus \{n(t)\}$ denotes the set of remaining users after t greedy search iterations, and the symbol '$\setminus$' stands for the set difference operator.

Note that, intuitively, $\mathcal{N}(t)$ can be interpreted as the 'best' set of $N - t$ users that requires the minimum total bandwidth among all possible sets of $N - t$ users from $\mathcal{N}$, and

[2] Note that the approach based on user removals appears in different contexts also in [1], [3], [13], [14], [36].

$G(\mathcal{N}(t))$ is the corresponding minimum total bandwidth requirement. Thus, the stopping rule for the greedy search iterations should be finding such t^* that $G(\mathcal{N}(t^*-1)) > W$ and $G(\mathcal{N}(t^*)) \leq W$. In other words, $N - t^*$ can be interpreted as the maximum number of admissible users.

2.4.2 Complexity of the greedy search algorithm

It can be seen from Proposition 2.4 that using the exhaustive search for finding the maximum number of admissible users is equivalent to checking $G(\mathcal{I})$ for all possible $\mathcal{I} \subseteq \mathcal{N}$ and, therefore, the number of times of solving the problem (2.21a)–(2.21c) is upper bounded by $\sum_{i=d^*}^{N} \binom{N}{i}$, where d^* denotes the optimal value of the problem (2.20a)–(2.20c). On the other hand, it can be seen from (2.22) that using the greedy search, the number of times of solving the problem (2.21a)–(2.21c) is upper bounded by $\sum_{i=0}^{t^*-1} N - i$. Therefore, the complexity of the proposed greedy search is significantly reduced as compared to that of the exhaustive search, especially if N is large and d^* is small. Moreover, the complexity of the greedy search can be further reduced. Then the lemma given below is in order.

Lemma 2.2: *The reduction of the total bandwidth requirement after removing a certain user is only coupled with the users served by the same source as this user, and is decoupled with the users served by other sources. Mathematically, it means that* $G(\mathcal{I}) - G(\mathcal{I} \setminus \{n\}) = G(\mathcal{I} \cap \mathcal{N}_{S_i}) - G(\mathcal{I} \cap \mathcal{N}_{S_i} \setminus \{n\})$ *for* $n \in \mathcal{N}_{S_i}, \forall \mathcal{I} \subseteq \mathcal{N}$.

Proof: This lemma follows directly from the decomposable structure of the problem (2.21a)–(2.21c), that is, $G(\mathcal{I}) = \sum_{i \in \mathcal{M}} G(\mathcal{I} \cap \mathcal{N}_{S_i})$. This completes the proof. □

Let $\mathcal{N}_{S_i}(t) \triangleq \mathcal{N}_{S_i} \cap \mathcal{N}(t)$ denote the set of remaining users served by S_i after t greedy search iterations. Then the following proposition is of interest.

Proposition 2.5: *The user to be removed at the t-th greedy search iteration according to* (2.22) *can be found by first finding the 'worst' user in each set of users served by each source, i.e.,*

$$n^*_{S_i}(t-1) \triangleq arg \max_{n \in \mathcal{N}_{S_i}(t-1)} \left(G(\mathcal{N}_{S_i}(t-1)) - G(\mathcal{N}_{S_i}(t-1) \setminus \{n\})\right)$$

and then determining the 'worst' user among all these 'worst' users. Mathematically, it means that $n(t) = n^*_{S_{i^*}}(t-1)$ *where*

$$i^* \triangleq \arg\max_{i \in \mathcal{M}} \left(G(\mathcal{N}_{S_i}(t-1)) - G(\mathcal{N}_{S_i}(t-1) \setminus \{n^*_{S_i}(t-1)\})\right).$$

Proof: This proposition follows from applying Lemma 2.2 directly to the removal criterion in (2.22). This completes the proof. □

Proposition 2.5 can be directly used to build an algorithm for searching for the user to be removed at each greedy search iteration. It is important that such algorithm has a reduced computational complexity compared to the direct use of (2.22). As a result, although the number of times that the problem (2.21a)–(2.21c) has to be solved remains the same, the number of variables of the problem (2.21a)–(2.21c) solved at each time is reduced, and is upper bounded by $2\max_{i\in\mathcal{M}} N_{S_i}$.

2.4.3 Optimality conditions of the greedy search algorithm

We also study the conditions under which the proposed greedy search algorithm is optimal. Specifically, the greedy search is optimal if the set of remaining users after each greedy search iteration is the 'best' set of users, i.e.,

$$\mathcal{N}(t) = \mathcal{N}^*_{N-t}, \forall 1 \leq t \leq N \tag{2.23}$$

where $\mathcal{N}^*_i \triangleq \arg\min_{|\mathcal{I}|=i} G(\mathcal{I})$ is the 'best' set of i users.

Let us apply the greedy search to the set of users $\mathcal{N}_{S_i}$ served by the source S_i. The 'worst' user, i.e., the user $\bar{n}_{S_i}(t) \triangleq \arg\max_{n\in\bar{\mathcal{N}}_{S_i}(t-1)} \big(G(\bar{\mathcal{N}}_{S_i}(t-1)) - G(\bar{\mathcal{N}}_{S_i}(t-1)\setminus\{n\})\big)$ is removed at the t-th greedy search iteration, where $\bar{\mathcal{N}}_{S_i}(t) \triangleq \bar{\mathcal{N}}_{S_i}(t-1)\setminus\{\bar{n}_{S_i}(t)\}$ denotes the set of remaining users in the set $\mathcal{N}_{S_i}$ after t greedy search iterations. Also let $\mathcal{N}^*_{S_i,j} \triangleq \arg\min_{\mathcal{I}\subseteq\mathcal{N}_{S_i},|\mathcal{I}|=j} G(\mathcal{I})$ denote the 'best' set of j users in $\mathcal{N}_{S_i}$. The following theorem decouples the optimality condition (2.23) into two equivalent conditions C1 and C2 per each set of users $\mathcal{N}_{S_i}$ and, therefore, allows us to focus on equivalent problems in which users are subject to the same power constraints. Specifically, the condition C1 of the theorem indicates that the set of remaining users in $\mathcal{N}_{S_i}$ after each greedy search iteration is the 'best' set of users, while the condition C2 of the theorem indicates that the reduction of the total bandwidth requirement is decreasing with the greedy search iterations.

Theorem 2.1: *The condition* (2.23) *holds if and only if the following two conditions hold:*

C1: $\bar{\mathcal{N}}_{S_i}(t) = \mathcal{N}^*_{S_i,N_{S_i}-t}$, $\forall 1 \leq t \leq N_{S_i}$, $\forall i \in \mathcal{M}$;

C2: $G(\bar{\mathcal{N}}_{S_i}(t-2)) - G(\bar{\mathcal{N}}_{S_i}(t-1)) > G(\bar{\mathcal{N}}_{S_i}(t-1)) - G(\bar{\mathcal{N}}_{S_i}(t))$, $\forall 2 \leq t \leq N_{S_i}$, $\forall i \in \mathcal{M}$.

Proof: We first show that C1 and C2 are sufficient conditions.

Define $V(n) \triangleq G(\mathcal{N}(t-1)) - G(\mathcal{N}(t))$ for $n = n(t)$, $1 \le t \le N$. It follows from C2 that $V(\bar{n}_{S_i}(1)) > V(\bar{n}_{S_i}(2)) > \cdots > V(\bar{n}_{S_i}(N_{S_i}))$, $\forall i \in \mathcal{M}$. Then using Proposition 2.2, we have $n(t) = \arg\max_{n\in\mathcal{N}(t-1)} V(n)$, $1 \le t \le N$. Therefore, we obtain

$$V(n(1)) > V(n(2)) > \cdots > V(n(N)). \tag{2.24}$$

It can be seen from C1 that $\mathcal{N} \setminus \mathcal{N}^*_{N-t} \cap \mathcal{N}_{S_i} = \arg\min_{\mathcal{I}\subseteq\mathcal{N}_{S_i}, |\mathcal{I}|=t_i} G(\mathcal{N}_{S_i} \setminus \mathcal{I}) = \{\bar{n}_{S_i}(j) | 1 \le j \le t_i\}$, $\forall i \in \mathcal{M}$, where $t_i \triangleq |\mathcal{N} \setminus \mathcal{N}^*_{N-t} \cap \mathcal{N}_{S_i}|$. Then we have $\mathcal{N} \setminus \mathcal{N}^*_{N-t} = \{\bar{n}_{S_i}(j) | 1 \le j \le t_i, i \in \mathcal{M}\}$ and $G(\mathcal{N}) - G(\mathcal{N}^*_{N-t}) = \sum_{i\in\mathcal{M}} \sum_{j=1}^{t_i} V(\bar{n}_{S_i}(j))$. Therefore, we obtain $\{t_i | i \in \mathcal{M}\} = \arg\max_{\{k_i\}; \sum_{i\in\mathcal{M}} k_i = t} \sum_{i\in\mathcal{M}} \sum_{j=1}^{k_i} V(\bar{n}_{S_i}(j))$. Since it follows from C2 that $V(\bar{n}_{S_i}(1)) > V(\bar{n}_{S_i}(2)) > \cdots > V(\bar{n}_{S_i}(N_{S_i}))$, $\forall i \in \mathcal{M}$, we have $\mathcal{N} \setminus \mathcal{N}^*_{N-t} = \arg\max_{\mathcal{I}\in\mathcal{N}, |\mathcal{I}|=t} \sum_{n\in\mathcal{I}} V(n) = \{n(i) | 1 \le i \le t\} = \mathcal{N} \setminus \mathcal{N}(t)$, where the second equality is from (2.24). This completes the proof for sufficiency of C1 and C2.

We next show that C1 and C2 are necessary conditions by giving two instructive counter examples.

Consider if C1 does not hold. Assume without loss of generality that $\mathcal{M} = \{1\}$. Then it can be seen that C1 is equivalent to the condition (2.23) and, therefore, the condition (2.23) does not hold, either.

Consider if C2 does not hold. Assume without loss of generality that $\mathcal{M} = \{2\}$, $N_{S_2} = 1$ and $G(\bar{\mathcal{N}}_{S_1}(1)) - G(\bar{\mathcal{N}}_{S_1}(2)) > G(\mathcal{N}_{S_2}) - G(\bar{\mathcal{N}}_{S_2}(1)) > G(\mathcal{N}_{S_1}) - G(\bar{\mathcal{N}}_{S_1}(1))$. Then we have $\mathcal{N}^*_{N-2} = \mathcal{N} \setminus \{\bar{n}_{S_1}(1), \bar{n}_{S_1}(2)\}$, while it follows from Proposition 2.2 that $\mathcal{N}(2) = \mathcal{N} \setminus \{\bar{n}_{S_1}(1), \bar{n}_{S_2}(1)\}$. Therefore, $\mathcal{N}^*_{N-2} \ne \mathcal{N}(2)$. This completes the proof for necessity of C1 and C2. □

Let $h_i \triangleq h_i^{\mathrm{SD}}/N_0$ denote the channel gain normalized by the noise PSD. Recall that c_i is the minimum acceptable capacity for U_i. Define $F_i(p)$ as the unique solution for w in the equation

$$c_i = w \log\left(1 + \frac{h_i p}{w}\right) \tag{2.25}$$

given h_i and c_i for any $p > 0$, which represents the minimum bandwidth required by a user for its allocated transmit power. Then the problem (2.21a)–(2.21c) for the set of users $\mathcal{N}_{S_i}$

can be rewritten as

$$G(\mathcal{N}_{S_i}) = \quad \min_{\{p_i\}} \quad \sum_{i\in\mathcal{N}_{S_i}} F_i(p_i) \tag{2.26a}$$

$$\text{s.t.} \sum_{i\in\mathcal{N}_{S_i}} p_i \leq P_{S_i}. \tag{2.26b}$$

The following proposition stands that the condition C2 of Theorem 2.1 always holds, which reduces the study on the optimality of the proposed greedy search only to checking the condition C1 of Theorem 2.1.

Proposition 2.6: *The condition C2 of Theorem 2.1 always holds true.*

Proof: The proof of this proposition is built upon the following two lemmas. It suffices to show that C2 holds for $i = 1$.

Lemma 2.3: *If* $p_1 > p_2 > \Delta p > 0$, *the following inequality holds*

$$F_i(p_1 - \Delta p) - F_i(p_1) < F_i(p_2 - \Delta p) - F_i(p_1). \tag{2.27}$$

Proof of Lemma 2.3: It can be shown that $F_i(p)$ is a strictly convex and decreasing function of p. Using the first order convexity condition, we have

$$F_i(p_2 - \Delta p) - F_i(p_2) > -F_i'(p_2)\Delta p \tag{2.28}$$

and

$$F_i(p_1 - \Delta p) - F_i(p_1) < -F_i'(p_1 - \Delta p)\Delta p \tag{2.29}$$

where F_i' is the first order derivative of F_i. Consider two cases. (i) If $p_2 \leq p_1 - \Delta p$, then $F_i'(p_2) \leq F_i'(p_1 - \Delta p)$ due to the convexity of $F_i(p_2)$. Therefore, using $\Delta p > 0$ together with (2.28) and (2.29), we obtain (2.27); (ii) If $p_2 \geq p_1 - \Delta p$, using $p_1 > p_2$ and a similar argument as in 1), we can show that $F_i(p_2) - F_i(p_1) < F_i(p_2 - \Delta p) - F_i(p_1 - \Delta p)$, which is equivalent to (2.27). This completes the proof. □

$G(\mathcal{N}_{S_1})$ can be extended to $G(\mathcal{N}_{S_1}, P_{S_1})$ if P_{S_1} is considered as a variable.

Lemma 2.4: p_i^*, $\forall i \in \mathcal{N}_{S_1}$, *is increasing with* P_{S_1}, *where* $\{p_i^* | i \in \mathcal{N}_{S_1}\}$ *denotes the optimal solution of the problem* (2.26a)–(2.26b) *for* $\mathcal{N}_{S_1}$ *and* P_{S_1}.

Proof of Lemma 2.4: The inverse function of $w = F_i(p)$ is $p = F_i^{-1}(w) = (e^{c_i/w} - 1)w/h_i$.

Then we have

$$G(\mathcal{N}_{S_1}, P_{S_1}) = \qquad \max_{w_i} \sum_{i \in \mathcal{N}_{S_1}} w_i \tag{2.30a}$$

$$\text{s.t.} \sum_{i \in \mathcal{N}_{S_1}} F_i^{-1}(w_i) \le P_{S_1}. \tag{2.30b}$$

Since the problem (2.30b)–(2.30b) is convex, using the KKT conditions, the optimal solution and the optimal Lagrange multiplier of this problem, denoted by $\{w_i^*|i \in \mathcal{N}_{S_1}\}$ and λ^*, respectively, satisfy the following equations

$$1 + \frac{\lambda^*}{h_i}\left(e^{\frac{c_i}{w_i^*}}\left(\frac{c_i}{w_i^*} - 1\right) + 1\right) = 0, \quad \forall i \in \mathcal{N}_{S_1}. \tag{2.31}$$

It can be shown that $(e^{c_i/w_i^*}(c_i/w_i^* - 1) + 1)/h_i$ is monotonically decreasing with w_i^*. Therefore, w_i^*, $\forall i \in \mathcal{N}_{S_1}$, and, correspondingly, $p_i^* = F_i^{-1}(w_i^*)$, $\forall i \in \mathcal{N}_{S_1}$, is decreasing and increasing, respectively, with λ^*. Then it follows from (2.30b) that p_i^*, $\forall i \in \mathcal{N}_{S_1}$, is increasing with P_{S_1}. This completes the proof. $\square$

We are now ready to prove Proposition 2.6. Let $P_1 > P_2$ and $\mathcal{N}_{S_1}^{-k} \triangleq \mathcal{N}_{S_1} \setminus \{k\}$ for some $k \in \mathcal{N}_{S_1}$. Let $\{p_i^\star|i \in \mathcal{N}_{S_1}^{-k}\}$ denote the optimal solution of the problem (2.26a)–(2.26b) for $\mathcal{N}_{S_1}^{-k}$ and P_2. Using Lemma 2.4, the optimal solution of the problem (2.26a)–(2.26b) for $G(\mathcal{N}_{S_1}, P_2)$ can be expressed as $\{p_i^\star - \Delta p_i\}$, $i \in \mathcal{N}_{S_1}^{-k}$, and $p_k^\star$, respectively, where $\Delta p_i > 0$ and $\sum_{i \in \mathcal{N}_{S_1}^{-k}} \Delta p_i = p_k^\star$. Then we have

$$G(\mathcal{N}_{S_1}, P_2) - G(\mathcal{N}_{S_1}^{-k}, P_2) = \sum_{i \in \mathcal{N}_{S_1}^{-k}} (F_i(p_i^\star - \Delta p_i) - F_i(p_i^\star)) + F_k(p_k^\star). \tag{2.32}$$

Let $\{p_i^+|i \in \mathcal{N}_{S_1}^{-k}\}$ denote the optimal solution of the problem (2.26a)–(2.26b) for $\mathcal{N}_{S_1}^{-k}$ and P_1. Then we have

$$\begin{aligned} G(\mathcal{N}_{S_1}, P_1) - G(\mathcal{N}_{S_1}^{-k}, P_1) &= \min_{\{p_i\}; \sum_{i \in \mathcal{N}_{S_1}} p_i \le P_1} \sum_{i \in \mathcal{N}_{S_1}} F_i(p_i) - \sum_{i \in \mathcal{N}_{S_1}^{-k}} F_i(p_i^+) \\ &\le \sum_{i \in \mathcal{N}_{S_1}^{-k}} (F_i(p_i^+ - \Delta p_i) - F_i(p_i^+)) + F_k(p_k^\star). \end{aligned} \tag{2.33}$$

Since $P_1 > P_2$, it follows from Lemma 2.4 that $p_i^+ > p_i^\star > \Delta p_i > 0$, $i \in \mathcal{N}_{S_1}^{-k}$. Using Lemma 2.3, we obtain $F_i(p_i^+ - \Delta p_i) - F_i(p_i^+) < F_i(p_i^\star - \Delta p_i) - F_i(p_i^\star)$, $j \in \mathcal{N}_{S_1}^{-k}$. Therefore, comparing (2.32) with (2.33), we have

$$G(\mathcal{N}_{S_1}, P_1) - G(\mathcal{N}_{S_1}^{-k}, P_1) < G(\mathcal{N}_{S_1}, P_2) - G(\mathcal{N}_{S_1}^{-k}, P_2). \tag{2.34}$$

which can be rewritten as

$$G(\mathcal{N}_{S_1}^{-k}, P_2) - G(\mathcal{N}_{S_1}^{-k}, P_1) < G(\mathcal{N}_{S_1}, P_2) - G(\mathcal{N}_{S_1}, P_1). \tag{2.35}$$

Let $\{p_i^*|i \in \mathcal{N}_{S_1}\}$, denote the optimal solution of the problem (2.26a)–(2.26b) for $\mathcal{N}_{S_1}$ and P_{S_1}. Then we have

$$\begin{aligned} G(\mathcal{N}_{S_1} \setminus \{\bar{n}_{S_1}(2)\}, P_{S_1}) - G(\bar{\mathcal{N}}_{S_1}(2), P_{S_1}) &\leq F_{\bar{n}_{S_1}(1)}(p^*_{\bar{n}_{S_1}(1)}) + G(\bar{\mathcal{N}}_{S_1}(2), P_{S_1} - p^*_{\bar{n}_{S_1}(1)}) \\ &\quad - G(\bar{\mathcal{N}}_{S_1}(2), P_{S_1}) \\ &< F_{\bar{n}_{S_1}(1)}(p^*_{\bar{n}_{S_1}(1)}) + G(\bar{\mathcal{N}}_{S_1}(1), P_{S_1} - p^*_{\bar{n}_{S_1}(1)}) \\ &\quad - G(\bar{\mathcal{N}}_{S_1}(1), P_{S_1}) \\ &= G(\mathcal{N}_{S_1}, P_{S_1}) - G(\bar{\mathcal{N}}_{S_1}(1), P_{S_1}) \end{aligned} \tag{2.36}$$

where the second inequality follows from (2.35). On the other hand, we have

$$\begin{aligned} G(\mathcal{N}_{S_1} \setminus \{\bar{n}_{S_1}(2)\}, P_{S_1}) - G(\bar{\mathcal{N}}_{S_1}(2), P_{S_1}) &\geq G(\mathcal{N}_{S_1} \setminus \{\bar{n}_{S_1}(1)\}, P_{S_1}) - G(\bar{\mathcal{N}}_{S_1}(2), P_{S_1}) \\ &= G(\bar{\mathcal{N}}_{S_1}(1), P_{S_1}) - G(\bar{\mathcal{N}}_{S_1}(2), P_{S_1}). \end{aligned} \tag{2.37}$$

Therefore, comparing (2.36) with (2.37), we complete the proof. □

The following lemma gives a condition under which C1 holds for a specific t.

Lemma 2.5: *If there exists $\mathcal{N}_{S_l,k} \subseteq \mathcal{N}_{S_l}$, $|\mathcal{N}_{S_l,k}| = k$ such that $F_i(p) < F_j(p)$, $\forall 0 < p < P_{S_i}$, $\forall i \in \mathcal{N}_{S_l,k}$ and $\forall j \in \mathcal{N} \setminus \mathcal{N}_{S_l,k}$, then $\mathcal{N}_{S_l,k} = \mathcal{N}^*_{S_l,k} = \bar{\mathcal{N}}_{N_{S_l}}(N_{S_l} - k)$.*

Proof: Assume $\mathcal{N}_{S_l,k} \neq \mathcal{N}^*_{S_l,k}$. Then there exist $a \in \mathcal{N}^*_{S_l,k}$ and $b \in \mathcal{N} \setminus \mathcal{N}^*_{S_l,k}$ such that $F_a(p) > F_b(p)$. Let $\{p_i^*|i \in \mathcal{N}^*_{S_1,k}\}$ denote the optimal solution of the problem (2.26a)–(2.26b) for $G(\mathcal{N}^*_{S_l,k})$. Then there always exists $\mathcal{N}'_{S_l,k} \triangleq \mathcal{N}^*_{S_l,k} \cup \{b\} \setminus \{a\}$ such that

$$\begin{aligned} G(\mathcal{N}^*_{S_l,k}) = \sum_{i \in \mathcal{N}^*_{S_l,k},\, i \neq a} F_i(p_i^*) + F_a(p_a^*) &> \sum_{i \in \mathcal{N}^*_{S_l,k},\, i \neq a} F_i(p_i^*) + F_b(p_a^*) \\ \geq \min_{\{p_i\};\, \sum_{i \in \mathcal{N}'_{S_l,k}} p_i \leq P_{S_1}} \sum_{i \in \mathcal{N}'_{S_l,k}} F_i(p_i) &= G(\mathcal{N}'_{S_l,k}) \end{aligned} \tag{2.38}$$

which contradicts the definition of $\mathcal{N}^*_{S_l,k}$. Then it follows that $\mathcal{N}_{S_l,k} = \mathcal{N}^*_{S_l,k}$. Using similar arguments, it can be shown that $\mathcal{N}_{S_l,k} = \bar{\mathcal{N}}_{N_{S_l}}(N_{S_l} - k)$. This completes the proof. □

It can be seen from Lemma 2.5 that since any user in $\mathcal{N}_{S_l,k}$ has a smaller bandwidth requirement than any user in $\mathcal{N} \setminus \mathcal{N}_{S_l,k}$ for the same allocated power over the available power range, the former is preferable to the latter in the sense of reducing the total bandwidth requirement. Therefore, $\mathcal{N}_{S_l,k}$ is the 'best' set of k users and the greedy search removes users in $\mathcal{N} \setminus \mathcal{N}_{S_l,k}$ before $\mathcal{N}_{S_l,k}$.

It is worth noting that C1 does not hold in general. Indeed, since the reduction of the total bandwidth requirement is maximized only at each single greedy search iteration, the greedy search does not guarantee that the reduction of the total bandwidth requirement is also maximized over multiple greedy search iterations. In other words, it does not guarantee that the set of remaining users is the 'best' set of users. In order to demonstrate this, we present the following counter example.

Example 1: *Let* $\mathcal{N}_{S_1} = \{1, 2, 3\}$*. Also let* $h_1 = 4$*,* $h_2 = 5$*,* $h_3 = 6$*,* $c_1 = 1$*,* $c_2 = 1.1$*,* $c_3 = 1.2$*, and* $P_{S_1} = 1.1$*. Then we have* $G(\{1,2\}) = 1.3849$*,* $G(\{1,3\}) = 1.3808$*,* $G(\{2,3\}) = 1.3573$*,* $G(\{1\}) = 0.4039$*,* $G(\{2\}) = 0.4135$*,* $G(\{3\}) = 0.4292$ *and, therefore,* $\bar{\mathcal{N}}_{S_1}(1) = \{2,3\}$*,* $\bar{\mathcal{N}}_{S_1}(2) = \{2\}$*,* $\mathcal{N}^*_{S_1,1} = \{1\}$*. This shows that* $\bar{\mathcal{N}}_{S_1}(2) \neq \mathcal{N}^*_{S_1,1}$*.*

Example 1 shows that the 'worst' user, which is removed first in the greedy search, may be among the 'best' set of users after more users are removed. An intuitive interpretation of this result is that the bandwidth required by the 'worst' user changes from being larger to being smaller compared to the bandwidth required by other users for the same allocated power. It is because the average available power to each user increases after some users are removed in the greedy search.

Using Lemma 2.5, the following proposition that gives a sufficient condition under which C1 holds is in order.

Proposition 2.7: *The condition C1 holds if for any* $i \in \mathcal{N}_{S_k}$*,* $\forall k \in \mathcal{M}$*, there exists no more than one* $j \in \mathcal{N}_{S_k}$*,* $j \neq i$*, such that*

C3: $F_i(p)$ *intersects* $F_j(p)$ *in the interval* $0 < p < P_{S_i}$*.*

Proof: It suffices to show that C1 holds for $i = 1$ if for any $j \in \mathcal{N}_{S_1}$, there exists no more than one $k \in \mathcal{N}_{S_1}$, $k \neq j$, such that C3 holds. It can be seen that for any $1 \leq k \leq N_{S_1}$, only two cases are under consideration: (i) there exists $\mathcal{N}_{S_1,k}$ that satisfies the condition given in Lemma 2.4 and, therefore, $\mathcal{N}^*_{S_1,k} = \mathcal{N}(N_{S_1} - k)$; (ii) there exist $\mathcal{N}_{S_1,k-1}$ and $\mathcal{N}_{S_1,k+1}$ that satisfy the condition given in Lemma 2.2 respectively and, therefore, $\mathcal{N}^*_{S_1,k-1} = \mathcal{N}(N_{S_1} - k + 1) \subseteq \mathcal{N}(N_{S_1} - k - 1) = \mathcal{N}^*_{S_1,k+1}$. Then it follows that $\mathcal{N}^*_{S_1,k} = \mathcal{N}(N_{S_1} - k)$. This completes the proof. □

It can be seen from Proposition 2.7 that the chance that C1 holds increases when the chance that C3 holds decreases. Moreover, the chance that C1 holds increases when N_{S_i} is large for all $i \in \mathcal{M}$ or M is large. The next lemma compares the bandwidth requirements of two users in terms of the ratio between their minimum acceptable capacities and the ratio between their channel gains.

Lemma 2.6: *If $i \neq j$ and $h_j/h_i \geq 1$, then*

(i) $\forall p$, $F_i(p)$ intersects $F_j(p)$ at a unique point p', if and only if $1 < c_j/c_i < h_j/h_i$; furthermore, p' increases as h_j/h_i increases or c_j/c_i decreases;

(ii) $F_i(p) > F_j(p)$, $\forall p > 0$, or $F_i(p) = F_j(p)$, $\forall p > 0$, if and only if $c_j/c_i \leq 1$;

(iii) $F_i(p) < F_j(p)$, $\forall p > 0$, if and only if $c_j/c_i \geq h_j/h_i$.

Proof: Consider if $F_i(p)$ intersects $F_j(p)$ at a point (p', w'). Then we obtain

$$\frac{c_j}{c_i} = \frac{w' \log\left(1 + \frac{h_j p'}{w'}\right)}{w' \log\left(1 + \frac{h_i p'}{w'}\right)} = q\left(\frac{p'}{w'}\right) \tag{2.39}$$

where $q(x) = \log(1+h_j x)/\log(1+h_i x)$, $0 < x < \infty$. It can be shown that $\lim_{x\to 0} q(x) = h_j/h_i$, $\lim_{x\to\infty} q(x) = 1$, and $q(x)$ is monotonically decreasing with x. Therefore, the range of $q(x)$ is $(1, h_j/h_i)$. If $c_j/c_i \in (1, h_j/h_i)$, there exists a unique solution x' such that $q(x') = c_j/c_i$. Hence, $F_i(p)$ and $F_j(p)$ have a unique intersection point given by $w' = c_j/\log(1 + h_j x')$, $p' = w'x'$, and the claim (i) follows. If $c_j/c_i \notin (1, h_j/h_i)$, there is a special case that $F_i(p) = F_j(p)$, $\forall p > 0$ if $h_j/h_i = c_j/c_i = 1$. Otherwise, the solution of (2.39) does not exist, i.e., $F_i(p)$ does not intersect $F_j(p)$ and, therefore, the claims (ii) and (iii) also follow. This completes the proof. □

It can be seen from Lemma 2.6 that the condition C3 of Proposition 2.7 holds if and only if the claim (i) of Lemma 2.6 holds with $0 < p' < P_{S_i}$. Then it follows from Proposition 2.7 and Lemma 2.6 that the condition C1 of Theorem 2.1 holds if for any pair $\{i, j\} \subseteq \mathcal{N}_{S_k}$, $\forall k$, the ratio c_j/c_i is in the grey range shown in Fig. 2.3. According to Lemma 2.6, the coordinate x in Fig. 2.3 satisfies $1 \leq x \leq \frac{h_j}{h_i}$ and $x \to \frac{h_j}{h_i}$ as $P_{S_k} \to 0$. It can be seen that the grey range in Fig. 2.3 is wide and, therefore, the condition C1 of Theorem 2.1 is, in fact, a mild condition to hold.

Applying Lemma 2.6, Proposition 2.6, and Proposition 2.7, the next corollary follows directly from Theorem 2.1.

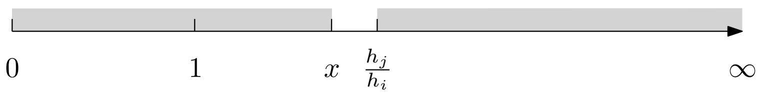


Fig. 2.3. The range of c_j/c_i to satisfy the condition C1.

Corollary 2.1: *The proposed greedy search is optimal, i.e.,* $\mathcal{N}(t) = \mathcal{N}^*_{N-t}$, $\forall 1 \leq t \leq N$, *if* $c_i = c_j$, $\forall i, j \in \mathcal{N}$, $i \neq j$.

In some wireless networks where data transmissions are conducted to support the same kind of application, e.g., voice application, users have the same capacity requirements and, thus, Corollary 1 applies.

Note that the optimality condition given in (2.23) is a sufficient condition under which $\mathcal{N}(t^*) = \mathcal{N}^*_{N-t^*} = \mathcal{N}^*_{d^*}$. Indeed, the greedy search is optimal if and only if $t^* = N - d^*$. Therefore, even if $\mathcal{N}(t^*) \neq \mathcal{N}^*_{d^*}$, the greedy search still gives the maximum number of admissible users if $G(\mathcal{N}^*_{d^*}) < G(\mathcal{N}(N - d^*)) \leq W$.

With Relaying

The admission control based joint bandwidth and power allocation problem in the network with relaying is given by

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}, P_i^{\mathrm{R}}, W_i^{\mathrm{R}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{2.40a}$$

$$\text{s.t. } c_i - C_i^{\mathrm{SR}} \leq 0, \ i \in \mathcal{I} \tag{2.40b}$$

$$c_i - C_i^{\mathrm{RD}} \leq 0, \ i \in \mathcal{I} \tag{2.40c}$$

$$\text{the constraint (2.12b)–(2.12e).} \tag{2.40d}$$

The proposed greedy search algorithm can also be used to reduce the complexity of solving the problem (2.40a)–(2.40d). Specifically, the problem (2.40a)–(2.40d) can be decomposed into

$$\max_{\{P_i^{\mathrm{S}}, W_i^{\mathrm{S}}\}, \mathcal{I} \subseteq \mathcal{N}} \quad |\mathcal{I}| \tag{2.41a}$$

$$\text{s.t. } c_i - C_i^{\mathrm{SR}} \leq 0, \ i \in \mathcal{I} \tag{2.41b}$$

$$\text{the constraint (2.12b), (2.12d)} \tag{2.41c}$$

and

$$\max_{\{P_i^{\mathrm{R}},W_i^{\mathrm{R}}\},\mathcal{I}\subseteq\mathcal{N}} \quad |\mathcal{I}| \tag{2.42a}$$

$$\text{s.t. } c_i - C_i^{\mathrm{RD}} \leq 0,\ \ i \in \mathcal{I} \tag{2.42b}$$

$$\text{the constraint } (2.12c), (2.12e). \tag{2.42c}$$

each of which has the same form as the problem (2.20a)–(2.20c). Therefore, the proposed greedy search can be applied for solving each of these two problems separately. As a result, the numbers of users removed by the greedy search in each transmission phase can be found as t_1^* and t_2^*, respectively. Let d^*, d_1^*, and d_2^* denote the optimal values of the problem (2.40a)–(2.40d), (2.41a)–(2.41c), and (2.42a)–(2.42c), respectively. Since the feasible set of the problem (2.40a)–(2.40d) is a subset of those of the problem (2.41a)–(2.41c) and (2.42a)–(2.42c), we have $d^* \leq \min\{d_1^*, d_2^*\}$. Therefore, d^* should be obtained by solving the problem

$$\max_{\{P_i^{\mathrm{S}},W_i^{\mathrm{S}},P_i^{\mathrm{R}},W_i^{\mathrm{R}}\},\mathcal{I}\subseteq\mathcal{N},|\mathcal{I}|\leq t'} \quad |\mathcal{I}| \tag{2.43a}$$

$$\text{s.t. } c_i - C_i^{\mathrm{SR}} \leq 0,\ \ i \in \mathcal{I} \tag{2.43b}$$

$$c_i - C_i^{\mathrm{RD}} \leq 0,\ \ i \in \mathcal{I} \tag{2.43c}$$

$$\text{the constraints } (2.12b)\text{–}(2.12e) \tag{2.43d}$$

where $d' \triangleq \min\{N - t_1^*, N - t_2^*\}$ and the feasible set is reduced as compared to that of the problem (2.40a)–(2.40d). The problem (2.43a)–(2.43d) can then be solved using exhaustive search with significantly reduced complexity compared to total exhaustive search over two transmission phases.

Using the exhaustive search, the number of times that the problem (2.21a)–(2.21c) has to be solved is upper bounded by $2\sum_{i=d^*}^{N}\binom{N}{i}$. Using the greedy search combined with the exhaustive search, this number of times significantly reduces and is upper bounded by $\sum_{i=0}^{t_1^*-1} N - i + \sum_{i=0}^{t_2^*-1} N - i + 2\sum_{i=d^*}^{d'}\binom{N}{i}$ if $d' \geq d^*$ and $\sum_{i=0}^{t_1^*-1} N - i + \sum_{i=0}^{t_2^*-1} N - i + 2\binom{N}{d'}$ if $d' < d^*$. This complexity reduction is especially pronounced when N is large and d', d^* are small.

It can be seen from comparing the problem (2.40a)–(2.40d) and (2.43a)–(2.43d) that the greedy search is optimal if and only if $d' \geq d^*$.

2.5 Simulation Results

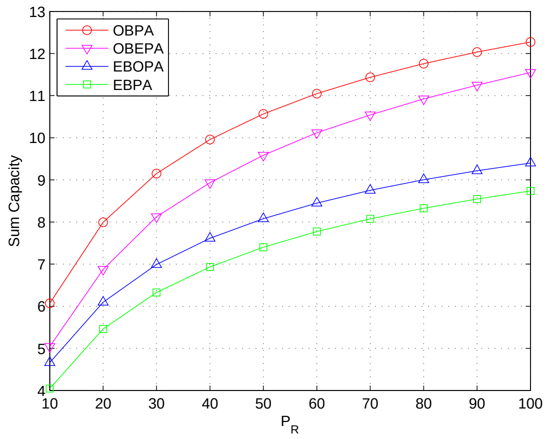
2.5.1 Joint bandwidth and power allocation

Consider a wireless network which consists of four users $\mathcal{N} = \{1, 2, 3, 4\}$, four sources, and two relays. The source and relay assignments to the users are the following: $\mathcal{N}_{S_1} = \{1\}$, $\mathcal{N}_{S_2} = \{2\}$, $\mathcal{N}_{S_3} = \{3\}$, $\mathcal{N}_{S_4} = \{4\}$, $\mathcal{N}_{R_1} = \{1, 2\}$, and $\mathcal{N}_{R_2} = \{3, 4\}$. The sources and destinations are randomly distributed inside a square area bounded by (0,0) and (10,10), and the relays are fixed at (5,3) and (5,7). The path loss and the Rayleigh fading effects are present in all links. The path loss gain is given by $g = (1/d)^2$, where d is the distance between two transmission ends, and the variance of the Rayleigh fading gain is denoted as σ^2. We set $P_{S_i} = 20$, $\forall i \in \{1, 2, 3, 4\}$, $P_{R_i} \triangleq P_R = 40$, $\forall i \in \{1, 2\}$, $W = 10$, $\sigma^2 = 5$, and $c_i = 1$, $\forall i \in \{1, 2, 3, 4\}$ as default values if no other values are indicated otherwise. The noise PSD N_0 equals to 1. All results are averaged over 1000 simulation runs for different instances of random channel realizations.

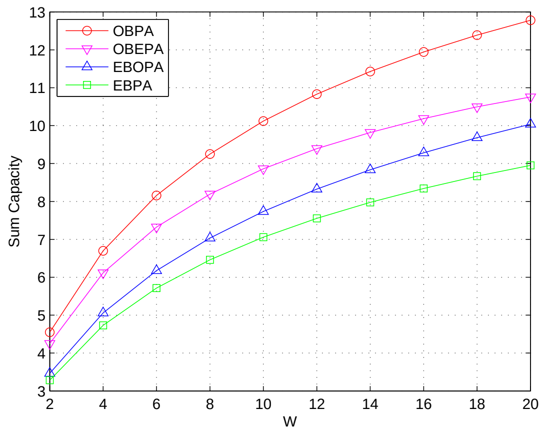
The following resource allocation schemes are compared to each other: the proposed optimal joint bandwidth and power allocation (OBPA), optimal bandwidth with equal power allocation (OBEPA), equal bandwidth with optimal power allocation (EBOPA), and equal bandwidth and power allocation (EBPA). Software package TOMLAB [9] is used to solve the corresponding convex optimization problems.

In Figs. 2.4 (a) and (b), the performance of the sum capacity maximization based allocation is shown versus P_R and W, respectively. These figures show that the OBPA scheme achieves significant performance improvement over the other three schemes for all parameter values. The performance improvement is higher when P_R or W is larger. The observed significant performance improvement for the OBPA can be partly attributed to the fact that the sum capacity maximization based joint bandwidth and power allocation can lead to highly unbalanced resource allocation, while bandwidth is equally allocated in the EBOPA and EBPA, and power is equally allocated in the OBEPA and EBPA.

Figs. 2.5 (a) and (b) demonstrate the performance of the worst user capacity maximization based allocation versus P_R and W, respectively. The performance improvement for the OBPA is still significant as compared to the other three schemes for all parameter values. The improvement provided by the OBPA, in this case, can be attributed to the fact that

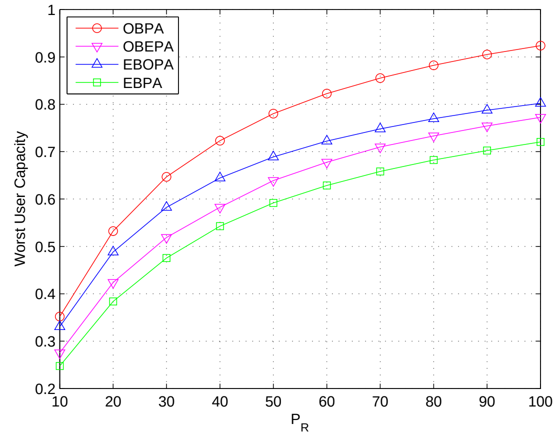


(a)

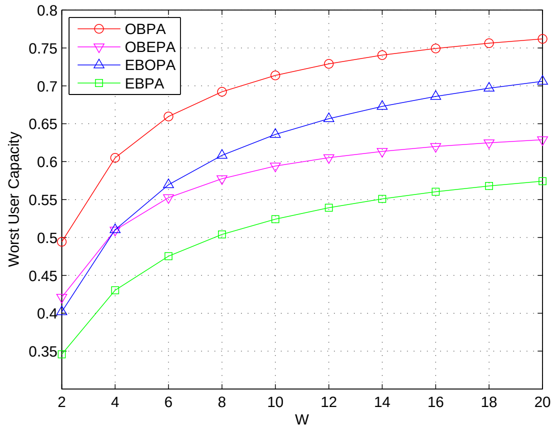


(b)

Fig. 2.4. Sum capacity vs P_R, W.



(a)

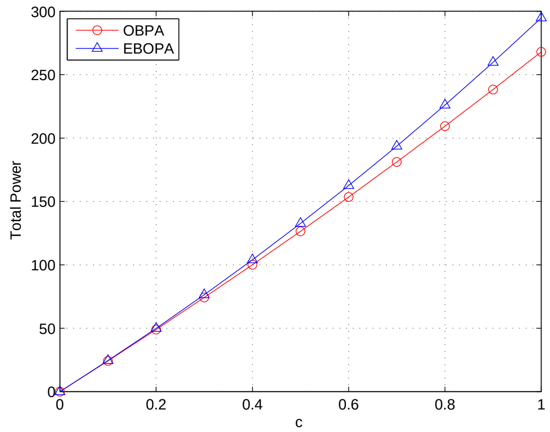


(b)

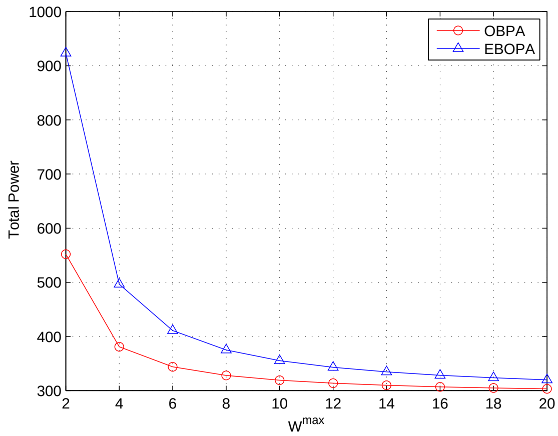
Fig. 2.5. Worst user capacity vs P_R, W.

the worst user capacity maximization based allocation results in balanced capacities among the users, while the EBOPA and EBPA are balanced bandwidth allocation schemes, and the OBEPA and EBPA are balanced power allocation schemes, respectively.

Figs. 2.6 (a) and (b) show the total power consumption of the sources and relays versus c and W for the power minimization based allocation, where $c_1 = c_2 = c_3 = c_4 \triangleq c$ is assumed. Note that the total power of the OBPA is always less than that of the EBOPA, and the total power difference between the two tested schemes is larger when c is larger, or when W is smaller. This shows that more power is saved when the parameters are unfavorable due to the flexible bandwidth allocation in the OBPA.



(a)



(b)

Fig. 2.6. Total network power vs c, W.

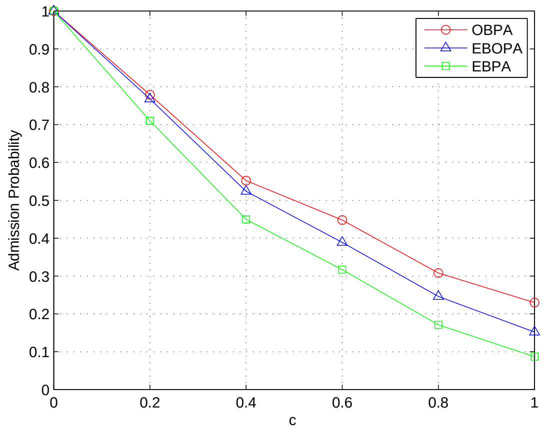


Fig. 2.7. Admission probability vs capacity threshold.

Fig. 2.7 depicts the admission probability versus c, where $c_1 = c_2 = c_3 = c_4 \triangleq c$ is assumed. The admission probability is defined as the probability that c can be satisfied for all the users under random channel realizations. The figure shows that the OBPA outperforms the other two schemes for all values of c, and the improvement is more significant when c is large. This shows that more users or users with higher rate requirements can be admitted into the network using the OBPA scheme.

2.5.2 Greedy search algorithm

In this example, the performance of the proposed greedy search algorithm is compared to that of the exhaustive search algorithm. We consider eight users $\mathcal{N} = \{1, 2, \cdots, 8\}$ requesting for admission. The sources and the destinations are randomly distributed inside a square area bounded by (0,0) and (10,10). We assume that c_i, $i \in \{1, 2, \cdots, 8\}$, is uniformly distributed over the interval $[c_0, c_0 + 4]$ where c_0 is a variable parameter. The channel model is the same as that given in the previous subsection. We set $W = 10$, $\sigma^2 = 10$ as default values. The results are averaged over 20 random channel realizations.

Without Relaying

We consider the following two network setups.

Setup 1: In this setup, the optimality condition of the greedy search is satisfied. Specifically, there are four sources. The source assignments to the users are the following: $\mathcal{N}_{S_1} = \{1, 2\}$, $\mathcal{N}_{S_2} = \{3, 4\}$, $\mathcal{N}_{S_3} = \{5, 6\}$, and $\mathcal{N}_{S_4} = \{7, 8\}$. We set $P_{S_i} = 40$, $\forall i \in \{1, 2, 3, 4\}$.

Fig. 2.8(a) shows the number of admitted users obtained by the greedy search and the corresponding computational complexity in terms of the running time versus c_0. The figure shows that the greedy search gives exactly the same number of admitted users as that of the exhaustive search for all values of c_0. This confirms that the optimal solution can be obtained when the optimality condition of the greedy search is satisfied. The time consumption of the greedy search is significantly less than that of the exhaustive search, especially when c_0 is large. This shows that the proposed algorithm is especially efficient when the number of candidate users is large and the number of admitted users is small.

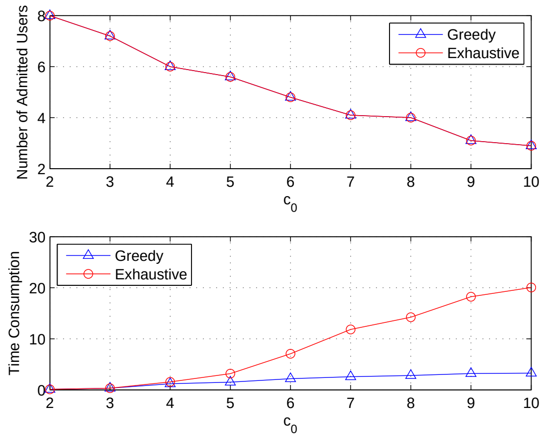
Setup 2: In this setup, the optimality condition of the greedy search may not be satisfied. There are two sources and the source assignments to the users are the following: $\mathcal{N}_{S_1} = \{1, 2, 3, 4\}$, and $\mathcal{N}_{S_2} = \{5, 6, 7, 8\}$. We set $P_{S_i} = 80$, $\forall i \in \{1, 2\}$. Fig. 2.8(b) demonstrates the performance of the greedy search. Similar conclusions can be drawn for this setup as those for Setup 1. This indicates that the proposed greedy search algorithm can still perform optimally even if the sufficient optimality condition may not be satisfied.

With Relaying

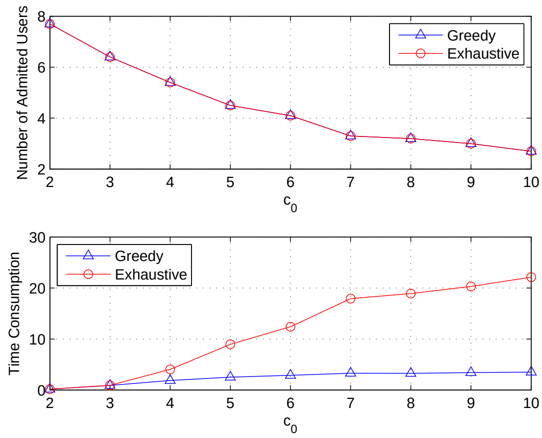
We also consider two network setups as follows.

Setup 3: In this setup, the optimality condition of the greedy search is satisfied. Specifically, in addition to the Setup 1 given in the case without relaying, four relays are included with the following user assignments: $\mathcal{N}_{R_1} = \{1, 2\}$, $\mathcal{N}_{R_2} = \{3, 4\}$, $\mathcal{N}_{R_3} = \{5, 6\}$, and $\mathcal{N}_{R_4} = \{7, 8\}$. The relays are fixed at (5,2), (5,4), (5,6), and (5,8), and we set $P_{R_i} = 40$, $\forall i \in \{1, 2, 3, 4\}$. Fig. 2.9(a) shows the number of admitted users obtained by the greedy search and the corresponding computational complexity in terms of the running time versus c_0. Similar observations can be obtained as those for Setup 1. However, it can be noted as expected that the time consumption of the greedy search for the network with relaying is much more than that for the network without relaying.

Setup 4: In this setup, the optimality condition of the greedy search may not be satisfied. Specifically, in addition to the Setup 2 given in the case without relaying, two relays are included with the following user assignments: $\mathcal{N}_{R_1} = \{1, 2, 7, 8\}$, $\mathcal{N}_{R_2} = \{3, 4, 5, 6\}$. The relays are fixed at (5,3) and (5,7) and we also set $P_{R_i} = 80$, $\forall i \in \{1, 2\}$. Fig. 2.9(b) demonstrates the performance of the greedy search. Similar conclusions can be obtained as those for Setup 3.

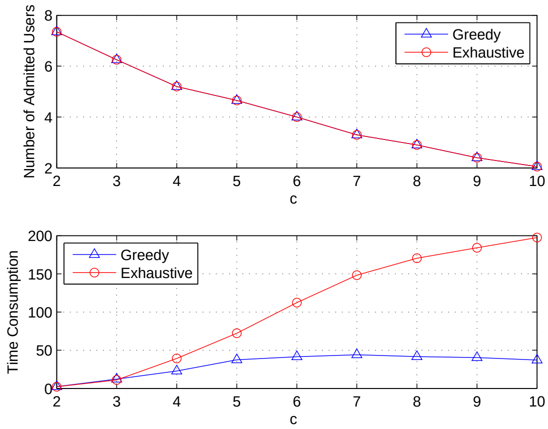


(a) setup 1

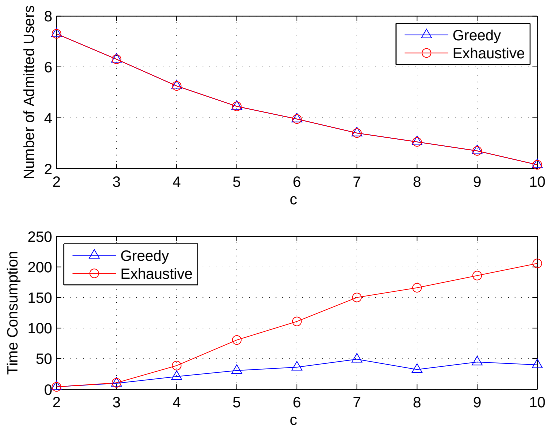


(b) setup 2

Fig. 2.8. Greedy search vs exhaustive search: without relaying.



(a) setup 3



(b) setup 4

Fig. 2.9. Greedy search vs exhaustive search: with relaying.

2.6 Conclusion

In this chapter, joint bandwidth and power allocation has been developed for wireless multi-user networks with and without relaying to (i) maximize the sum capacity of all users; (ii) maximize the capacity of the worst user; (iii) minimize the total power consumption of all users. It has been shown that the corresponding resource allocation problems are convex and, thus, can be solved efficiently. Moreover, admission control based on joint bandwidth and power allocation has been considered. Because of the high complexity of the admission control problem, a suboptimal greedy search algorithm with significantly reduced complexity has been developed. The optimality conditions of the proposed greedy search have been derived and shown to be mild. Simulation results demonstrated the efficiency of the proposed allocation schemes and the advantages of the greedy search.

Chapter 3

Optimal Bandwidth and Power Allocation for Sum Ergodic Capacity under Fading Channels in Cognitive Radio Networks

This chapter studies a cognitive radio network where multiple secondary users (SUs) share the licensed spectrum of a primary user (PU) under fading channels using the frequency division multiple access (FDMA) scheme. The sum ergodic capacity of all the SUs is taken as the performance metric of the network. Besides all combinations of the peak/average transmit power constraints at the SUs and the peak/average interference power constraint imposed by the PU, total bandwidth constraint of the licensed spectrum is also taken into account. Optimal bandwidth allocation is derived in closed-form for any given power allocation. The structures of the optimal power allocations are also derived under all possible combinations of the aforementioned power constraints. These structures indicate the possible numbers of users that transmit at nonzero power but below their corresponding peak power, and show that other users do not transmit or transmit at their corresponding peak power. Based on these structures, efficient algorithms are developed for finding the optimal power allocations.

The rest of this chapter is organized as follows. Section 3.2 summarizes the system model and formulates corresponding sum ergodic capacity maximization problems. Section 3.3 derives the optimal bandwidth allocation for the problems formulated in Section 3.2 subject to the bandwidth constraint. Section 3.4 obtains the optimal power allocations from the resultant problems in Section 3.3 under all combinations of the transmit power constraints and interference power constraints. Numerical results for the maximum sum ergodic capacity under different combinations of the power constraints and the bandwidth constraint are shown in Section 3.5. Section 3.6 concludes this chapter.

3.1 Introduction

The underutilization of licensed spectrum reported by FCC [40] has motivated intensive research efforts on improving the efficiency of spectrum access. Cognitive radio is a promising technology [41] for the implementation of dynamic spectrum access strategies, which improve spectrum utilization by allowing secondary users (SUs) to communicate over the licensed spectrum allocated to existing primary users (PUs). In cognitive radio networks, there exists fundamental tradeoff between enhancing the performance of SUs and reducing the performance degradation resulted from SUs to PUs. One commonly used spectrum sharing strategy to protect PUs is referred to as *spectrum overlay* or *opportunistic spectrum access* (OSA) [42], where SUs are allowed to access licensed spectrum only when the spectrum is not utilized by PUs. Such a strategy requires spectrum opportunity detection by employing spectrum sensing techniques [43]. Existing works on spectrum overlay have mainly studied spectrum sensing and access policies at the medium access control (MAC) layer [44]- [50].

An alternative strategy, which is known as *spectrum underlay* [51]- [53], enables PUs and SUs to transmit simultaneously, provided that the received interference power level by the PUs is below a prescribed threshold level. A number of works have recently studied information theoretic limits for resource allocation in the context of spectrum underlay. In [54], the optimal power allocation which aims at maximizing the ergodic capacity achieved by an SU is derived for various channel fading models subject to the peak interference power (PIP) constraint or average interference power (AIP) constraint imposed by a PU. In [55], the authors derive the optimal power allocation for the ergodic capacity, outage capacity, and minimum-rate capacity of an SU under both the PIP and AIP constraints from a PU. The ergodic capacity, delay-limited capacity, and outage capacity of an SU is studied in [56] under different combinations of the peak transmit power (PTP) constraint or average transmit power (ATP) constraint at the SU and the PIP constraint or AIP constraint from a PU. However, all the papers mentioned above consider the setup of a single SU. The most recent work [57] studies a cognitive radio network of multiple SUs under multiple access channel and broadcast channel models, where the optimal power allocation is derived to achieve the maximum sum ergodic capacity of the SUs subject to various mixed transmit and interference power constraints. The optimality conditions for the dynamic time division multiple access scheme are also derived.

In this chapter[1], we focus on a cognitive radio network where multiple SUs share the licensed spectrum of a PU using the frequency division multiple access (FDMA) scheme. The sum ergodic capacity of the SUs, which is a relevant network performance metric for delay-tolerant traffics, is studied. Besides the transmit power constraints at the SUs and the interference power constraint imposed by the PU, which are also considered in [54]- [57], we also take into account the total bandwidth constraint of the shared spectrum. Joint bandwidth and power allocation strategies for different applications have been studied in only a few works [10]– [38]. Thus, in this paper, instead of conventional fixed and equal bandwidth allocation used in FDMA, we investigate dynamic and unequal bandwidth allocation, where the bandwidth allocation varies for different SUs at different channel fading states. Moreover, different from the existing works [54]- [57], all combinations of the transmit power constraints and the interference power constraints are considered, including both PTP and ATP constraints combined with both PIP and AIP constraints.

We first derive the optimal bandwidth allocation for any given power allocation, which results in equivalent problems that only involve power allocation. Using the convexity of the resultant power allocation problems, we apply dual decomposition which transforms these problems into equivalent dual problems, where each dual function involves a power allocation subproblem associated with a specific channel fading state. The dual problems can be solved using standard subgradient algorithms. For the power allocation subproblem under all possible combinations of the power constraints, we derive the structures of the

[1]This work has been presented in [58].

optimal power allocations. These structures indicate the possible numbers of users that transmit at nonzero power but below their corresponding peak power, and show that other users do not transmit or transmit at their corresponding peak power. Based on these structures, we develop algorithms for finding the optimal power allocations in each channel fading state.

3.2 System Model

Consider a cognitive radio network of N SUs and one PU, as shown in Fig. 3.1. The PU occupies a spectrum of bandwidth W for its transmission, while the same spectrum is shared by the SUs. The spectrum is assumed to be divided into distinct and nonoverlapping flat fading channels with different bandwidth, so that the SUs share the spectrum through FDMA to avoid interferences with each other. The channel power gains between the ith SU transmitter (SU-Tx) and the ith SU receiver (SU-Rx) and between the ith SU-Tx and the PU receiver (PU-Rx) are denoted by h_i and g_i, respectively. The channel power gains, i.e., $\boldsymbol{g} \triangleq [g_1 \ g_2 \ \cdots \ g_N]$ and $\boldsymbol{h} \triangleq [h_1 \ h_2 \ \cdots \ h_N]$, are assumed to be drawn from an ergodic and stationary vector random process. We further assume that full channel state information (CSI), i.e., the joint probability density function (PDF) of the channel power gains and the instantaneous channel power gains, are known at the SUs. [2] The noise at each SU-Rx plus the interference from the PU transmitter (PU-Tx), is assumed to be additive white Gaussian noise (AWGN) with unit power spectral density (PSD).

We denote the transmit power of the ith SU-Tx and the channel bandwidth allocated to the ith SU-Tx as $p_i(\boldsymbol{g}, \boldsymbol{h})$ and $w_i(\boldsymbol{h}, \boldsymbol{g})$, respectively, for the instantaneous channel power gains $\boldsymbol{g}$ and $\boldsymbol{h}$. Then the total bandwidth constraint can be expressed as

$$\sum_{i=1}^{N} w_i(\boldsymbol{h}, \boldsymbol{g}) \leq W, \ \forall \ \boldsymbol{h}, \boldsymbol{g}. \tag{3.1}$$

The PTP constraints are given by

$$p_i(\boldsymbol{h}, \boldsymbol{g}) \leq P_i^{pk}, \ \forall \ i, \boldsymbol{h}, \boldsymbol{g} \tag{3.2}$$

[2] Note that full CSI assumption is typically in the context of cognitive radio and is also made in other works, such as [54]- [57]

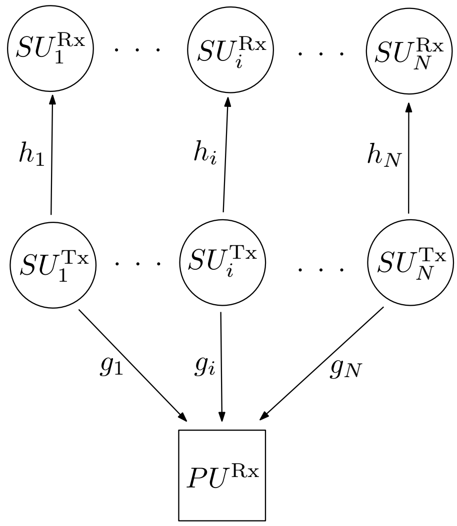


Fig. 3.1. Cognitive radio network.

where P_i^{pk} denotes the maximum peak transmit power of the ith SU-Tx. The PIP constraint is given by

$$\sum_{i=1}^{N} g_i p_i(\boldsymbol{h}, \boldsymbol{g}) \leq Q^{pk}, \ \forall \ \boldsymbol{h}, \boldsymbol{g} \tag{3.3}$$

where Q^{pk} denotes the maximum peak interference power allowed at the PU-Rx. The ATP constraints are given by

$$\mathrm{E}\left\{p_i(\boldsymbol{h}, \boldsymbol{g})\right\} \leq P_i^{av}, \ \forall \ i \tag{3.4}$$

where the expectation is taken over $\boldsymbol{h}$ and $\boldsymbol{g}$, and P_i^{av} denotes the maximum average transmit power of the ith SU-Tx. The AIP constraint is given by

$$\mathrm{E}\left\{\sum_{i=1}^{N} g_i p_i(\boldsymbol{h}, \boldsymbol{g})\right\} \leq Q^{av} \tag{3.5}$$

where Q^{av} denotes the maximum average interference power allowed at the PU-Rx.

The objective is to maximize the sum ergodic capacity of the SUs, which can be written as

$$\max_{\{w_i(\boldsymbol{h},\boldsymbol{g}), p_i(\boldsymbol{h},\boldsymbol{g})\} \in \mathcal{F}} \mathrm{E}\left\{\sum_{i=1}^{N} w_i(\boldsymbol{h}, \boldsymbol{g}) \log\left(1 + \frac{h_i p_i(\boldsymbol{h}, \boldsymbol{g})}{w_i(\boldsymbol{h}, \boldsymbol{g})}\right)\right\} \tag{3.6}$$

where $\mathcal{F}$ is a feasible set specified by the bandwidth constraints (3.1) and a particular combination of the transmit power constraints $\{(3.2), (3.4)\}$ and the interference power constraints $\{(3.3), (3.5)\}$. Note that the constraints on nonnegativity of the bandwidth and power allocations, i.e., $w_i(\boldsymbol{h},\boldsymbol{g}) \geq 0$ and $p_i(\boldsymbol{h},\boldsymbol{g}) \geq 0$, $\forall i, \boldsymbol{h}, \boldsymbol{g}$, are natural and, thus, omitted through out the paper for brevity.

It can be shown that the objective function in the problem (3.6) is concave, since the function $w_i(\boldsymbol{h},\boldsymbol{g}) \log(1 + h_i p_i(\boldsymbol{h},\boldsymbol{g})/w_i(\boldsymbol{h},\boldsymbol{g}))$ is concave with respect to $\{w_i(\boldsymbol{h},\boldsymbol{g}), p_i(\boldsymbol{h},\boldsymbol{g})\}$, $\forall i, \boldsymbol{h}, \boldsymbol{g}$ [37] and [38]. It can also be seen that the bandwidth and power constraints (3.1)–(3.5) are linear and, thus, convex. Therefore, the sum ergodic capacity maximization problem (3.6) under different combinations of the constraints (3.1)–(3.5) is a convex optimization problem.

3.3 Optimal Bandwidth Allocation

Given that the power allocation $p_i(\boldsymbol{h},\boldsymbol{g})$, $\forall i, \boldsymbol{h}, \boldsymbol{g}$, is fixed, the maximum sum ergodic capacity can be expressed as $\mathrm{E}\{f_0(\boldsymbol{h},\boldsymbol{g})\}$, where $f_0(\boldsymbol{h},\boldsymbol{g})$ is given by

$$f_0(\boldsymbol{h},\boldsymbol{g}) \triangleq \max_{\{w_i(\boldsymbol{h},\boldsymbol{g})\}} \sum_{i=1}^{N} G_i\left(w_i(\boldsymbol{h},\boldsymbol{g})\right) \tag{3.7a}$$

$$\text{s.t.} \sum_{i=1}^{N} w_i(\boldsymbol{h},\boldsymbol{g}) \leq W \tag{3.7b}$$

where $G_i(w_i(\boldsymbol{h},\boldsymbol{g})) \triangleq w_i(\boldsymbol{h},\boldsymbol{g}) \log(1 + h_i p_i(\boldsymbol{h},\boldsymbol{g})/w_i(\boldsymbol{h},\boldsymbol{g}))$ is an increasing and concave function of $w_i(\boldsymbol{h},\boldsymbol{g})$. The problem (3.7a)–(3.7b) is similar to the classical water-filling power allocation problem. Thus, the optimal solution of the problem (3.7a)–(3.7b), denoted by $\{w_i'(\boldsymbol{h},\boldsymbol{g})\}$, must satisfy

$$\left.\frac{\partial G_i(w_i(\boldsymbol{h},\boldsymbol{g}))}{\partial w_i(\boldsymbol{h},\boldsymbol{g})}\right|_{w_i(\boldsymbol{h},\boldsymbol{g})=w_i'(\boldsymbol{h},\boldsymbol{g})} = \left.\frac{\partial G_j(w_j(\boldsymbol{h},\boldsymbol{g}))}{\partial w_j(\boldsymbol{h},\boldsymbol{g})}\right|_{w_j(\boldsymbol{h},\boldsymbol{g})=w_j'(\boldsymbol{h},\boldsymbol{g})}, \ \forall\, i \neq j. \tag{3.8}$$

Since we have

$$\begin{aligned}\left.\frac{\partial G_i(w_i(\boldsymbol{h},\boldsymbol{g}))}{\partial w_i(\boldsymbol{h},\boldsymbol{g})}\right|_{w_i(\boldsymbol{h},\boldsymbol{g})=w_i'(\boldsymbol{h},\boldsymbol{g})} &= \log\left(1 + \frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{w_i'(\boldsymbol{h},\boldsymbol{g})}\right) - \frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{w_i'(\boldsymbol{h},\boldsymbol{g}) + h_i p_i(\boldsymbol{h},\boldsymbol{g})} \\ &= Y\left(\frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{w_i'(\boldsymbol{h},\boldsymbol{g})}\right)\end{aligned} \tag{3.9}$$

where $Y(x) \triangleq \log(1+x) - x/(1+x)$ is a monotonically increasing function, we can obtain from (3.8) that

$$\frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{w_i'(\boldsymbol{h},\boldsymbol{g})} = \frac{h_j p_j(\boldsymbol{h},\boldsymbol{g})}{w_j'(\boldsymbol{h},\boldsymbol{g})}, \quad \forall\, i \neq j \tag{3.10}$$

It follows from (3.7b) that at optimality we have $\sum_{i=1}^{N} w_i'(\boldsymbol{h},\boldsymbol{g}) = W$. Furthermore, using (3.10), we can obtain that

$$w_i'(\boldsymbol{h},\boldsymbol{g}) = W \frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{\sum_{i=1}^{N} h_i p_i(\boldsymbol{h},\boldsymbol{g})}. \tag{3.11}$$

Substituting the optimal $w_i(\boldsymbol{h},\boldsymbol{g})$ given by (3.11) into (3.6), we can equivalently rewrite (3.6) as

$$\max_{\{p_i(\boldsymbol{h},\boldsymbol{g})\}\in\mathcal{F}'} \mathrm{E}\left\{ W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{W}\right)\right\} \tag{3.12}$$

where $\mathcal{F}'$ is a feasible set specified only by a particular combination of the power constraints $\{(3.2), (3.3), (3.4), (3.5)\}$. Therefore, the optimal power allocation obtained from the problem (3.6) and denoted by $\{p_i^*(\boldsymbol{h},\boldsymbol{g})\}$, can also be obtained by solving the equivalent problem (3.12). Then the optimal bandwidth allocation obtained from the problem (3.6) and denoted by $\{w_i^*(\boldsymbol{h},\boldsymbol{g})\}$, can be found as

$$w_i^*(\boldsymbol{h},\boldsymbol{g}) = W \frac{h_i p_i^*(\boldsymbol{h},\boldsymbol{g})}{\sum_{i=1}^{N} h_i p_i^*(\boldsymbol{h},\boldsymbol{g})}. \tag{3.13}$$

3.4 Optimal Power Allocation

In this section, we study the optimal power allocation obtained from the problem (3.12) with $\mathcal{F}'$ specified by different combinations of the power constraints.

3.4.1 Peak transmit power with peak interference power constraints

Consider $\mathcal{F}' = \{\text{the constraints (3.2) and (3.3)}\}$. Then the optimal value of the problem (3.12) can be expressed as $\mathrm{E}\{f_1(\boldsymbol{h},\boldsymbol{g})\}$, where $f_1(\boldsymbol{h},\boldsymbol{g})$ is given by

$$f_1(\boldsymbol{h},\boldsymbol{g}) \triangleq \max_{\{p_i(\boldsymbol{h},\boldsymbol{g})\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{W}\right) \tag{3.14a}$$

$$\text{s.t. } p_i(\boldsymbol{h},\boldsymbol{g}) \le P_i^{pk}, \ \forall\, i \tag{3.14b}$$

$$\sum_{i=1}^{N} g_i p_i(\boldsymbol{h},\boldsymbol{g}) \le Q^{pk}. \tag{3.14c}$$

For brevity, we drop the dependence on $\boldsymbol{h}$ and $\boldsymbol{g}$ that specifies instantaneous channel power gains. Also let $\{p_i^*\}$ denote the optimal solution of the problem (3.14a)–(3.14c). Introducing

$q_i \triangleq g_i p_i$, the problem (3.14a)–(3.14c) can be equivalently rewritten as

$$\max_{\{q_i\}} \sum_{i=1}^{N} \frac{h_i}{g_i} q_i \tag{3.15a}$$

$$\text{s.t. } q_i \leq g_i P_i^{pk}, \ \forall\, i \tag{3.15b}$$

$$\sum_{i=1}^{N} q_i \leq Q^{pk}. \tag{3.15c}$$

Let $\{q_i^*\}$ denote the optimal solution of the problem (3.15a)–(3.15c) and $(s_1, s_2, \cdots, s_N)$ denote a permutation of the SU indexes such that $h_{s_1}/g_{s_1} > h_{s_2}/g_{s_2} > \cdots > h_{s_N}/g_{s_N}$. It is assumed that $h_i/g_i \neq h_j/g_j$, $\forall i \neq j$, since h_i, g_i, h_j, and g_j are drawn from a continuous-valued random process. Then the following lemma is in order.

Lemma 3.1: *There exists k, $1 \leq k \leq N$, such that $q_{s_i}^* = g_{s_i} P_{s_i}^{pk}$, $\forall i$, $1 \leq i \leq k-1$, $0 < q_{s_k}^* \leq g_{s_k} P_{s_k}^{pk}$, and $q_{s_i}^* = 0$, $\forall i$, $k+1 \leq i \leq N$.*

Proof: Let $q_{s_j}^* > 0$ for some j and let $l < j$ for some l. First we prove that $q_{s_l}^* = g_{s_l} P_{s_l}^{pk}$ by contradiction. If $q_{s_l}^* < g_{s_l} P_{s_l}^{pk}$, then we can always find $\Delta q > 0$ and define a feasible solution $\{q'_{s_i}\}$ of the problem (3.15a)–(3.15c) $q'_{s_j} \triangleq q_{s_j}^* - \Delta q, q'_{s_l} \triangleq q_{s_l}^* + \Delta q, q'_{s_i} \triangleq q_{s_i}^*, \forall i, i \neq j, i \neq l$ such that the objective function in (3.15a) achieves larger value for $\{q'_{s_i}\}$ than for the optimal solution $\{q_i^*\}$, since we have

$$\sum_{i=1}^{N} \frac{h_{s_i}}{g_{s_i}} q'_{s_i} - \sum_{i=1}^{N} \frac{h_{s_i}}{g_{s_i}} q_{s_i}^* = \left(\frac{h_{s_l}}{g_{s_l}} - \frac{h_{s_j}}{g_{s_j}} \right) \Delta q > 0. \tag{3.16}$$

Therefore, it contradicts the fact that $\{q_{s_i}^*\}$ is the optimal solution of the problem (3.15a)–(3.15c).

Let $q_{s_j}^* < g_{s_j} P_{s_j}^{pk}$ for some j and let $l > j$ for some l. Using the result obtained above, it can be proved also by contradiction that $q_{s_l}^* = 0$. This completes the proof. $\square$

Lemma 3.1 shows that for the optimal power allocation under the constraints (3.2) and (3.3), as demonstrated in Fig 3.2, there exists at most one user that transmits at nonzero power and below its peak power, while any other user either does not transmit or transmits at its peak power.

Note that either the constraints (3.15b) or the constraint (3.15c) must be active at optimality. Using the structure of $\{q_i^*\}$ given in Lemma 3.1, k can be found by Algorithm 1.

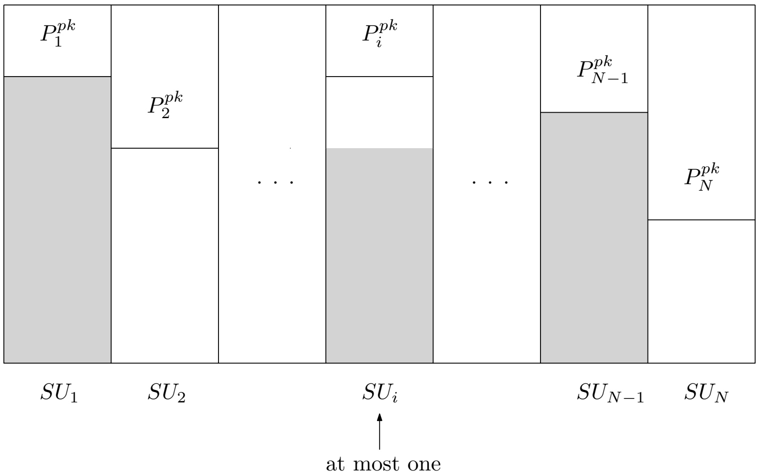


Fig. 3.2. Optimal power allocation under PTP + PIP constraints.

Algorithm 1 Algorithm for finding k in Lemma 3.1

Initialize: $k = 1$

while $\sum_{i=1}^{k} g_{s_i} P_{s_i}^{pk} < Q^{pk}$ and $k \leq N-1$ **do**

  $k = k + 1$

end while

Output: k

Since $p_{s_i}^* = q_{s_i}^* / g_{s_i}$, we obtain

$$p_{s_i}^* = \begin{cases} P_{s_i}^{pk}, & 1 \leq i \leq k-1 \\ \min\{P_{s_i}^{pk}, (Q^{pk} - \sum_{i=1}^{k-1} g_{s_i} P_{s_i}^{pk}) / g_{s_i}\}, & i = k \\ 0, & k+1 \leq i \leq N. \end{cases} \tag{3.17}$$

Note that for brevity, we say in this paper that $\sum_{i=1}^{n} x_i = 0$ if $n = 0$ with a little abuse of notation.

3.4.2 Average transmit power with average interference power constraints

Consider $\mathcal{F}' = \{$the constraints (3.4) and (3.5)$\}$. Then the dual function of the problem (3.12) can be written as

$$f_2(\{\lambda_i\}, \mu) \triangleq \mathrm{E}\left\{f_2'(\boldsymbol{h}, \boldsymbol{g})\right\} + \sum_{i=1}^{N} \lambda_i P_i^{av} + \mu Q^{av} \tag{3.18}$$

where $\{\lambda_i | 1 \le i \le N\}$ and μ are the nonnegative dual variables associated with the corresponding constraints in (3.4) and (3.5) and $f_2'(\boldsymbol{h}, \boldsymbol{g})$ is given by

$$f_2'(\boldsymbol{h}, \boldsymbol{g}) \triangleq \max_{\{p_i(\boldsymbol{h},\boldsymbol{g})\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{W}\right) - \sum_{i=1}^{N} \gamma_i p_i(\boldsymbol{h},\boldsymbol{g}) \tag{3.19}$$

with $\gamma_i \triangleq \lambda_i + \mu g_i$. Let $\{p_i^*\}$ denote the optimal solution of the problem (3.19), where we drop the dependence on $\boldsymbol{h}$ and $\boldsymbol{g}$ for brevity. Also let $F(\{p_i\})$ denote the objective function in (3.19). If $p_i^* > 0$ for some i, the following must hold

$$\left.\frac{\partial F(\{p_i\})}{\partial p_i}\right|_{\{p_i\}=\{p_i^*\}} = \frac{h_i}{1 + \sum_{i=1}^{N} \frac{h_i p_i^*}{W}} - \gamma_i = 0. \tag{3.20}$$

Then the following lemma is of interest.

Lemma 3.2: *If $h_i \le \gamma_i$ for some i, then $p_i^* = 0$.*

Proof: If $p_j^* = 0$, $\forall j$, then $p_i^* = 0$. If $p_j^* \ne 0$ for some j, it can be seen that (3.20) can not be satisfied since $h_i \le \gamma_i$. Thus, $p_i^* = 0$. ☐

If $p_i^* = 0$ for some i, the following must hold

$$\left.\frac{\partial F(\{p_i\})}{\partial p_i}\right|_{\{p_i\}=\{p_i^*\}} = \frac{h_i}{1 + \sum_{i=1}^{N} \frac{h_i p_i^*}{W}} - \gamma_i \le 0. \tag{3.21}$$

Then the next lemma is in order.

Lemma 3.3: *$p_i^* = 0$, $\forall i$, if and only if $h_i \le \gamma_i$, $\forall i$.*

Proof: It can be seen from Lemma 3.2 that if $h_i \le \gamma_i$, $\forall i$, then $p_i^* = 0$, $\forall i$. Moreover, it can be seen from (3.21) that if $p_i^* = 0$, $\forall i$, then $h_i \le \gamma_i$, $\forall i$. ☐

Let $(s_1, s_2, \cdots, s_N)$ denote a permutation of the SU indexes such that $h_{s_1}/\gamma_{s_1} > h_{s_2}/\gamma_{s_2} > \cdots > h_{s_N}/\gamma_{s_N}$. Then we can also prove the following lemma.

Lemma 3.4: *There exists at most one k such that $p_k^* > 0$. Moreover, $k = s_1$.*

Proof: We prove it by contradiction. It can be seen from (3.20) that if $p_i^* > 0$ and $p_j^* > 0$ for some $i \ne j$, the following must hold

$$\frac{h_i}{\gamma_i} = \frac{h_j}{\gamma_j}. \tag{3.22}$$

Since h_i, γ_i, h_j, and γ_j are independent constants given in the problem (3.19), (3.22) can not be satisfied. Let $p_k^* > 0$ and $p_i^* = 0$, $\forall i$, $i \ne k$. Then it follows from (3.20) and (3.21) that the following must hold

$$\frac{h_k}{\gamma_k} \geq \frac{h_i}{\gamma_i},\ \forall\ i \neq k. \tag{3.23}$$

Therefore, we must have $k = s_1$. □

Lemma 3.4 shows that for the optimal power allocation under the constraints (3.4) and (3.5), as demonstrated in Fig. 3.3, there exists at most one user that transmits at nonzero power, while any other user does not transmit.

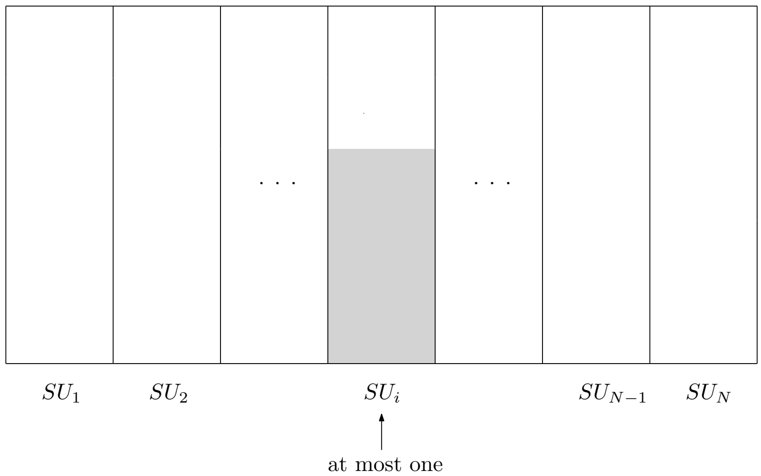


Fig. 3.3. Optimal power allocation under ATP + AIP constraints.

Case 1: Consider the case when $h_i \leq \gamma_i$, $\forall i$. It follows from Lemma 3.3 that $p_i^* = 0$, $\forall i$.

Case 2: Consider the case when $h_i \leq \gamma_i$ does not hold for some i. Using Lemma 3.4, let $p_k^* > 0$ and $p_i^* = 0$, $\forall i$, $i \neq k$. Substituting $\{p_i^*\}$ into (3.20), we have $p_{s_1}^* = W(1/\gamma_{s_1} - 1/h_{s_1})$. Therefore, we obtain

$$p_{s_i}^* = \begin{cases} W\left(1/\left(\lambda_{s_1} + \mu g_{s_1}\right) - 1/h_{s_1}\right), & i = 1 \\ 0, & 2 \leq i \leq N. \end{cases} \tag{3.24}$$

3.4.3 Peak transmit power with average interference power constraints

Consider $\mathcal{F}' = \{$the constraints (3.2) and (3.5)$\}$. Then the dual function of the problem (3.12) can be written as

$$f_3(\mu) \triangleq \mathrm{E}\left\{f_3'(\boldsymbol{h}, \boldsymbol{g})\right\} + \mu Q^{av} \tag{3.25}$$

where μ is the nonnegative dual variable associated with the constraint (3.5), and $f_3'(\boldsymbol{h}, \boldsymbol{g})$ is given by

$$f_3'(\boldsymbol{h}, \boldsymbol{g}) \triangleq \max_{\{p_i(\boldsymbol{h},\boldsymbol{g})\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i(\boldsymbol{h}, \boldsymbol{g})}{W}\right) - \mu \sum_{i=1}^{N} g_i p_i(\boldsymbol{h}, \boldsymbol{g}) \quad (3.26\text{a})$$

$$\text{s.t. } p_i(\boldsymbol{h}, \boldsymbol{g}) \le P_i^{pk}, \quad \forall\, i. \quad (3.26\text{b})$$

Let $\{p_i^*\}$ denote the optimal solution of the problem (3.26a)–(3.26b) after dropping the dependence on $\boldsymbol{h}$ and $\boldsymbol{g}$ for brevity. The following cases are of interest.

Case 1: Consider the case when $h_i \le \mu g_i$, $\forall i$. Since the problem (3.26a)–(3.26b) without the constraints (3.26b) has the same form as the problem (3.19), and $p_i = 0$, $\forall i$, satisfies the constraint (3.26b), it can be seen from Lemma 3.3 that $p_i^* = 0$, $\forall i$.

Case 2: Consider the case when $h_i \le \mu g_i$ does not hold for some i. The problem (3.26a)–(3.26b) is equivalent to

$$\max_{\{q_i\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i q_i}{\mu g_i W}\right) - \sum_{i=1}^{N} q_i \quad (3.27\text{a})$$

$$\text{s.t. } q_i \le \mu g_i P_i^{pk}, \ \forall\, i \quad (3.27\text{b})$$

where $q_i \triangleq \mu g_i p_i$. Let $\{q_i^*\}$ denote the optimal solution of the problem (3.27a)–(3.27b) and $(s_1, s_2, \cdots, s_N)$ denote a permutation of the SU indexes such that $h_{s_1}/\mu g_{s_1} > h_{s_2}/\mu g_{s_2} > \cdots > h_{s_N}/\mu g_{s_N}$. Then the following lemma is in order.

Lemma 3.5: *There exists k, $1 \le k \le N$, such that $q_{s_i}^* = g_{s_i} P_{s_i}^{pk}$, $\forall i$, $1 \le i \le k-1$, $0 < q_{s_k}^* \le g_{s_k} P_{s_k}^{pk}$, and $q_{s_i}^* = 0$, $\forall i$, $k+1 \le i \le N$.*

Proof: Consider the following intermediate problem

$$\max_{\{q_i\}} \sum_{i=1}^{N} \frac{h_i}{\mu g_i} q_i \quad (3.28\text{a})$$

$$\text{s.t. } q_i \le \mu g_i P_i^{pk}, \ \forall\, i \quad (3.28\text{b})$$

$$\sum_{i=1}^{N} q_i = Q \quad (3.28\text{c})$$

where Q is defined as $Q \triangleq \sum_{i=1}^{N} q_i^*$ and it is unknown since $\{q_i^*\}$ is unknown. Let $\{q_i'\}$ denote the optimal solution of the problem (3.28a)–(3.28c). If $\{q_i'\} \ne \{q_i^*\}$, we have $\sum_{i=1}^{N} h_i q_i'/\mu g_i \ge \sum_{i=1}^{N} h_i q_i^*/\mu g_i$ since $\{q_i^*\}$ is a feasible solution of the problem (3.28a)–(3.28c). Then we have

$$F\left(\{q_i'\}\right) - F\left(\{q_i^*\}\right) = W \log\left(1 + \sum_{i=1}^{N} \frac{h_i q_i'}{\mu g_i W}\right) - W \log\left(1 + \sum_{i=1}^{N} \frac{h_i q_i^*}{\mu g_i W}\right) \ge 0 \quad (3.29)$$

where $F(\{q_i\})$ denotes the objective function in the problem (3.27a)–(3.27b). Since $\{q_i'\}$ is a feasible solution of the problem (3.27a)–(3.27b), it contradicts the fact that $\{q_i^*\}$ is the optimal solution of the problem (3.27a)–(3.27b). Therefore, it must be true that $\{q_i'\} = \{q_i^*\}$.

It can be seen from the constraints (3.27b) that $\sum_{i=1}^{N} q_i' = \sum_{i=1}^{N} q_i^* = Q \leq \sum_{i=1}^{N} \mu g_i P_i^{pk}$. Then the problem (3.28a)–(3.28c) is equivalent to the following problem

$$\max_{\{q_i\}} \sum_{i=1}^{N} \frac{h_i}{\mu g_i} q_i \tag{3.30a}$$

$$\text{s.t. } q_i \leq \mu g_i P_i^{pk}, \ \forall \ i \tag{3.30b}$$

$$\sum_{i=1}^{N} q_i \leq Q \tag{3.30c}$$

since the constraint (3.30c) is active at optimality. Therefore, the problem (3.27a)–(3.27b) is equivalent to the problem (3.30a)–(3.30c). Since the problem (3.30a)–(3.30c) is similar to the problem (3.15a)–(3.15c) in Section 3.4.1, we conclude that $\{q_i^*\}$ has the same structure as that given in Lemma 3.1. □

The result of Lemma 3.5 is similar to that of Lemma 3.1. Specifically, it shows that for the optimal power allocation under the constraints (3.2) and (3.5), as demonstrated in Fig. 3.4, there exists at most one user that transmits at nonzero power and below its peak power, while any other user either does not transmit or transmits at its peak power.

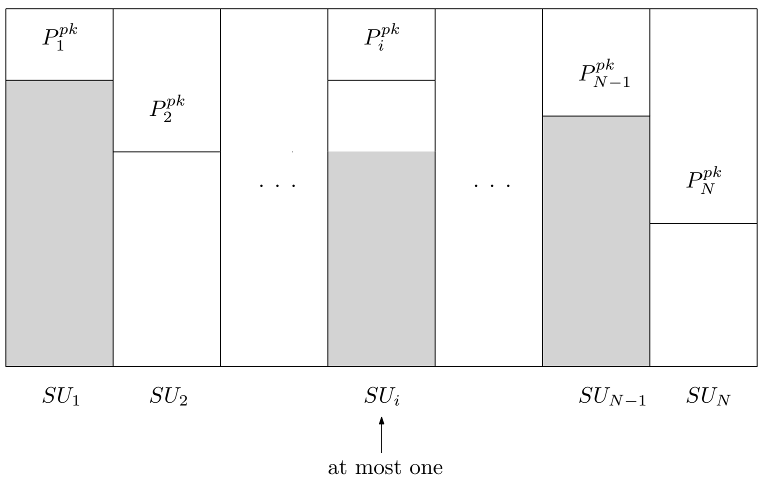


Fig. 3.4. Optimal power allocation under PTP + AIP constraints.

Using Lemma 3.5, let $q_{s_i}^* = \mu g_{s_i} P_{s_i}^{pk}$, $\forall i$, $1 \leq i \leq k-1$, $0 < q_{s_k}^* \leq \mu g_{s_i} P_{s_i}^{pk}$, and $q_{s_i}^* = 0$, $\forall i$, $k+1 \leq i \leq N$. Then we only need to find k and $q_{s_k}^*$ to determine $\{q_i^*\}$.

Consider the case when $0 < q_{s_k}^* < \mu g_{s_k} P_{s_k}^{pk}$, $1 \leq k \leq N$. Then the following must be true

$$\left.\frac{\partial H(q_{s_k})}{\partial q_{s_k}}\right|_{q_{s_k}=q_{s_k}^*} = \frac{\dfrac{h_{s_k}}{\mu g_{s_k}}}{1 + \displaystyle\sum_{i=1, i\neq k}^{N} \frac{h_{s_i} q_{s_i}^*}{\mu g_{s_i} W} + \frac{h_{s_k} q_{s_k}^*}{\mu g_{s_k} W}} - 1 = 0 \tag{3.31}$$

where

$$H(q_{s_k}) \triangleq W \log\left(1 + \sum_{i=1, i\neq k}^{N} \frac{h_{s_i} q_{s_i}^*}{\mu g_{s_i} W} + \frac{h_{s_k} q_{s_k}}{\mu g_{s_k} W}\right) - \sum_{i=1, i\neq k}^{N} q_{s_i}^* - q_{s_k}. \tag{3.32}$$

Substituting $\{q_{s_i}^*\}$ into (3.31), we obtain $q_{s_k}^* = W(1 - \mu g_{s_k}/h_{s_k}) - \mu g_{s_k} \sum_{i=1}^{k-1} h_{s_i} P_{s_i}^{pk}/h_{s_k}$. Since $q_{s_k}^*$ must satisfy $0 < q_{s_k}^* < \mu g_{s_i} P_{s_i}^{pk}$, it must be true that

$$\sum_{i=1}^{k-1} h_{s_i} P_{s_i}^{pk} < W\left(\frac{h_{s_k}}{\mu g_{s_k}} - 1\right) < \sum_{i=1}^{k} h_{s_i} P_{s_i}^{pk}. \tag{3.33}$$

Consider the case when $q_{s_k}^* = \mu g_{s_k} P_{s_k}^{pk}$, $1 \leq k \leq N-1$. Then the following must hold

$$\left.\frac{\partial H(q_{s_k})}{\partial q_{s_k}}\right|_{q_{s_k}=q_{s_k}^*} = \frac{\dfrac{h_{s_k}}{\mu g_{s_k}}}{1 + \displaystyle\sum_{i=1, i\neq k}^{N} \frac{h_{s_i} q_{s_i}^*}{\mu g_{s_i} W} + \frac{h_{s_k} q_{s_k}^*}{\mu g_{s_k} W}} - 1 \geq 0 \tag{3.34}$$

and

$$\left.\frac{\partial H(q_{s_{k+1}})}{\partial q_{s_{k+1}}}\right|_{q_{s_{k+1}}=q_{s_{k+1}}^*} = \frac{\dfrac{h_{s_{k+1}}}{\mu g_{s_{k+1}}}}{1 + \displaystyle\sum_{i=1, i\neq k+1}^{N} \frac{h_{s_i} q_{s_i}^*}{\mu g_{s_i} W} + \frac{h_{s_{k+1}} q_{s_{k+1}}^*}{\mu g_{s_{k+1}} W}} - 1 \leq 0. \tag{3.35}$$

Substituting $\{q_i^*\}$ into (3.34) and (3.35), we obtain

$$W\left(\frac{h_{s_{k+1}}}{\mu g_{s_{k+1}}} - 1\right) \leq \sum_{i=1}^{k} h_{s_i} P_{s_i}^{pk} \leq W\left(\frac{h_{s_k}}{\mu g_{s_k}} - 1\right),\ 1 \leq k \leq N-1. \tag{3.36}$$

If $q_{s_k}^* = \mu g_{s_k} P_{s_k}^{pk}$, $k = N$, then only (3.34) must be true and it follows that

$$\sum_{i=1}^{k} h_{s_i} P_{s_i}^{pk} \leq W\left(\frac{h_{s_k}}{\mu g_{s_k}} - 1\right),\ k = N. \tag{3.37}$$

Lemma 3.6: *There exists only one set of values for* $\{q_i^*\}$ *that satisfies only one of the necessary conditions* (3.31)*,* (3.34) *or* (3.35)*.*

Proof: It is equivalent to prove that there exists only one k that satisfies only one of (3.33), (3.36) or (3.37). Let $L_j \triangleq \sum_{i=1}^{j} h_{s_i} P_{s_i}^{pk}$ and $M_j \triangleq W(h_{s_j}/\mu g_{s_j} - 1)$ for brevity. Then it must be true that $L_0 < L_1 < \cdots < L_N$, $M_1 > M_2 > \cdots > M_N$ and $L_0 < M_1$. It can be seen that if (3.37) holds, i.e., if $L_i < M_i$, $\forall i$, $1 \leq i \leq N$, then (3.33) and (3.36) do not hold.

If (3.37) does not hold, then these exists such l that $L_i < M_i$, $\forall i$, $1 \leq i \leq l-1$ and $L_i > M_i$, $\forall i$, $1 \leq i \leq N$. The following two cases should be considered. (i) If $L_{l-1} < M_l < L_l$, (3.33) holds for $k = l$. Since $L_i < M_i$, $\forall i$, $1 \leq i \leq l-1$, (3.33) does not hold for $k < l$ as well. Since $M_i < M_l < L_l \leq L_{i-1}$, $\forall i$, $l+1 \leq i$, (3.33) does not hold for $k > l$. Since $L_i < L_{i+1} < M_{i+1}$, $\forall i$, $1 \leq i \leq l-2$, (3.36) does not hold for $k < l-1$. Since $L_{l-1} < M_l$, (3.36) does not hold also for $k = l-1$. Moreover, since $M_i < L_i$, $\forall i$, $l \leq i$, (3.36) does not hold for $k > l-1$. Therefore, only (3.33) holds for only $k = l$. (ii) If $M_l < L_{l-1} < M_{l-1}$, (3.36) holds for $k = l-1$. Similar to the case (i), it can be proved that only (3.36) holds for only $k = l-1$. This completes the proof. □

Using Lemma 3.6, Algorithm 2 is developed to find the unique k in Lemma 3.5. Note

Algorithm 2 Algorithm for finding k in Lemma 3.5

Initialize: $k = 0$, $c = 0$
while $c = 0$ **do**
  $k = k + 1$
  if $\sum_{i=1}^{k-1} h_{s_i} P_{s_i}^{pk} < W(h_{s_k}/\mu g_{s_k} - 1) < \sum_{i=1}^{k} h_{s_i} P_{s_i}^{pk}$ **then**
    $c = 1$
  end if
  if $\{W(h_{s_{k+1}}/\mu g_{s_{k+1}} - 1) \leq \sum_{i=1}^{k} h_{s_i} P_{s_i}^{pk} \leq W(h_{s_k}/\mu g_{s_k} - 1)$ and $k \leq N-1\}$ or $\{\sum_{i=1}^{k} h_{s_i} P_{s_i}^{pk} \leq W(h_{s_k}/\mu g_{s_k} - 1)$ and $k = N\}$ **then**
    $c = 2$
  end if
end while
Output: k, c

that k satisfies (3.33) and (3.36) or (3.37) if the output of Algorithm 2 is $c = 1$ and $c = 2$, respectively. Since $p^*_{s_i} = q^*_{s_i}/\mu g_{s_i}$, when $c = 1$, we obtain

$$p^*_{s_i} = \begin{cases} P^{pk}_{s_i}, & 1 \le i \le k-1 \\ W(1/\mu g_{s_k} - 1/h_{s_k}) - \sum_{i=1}^{k-1} h_{s_i} P^{pk}_{s_i}/h_{s_k}, & i = k \\ 0, & k+1 \le i \le N \end{cases}, \quad 1 \le i \le N \tag{3.38}$$

and when $c = 2$, we obtain

$$p^*_{s_i} = \begin{cases} P^{pk}_{s_i}, & 1 \le i \le k \\ 0, & k+1 \le i \le N \end{cases}, \quad 1 \le i \le N. \tag{3.39}$$

3.4.4 Average transmit power with peak interference power constraints

Consider $\mathcal{F}' = \{\text{the constraints (3.3) and (3.4)}\}$. Then the dual function of the problem (3.12) can be written as

$$f_4(\{\lambda_i\}) \triangleq \mathrm{E}\left\{f'_4(\boldsymbol{h}, \boldsymbol{g})\right\} + \sum_{i=1}^{N} \lambda_i P^{av}_i \tag{3.40}$$

where $\{\lambda_i | 1 \le i \le N\}$ are the nonnegative dual variables associated with the corresponding constraints (3.4) and $f'_4(\boldsymbol{h}, \boldsymbol{g})$ is given by

$$f'_4(\boldsymbol{h}, \boldsymbol{g}) \triangleq \max_{\{p_i(\boldsymbol{h},\boldsymbol{g})\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i(\boldsymbol{h}, \boldsymbol{g})}{W}\right) - \sum_{i=1}^{N} \lambda_i p_i(\boldsymbol{h}, \boldsymbol{g}) \tag{3.41a}$$

$$\text{s.t.} \sum_{i=1}^{N} g_i p_i(\boldsymbol{h}, \boldsymbol{g}) \le Q^{pk}. \tag{3.41b}$$

Let $\{p^*_i\}$ denote the optimal solution of the problem (3.41a)–(3.41b) where the dependence on $\boldsymbol{h}$ and $\boldsymbol{g}$ is dropped for brevity. The following three cases are of interest.

Case 1: Consider the case when $h_i \le \lambda_i$, $\forall i$. Similar to Case 1 in Section 3.4.3, it can be seen from Lemma 3.3 that $p^*_i = 0$, $\forall i$.

Case 2: Consider the case when $h_i \le \lambda_i$ does not hold for some i and the constraint (3.41b) is inactive at optimality. Let $(s_1, s_2, \cdots, s_N)$ denote a permutation of the SU indexes such that $h_{s_1}/\lambda_{s_1} > h_{s_2}/\lambda_{s_2} > \cdots > h_{s_N}/\lambda_{s_N}$. Since the problem (3.41a)–(3.41b) without the constraint (3.41b) has the same form as the problem (3.19), it can be seen from (3.24) that $p^*_{s_1} = W(1/\lambda_{s_1} - 1/h_{s_1})$ and $p^*_{s_i} = 0$, $\forall i$, $2 \le i \le N$, if it satisfies the constraint (3.41b), i.e., $\sum_{i=1}^{N} g_{s_i} p^*_{s_i} = g_{s_1} W(1/\lambda_{s_1} - 1/h_{s_1}) < Q^{pk}$.

Case 3: Consider the case when $h_i \leq \lambda_i$ does not hold for some i and the constraint (3.41b) is active at optimality, i.e., $g_{s_1} W(1/\lambda_{s_1} - 1/h_{s_1}) \geq Q^{pk}$. The dual function of the problem (3.41a)–(3.41b) can be written as $f_4''(\mu) \triangleq f_4''' + \mu Q^{pk}$, where μ is the nonnegative dual variable associated with the constraint (3.41b), and f_4''' is given by

$$f_4''' \triangleq \max_{\{p_i\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i}{W}\right) - \sum_{i=1}^{N} \lambda_i p_i - \mu \sum_{i=1}^{N} g_i p_i. \tag{3.42}$$

Let μ^* denote the optimal dual variable. Also let $F(\{p_i\})$ denote the objective function in the problem (3.42). If $p_i^* > 0$ for some i, the following must hold

$$\left.\frac{\partial F(\{p_i\})}{\partial p_i}\right|_{\{p_i\}=\{p_i^*\}} = \frac{h_i}{1 + \sum_{i=1}^{N} \frac{h_i p_i^*}{W}} - \lambda_i - \mu^* g_i = 0. \tag{3.43}$$

If $p_i^* = 0$ for some i, the following must hold

$$\left.\frac{\partial F(\{p_i\})}{\partial p_i}\right|_{\{p_i\}=\{p_i^*\}} = \frac{h_i}{1 + \sum_{i=1}^{N} \frac{h_i p_i^*}{W}} - \lambda_i - \mu^* g_i \leq 0. \tag{3.44}$$

Note that since the problem (3.41a)–(3.41b) is convex, the necessary conditions (3.43) and (3.44) for the optimal solution $\{p_i^*\}$ are also sufficient conditions.

Lemma 3.7: *There exists at most two $j \neq k$ such that $p_j^* > 0$ and $p_k^* > 0$.*

Proof: We prove it by contradiction. It can be seen from (3.43) that if $p_i^* > 0$, $p_j^* > 0$, and $p_k^* > 0$ for some $i \neq j$, $j \neq k$, $i \neq k$, the following must hold

$$\frac{h_i}{\lambda_i + \mu^* g_i} = \frac{h_j}{\lambda_j + \mu^* g_j} = \frac{h_k}{\lambda_k + \mu^* g_k}. \tag{3.45}$$

Since h_i, λ_i, g_i, h_j, λ_j, g_j, h_k, λ_k, and g_k are independent constants given in the problem (3.41a)–(3.41b), and only μ^* is a variable, (3.45) can not be satisfied. □

Lemma 3.7 shows that for the optimal power allocation under the constraints (3.3) and (3.4), as demonstrated in Fig. 3.5, there exists at most two users that transmit at nonzero power, while any other user does not transmit.

Then Case 3 can be further divided into the following two subcases.

Case 3.1: Consider the subcase when $p_k^* > 0$ and $p_i^* = 0$, $\forall i \neq k$. Since the constraint (3.41b) is active at optimality, i.e., $\sum_{i=1}^{N} g_i p_i^* = g_k p_k^* = Q^{pk}$, we obtain that $p_k^* = Q^{pk}/g_k$. Then substituting $\{p_i^*\}$ into (3.43) we have

$$\mu^* = \frac{1}{g_k/h_k + Q^{pk}/W} - \frac{\lambda_k}{g_k}. \tag{3.46}$$

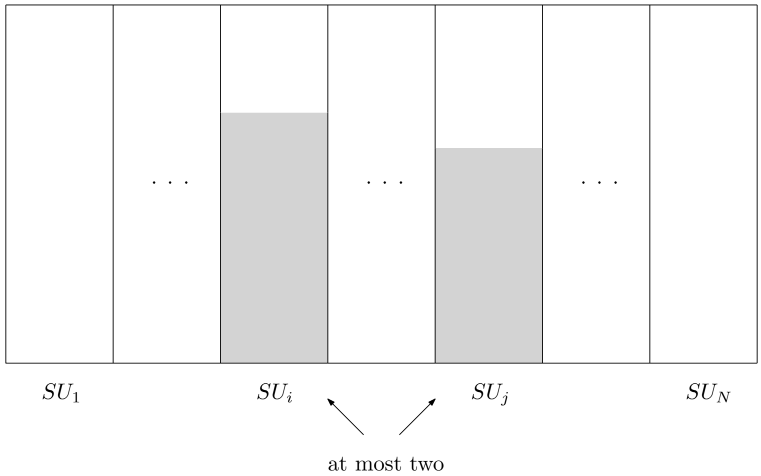


Fig. 3.5. Optimal power allocation under ATP + PIP constraints.

Note that μ^* given in (3.46) must satisfy $\mu^* \geq 0$. Substituting $\{p_i^*\}$ into (3.44), we can see that μ^* given in (3.46) also must satisfy

$$\mu^* \geq \frac{h_i/g_i}{1 + h_k Q^{pk}/g_k W} - \frac{\lambda_i}{g_i}, \quad \forall\, i, i \neq k. \tag{3.47}$$

Then Algorithm 3 can be used to find k. Note that $\{p_i^*\}$ does not exist in Case 3.1 if the

---

Algorithm 3 Algorithm for finding k in Case 3.1

$k = \arg\max_{\{i\}} W \log\left(1 + \frac{h_i Q^{pk}}{g_i W}\right) - \frac{\lambda_i Q^{pk}}{g_i}$

$\mu^* = \frac{1}{g_k/h_k + Q^{pk}/W} - \frac{\lambda_k}{g_k}$

if $\mu^* < \max_{\{i \neq k\}} \frac{h_i/g_i}{1 + h_k Q^{pk}/g_k W} - \frac{\lambda_i}{g_i}$ or $\mu^* < 0$ **then**

  $k = 0$

end if

Output: k

---

output of Algorithm 3 is $k = 0$.

Case 3.2: Consider the subcase when $p_j^* > 0$, $p_k^* > 0$, $j \neq k$ and $p_i^* = 0$, $\forall i, i \neq j, i \neq k$. It follows from (3.43) that

$$\frac{h_j}{\lambda_j + \mu^* g_j} = \frac{h_k}{\lambda_k + \mu^* g_k}. \tag{3.48}$$

Therefore, we obtain that

$$\mu^* = \frac{\lambda_j/h_j - \lambda_k/h_k}{g_k/h_k - g_j/h_j}. \tag{3.49}$$

Note that μ^* given in (3.49) must satisfy $\mu^* \geq 0$. Using (3.43) and the fact that the constraint (3.41b) is active at optimality, we have

$$\begin{cases} h_j p_j^* + h_k p_k^* = W h_j / (\lambda_j + \mu^* g_j) - W \\ g_j p_j^* + g_k p_k^* = Q^{pk}. \end{cases} \tag{3.50}$$

Solving the system of equation (3.50), we obtain

$$p_j^* = \frac{Q^{pk}/g_k - a/h_k}{g_j/g_k - h_j/h_k}, \quad p_k^* = \frac{a/h_j - Q^{pk}/g_j}{h_k/h_j - g_k/g_j} \tag{3.51}$$

where $a \triangleq W h_j / (\lambda_j + \mu^* g_j) - W$. Note that p_j^* and p_k^* given in (3.51) must satisfy $p_j^* > 0$ and $p_k^* > 0$. Substituting $\{p_i^*\}$ and μ^* into (3.44), we can see that j and k must satisfy

$$\frac{\lambda_j/h_j - \lambda_k/h_k}{g_k/h_k - g_j/h_j} \geq \frac{\lambda_j/h_j - \lambda_i/h_i}{g_i/h_i - g_j/h_j}, \quad \forall\, i, i \neq j, i \neq k. \tag{3.52}$$

Then Algorithm 4 can be used to find j and k. Note that $\{p_i^*\}$ does not exist if the output of Algorithm 4 is $j = 0$ and $k = 0$.

3.4.5 Combinations of more than two power constraints

Consider $\mathcal{F}' = \{$the constraints (3.2), (3.4), and (3.5)$\}$ or $\mathcal{F}' = \{$the constraints (3.3), (3.4), and(3.5)$\}$. It can be shown that the corresponding dual functions of the problem (3.12) under these two combinations of the power constraints have the same form as those in Subsections 3.4.3 and 3.4.4, respectively. Therefore, optimal solutions can be found similarly therein and, thus, are omitted here.

Consider $\mathcal{F}' = \{$the constraints (3.2), (3.3), and (3.4)$\}$ or $\mathcal{F}' = \{$the constraints(3.2), (3.3), and(3.5)$\}$ or $\mathcal{F}' = \{$the constraints (3.2), (3.3), (3.4), and (3.5)$\}$. It can be shown that the corresponding dual functions of the problem (3.12) under the first two combinations of the power constraints have the same form as that under the third combination. Therefore, we only focus on $\mathcal{F}' = \{$the constraints(3.2), (3.3), (3.4), and (3.5)$\}$. Then the dual function of the problem (3.12) can be written as

$$f_5(\{\lambda_i\}, \mu) \triangleq \mathrm{E}\left\{f_5'(\boldsymbol{h}, \boldsymbol{g})\right\} + \sum_{i=1}^{N} \lambda_i P_i^{av} + \mu Q^{av} \tag{3.53}$$

Algorithm 4 Algorithm for finding j and k in Case 3.2

Initialize: $\mathcal{I} = \varnothing$

for $j = 1, \cdots, N-1$ **do**

  for $k = j+1, \cdots, N$ **do**

    $\mu^* = \frac{\lambda_j/h_j - \lambda_k/h_k}{g_k/h_k - g_j/h_j}$

    if $\mu^* \geq 0$ **then**

      $a = Wh_j/(\lambda_j + \mu^* g_j) - W$

      $p_j^* = \frac{Q^{pk}/g_k - a/h_k}{g_j/g_k - h_j/h_k}$, $p_k^* = \frac{a/h_j - Q^{pk}/g_j}{h_k/h_j - g_k/g_j}$

      if $p_j^* > 0$ and $p_k^* > 0$ **then**

        $\mathcal{I} = \mathcal{I} \cup \{(j,k)\}$

        $v_{j,k} = W \log\left(1 + \frac{h_j p_j^* + h_k p_k^*}{W}\right) - \lambda_j p_j^* - \lambda_k p_k^*$

      end if

    end if

  end for

end for

$(j,k) = \arg\max_{\{(i,l)\in\mathcal{I}\}} v_{i,l}$

if $\frac{\lambda_j/h_j - \lambda_k/h_k}{g_k/h_k - g_j/h_j} < \max_{\{i \neq j,k\}} \frac{\lambda_j/h_j - \lambda_i/h_i}{g_i/h_i - g_j/h_j}$ **then**

  $j = 0$, $k = 0$

end if

Output: j, k

where $\{\lambda_i | 1 \leq i \leq N\}$ and μ are the nonnegative dual variables associated with the corresponding constraints in (3.4) and (3.5) and $f_5'(\boldsymbol{h}, \boldsymbol{g})$ is given by

$$f_5'(\boldsymbol{h},\boldsymbol{g}) \triangleq \max_{\{p_i(\boldsymbol{h},\boldsymbol{g})\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i(\boldsymbol{h},\boldsymbol{g})}{W}\right) - \sum_{i=1}^{N} \lambda_i p_i(\boldsymbol{h},\boldsymbol{g}) - \mu \sum_{i=1}^{N} g_i p_i(\boldsymbol{h},\boldsymbol{g}) \quad (3.54a)$$

$$\text{s.t.} \sum_{i=1}^{N} g_i p_i(\boldsymbol{h},\boldsymbol{g}) \leq Q^{pk} \quad (3.54b)$$

$$p_i(\boldsymbol{h},\boldsymbol{g}) \leq P_i^{pk}, \ \forall\, i. \quad (3.54c)$$

Let $\{p_i^*\}$ denote the optimal solution of the problem (3.54a)–(3.54c) where the dependence on $\boldsymbol{h}$ and $\boldsymbol{g}$ is dropped for brevity. The following cases are of interest.

Case 1: Consider the case when $h_i \leq \lambda_i + \mu g_i$, $\forall i$. Similar to Case 1 in Subsections 3.4.3

and 3.4.4, it can be seen from Lemma 3.3 that $p_i^* = 0$, $\forall i$.

Case 2: Consider the case when $h_i \leq \lambda_i + \mu g_i$ does not hold for some i and the constraint (3.54b) is inactive at optimality. Since the problem (3.54a)–(3.54c) without the constraint (3.54b) has the same form as the problem (3.26a)–(3.26b), $\{p_i^*\}$ can be found using Algorithm 2 and (3.38) or (3.39) if it satisfies the constraint (3.54b).

Case 3: Consider the case when $h_i \leq \lambda_i + \mu g_i$ does not hold for some i and the constraint (3.54b) is active at optimality. The dual function of the problem (3.54a)–(3.54c) can be written as $f_5''(\beta) \triangleq f_5''' + \beta Q^{pk}$, where β is the nonnegative dual variable associated with the constraint (3.54b) and f_5''' is given by

$$f_5''' \triangleq \quad \max_{\{p_i\}} W \log\left(1 + \sum_{i=1}^{N} \frac{h_i p_i}{W}\right) - \sum_{i=1}^{N} \gamma_i p_i - \beta \sum_{i=1}^{N} g_i p_i \tag{3.55a}$$

$$\text{s.t. } p_i \leq P_i^{pk}, \ \forall\, i. \tag{3.55b}$$

where $\gamma_i \triangleq \lambda_i + \mu g_i$. Let β^* denote the optimal dual variable and $F(\{p_i\})$ stands for the objective function in the problem (3.55a). If $P_i^{pk} > p_i^* > 0$ for some i, the following must hold

$$\left.\frac{\partial F(\{p_i\})}{\partial p_i}\right|_{\{p_i\}=\{p_i^*\}} = \frac{h_i}{1 + \sum_{i=1}^{N} \frac{h_i p_i^*}{W}} - \gamma_i - \beta^* g_i = 0. \tag{3.56}$$

If $p_i^* = P_i^{pk}$ for some i, the following must hold

$$\left.\frac{\partial F(\{p_i\})}{\partial p_i}\right|_{\{p_i\}=\{p_i^*\}} = \frac{h_i}{1 + \sum_{i=1}^{N} \frac{h_i p_i^*}{W}} - \gamma_i - \beta^* g_i \geq 0. \tag{3.57}$$

Moreover, if $p_i^* = 0$ for some i, the following must hold

$$\left.\frac{\partial F(\{p_i\})}{\partial p_i}\right|_{\{p_i\}=\{p_i^*\}} = \frac{h_i}{1 + \sum_{i=1}^{N} \frac{h_i p_i^*}{W}} - \gamma_i - \beta^* g_i \leq 0. \tag{3.58}$$

Note that since the problem (3.54a)–(3.54c) is convex, the necessary conditions (3.56), (3.57) and (3.58) for the optimal solution $\{p_i^*\}$ are also sufficient conditions.

Lemma 3.8: *There exists at most two j and k, $j \neq k$ such that $P_j^{pk} > p_j^* > 0$ and $P_k^{pk} > p_k^* > 0$.*

Proof: We prove it by contradiction. It can be seen from (3.56) that if $P_i^{pk} > p_i^* > 0$, $P_j^{pk} > p_j^* > 0$, and $P_k^{pk} > p_k^* > 0$ for some $i \neq j$, $j \neq k$, $i \neq k$, the following must be true

$$\frac{h_i}{\gamma_i + \beta^* g_i} = \frac{h_j}{\gamma_j + \beta^* g_j} = \frac{h_k}{\gamma_k + \beta^* g_k}. \tag{3.59}$$

Since h_i, γ_i, g_i, h_j, γ_j, g_j, h_k, γ_k, and g_k are independent constants given in the problem (3.54a)–(3.54c), and only β^* is a variable, (3.59) can not be satisfied. □

Lemma 3.8 shows that for the optimal power allocation under the constraints (3.2), (3.3), (3.4) and (3.5), as demonstrated in Fig. 3.6, there exists at most two user that transmit at nonzero power and below their peak power, while any other user either does not transmit or transmits at its peak power.

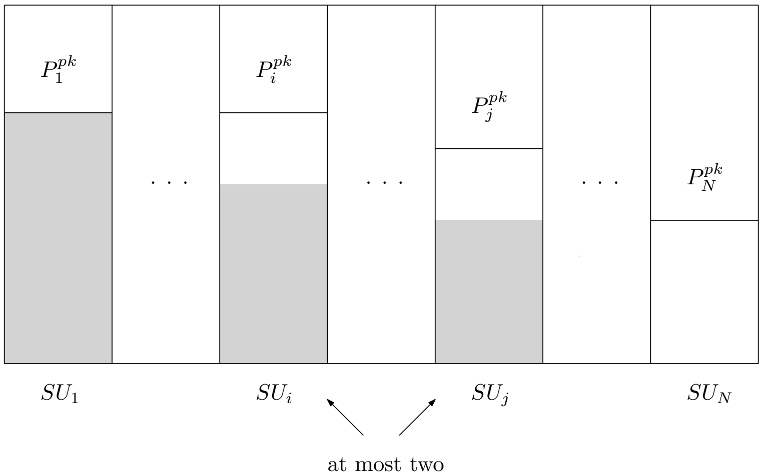


Fig. 3.6. Optimal power allocation under PTP + PIP + ATP + AIP constraints.

Then Case 3 can be further divided into the following two subcases.

Case 3.1: Consider the subcase when $P_k^{pk} > p_k^* > 0$ and $p_i^* \in \{P_i^{pk}, 0\}$, $\forall i \neq k$. Let $\mathcal{N}_1$ and $\mathcal{N}_0$ denote the sets of SU indexes such that $p_i^* = P_i^{pk}, \forall i \in \mathcal{N}_1$ and $p_i^* = 0, \forall i \in \mathcal{N}_0$. Since the constraint (3.54b) is active at optimality, i.e., $\sum_{i=1}^N g_i p_i^* = g_k p_k^* + \sum_{i \in \mathcal{N}_1} g_i P_i^{pk} = Q^{pk}$, we obtain $p_k^* = (Q^{pk} - \sum_{i \in \mathcal{N}_1} g_i P_i^{pk})/g_k$. Note that p_k^* given here must satisfy $P_k^{pk} > p_k^* > 0$. Then substituting $\{p_i^*\}$ into (3.56) we obtain

$$\beta^* = \frac{h_k/g_k}{1 + \left(h_k(Q^{pk} - \sum_{i \in \mathcal{N}_1} g_i P_i^{pk})/g_k + \sum_{i \in \mathcal{N}_1} h_i P_i^{pk}\right)/W} - \frac{\gamma_k}{g_k}. \tag{3.60}$$

Note that β^* given by (3.60) must satisfy $\beta^* \geq 0$. Substituting $\{p_i^*\}$ into (3.57) we can see that β^* given by (3.60) must satisfy

$$\beta^* \leq \frac{h_i/g_i}{1+\left(h_k(Q^{pk}-\sum_{i\in\mathcal{N}_1} g_i P_i^{pk})/g_k+\sum_{i\in\mathcal{N}_1} h_i P_i^{pk}\right)/W} - \frac{\gamma_i}{g_i},\ \forall\ i \in \mathcal{N}_1. \tag{3.61}$$

Substituting $\{p_i^*\}$ into (3.58), we can see that β^* given in (3.60) also must satisfy

$$\beta^* \geq \frac{h_i/g_i}{1+\left(h_k(Q^{pk}-\sum_{i\in\mathcal{N}_1} g_i P_i^{pk})/g_k+\sum_{i\in\mathcal{N}_1} h_i P_i^{pk}\right)/W} - \frac{\gamma_i}{g_i},\ \forall\ i \in \mathcal{N}_0. \tag{3.62}$$

Let $\mathcal{S}_i^{(1)}, \mathcal{S}_i^{(2)}, \cdots, \mathcal{S}_i^{(2^{N-1})}$ denote all the subsets of the set $\mathcal{N}\backslash\{i\}$ where $\backslash$ denotes the set difference operator. Then Algorithm 5 can be used to find k, $\mathcal{N}_1$, and $\mathcal{N}_0$. Note that $\{p_i^*\}$ does not exist if the output of Algorithm 5 is $k=0$.

Case 3.2: Consider the subcase when $P_j^{pk} > p_j^* > 0$, $P_k^{pk} > p_k^* > 0$ and $p_i^* \in \{P_i^{pk}, 0\}$, $\forall i \neq j,k$. Let $\mathcal{N}_1$ and $\mathcal{N}_0$ denote the sets of SU indexes such that $p_i^* = P_i^{pk}$, $\forall i \in \mathcal{N}_1$ and $p_i^* = 0$, $\forall i \in \mathcal{N}_0$, respectively. It follows from (3.56) that

$$\frac{h_j}{\gamma_j+\beta^* g_j} = \frac{h_k}{\gamma_k+\beta^* g_k}. \tag{3.63}$$

Therefore, we obtain that

$$\beta^* = \frac{\gamma_j/h_j - \gamma_k/h_k}{g_k/h_k - g_j/h_j}. \tag{3.64}$$

Note that β^* given in (3.64) must satisfy $\beta^* \geq 0$. Following (3.56) and the fact that the constraint (3.54b) is active at optimality, we have

$$\begin{cases} h_j p_j^* + h_k p_k^* = W h_j/(\gamma_j+\beta^* g_j) - W - \sum_{i\in\mathcal{N}_1} h_i P_i^{pk} \\ g_j p_j^* + g_k p_k^* = Q^{pk} - \sum_{i\in\mathcal{N}_1} g_i P_i^{pk}. \end{cases} \tag{3.65}$$

Solving the system of equation (3.65), we obtain

$$p_j^* = \frac{a/g_k - b/h_k}{g_j/g_k - h_j/h_k}, \quad p_k^* = \frac{b/h_j - a/g_j}{h_k/h_j - g_k/g_j} \tag{3.66}$$

where $a \triangleq Q^{pk} - \sum_{i\in\mathcal{N}_1} g_i P_i^{pk}$ and $b \triangleq W h_j/(\gamma_j+\beta^* g_j) - W - \sum_{i\in\mathcal{N}_1} h_i P_i^{pk}$. Note that p_j^* and p_k^* given in (3.66) must satisfy $P_j^{pk} > p_j^* > 0$ and $P_k^{pk} > p_k^* > 0$. Substituting $\{p_i^*\}$ and β^* given by (3.64) into (3.57), we obtain

$$\frac{\gamma_j/h_j - \gamma_k/h_k}{g_k/h_k - g_j/h_j} \leq \frac{\gamma_j/h_j - \gamma_i/h_i}{g_i/h_i - g_j/h_j}, \quad \forall\ i \in \mathcal{N}_1. \tag{3.67}$$

Algorithm 5 Algorithm for finding k, $\mathcal{N}_1$, $\mathcal{N}_0$ in Case 3.1

Initialize: $\mathcal{I} = \varnothing$

for $k = 1, 2, \cdots, N$ **do**

  for $l = 1, 2, \cdots, 2^{N-1}$ **do**

    $\mathcal{N}_1 = \mathcal{S}_k^{(l)}$

    $p_k^* = (Q^{pk} - \sum_{i \in \mathcal{N}_1} g_i P_i^{pk})/g_k$

    if $P_k^{pk} > p_k^* > 0$ **then**

      $\mathcal{I} = \mathcal{I} \cup \{l\}$

      $r_l = W \log\left(1 + \frac{h_k p_k^* + \sum_{i \in \mathcal{N}_1} h_i P_i^{pk}}{W}\right) - \gamma_k p_k^* - \sum_{i \in \mathcal{N}_1} \gamma_i P_i^{pk}$

    end if

  end for

  $v_k = \max_{\{i \in \mathcal{I}\}} r_i$, $t = \arg\max_{\{i \in \mathcal{I}\}} r_i$

  $\mathcal{S}_k^* = \mathcal{S}_k^{(t)}$

  $\mathcal{I} = \varnothing$

end for

$k = \arg\max_{\{i\}} v_i$

$\mathcal{N}_1 = \mathcal{S}_k^*$

$\mathcal{N}_0 = \mathcal{N} \backslash \mathcal{N}_1 \backslash \{k\}$

$\beta^* = \frac{h_k/g_k}{1+\left(h_k(Q^{pk} - \sum_{i \in \mathcal{N}_1} g_i P_i^{pk})/g_k + \sum_{i \in \mathcal{N}_1} h_i P_i^{pk}\right)/W} - \frac{\gamma_k}{g_k}$

if $\beta^* < 0$ or $\beta^* > \frac{h_i/g_i}{1+\left(h_k(Q^{pk} - \sum_{i \in \mathcal{N}_1} g_i P_i^{pk})/g_k + \sum_{i \in \mathcal{N}_1} h_i P_i^{pk}\right)/W} - \frac{\gamma_i}{g_i}, \exists i \in \mathcal{N}_1$
or $\beta^* < \frac{h_i/g_i}{1+\left(h_k(Q^{pk} - \sum_{i \in \mathcal{N}_1} g_i P_i^{pk})/g_k + \sum_{i \in \mathcal{N}_1} h_i P_i^{pk}\right)/W} - \frac{\gamma_i}{g_i}, \exists i \in \mathcal{N}_0$ **then**

  $k = 0$

end if

Output: k, $\mathcal{N}_1$, $\mathcal{N}_0$

Moreover, substituting $\{p_i^*\}$ and β^* given by (3.64) into (3.58), we also obtain

$$\frac{\gamma_j/h_j - \gamma_k/h_k}{g_k/h_k - g_j/h_j} \geq \frac{\gamma_j/h_j - \gamma_i/h_i}{g_i/h_i - g_j/h_j}, \quad \forall\, i \in \mathcal{N}_0. \tag{3.68}$$

Let $\mathcal{S}_{i,j}^{(1)}, \mathcal{S}_{i,j}^{(2)}, \cdots, \mathcal{S}_{i,j}^{(2^{N-2})}$ denote all the subsets of the set $\mathcal{N}\backslash\{i,j\}$. Then Algorithm 6 can be used to find j, k, $\mathcal{N}_1$, and $\mathcal{N}_0$. Note that $\{p_i^*\}$ does not exist if the output of Algorithm 6 is $j = 0$ and $k = 0$.

3.5 Simulation Results

Consider a cognitive radio network which consists of one PU and four SUs. For simplicity, we assume that only Rayleigh fading is present in all links. The variance of the channel power gain is set to $\sigma^2 = 1$. We also set $W = 1$, $P_i^{pk} = 10$, $\forall i$, $P_i^{av} = 10$, $\forall i$, $Q^{pk} = 1$, and $Q^{av} = 1$ as default values if no other values are specified otherwise. The AWGN with unit PSD is assumed. We use 1000 randomly generated channel power gains for $\boldsymbol{h}$ and $\boldsymbol{g}$ in our simulations. The results are compared under the following five combinations of the power constraints: the PTP with PIP constraints (PTP+PIP), the PTP with AIP constraints (PTP+AIP), the ATP with PIP constraints (ATP+PIP), the ATP with AIP constraints (ATP+AIP), the PTP and ATP with PIP and AIP constraints (PTP+ATP+PIP+AIP).

Fig. 3.7 shows the maximum sum ergodic capacity under PTP+PIP, PTP+AIP and PTP+ATP+
PIP+AIP constraints versus P^{pk} where $P^{pk} = P_i^{pk}$, $\forall i$ is assumed. It can be seen from the figure that the maximum sum ergodic capacity achieved under PTP+AIP is larger than that achieved under PTP+PIP for any given P^{pk}. This is due to the fact that the AIP constraint is more favorable than the PIP constraint from SUs' perspective, since the former allows for more flexibility for SUs to allocate transmit power over different channel fading states. It is also observed that the performance under PTP+ATP+PIP+AIP is very close to that under PTP+PIP that is because the PTP constraint dominates over the ATP, PIP, and AIP constraints for all values of P^{pk}.

Fig. 3.8 shows the maximum sum ergodic capacity under ATP+PIP, ATP+AIP and PTP+ATP+
PIP+AIP constraints versus P^{av} where $P^{av} = P_i^{av}$, $\forall i$ is assumed. The maximum achiev-

Algorithm 6 Algorithm for finding j, k, $\mathcal{N}_1$, $\mathcal{N}_0$ in Case 3.2

Initialize: $\mathcal{I} = \varnothing$

for $j = 1, 2, \cdots, N-1$ **do**

  for $k = j+1, \cdots, N$ **do**

    for $l = 1, 2, \cdots, 2^{N-2}$ **do**

      $\mathcal{N}_1 = \mathcal{S}_{j,k}^{(l)}$

      $\beta^* = \frac{\gamma_j/h_j - \gamma_k/h_k}{g_k/h_k - g_j/h_j}$

      if $\beta^* \geq 0$ **then**

        $a \triangleq Q^{pk} - \sum_{i\in\mathcal{N}_1} g_i P_i^{pk}$, $b \triangleq Wh_j/(\gamma_j + \beta^* g_j) - W - \sum_{i\in\mathcal{N}_1} h_i P_i^{pk}$

        $p_j^* = \frac{a/g_k - b/h_k}{g_j/g_k - h_j/h_k}$, $p_k^* = \frac{b/h_j - a/g_j}{h_k/h_j - g_k/g_j}$

        if $P_j^{pk} > p_j^* > 0$ and $P_k^{pk} > p_k^* > 0$ **then**

          $\mathcal{I} = \mathcal{I} \cup \{l\}$

          $r_l = W \log\left(1 + \frac{h_j p_j^* + h_k p_k^* + \sum_{i\in\mathcal{N}_1} h_i P_i^{pk}}{W}\right) - \gamma_j p_j^* - \gamma_k p_k^* - \sum_{i\in\mathcal{N}_1} \gamma_i P_i^{pk}$

        end if

      end if

    end for

    $v_{j,k} = \max_{\{i\in\mathcal{I}\}} r_i$, $t = \arg\max_{\{i\in\mathcal{I}\}} r_i$

    $\mathcal{S}_{j,k}^* = \mathcal{S}_{j,k}^{(t)}$

    $\mathcal{I} = \varnothing$

  end for

end for

$(j,k) = \arg\max_{\{(i,l)\}} v_{i,l}$

$\mathcal{N}_1 = \mathcal{S}_{j,k}^*$

$\mathcal{N}_0 = \mathcal{N} \backslash \mathcal{N}_1 \backslash \{j,k\}$

if $\frac{\gamma_j/h_j - \gamma_k/h_k}{g_k/h_k - g_j/h_j} > \frac{\gamma_j/h_j - \gamma_i/h_i}{g_i/h_i - g_j/h_j}, \exists i \in \mathcal{N}_1$ or $\frac{\gamma_j/h_j - \gamma_k/h_k}{g_k/h_k - g_j/h_j} < \frac{\gamma_j/h_j - \gamma_i/h_i}{g_i/h_i - g_j/h_j}, \exists i \in \mathcal{N}_0$ **then**

  $j = 0$, $k = 0$

end if

Output: j, k, $\mathcal{N}_1$, $\mathcal{N}_0$

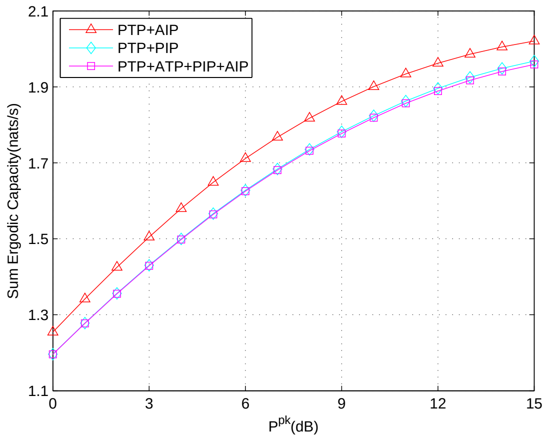


Fig. 3.7. Sum ergodic capacity vs P^{pk}.

able sum ergodic capacity achieved under ATP+AIP is larger than that achieved under ATP+PIP for all values of P^{av} since the PIP constraint is stricter than the AIP constraint. The sum ergodic capacity under PTP+ATP+PIP+AIP is much smaller than that under ATP+PIP and ATP+AIP due to the fact that the PTP constraint is dominant over other constraints for all values of P^{av}.

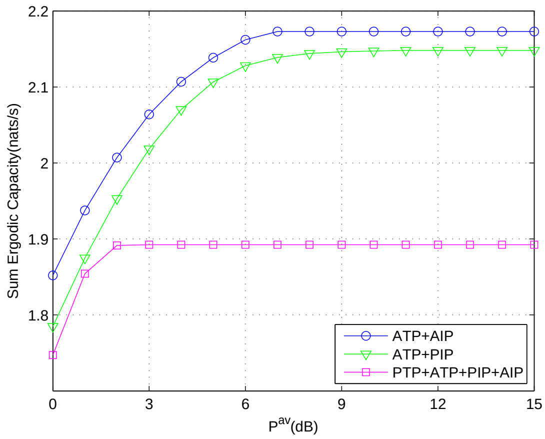


Fig. 3.8. Sum ergodic capacity vs P^{av}.

Fig. 3.9 shows the maximum sum ergodic capacity under PTP+PIP, ATP+PIP and PTP+ATP+

PIP+AIP constraints versus Q^{pk}. It can be seen from the figure that the maximum sum ergodic capacity achieved under ATP+PIP is larger than that achieved under PTP+PIP for any given Q^{pk}. This is because the power allocation is more flexible for SUs under the ATP constraint than under the PTP constraint. The sum ergodic capacity under PTP+ATP+PIP+AIP saturates earlier than that under PTP+PIP and ATP+PIP, because it is restricted by the AIP constraint.

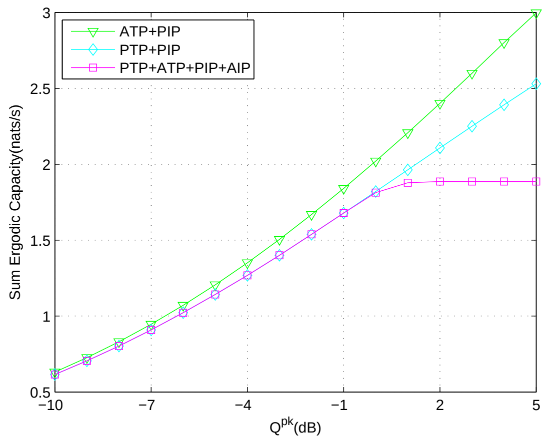


Fig. 3.9. Sum ergodic capacity vs Q^{pk}.

Fig. 3.10 shows the maximum sum ergodic capacity under PTP+AIP, ATP+AIP and PTP+ATP+
PIP+AIP constraints versus Q^{av}. Due to the same reasons as for the results in Fig. 3.9, the sum ergodic capacity achieved under ATP+AIP is larger than that achieved under PTP+AIP. The sum ergodic capacity under PTP+ATP+PIP+AIP saturates earlier than that for PTP+AIP and ATP+AIP because of the presence of the PIP constraint.

Finally, Fig. 3.11 shows the maximum sum ergodic capacity under PTP+PIP, PTP+AIP, ATP+PIP, ATP+AIP and PTP+ATP+PIP+AIP versus W. Similar performance comparison results as in the previous figures can be observed. One difference is that the sum ergodic capacities do not saturate with the increase of W.

3.6 Conclusion

A cognitive radio network where multiple SUs share the licensed spectrum of a PU using the FDMA scheme has been considered. The maximum achievable sum ergodic capacity of all the SUs has been studied subject to the total bandwidth constraint of the licensed spectrum

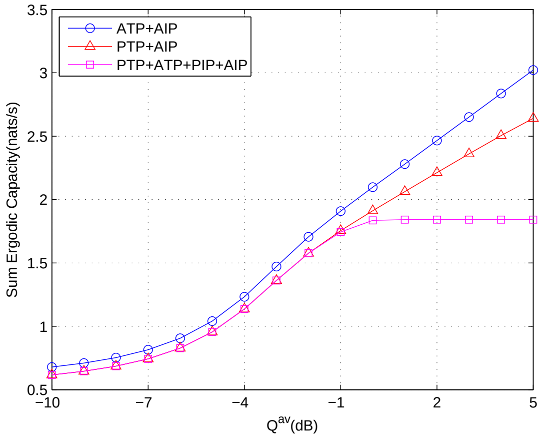


Fig. 3.10. Sum ergodic capacity vs Q^{av}.

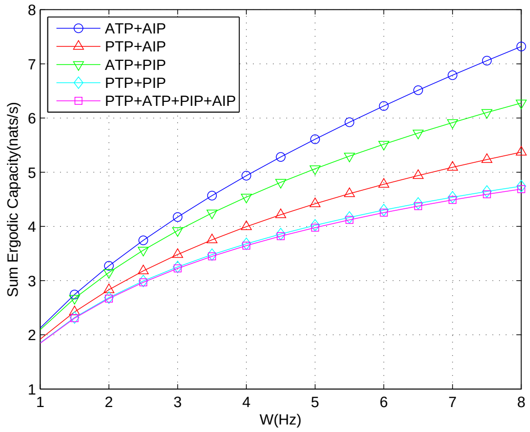


Fig. 3.11. Sum ergodic capacity vs W.

and all possible combinations of the peak/average transmit power constraints at the SUs and interference power constraint imposed by the PU. Optimal bandwidth allocation has been derived in each channel fading state for any given power allocation. Using the structures of the optimal power allocations under each combination of the power constraints, algorithms for finding the optimal power allocations in each channel fading state have been developed.

Chapter 4

Conclusion and Future Work

This thesis has studied several resource allocation problems for different wireless communication networks. Each chapter is summarized as follows.

4.1 Conclusion

Chapter 1 has presented the motivation, contribution, and outline of the thesis, and also the brief overview of convex optimization theory.

Chapter 2 has focused on joint bandwidth and power allocation strategy for wireless multi-user network without relaying and with decode-and-forward relaying. Joint bandwidth and power allocation problems have been formulated and solved to (i) maximize the sum capacity of all users; (ii) maximize the worst user capacity; (iii) minimize the total power consumption of all users. It has been shown that the formulated optimization problems are convex and, thus, optimal bandwidth and power allocation can be efficiently obtained using convex optimization techniques. Furthermore, admission control for the joint bandwidth and power allocation strategy has been considered. A greedy search algorithm has been developed to solve the admission control problem efficiently. The optimality conditions of the greedy search algorithm have been derived and shown to be mild.

In Chapter 3, the sum ergodic capacity of SUs under fading channels in cognitive radio networks has been investigated based on joint bandwidth and power allocation strategy. It has been shown that optimal bandwidth allocation can be derived for any given power allocation in any channel fading state. Then the structure of optimal power allocation in any channel fading state has been derived subject to each combination of four types of power constraints. Efficient algorithms have been developed for finding the optimal power allocations by employing these structures.

4.2 Future Work

A number of future research directions have been recognized for extending the studies in this thesis.

In Chapter 2, it has been assumed that only one relay is assigned to each user and the relay assignment is fixed. Since multiple relays are available in the network, it is interesting and also challenging to study the case where multiple relays can be employed to assist one user. Then the related question is also which set of relay assignments for all users can achieve the optimal network performance? Therefore, resource allocation and relay selection can be carried out jointly. Note that similar joint strategies have been studied in other contexts [62].

In Chapter 3, it has been assumed that perfect CSI of all channels is available at a central point. However, it may be impossible for SUs to obtain perfect CSI of interference channels from a PU to the SUs, and it is impractical to exchange CSI among all the SUs due to high complexity. Therefore, imperfect CSI can be taken into account for obtaining optimal bandwidth and power allocation strategies. Indeed, a distributed resource allocation scheme that only employs local CSI for each SU is more practically desirable.

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