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Algorithm for Adaptive Identification of Dynamical Parametrically Nonstationary Objects

E. V. Bodyanskii*, S. A. Vorob'ev*, and A. Shtefan**

* *Khar'kov State Technical University of Radioelectronics, Khar'kov, Ukraine*
** *Il'menau, Germany*

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Abstract—The problem of the adaptive identification of dynamic nonstationary objects under uncertain conditions, with respect to the drifting parameters, is considered. An adaptive algorithm for multiple identification is proposed; this algorithm is constructed by using the exponentially weighted recursive least-squares method with tunable coefficients for discounting obsolete information. A set of discounting coefficients is used when estimating the parameters: some weight is assigned to each of these coefficients. The optimal algorithms for the tuning of weights are synthesized. The proposed algorithm ensures the quality of identification that is higher than that ensured by the common exponentially weighted recursive least-squares method or by the algorithm for competitive identification [1].

INTRODUCTION

One of the most intensively developing fields of control theory is the theory of adaptive systems which develops and studies the methods for controlling dynamic objects under the conditions where the number of parameters determining the behavior of these objects is unknown.

The self-tuning systems are most widely applied in the class of adaptive systems. The former includes the loop of adaptive identification which refines, in real time, the parameters of the tuned model of the object with the help of this or that recursive procedure. The functioning quality of an entire self-tuning control system depends on the quality of solution of the identification problem. The most widely applied among the identification algorithms are different modifications of the recursive least-squares method and those of the method of stochastic approximation [2, 3], which provide under certain conditions for the convergence of the estimates of the unknown constant parameters of the control object to the true values of these parameters.

The problem becomes considerably more complex if the parameters of the object change in time in an unpredictable manner; in this case, the identification algorithm should have both the filtering and tracking properties, i.e., it should provide for the high accuracy of the parameter estimation in conditions of the noise action at the output of the object and to track the drift of parameters. Here, the exponentially weighted recursive least-squares method (EWRLSM) is widely applied [2]. Moreover, the choice of the numerical value of the parameter regulating the process of forgetting the "obsolete" information in this algorithm is a nontrivial problem for the real practical applications.

The main idea of this paper consists in the combined use of several parameter values for discounting the "obsolete" information to which certain weights are assigned. Moreover, the estimator of parameters of the object is presented in the form of a linear combination of weight products and corresponding partial estimators for given values of the discounting parameter. The proposed algorithm has a number of essential distinctions from the algorithm proposed in [1] in the aspect of tuning weights and ensures a higher quality of identification.

1. STATEMENT OF THE PROBLEM

Consider a nonstationary dynamical stochastic object described by the difference equation

$$\begin{aligned} y(k) &= -a_1(k)y(k-1) - a_2(k)y(k-2) - \dots \\ &- a_{n_a}(k)y(k-n_a) + b_1(k)u(k-1) \\ &+ b_2(k)u(k-2) + \dots + b_{n_b}(k)u(k-n_b) + \omega(k), \end{aligned} \quad (1.1)$$

where parameters $a_1(k)$, $a_2(k)$, …, $a_{n_a}(k)$, $b_1(k)$, $b_2(k)$, …, $b_{n_b}(k)$ change during unpredictably the course of the process, $u(k)$ and $y(k)$ are the control and output sequences, respectively, $k = 0, 1, 2, \dots$, is the current discrete time, $\omega(k)$ is the disturbing random sequence with $E\{\omega(k)\} = 0$, $E\{\omega^2(k)\} = \sigma_\omega^2 < \infty$, and $E\{\cdot\}$ is the symbol of expectation.

Let us introduce into consideration the vector of unknown nonstationary parameters $\theta(k) = (a_1(k), a_2(k), \ldots, a_{n_a}(k), b_1(k), b_2(k), \ldots, b_{n_b}(k))^T$ and the vector of the previous history of the process $\varphi(k) = (-y(k-1), -y(k-2), \ldots, -y(k-n_a), u(k-1), u(k-2), \ldots, u(k-n_b))^T$. We can now rewrite (1.1) in the form

$$y(k) = \theta^T(k)\varphi(k) + \omega(k). \quad (1.2)$$

Put the tuned model in correspondence to (1.2) in the form

$$\hat{y}(k) = \hat{\theta}^T(k-1)\varphi(k), \quad (1.3)$$

where $\hat{\theta}(k-1) = (\hat{a}_1(k-1), \hat{a}_2(k-1), \ldots, \hat{a}_{n_a}(k-1), \hat{b}_1(k-1), \hat{b}_2(k-1), \ldots, \hat{b}_{n_b}(k-1))^T$ is the vector of unknown tuned parameters.

The EWRLSM is written in the form

$$\begin{cases} \hat{\theta}(k) = \hat{\theta}(k-1) + \dfrac{P(k-1)\varphi(k)\varepsilon(k)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}, \\ \varepsilon(k) = y(k) - \hat{\theta}^T(k-1)\varphi(k), \\ P(k) = \dfrac{1}{\lambda}\left(P(k-1) - \dfrac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}\right), \\ 0 < \lambda \le 1, \end{cases} \quad (1.4)$$

where λ is the discounting parameter of the "obsolete" information.

The principal problem encountered within the use of (1.4) consists in choosing parameter λ. It is clear that λ should be less than 1, in order that (1.4) should track the nonstationarities. But, under uncertainty conditions with respect to the drift of parameters, it is difficult to choose beforehand the constant optimal value of λ that would realize the compromise between the filtering and tracking properties of the algorithm on the whole interval of estimation. This problem can be solved either by using along with the estimation procedure different procedures of controlling λ [4–7] that provide additional sluggishness to the loop of identification, which results in the impossibility of applying these procedures in conditions of fast drift, or by the parallel use of a set of different values of λ.

2. COMPETITIVE IDENTIFICATION ALGORITHM BY L. KOWALCZUK [1]

In [1], it is proposed to estimate the parameters of model (1.3) being tuned with the use of J different values $\lambda_1, \lambda_2, \ldots, \lambda_j, \ldots, \lambda_J$. Under this procedure, we have J models, and an algorithm for tuning the parameters of the models is respectively written in the form

$$\begin{cases} \hat{\theta}_j(k) = \hat{\theta}_j(k-1) + \dfrac{P_j(k-1)\varphi(k)\varepsilon_j(k)}{\lambda_j + \varphi^T(k)P_j(k-1)\varphi(k)}, \\ \varepsilon_j(k) = y(k) - \hat{\theta}_j^T(k-1)\varphi(k), \\ P_j(k) = \dfrac{1}{\lambda_j}\left(P_j(k-1) - \dfrac{P_j(k-1)\varphi(k)\varphi^T(k)P_j(k-1)}{\lambda_j + \varphi^T(k)P_j(k-1)\varphi(k)}\right), \\ 0 < \lambda_j \le 1, \quad j = \overline{1, J} \end{cases} \quad (2.1)$$

where

$$P_j(k) = \left(\sum_{i=1}^{k} \varphi(i)\varphi^T(i)\lambda_i^{k-i}\right)^+ \quad (2.2)$$

$$\text{for } \det P_j^{-1}(k) \ne 0.$$

Algorithm (2.1) is obtained by means of minimizing the following test:

$$I_j = \sum_{i=1}^{k} \varepsilon_j^2(i)\lambda_j^{k-i}. \quad (2.3)$$

The degenerate case arises for $\lambda_1 = 0$ when test (2.3) and matrix (2.2) take the form

$$I_1 = \varepsilon^2(k),$$
$$P_1(k) = (\varphi(k)\varphi^T(k))^+ = \frac{\varphi(k)\varphi^T(k)}{\|\varphi(k)\|^4}. \quad (2.4)$$

Substituting the value for P_1 from (2.4) into (2.1) we obtain the following algorithm for tuning the parameters:

$$\hat{\theta}_1(k) = \hat{\theta}_1(k-1) + \frac{\varphi(k)\varepsilon_1(k)}{\|\varphi(k)\|^2}, \quad (2.5)$$

this algorithm corresponds with the Kaczmarz algorithm [8].

On the basis of estimators $\hat{\theta}_j(k)$, $j = \overline{1, J}$ we estimate the generalized estimator

$$\hat{\theta}(k) = \sum_{j=1}^{J} \mu_j(k)\hat{\theta}_j(k), \quad (2.6)$$

where the coefficients $\mu_j(k)$ are computed according to the following rule:

$$\mu_j(k) = \frac{\left(\sum_{i=0}^{M-1} \varepsilon_j^2(k-i)\right)^{-M/2}}{\sum_{j=1}^{J}\left(\sum_{i=0}^{M-1} \varepsilon_j^2(k-i)\right)^{-M/2}} = \frac{\alpha_j(k)}{\sum_{j=1}^{J}\alpha_j(k)}, \quad (2.7)$$

here M is the interval of smoothing.

The set of weights $\mu_j(k)$ can be represented in the form of the weight vector $\boldsymbol{\mu}(k) = (\mu_1(k), \mu_2(k), \ldots, \mu_J(k))^T$. It is seen from (2.7) that

$$\boldsymbol{\mu}^T(k)E = 1, \quad (2.8)$$

where $E = (1, 1, \ldots 1)^T$ is the vector of dimension $(J \times 1)$ consisting of unities.

It is easy to see that $0 \le \mu_j(k) \le 1$. But it is fairly difficult to analytically estimate the accuracy of the model based on (2.6) with the computation of $\mu_j(k)$ according to (2.7).

3. ADAPTIVE IDENTIFICATION ALGORITHM. TUNING THE WEIGHT VECTOR

The following algorithm is proposed as an alternative to the heuristic algorithm (2.7) for computing the weight vector.

Let us introduce into consideration the generalized model [9], i.e.,

$$\hat{y}(k) = \boldsymbol{\mu}^T(k)\tilde{\mathbf{y}}(k) = \boldsymbol{\mu}^T(k)\hat{\boldsymbol{\Theta}}^T(k)\varphi(k), \quad (3.1)$$

where $\tilde{\mathbf{y}}(k) = (\hat{y}_1(k), \hat{y}_2(k), \ldots, \hat{y}_J(k))^T$ is the $(J \times 1)$-vector of outputs, $\hat{\boldsymbol{\Theta}}(k) = (\hat{\theta}_1(k), \hat{\theta}_2(k), \ldots, \hat{\theta}_J(k))$ – $((n_a + n_b) \times J)$ is the matrix of coefficients of models $\hat{y}_j(k)$.

The weight vector $\boldsymbol{\mu}(k)$ should be chosen in such a way that the accuracy of generalized model (3.1) would be optimal in some sense. Let us use the penalty function method for the synthesis of the procedures of tuning the weight vector $\boldsymbol{\mu}(k)$.

Denote by $\mathbf{Y}(k) = (y(k - M + 1), y(k - M + 2), \ldots, y(k))^T$ the $(M \times 1)$-vector of outputs of the object on the interval of smoothing; $\tilde{\mathbf{Y}}(k) = (\tilde{\mathbf{y}}(k - M + 1), \tilde{\mathbf{y}}(k - M + 2), \ldots, \tilde{\mathbf{y}}(k))^T$ is the $(M \times J)$-matrix of predictions with respect to models from $(k - M + 1)$th to kth.

Then, by using the penalty function method with regards to (2.8), we write the test of the following form:

$$I(\boldsymbol{\mu}, \rho) = (\mathbf{Y}(k) - \tilde{\mathbf{Y}}(k)\boldsymbol{\mu})^T(\mathbf{Y}(k) - \tilde{\mathbf{Y}}(k)\boldsymbol{\mu}) + \rho^{-2}(1 - \boldsymbol{\mu}^T E),$$

from which we obtain the following expression for $\boldsymbol{\mu}$:

$$\boldsymbol{\mu}(\rho) = (\tilde{\mathbf{Y}}^T(k)\tilde{\mathbf{Y}}(k) + \rho^{-2}EE^T)^{-1} \times (\tilde{\mathbf{Y}}^T(k)\mathbf{Y}(k) + \rho^{-2}E).$$

The final estimator $\boldsymbol{\mu}(k)$ is obtained for $\rho \longrightarrow 0$:

$$\boldsymbol{\mu}(k) = \lim_{\rho \to 0}\boldsymbol{\mu}(\rho) = \boldsymbol{\mu}^*(k) + S(k)\frac{1 - E^T\boldsymbol{\mu}^*(k)}{E^T S(k) E}E, \quad (3.2)$$

$$\boldsymbol{\mu}^*(k) = (\tilde{\mathbf{Y}}^T(k)\tilde{\mathbf{Y}}(k))^{-1}\tilde{\mathbf{Y}}^T(k)\mathbf{Y}(k) = S(k)\tilde{\mathbf{Y}}^T(k)\mathbf{Y}(k), \quad (3.3)$$

where $\boldsymbol{\mu}^*(k)$ is the estimator obtained by using the least-squares method and ρ is the penalty parameter.

A similar result can be obtained if one uses the method of indefinite Lagrangian multipliers. Let us introduce into consideration the estimation error

$$\varepsilon(k) = y(k) - \hat{y}(k) = y(k) - \boldsymbol{\mu}^T\tilde{\mathbf{y}}(k) = \boldsymbol{\mu}^T E y(k) - \boldsymbol{\mu}^T\tilde{\mathbf{y}}(k) = \boldsymbol{\mu}^T(Ey(k) - \tilde{\mathbf{y}}(k)) = \boldsymbol{\mu}^T\tilde{\varepsilon}(k).$$

and let us write the Lagrangian with respect to $\boldsymbol{\varepsilon}(k)$ with due regard for condition (2.8):

$$L(\boldsymbol{\mu}, \eta) = \sum_{i=0}^{M-1}\varepsilon^2(k-i) + \eta(\boldsymbol{\mu}^T E - 1) = \sum_{i=0}^{M-1}\boldsymbol{\mu}^T\tilde{\varepsilon}(k-i)\tilde{\varepsilon}^T(k-i)\boldsymbol{\mu} + \eta(\boldsymbol{\mu}^T E - 1) = \boldsymbol{\mu}^T R(k)\boldsymbol{\mu} + \eta(\boldsymbol{\mu}^T E - 1), \quad (3.4)$$

where η is the indefinite Lagrangian multiplier, $R(k)$ is the covariance matrix of estimation errors:

$$R(k) = \sum_{i=0}^{M-1}\tilde{\varepsilon}(k-i)\tilde{\varepsilon}^T(k-i). \quad (3.5)$$

Solving the Kuhn–Tucker system of equations

$$\begin{cases}\nabla_{\mu}L(\boldsymbol{\mu}, \eta) = 2R(k)\boldsymbol{\mu} + \eta E = 0, \\ \partial L(\boldsymbol{\mu}, \eta)/\partial\eta = \boldsymbol{\mu}^T E - 1 = 0,\end{cases}$$

we obtain the expression for $\boldsymbol{\mu}(k)$:

$$\begin{cases}\boldsymbol{\mu}(k) = R^{-1}(k)E(E^T R^{-1}(k)E)^{-1}, \\ \eta(k) = -2E^T R^{-1}(k)E.\end{cases} \quad (3.6)$$

Moreover, the Lagrangian (3.4) at the saddle point has the value

$$L^*(\boldsymbol{\mu}, \eta) = (E^T R^{-1}(k)E)^{-1}. \quad (3.7)$$

Proving the optimality of the obtained procedure for the computation of the weight vector, let us consider the pair of vectors X and Z. On the basis of the Kantorovich–Bergstrem inequality [11] the following relations are valid:

$$(X^T Z)^2 = (X^T R^{1/2}(k)R^{-1/2}Z)^2 = ((R^{1/2}(k)X)^T(R^{-1/2}(k)Z))^2 \le \|R^{1/2}(k)X\|^2 \times \|R^{-1/2}(k)Z\|^2 = (X^T R(k)X)(Z^T R^{-1}(k)Z). \quad (3.8)$$

Further, let us bring into consideration the $(J \times 1)$-vector E_j consisting of zeros, except for the jth position which is a unit one, $E_j = (0, 0, \ldots, 1(j), \ldots, 0)^T$, and let us rewrite (3.8) in the form

$$(E^T E_j)^2 \le (E_j^T R(k) E_j)(E^T R^{-1}(k) E). \quad (3.9)$$

In (3.9), $(E^T E_j)^2 = 1$ and $E_j^T R(k) E_j = R_{jj}(k)$ is the jth diagonal entry of the covariance error matrix $R(k)$; this entry characterizes the accuracy of the prediction according to the jth model.

Hence, the following inequality is valid:

$$1 \le R_{jj}(k)(E^T R^{-1}(k) E),$$

from where we finally obtain

$$R_{jj}(k) = \sum_{i=0}^{M-1} (y(k-i) - \hat{y}_j(k-i))^2$$
$$= \sum_{i=0}^{M-1} \varepsilon_j^2(k-i) \ge (E^T R^{-1}(k) E)^{-1} = L^*(\mu, \eta). \quad (3.10)$$

From (3.10) it follows that the generalized prediction $\hat{y}(k)$ compares well in its accuracy with all predictions according to the models $\hat{y}_j(k)$.

In order to guarantee the control over the measurements of the output of $y(k)$ induced by the parametric changes of the model of the object in real time, let us make use of the fact that $R(k) = R(k-1) + \tilde{\varepsilon}(k)\tilde{\varepsilon}^T(k)$. By using the well-known Sherman–Morrison lemma on inversion of matrices of this type, procedure (3.6) can be written in the following recursive form:

$$\begin{cases} \tilde{R}^{-1}(k) = \left(\sum_{i=0}^{M} \tilde{\varepsilon}(k-i)\tilde{\varepsilon}^T(k-i)\right)^{-1} \\ = R^{-1}(k-1) - \dfrac{R^{-1}(k-1)\tilde{\varepsilon}(k)\tilde{\varepsilon}^T(k)R^{-1}(k-1)}{1 + \tilde{\varepsilon}^T(k)R^{-1}(k-1)\tilde{\varepsilon}(k)}, \\ R^{-1}(k) = \left(\sum_{i=0}^{M-1} \tilde{\varepsilon}(k-i)\tilde{\varepsilon}^T(k-i)\right)^{-1} \\ = \tilde{R}^{-1}(k) + \dfrac{\tilde{R}^{-1}(k)\tilde{\varepsilon}(k-M)\tilde{\varepsilon}^T(k-M)\tilde{R}^{-1}(k)}{1 - \tilde{\varepsilon}^T(k-M)\tilde{R}^{-1}(k)\tilde{\varepsilon}(k-M)}, \\ \mu(k) = R^{-1}(k)E(E^T R^{-1}(k)E)^{-1}. \end{cases} \quad (3.11)$$

Similarly, making use of the fact that $S(k) = S(k-1) + \tilde{y}(k)\tilde{y}^T(k)$, let us rewrite procedure (3.2), (3.3) in the recursive form:

$$\begin{cases} \tilde{S}(k) = \left(\sum_{i=0}^{M} \tilde{y}(k-i)\tilde{y}^T(k-i)\right)^{-1} \\ = S(k-1) - \dfrac{S(k-1)\tilde{y}(k)\tilde{y}^T(k)S(k-1)}{1 + \tilde{y}^T(k)S(k-1)\tilde{y}(k)}, \\ S(k) = \left(\sum_{i=0}^{M-1} \tilde{y}(k-i)\tilde{y}^T(k-i)\right)^{-1} \\ = \tilde{S}(k) + \dfrac{\tilde{S}(k)\tilde{y}(k-M)\tilde{y}^T(k-M)\tilde{S}(k)}{1 - \tilde{y}^T(k-M)\tilde{S}(k)\tilde{y}(k-M)}, \\ \mu^*(k) = \mu^*(k-1) \\ + S(k)(y(k) - \tilde{y}^T(k)\mu^*(k-1))\tilde{y}(k), \\ \mu(k) = \mu^*(k) + S(k)(1 - E^T\mu^*(k))(E^T S(k) E)^{-1}, \\ \mu_j(0) = J^{-1}. \end{cases} \quad (3.12)$$

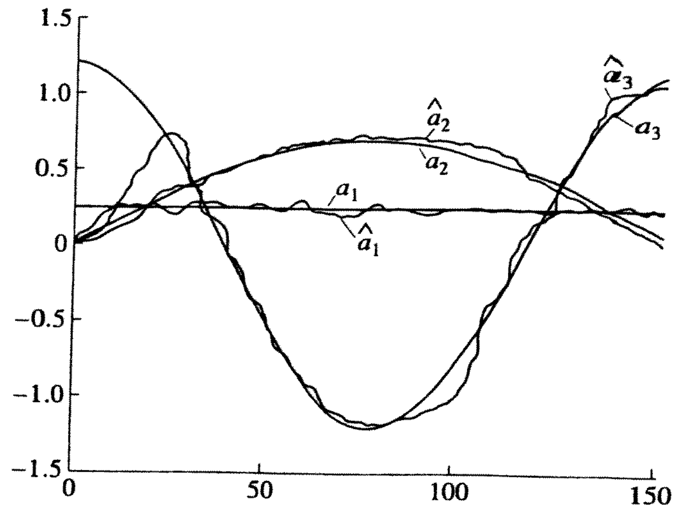

Fig. 1. Estimators of drifting parameters obtained according to the method developed by L. Kowalczuk.

Moreover, the first three relations in procedures (3.11) and (3.12) are the recursive least-squares method on the sliding window [12].

It should be noted that (3.11) or (3.12) is used jointly with (2.1) and (2.6). To this end, the grid of values of the parameter of discounting the "obsolete" information $\lambda_1, \lambda_2, \ldots, \lambda_j, \ldots, \lambda_J$ is assigned; the estimators $\hat{\theta}_j(k)$ are computed according to (2.1) on the basis of these values; after that the generalized estimator $\hat{\theta}(k)$ is computed according to (2.6), where the vector of weight parameters $\mu(k)$ is present; this vector is obtained from (3.11) or (3.12). When choosing the grid

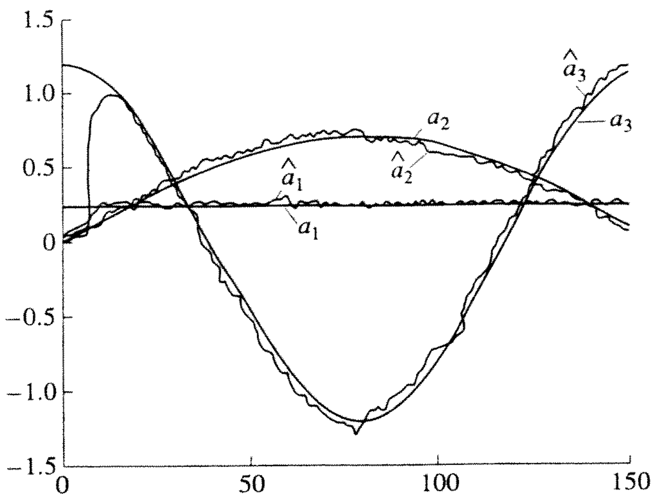

Fig. 2. Estimators of drifting parameters obtained with the use of the algorithm proposed in the paper.

of values $\lambda_1, \lambda_2, \ldots, \lambda_j, \ldots, \lambda_J$ one should take into account the constraints on the computational costs.

4. EXAMPLE

As an example, let us consider the problem of the parametric identification of the nonstationary stochastic object of the form

$$y(k) = a_1(k)y(k-1) + a_2(k)y(k-2)$$
$$+ a_3(k)y(k-3) + \omega(k) = \hat{\theta}^{T}(k)\varphi(k) + \omega(k)$$

with the use of the algorithms (2.1), (2.6), (2.7), and (2.1), (2.6), (3.11). Moreover, it is assumed that $\sigma_\omega = 0.1$, the parameter $a_1 = -0.15$ is constant, and the other parameters $a_2(k) = 0.7\sin 0.02k$, $a_3(k) = 1.2\cos 0.04k$ are changing according to the harmonic law. The following conditions are taken as the initial ones: $\hat{\theta}^{T}(0) = (0, 0, 0)$; $\varphi^{T}(0) = (y(2), y(1), y(0)) = (1, 1, 1)$; $J = 3$; $\lambda_1 =$ r $= 0.5$; $\lambda_3 = 1$;

$$P_j(0) = \begin{pmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{pmatrix}; \quad j = 1, 2, 3;$$

$$R(0) = \begin{pmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{pmatrix}.$$

The identification algorithm should track the drift of the parameters of the object.

The results of the computation are presented in the form of graphs in Figs. 1 and 2.

The maximal estimation error of parameters when using both the algorithm (2.1), (2.6), (2.7) and the algorithm (2.1), (2.6), (3.11) corresponds to the initial tuning of the parameters. But the maximal deviation of estimators of parameters from their true values at the instants of changing the direction of the drift is larger for the algorithm from [1] and makes up 8–10 per cent of the range of change of parameters. Meanwhile, the maximal deviation of parameters estimators obtained with the use of the algorithm proposed in this paper is no more than 6 per cent in the range of change of parameter values.

Thus, for a given level of disturbances in the case of the arbitrary drift of parameters, the algorithm for parametric identification proposed in this paper guarantees more exact estimation than the algorithm from [1] similar to it, which, in turn, operates better than the EWRLSM in conditions of drift of parameters.

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