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Simple Adaptive Controller for Objects with Constraints

Abstract: The methods of adaptive PID controller design for industrial plants with unknown parameters are proposed. It is shown that proposed PID controller is identical to the adaptive minimum variance controller. Much attention had been given to the tuning procedures in the case of constraints on the plant control and output variables.

Introduction

During the last 30 years various kinds of direct digital control (DDC) systems have been implemented in the different process industries, and almost all of them include PID controllers as standard facilities. Industry people prefer PID controllers due to their high effectiveness for a wide range of industrial processes and simple and well-understood structure [1,2]. However, the capabilities of the traditional PID controller are limited. Tuning of this controller is usually an empirical procedure based on knowledge and skill of control engineer or some simple tuning guidelines (Ziegler-Nickols method [3], for example) rather than particular optimality criterion. As a result many plants control loops are often poorly tuned and do not account for changes in the controlled process parameters that prevent to achieve full potential of the digital control. To overcome this problem in previous papers some adaptive and self-tuning procedures have been presented — one based on the generalized minimum variance (GMV) approach [4-7] and other based on the pole-placement (PP) technique [8] and expert system concept [3,9,10]. In this paper we propose new on-line recursive tuning procedures that were obtained by applying of non-linear programming techniques to the GMV controller design problem. The most attractive feature of the proposed procedure is the capability of the PID controller parameter tuning regarding constraints on the control input and output values and output error dynamics.

Problem formulation and controller tuning procedures

1 Problem formulation. Consider adaptive control problem for stochastic single input/ single output plant with time-varying dynamics that can be described by the linear stochastic difference equation

$$y_t = a_1 y_{t-1} + \ldots + a_n y_{n-1} + b_1 u_{t-1} + \ldots + b_m u_{t-m} + w_t \qquad (1)$$

where y_t is measured plant output, u_t is input control signal, w_t is independent random disturbance with zero mean $M\{w_t\} = 0$ and bounded variance $M\{w_t^2\} = \sigma^2$ (here $M\{.\}$ denotes expected value). Plant parameters a_i and b_i are supposed to be unknown. Such a plant can be successfully controlled by the well known GMV controller [1,2] designed with using the following cost function minimization

$$J_t = M\left\{\left(y^*_{t+1} - y_{t+1}\right)^2 + \lambda u_t^2\right\} = M\left\{v_{t+1}^2 + \lambda u_t^2\right\} \qquad (2)$$

where y_t^* is desired plant output trajectory and λ is non-negative weighting multiplier.

In spite of many successful applications of such controllers that have rather complex structure the industry people prefer PID controllers but they do not associate tuning of the controller parameters with any control criterion such as (2). In this paper the adaptive PID controller tuning procedure is proposed that can be applied to plants described by equation. (1) with arbitrary analytical optimality criterion.. It should be noted that this problem had been considered in some previous papers [4-7] and it had been shown that adaptive PID controller is equivalent to the adaptive GMV controller if the plant equation order does not exceed 2.

2 Control for a plant with known parameters. First let us consider plant with known parameters (stochastic control problem). Let us transform (1) to the form

$$y_t = b_1 u_{t-1} + a^T \phi_{t-1} + w_t \tag{3}$$

where $a = [b_2, \dots, b_m, a_1, \dots, a_n]^T$, $\phi_{t-1} = [u_{t-2}, \dots, u_{t-m}, y_{t-1}, \dots, y_{t-n}]^T$, and introduce into consideration discrete PID controller

$$u_t = u_{t-1} + g_0 v_t + g_1 v_{t-1} + g_2 v_{t-2} = u_{t-1} + g_t^T V_t \tag{4}$$

where $g = [g_0, g_1, g_2]^T$, $V_t = [v_t, v_{t-1}, v_{t-2}]^T$. The control error at the time $(t+1)$ can be written as

$$v_{t+1} = y^*_{t+1} - b_1 u_{t-1} - b_1 g_t^T V_t - a^T \phi_t - w_{t+1} = \bar{v}_{t+1} - b_1 g^T V_t - w_{t+1} \tag{5}$$

where $\bar{v}_{t+1} = y^*_{t+1} - b_1 u_{t-1} - a_t^T \phi_t$. It is relevant to remark that error value $\bar{v}_{t+1}$ can be calculated recurrently using the following relation

$$\bar{v}_{t+1} = \bar{v}_t - a^T \Delta\phi_t - b_1 \Delta u_{t-1} + \Delta y^*_{t+1} \tag{6}$$

where $\Delta x_t = x_t - x_{t-1}$ is the first difference of appropriate variable.
Let us use the cost function

$$J_t = M\left\{v_{t+1}^2 + \lambda(u_t - u_{t-1})^2\right\} \tag{7}$$

as a control optimality criterion and minimize it with using of the Gauss-Newton type procedure

$$g_t = g_{t-1} - [J_t]^+ \nabla_g J_t \tag{8}$$

where g_t is the vector of tuned PID controller parameters, $\nabla_g J_t = -2\bar{v}_{t+1} b_1 V_t + 2(b_1^2 + \lambda) V_t V_t^T g_{t-1}$ is the gradient vector of J_t regarding g_t and $[J_t]^+ = 0.5(b_1^2 + \lambda)^{-1} V_t V_t^T \|V_t\|^{-4}$ is pseudo-inverse to the matrix $[J_t]$ of second derivatives of J_t with respect to g_t. Substituting all this expressions into (8) we obtain the following parameter tuning algorithm

$$g_t = g_{t-1} + \frac{\bar{v}_{t+1} b_1 - (b_1^2 + \lambda) g_{t-1}^T V_t}{(b_1 + \lambda)\|V_t\|^2} V_t, \tag{9}$$

It is obvious that with $\lambda = 0$ the cost function (7) is transformed into the minimum variance criterion and the parameter tuning algorithm becomes

$$g_t = g_{t-1} + \left(\bar{v}_{t+1} b_1^{-1} - g_{t-1}^T V_t\right) \frac{V_t}{\|V_t\|^2} \tag{10}$$

that is identical to one-step Kaczmarz algorithm. Note that the multiplier $V_t\|V_t\|^{-2}$ in (9) and (10) is equal to $[V^T]^+$ if V_t is non-zero vector. In the opposite case if $\|V_t\| \approx 0$ the value $V_t(V_t^T V_t + \delta^2)^{-1}$ must be taken as pseudoinverse to V_t^T, where δ is small positive value defined by the word length of the computer [11].
To prevent from jumps of the controller parameters the following criterion must be used

$$J_t = M\left\{v_{t+1}^2 + \lambda(g_t - g_{t-1})^T (g_t - g_{t-1})\right\} \tag{11}$$

that results in another tuning formula

$$g_t = g_{t-1} + \frac{b_1\left(\bar{v}_{t+1} - b_1 g_{t-1}^T V\right)_t}{\lambda + b_1^2\|V_t\|} V_t, \tag{12}$$

The direct minimization of $M\{v_{t+1}^2\}$ by g_t gives

$$g_t = -0.5[J_t]^+ \nabla_g J_t = \frac{\bar{v}_{t+1} V_t}{b_1\|V_t\|^2} \tag{13}$$

If we substitute (13) and (4) to (1) we obtain the plant output value $y_{t+1} = y_{t+1}^* + w_{t+1}$ This means that proposed PID controller is identical to GMV controller and the control low can be written as following

$$u_t = u_{t-1} + g_{0,t} v_t + g_{1,t} v_{t-1} + g_{2,t} v_{t-2} = u_{t-1} + g_t^T V_t \tag{14}$$

3 Control for the plant with unknown parameters. In general case the plant parameters a_i, b_i are unknown, so the real-time identification loop must be included into control system for the plant parameter estimation using observed values of input and output variables. For this purpose let us rewrite the plant equation (1) as $y_t = \Theta^T \varphi_t + w_t$, where $\Theta = [b_1, a^T]^T$, $\varphi_t = [u_{t-1}, \phi_{t-1}^T]^T$. The model parameters tuning can be performed using any adaptive identification algorithm, for example, exponentially weighted recursive least square method

$$\Theta_t = \Theta_{t-1} + \frac{P_{t-1}\varphi_t\left(y_t - \Theta_{t-1}^T\varphi_t\right)}{\alpha + \varphi_t^T P_{t-1}\varphi_t} \tag{15}$$

$$P_t = \frac{1}{\alpha}\left[P_{t-1} - \frac{P_{t-1}\varphi_t\varphi_t^T P_{t-1}}{\alpha + \varphi_t^T P_{t-1}\varphi_t}\right]$$

or Kaczmarz algorithm

$$\hat{\Theta}_t = \hat{\Theta}_{t-1} + \frac{y_t - \hat{\Theta}_{t-1}^T\varphi_t}{\|\varphi_t\|^2}\varphi_t \tag{16}$$

Both algorithms provide estimate convergence in criterion when identification unit works in respect to the closed loop adaptive control system [12-14]. In this case the PID controller parameter vector is replaced by its estimates

$$\hat{g}_t = \hat{g}_{t-1} + \frac{\hat{\bar{v}}_t \hat{b}_{1,t} - (\hat{b}_{1,t}^2 + \lambda)\hat{g}_{t-1}^T V_t}{\hat{b}_{1,t}^2 + \lambda} \frac{V_t}{\|V_t\|^2} \tag{17}$$

where $\hat{\bar{v}}_{t+1} = y^{*}_{t+1} - \hat{b}_{1,t}u_{t-1} - \hat{a}_t^T \phi_t$ and adaptive PID control low is transformed to the form

$$\hat{u}_t = \hat{u}_{t-1} + \hat{g}_{0,t}v_t + \hat{g}_{1,t}v_{t-1} + \hat{g}_{2,t}v_{t-2} = \hat{u}_{t-1} + \hat{g}_t^T V_t \tag{18}$$

Thus, the plant parameters estimates provided by the identification loop are used for $\hat{u}_t$ calculation rather than true plant parameter values.

4 PID controller tuning for the plant with constrained variables. GMV approach to the controller design does not take into consideration any constraints on the plant variables. However, when designing controller for the particular plant the system engineer must deal with various constraints on the plant output and control signal. In this case adaptive PID controller design problem can be reduced to the non-linear programming problem consisting in the minimization of the output error variance subject to constraints on the plant input and output variables. Let the value of $M\{v^2_{t+1}\}$ be considered as the cost function and constraints on the plant variables be in the form $u_t^2 \le U^2$ and $M\{v^2_{t+1}\} \le V^2$ (that is identical to the $-U \le u_t \le U$ and $-V \le v_{t+1} \le V$). Writing the Lagrangian function

$$L_t = M\left\{v^2_{t+1} + \lambda_1\left(u_t^2 - U^2\right) + \lambda_2\left(v_t^2 - V^2\right)\right\} \tag{19}$$

where λ_1 and λ_2 are Lagrange multipliers, and minimizing it using the Arrow-Hurwits-Udzava [15,16] algorithm the adaptive PID controller is obtained

$$\begin{aligned}
\hat{u}_t &= \hat{u}_{t-1} + \hat{g}_{0,t}v_t + \hat{g}_{1,t}v_{t-1} + \hat{g}_{2,t}v_{t-2} = \hat{u}_{t-1} + \hat{g}_t^T V_t,\\
\hat{g}_t &= \frac{\hat{\bar{v}}_{t+1}\hat{b}_{1,t}\left(1+\lambda_{2,t}\right) - \lambda_{1,t}^T\hat{u}_{t-1}}{\hat{b}^2_{1,t}\left(1+\lambda_{2,t}\right)+\lambda_{1,t}}\frac{V_t}{\|V_2\|^2},\\
\lambda_{1,t+1} &= \lambda_{1,t} + \frac{\mu_{1,t+1}\lambda_{1,t}\left(\hat{u}_t^2 - U^2\right)}{U^2}, 0 < \mu_{1,t+1} < 1,\\
\lambda_{2,t+1} &= \lambda_{2,t} + \frac{\mu_{2,t+1}\lambda_{2,t}\left(v^2_{t+1} - V^2\right)}{V^2}, 0 < \mu_{2,t+1} < 1.
\end{aligned} \tag{20}$$

In the same way we can take into consideration constraint on the output error dynamics that can be written as $M\left\{\left(v_{t+1} - v_t\right)^2\right\} \le W^2$. In this case minimization of the Lagrangian function

$$L_t = M\left\{v^2_{t+1} + \lambda_1\left(u_t^2 - U^2\right) + \lambda_2\left[\left(v_{t+1} - v_t\right)^2 - W^2\right]\right\} \tag{21}$$

yields the following controller

$$\begin{aligned}
\hat{u}_t &= \hat{u}_{t-1} + \hat{g}_{0,t}v_t + \hat{g}_{1,t}v_{t-1} + \hat{g}_{2,t}v_{t-2} = \hat{u}_{t-1} + \hat{g}_t^T V_t,\\
\hat{g}_t &= \frac{\hat{\bar{v}}_{t+1}\hat{b}_{1,t}\left(1+\lambda_{2,t}\right) - \lambda_{1,t}^T\hat{u}_{t-1} - \hat{b}_{1,t}\lambda_{2,t}V_t}{\hat{b}^2_{1,t}\left(1+\lambda_{2,t}\right)+\lambda_{1,t}}\frac{V_t}{\|V_2\|^2},\\
\lambda_{1,t+1} &= \lambda_{1,t} + \frac{\mu_{1,t+1}\lambda_{1,t}\left(\hat{u}_t^2 - U^2\right)}{U^2}, 0 < \mu_{1,t+1} < 1,\\
\lambda_{2,t+1} &= \lambda_{2,t} + \frac{\mu_{2,t+1}\lambda_{2,t}\left(\left(v_{t+1} - v_t\right)^2 - W^2\right)}{W^2}, 0 < \mu_{2,t+1} < 1.
\end{aligned} \tag{22}$$

that is generalization of all previous tuning procedures.

Simulation results

Let us consider the control problem for the plant described by the difference equation [2]

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + b_1 u_{t-1} + b_2 u_{t-2}$$

where $a_1 = 1.5$, $a_2 = -0.7$, $b_1 = 1$, $b_2 = 0.5$ and use as control criterion the Lagrangian function

$$L_t = M\left\{v_{t+1}^2 + \lambda_1\left(u_t^2 - U^2\right) + \lambda_2\left(v_t^2 - V^2\right)\right\}$$

where $v_t = y^* - y_t, y^* = 1$. The plant and PI-controller parameter values are suppoused to be unknown and initial values of their estimates were the following: $\hat{a}_{1,0} = \hat{a}_{2,0} = \hat{b}_{1,0} = \hat{b}_{2,0} = \hat{g}_{0,0} = \hat{g}_{1,0} = 0.1$. The controller parameters were tuned using algorithm (17) with $\lambda = 20$ (fig.1). For the purpose of improving of the control quality, the constraints on the control magnitude and the output error dynamics were introduced with $U = 2$ and $W = 0.1$ and the controller parameters were tuned using algorithm (22) with $\lambda_{1,0} = 1$, $\lambda_{2,0} = 5$, $\mu_{1,t} = \mu_{2,t} = 0.1$. The fig.2 shows that overshoot, the settle time and number of peaks were considerably redused.

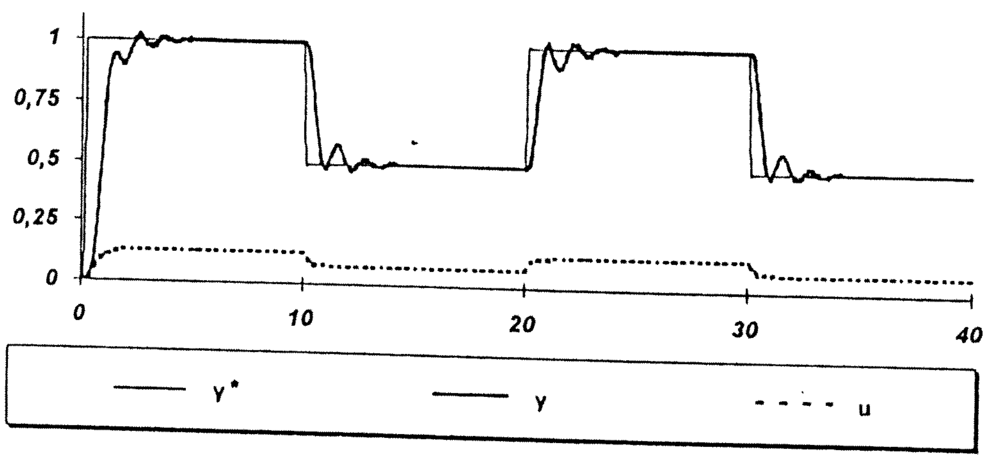

Fig.1

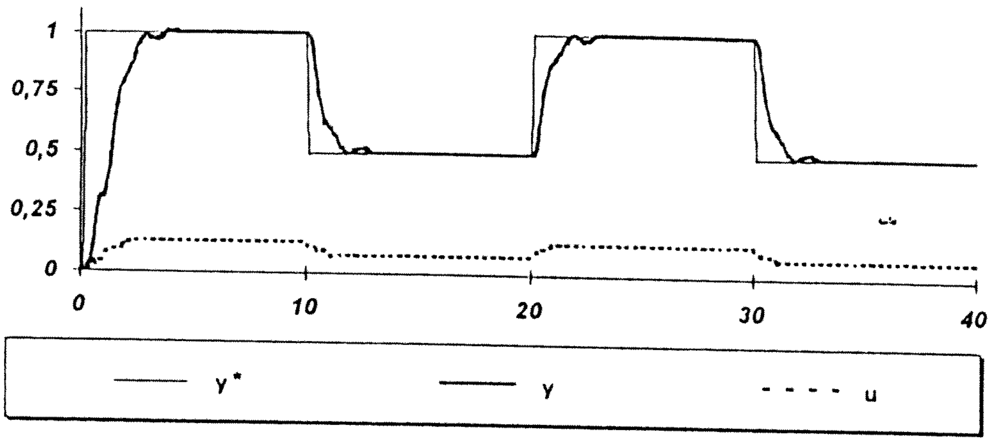

Fig. 2

Conclusion

An adaptive discrete PID controller for a plant described by linear stochastic difference equation is obtained. The on-line recurrent procedure for the controller parameter tuning with respect to constraints on control signal and output error value had been proposed. The performance of the adaptive PID controller had been demonstrated by computer simulation. The presented procedures make possible to implement generalized minimum variance control strategy using traditional PID controllers.

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