Chapter 1

Game Theory in Multiuser Wireless Communications

Jie Gao, Sergiy A. Vorobyov, and Hai Jiang

Contents

Wireless communications are always resource limited. In multiuser wireless systems, all users compete for resources and can interfere with each other. The conflicting objectives of users make it highly unlikely for any user to gain more profit without harming other users. The possibility of exploiting interactions among wireless users was not considered in traditional information-theoretic studies of the multiuser systems. Therefore, the results obtained from the information-theoretic perspectives might lead to unstable or even infeasible solutions for multiuser systems when the selfish nature of the users is taken into account. Indeed, it is reasonable to assume that all users compete for the maximum achievable benefit at all time. Then, the competition among the users should be analyzed and regulated to improve the overall performance of the whole multiuser system.

For a multiuser system, the resource-sharing problem can be investigated from a game-theoretic perspective. Without coordination among users, the existence of stable outcomes, corresponding to the so-called *Nash Equilibria* (NE), can be analyzed. On the other hand, if there is a voluntary cooperation among users, extra benefits for all users can be gained and optimally distributed among users. In both the noncooperative and cooperative cases, the efficiency of resource utilization can be boosted and the system stability can be guaranteed.

Due to the aforementioned advantages, there is an increasing amount of research efforts studying multiuser wireless systems from the game-theoretic perspective [1–6]. In the literature, game theory has been exploited in various systems such as code division multiple access (CDMA) [7,8], orthogonal frequency-division multiplexing (OFDM) systems [9,10], ad hoc [11,12], and cognitive radio networks [13–15]. The purpose of this chapter is to discuss applications of game theory for designing multiuser strategies in wireless communications. A brief introduction to the basics of game theory will be given first. Power allocation, beamforming, and precoding problems will then be discussed. These problems will be formulated as scalar-, vector-, and matrix-valued games, respectively, and analyzed from a game-theoretic perspective. Both the noncooperative and cooperative cases will be considered. Performance metrics such as efficiency, fairness, uniqueness of the solution, and complexity will be compared and discussed. One conclusion of the studies will be the demonstration of the advantages associated to the cooperative strategies over the noncooperative ones. The cooperative strategies, however, require coordination among users and, thus, can lead to an increase of system overhead.

1.1 A Brief Survey: Multiuser Games in Wireless Communications

Mathematically, an M-player game can be modeled as

$$\Gamma = \left\{\Omega, \{s_i | i \in \Omega\}, \{u_i | i \in \Omega\}\right\} \tag{1.1}$$

where
$\Omega = \{1, 2, \ldots, M\}$ is the set of all players
s_i is the strategy of player i
u_i is the utility (payoff) for player i as a function of $\{s_1, s_2, \ldots, s_M\}$

Depending on whether players collaborate or not, a game can be cooperative or noncooperative. In the following sections, some basic concepts of noncooperative and cooperative games will be reviewed, and examples based on simplified wireless systems will be given.

1.1.1 Noncooperative Games and Nash Equilibria

In a noncooperative game, there is no collaboration among users and the existence of an *equilibrium* is the main concern. An equilibrium $\{s_i | i = 1, \ldots, M\}$ is the strategy set composed of such strategies from which none of the players wants to deviate [16]. The most popular example of an equilibrium is the *Nash equilibrium* (NE), which can be mathematically expressed as

$$u_i\left(s_i^{\text{NE}}, s_{\bar{i}}^{\text{NE}}\right) \geq u_i\left(s_i{}', s_{\bar{i}}^{\text{NE}}\right) \quad \forall s_i{}' \tag{1.2}$$

where
s_i^{NE} is the strategy of player i in the NE
$s_{\bar{i}}^{\text{NE}}$ are the corresponding strategies of all players but player i in the NE
$s_i{}'$ stands for any possible strategy but s_i^{NE} for player i

An NE is a stable combination of all users' strategies such that no player can increase its utility by deviating from his current strategy given that other players do not deviate as well.

Generally, there are two typical issues with NE solutions. First, more than one NE may exist for a game, which renders difficulty in predicting the final outcome of the game. Second, an NE can lead and even usually leads to an inefficient outcome for all the players. In order to show it, the following two-user two-channel communication system is explored as an example.*

Example 1.1

Assume that there are two users and two communication channels, and each user has to make a decision about which channel to transmit on. First consider a simplified case: each user has a fixed power budget and is only allowed to transmit on one channel. Let c_1 denote the strategy that a user transmits on channel 1 and let c_2 denote the strategy that a user transmits on channel 2, respectively. Thus, the players are the two users and the strategy space is $\{c_1, c_2\}$. The information rates of the players are chosen to be their utilities. The combination of users' strategies and corresponding utilities are organized in Table 1.1.

In the table, the utilities of the two users are ordered as (u_1, u_2) with u_i representing the utility of user i, and $\hat{r}_{ij}$ and r_{ij} represent the information rate user i can obtain on channel j with and without interference from the other user, respectively (obviously $\hat{r}_{ij} < r_{ij}$). Although it is a simple two-user game, the uniqueness of NE† depends on the channel conditions (including desired channels and interference channels) and the noise power on each channel for each user. Let us assume that the noise power is identical on each channel for each user and study the impact of channel conditions on the game. We stress on the three following possible cases:

* The discussion of NE in this chapter is limited to pure strategy NE only.
† The abbreviation NE is used for both Nash equilibrium and Nash equilibria.

Table 1.1 Simplified Two-User Two-Channel Game

		User 1	
		c_1	c_2
User 2	c_1	$(\hat{r}_{11}, \hat{r}_{21})$	(r_{12}, r_{21})
	c_2	(r_{11}, r_{22})	$(\hat{r}_{12}, \hat{r}_{22})$

- *Case 1—Symmetric channels.* If the desired channels are identical for each user ($r_{i1} = r_{i2}, \forall i \in \{1, 2\}$), then the game has two NEs, which are $\{s_1 = c_1, s_2 = c_2\}$ and $\{s_1 = c_2, s_2 = c_1\}$. Note that in this case, the interference channels do not have to be identical.
- *Case 2—Asymmetric channels and high interference.* If channel 1 is better for both users than channel 2 ($r_{i1} > r_{i2}, \forall i \in \{1, 2\}$), and the interference channels are strong for user 1 but weak for user 2 such that $\hat{r}_{11} < r_{12}, \hat{r}_{21} > r_{22}$, then the game has a unique NE, which is $\{s_1 = c_2, s_2 = c_1\}$.
- *Case 3—Asymmetric channels and low interference.* If channel 1 is better for both users than channel 2 ($r_{i1} > r_{i2}, \forall i \in \{1, 2\}$), and the interference channels are weak such that $\hat{r}_{11} > r_{12}, \hat{r}_{21} > r_{22}$, then the game has a unique NE, which is $\{s_1 = c_1, s_2 = c_1\}$.

There are some other cases to discuss, which are left to the readers.

There is a special case of NE, which can be shown in the generalized two-user two-channel game. Assume that the number of channels that each user can transmit on is not limited. However, the power budget on each channel is fixed for each user (power is nontransferable between users or channels). Then the game can be described as in Table 1.2, where strategy 0 corresponds to allocating no power on any channel and strategy $c_1 + c_2$ corresponds to allocating power on both channels.

One feature of this game is that there is a unique NE regardless of the channel conditions. Indeed, given any choice that one player makes, the other player will choose the strategy of using all the channels, that is, it has a fixed best strategy. Such an NE is recognized as a dominant strategy equilibrium and can be formally described as

$$u_i\left(s_i^{\mathrm{D}}, s_{\bar{i}}\right) > u_i\left(s_i', s_{\bar{i}}\right) \quad \forall s_{\bar{i}} \quad \forall s_i' \neq s_i^{\mathrm{D}} \tag{1.3}$$

Table 1.2 Generalized Two-User Two-Channel Game

		User 1			
		0	c_1	c_2	$c_1 + c_2$
User 2	0	$(0, 0)$	$(r_{11}, 0)$	$(r_{12}, 0)$	$(r_{11} + r_{12}, 0)$
	c_1	$(0, r_{21})$	$(\hat{r}_{11}, \hat{r}_{21})$	(r_{12}, r_{21})	$(\hat{r}_{11} + r_{12}, \hat{r}_{21})$
	c_2	$(0, r_{22})$	(r_{11}, r_{22})	$(\hat{r}_{12}, \hat{r}_{22})$	$(r_{11} + \hat{r}_{12}, \hat{r}_{22})$
	$c_1 + c_2$	$(0, r_{21} + r_{22})$	$(\hat{r}_{11}, \hat{r}_{21} + r_{22})$	$(\hat{r}_{12}, r_{21} + \hat{r}_{22})$	$(\hat{r}_{11} + \hat{r}_{12}, \hat{r}_{21} + \hat{r}_{22})$

A dominant strategy equilibrium, if exists, is the unique NE in the game. However, the utilities resulted for the players may not be "dominant." Consider the same example as in Table 1.2. If the interference between users are very strong on both channels such that ($\hat{r}_{11} + \hat{r}_{12} < r_{11}$ and $\hat{r}_{21} + \hat{r}_{22} < r_{22}$), the strategy set $\{s_1 = c_1 + c_2, s_2 = c_1 + c_2\}$ will be inferior to the strategy set $\{s_1 = c_1, s_2 = c_2\}$.

The inefficiency of the NE is due to the fact that there is no cooperation among the players. The lack of cooperation usually leads to inefficient resource allocation in multiuser systems. We will next introduce the so-called Nash bargaining (NB) games in multiuser systems in order to improve the system performance.

1.1.2 Cooperative Games: Nash Bargaining and Other Bargaining Solutions

To achieve better payoffs, users may resort to cooperation. By sharing some information, players can determine whether there are potentially extra utilities for everyone if they cooperate. If there are such extra utilities, players may bargain with each other to decide how to share them. Otherwise, they come back to the noncooperative state.

For cooperative games, the Nash axiomatic bargaining theory states that, in a convex utility space, there is a unique point that satisfies four specific axioms and maximizes the Nash function defined as

$$F = \prod_{i \in \Omega} \left(u_i - u_i^{\text{NC}}\right) \tag{1.4}$$

where

u_i^{NC} is the utility that user i obtains in the noncooperative case

the point $\left(u_1^{\text{NC}}, \ldots, u_M^{\text{NC}}\right)$ is known as the disagreement point with M being the number of players

Readers are referred to [17] for details of the four axioms.

Nash axiomatic bargaining focuses on describing the properties of the final solution of a cooperative game. However, the manner of cooperation based on which the users cooperate to reach the solution is not specified. Thus, the NB solution in a specific game depends on the manner of cooperation. For example, wireless users may perform time division multiplexing (TDM) or frequency division multiplexing (FDM) in a cooperative game.

Example 1.2

Assume that there is one communication channel and two users, and denote the information rate of user i (user i's utility) that can be achieved by using the channel exclusively as R_i. Now let the users cooperate by using the channel alternatively in time. User 1 uses a fraction α of the time and obtains information rate αR_1 as its utility, while user 2 uses a fraction $1 - \alpha$ of the time and obtains information rate $(1 - \alpha)R_2$ as its utility. There are three following possible cases.

- *Case 1*. If the disagreement point is (0, 0), then the maximization of the Nash function (1.4) satisfies the so-called proportional fairness principle [18].
- *Case 2*. If the disagreement point is (R_1', R_2') and there exists $0 \le \alpha \le 1$ such that $\alpha R_1 > R_1'$ and $(1 - \alpha)R_2 > R_2'$, then the game has an NB solution.

Table 1.3 The "Fairness"of NB Solution

(R_1, R_2)	*Disagreement Point*	*α in NB Solution (%)*
(10,10)	(3,3)	50
(10,8)	(3,2)	52.5
(10,5)	(3,1)	55

- *Case 3*. If the disagreement point is (R_1', R_2') and there does not exist $0 \leq \alpha \leq 1$ such that $\alpha R_1 > R_1'$ and $(1-\alpha)R_2 > R_2'$, then the game does not have an NB solution if the TDM manner of cooperation is assumed.

Case 1 shows that the NB is related to a certain fairness principle, which justifies the necessity of cooperation among users. Case 2 states that all users should be able to improve their utilities to achieve a cooperative solution. Case 3 illustrates that the existence of the NB solution is not guaranteed in a cooperative game under a specific manner of cooperation.

Note that the proportional fairness of the NB solution does not mean that it must be always fair. In Table 1.3, the parameters in Example 1.2 are given different values and it is shown that the NB solution sometimes favors the user with better utility in the disagreement point.

One main limitation of the NB approach is that it requires convex utility spaces. In multiuser wireless systems, information rates are usually chosen as users' utilities. However, the interference among the users always renders the utility space (rate region) non-convex. The most popular approach to transfer the non-convex utility space to a convex one in multiuser wireless communication systems is to use orthogonal signaling such as time division multiple access (TDMA), frequency division multiple access (FDMA), or both. The efficiency of such methods will be discussed later.

There are several other important results on cooperative games, such as Kalai–Smorodinsky and Egalitarian solutions, which also deal only with convex games [19]. In the literature, there are limited research efforts that extend the Nash axiomatic bargaining, Kalai–Smorodinsky, and Egalitarian solutions to certain non-convex games [20,21].

1.1.3 Multiuser Systems: Generalized Signal Model and Game Model

In this section, a generalized model for many games played in multiuser systems is given. It will be specified and investigated in details in subsequent sections. Our focuses are the resource-sharing games, which are well studied in the literature in wireless communications. The following assumptions are adopted:

1. Players are the wireless users in a communication system. Codebook at each transmitter is assumed to be Gaussian codebook, and the achievable information rate between each transceiver pair is selected as the user's utility. Players' strategies depend on specific setups of the problem.
2. Users have to interact with each other due to the existence of interference among them, which makes game theory applicable. Interference perceived by a user at its receiver side is treated as noise. No interference canceling decoding is adopted for neither noncooperative games nor cooperative games.

3. The transmission channels of each user are known at both receiver and transmitter sides, that is, receivers are able to transmit the channel information back to their transmitters without errors.

Generalized Signal Model
Assume that there are M wireless users in a communication system. We have the following model:

$$\mathcal{Y}_i = \mathcal{H}_{ii}\mathcal{Q}_i\mathcal{S}_i + \sum_{j \neq i} \mathcal{H}_{ji}\mathcal{Q}_j\mathcal{S}_j + \mathcal{N}_i \quad \forall i \in \Omega = \{1, 2, \ldots, M\} \tag{1.5}$$

where
$\mathcal{Y}_i$ is the received symbol (or symbol vector) for user i
$\mathcal{S}_i$ is the information symbol (or symbol vector) to be transmitted for user i
$\mathcal{Q}_i$ can be the power allocation parameter/beamforming vector/precoding matrix of user i depending on the setup of the problem
$\mathcal{H}_{ji}$ is the channel between transmitter of user j and receiver of user i
$\mathcal{N}_i$ is the additive white Gaussian noise

It is further assumed that $E\{\mathcal{S}_i\mathcal{S}_i^H\} = \mathbf{I}$ (information symbols are uncorrelated and have unit-energy) and $E\{\mathcal{N}_i\mathcal{N}_i^H\} = \sigma_i^2\mathbf{I}$ (noise is white with variance σ_i^2), where $\mathbf{I}$ and $(\cdot)^H$ stand for the identity matrix and the Hermitian transpose, respectively. The dimensions of $\mathcal{Y}_i$, $\mathcal{S}_i$, $\mathcal{Q}_i$, $\mathcal{H}_{ji}$, and $\mathcal{N}_i$ depend on the specific problem setup, such as the channel fading conditions or number of antennas at the transceiver pairs. These parameters will be specified later in each example considered.

The corresponding game model for the generalized signal model is as follows:

Generalized Game Model

$$\Gamma = \{\Omega = \{1, 2, \ldots, M\}, \{\mathcal{Q}_i | i \in \Omega\}, \{R_i | i \in \Omega\}\} \tag{1.6}$$

where R_i is the achievable information rate for user i under the strategy set $\{\mathcal{Q}_i | i \in \Omega\}$.

The generalized signal and game models provide a unified structure for many games in wireless systems. In subsequent sections, the generalized models (1.5) and (1.6) will be specified to investigate the power allocation, beamforming, precoding games, etc. We start from the most basic games—power allocation games.

1.2 Power Allocation Games: Competition versus Cooperation

Power allocation games are the most fundamental and well-studied games in wireless communications. Games involving more complex signal-processing techniques can be transferred to power allocation games, as we will show later. In this section, several power allocation games are studied for both flat fading and frequency selective fading channels. Both the noncooperative and cooperative cases are covered.

1.2.1 Power Allocation on Flat Fading Channels

Assume that there are M wireless users sharing a channel with bandwidth W, which is flat fading for each user. The channel gain from transmitter j to receiver i is h_{ji}, and the power budget for user i is p_i. Then the generalized signal model (1.5) in this case can be specified as follows:

Signal Model for the M-User Flat Fading Channel

$$y_i = h_{ii}\sqrt{p_i}s_i + \sum_{j\neq i} h_{ji}\sqrt{p_j}s_j + n_i \quad \forall i \in \Omega \tag{1.7}$$

Note that the users' strategies are not shown in the signal model. The strategy is characterized by the bandwidth w_i occupied by each user. The corresponding game model is as follows.

Game Model for the M-User Flat Fading Channel

$$\Gamma = \left\{\Omega = \{1,2,\ldots,M\}, \{w_i | 0 < w_i \le W\}, \{R_i\}\right\} \tag{1.8}$$

Unlike the discrete games in the brief survey, this game is a continuous game. First, consider the game in the noncooperative case. Similar to the discussion in the preceding section, the result of the noncooperative game depends on the channel parameters. Following cases summarize some typical results. The detailed proof is omitted and the readers are referred to [22–24].

- *Case 1.* If the interference channels h_{ji} are weak, there exists a unique NE in which user i's strategy is $w_i = W, \forall i \in \Omega$. The utility for user i in this case is

$$R_i = \frac{W}{2}\log_2\left(1 + \frac{|h_{ii}|^2 p_i}{W\sigma^2 + \sum_{j\neq i}|h_{ji}|^2 p_j}\right) \tag{1.9}$$

- *Case 2.* When the interference channels become stronger, there may exist more than one NE. As the interference keeps increasing, the competitive solution converges to FDM.

It is straightforward to see that the NE in Case 1 is actually a dominant strategy equilibrium. The intuition behind these two cases is that it is more beneficial for the players to use a larger bandwidth when the interference from other users is weak. On the other hand, avoiding the interference leads to a better payoff when the potential interference is very high.

Next, we consider the cooperative case. A simple manner for the users to cooperate is FDM [24]. The assumption of FDM adds additional constraints to the utility set of the game. Practically, the users should use nonoverlapping frequency bands and $\sum_i w_i \le 1$. Equivalently, a user's strategy can be defined as the portion of the whole bandwidth that the user obtains. If user i obtains a fraction α_i of W, then $R_i(\alpha_i) = (\alpha_i W/2)\log_2(1 + |h_{ii}|^2 p_i/\alpha_i W\sigma^2)$ in the cooperative game case.

In this specific game, the Nash function is

$$F(\boldsymbol{\alpha}) = \prod_i \left(R_i(\alpha_i) - R_i^{\mathrm{NC}}\right) \tag{1.10}$$

where $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_M]$ and R_i^{NC} is decided by the disagreement point. The NB solution can be derived by solving the following optimization problem:

$$\begin{aligned} \max_{\boldsymbol{\alpha}} \quad & \prod_i \left(R_i(\alpha_i) - R_i^{\mathrm{NC}}\right) \\ \text{s.t.} \quad & 0 < \alpha_i < 1 \ \forall i \\ & \sum_i \alpha_i = 1 \ \forall i \end{aligned} \tag{1.11}$$

Table 1.4 Users' Utilities (R_1, R_2) in Two-User Flat Fading Channel Games

	$h=0.1$	$h=0.3675$	$h=1$
$w_1 = w_2 = 1$	(2.0715, 2.0715)	(1.3394, 1.3394)	(0.4826, 0.4826)
$w_1 = w_2 = 0.5$	(1.3394, 1.3394)		

Note that the equality in the second constraint follows from the fact that the objective $F(\boldsymbol{\alpha})$ can be further increased if $\sum_i \alpha_i < 1$.

The NB solution exists in this game if and only if there exists $\boldsymbol{\alpha} \succ 0$ such that $\sum_i \alpha_i \leq 1$ and $R_i(\alpha_i) \succ R_i^{\mathrm{NC}}$, where $\succ$ denotes "larger than" in element-wise comparison. In other words, the NB solution exists if and only if all users are able to improve their utilities using FDM.

It may appear contradictory that the previously discussed noncooperative NE solution may converge to FDM, which is assumed in the cooperative NB game. However, unlike the cooperative case, the users are driven to FDM in their pursuits of individual benefits when the interference is high in the noncooperative game. Moreover, generally the partitions of the whole bandwidth are different for the FDM-based NB and the NE in FDM.

The efficiency of the NB solution depends on the manner of cooperation assumed. For example, FDM is highly inefficient in a low-interference multiuser system.

Example 1.3

A comparison between the strategy in which both users choose to use the whole bandwidth and interfere with each other and the strategy in which the users share the bandwidth using FDM is listed in Table 1.4. Particularly, assuming a symmetric system in which $M = 2$, $W = 1$, $p_1 = p_2 = 10$, $\sigma^2 = 1$, $|h_{11}| = |h_{22}| = 1$, $|h_{12}| = |h_{21}| = h$, the users' utilities under different strategy set $\{w_i\}$ are shown in Table 1.4.

In the table, the noncooperative strategy $w_1 = w_2 = 1$ is the dominant strategy equilibrium when $h = 0.1$. The FDM-based NB solution does not exist when the interference channels are very weak (e.g., when $(2.0715, 2.0715)$ is the noncooperative NE solution for the case $h = 0.1$). The FDM-based cooperative solution becomes comparatively more and more efficient when the interference channel becomes stronger. For the case when the interference between the users is strong enough, for example, $h = 1$, the NB solution of the FDM cooperative game exists under strategies $w_1 = w_2 = 0.5$.

The fundamental reason behind the fact that FDM (and some other manners of cooperation) can be inefficient is that it reduces the utility space of the game while making it convex and, thus, limits the solution to belong to only a subset of the original utility space. We will introduce a different manner of cooperation, which enlarges the utility space and produces a "convex hull" in the subsequent sections.

1.2.2 Power Allocation on Frequency Selective Fading Channels

On frequency selective fading channels or equivalently inter-symbol interference (ISI) channels, multiple users have to spread their power over frequency bins of the wideband channel. The optimal power allocation scheme on a frequency selective fading channel for a single user can be derived using the well-known water-filling algorithm. However, the competition for resource arises when there are more than one user in the system.

Assume that there are M users, and the frequency selective fading channel can be decoupled into N frequency bins, each of which is flat fading for all users. The signal model (1.5) in this case can be specified as follows:

Signal Model for the M-User Frequency Selective Fading Channel

$$\mathbf{y}_i = \mathbf{H}_{ii}\sqrt{\mathbf{p}_i} \odot \mathbf{s}_i + \sum_{j \neq i} \mathbf{H}_{ji}\sqrt{\mathbf{p}_j} \odot \mathbf{s}_j + \mathbf{n}_i, \quad \forall i \in \Omega \tag{1.12}$$

where

$\mathbf{y}_i = [y_i(1), \ldots, y_i(N)]$ is the received symbol vector for user i on the frequency bins 1 to N

$\mathbf{s}_i = [s_i(1), \ldots, s_i(N)]$ is the information symbol vector transmitted by user i

$\mathbf{p}_i = [p_{i1}, \ldots, p_{iN}]$ is the power allocation vector of user i on the N frequency bins

$\mathbf{H}_{ji}$ is the $N \times N$ diagonal channel matrix with its kth diagonal element $h_{ji}(k)$ denoting the sampled channel gain of the kth frequency bin between transmitter of user j and receiver of user i

$\mathbf{n}_i$ is the additive white Gaussian noise for user i

$\odot$ denotes the Hadamard product

Note that the assumptions made on $\mathbf{s}_i$ and $\mathbf{n}_i$ in the generalized signal model in Section 1.1.3 are inherited here.

Power Allocation Game under Spectral Mask Constraints

On frequency selective fading channels, the so-called spectral mask (also known as power spectral density [PSD] mask) constraints are typically adopted. These constraints can be written as

$$\mathbf{p}_i \preceq \mathbf{p}_i^{\max} \quad \forall i \in \Omega \tag{1.13}$$

where

$\mathbf{p}_i^{\max} = \left[p_{i1}^{\max}, \ldots, p_{iN}^{\max}\right]$ is the spectral mask for user i

$\preceq$ denotes "less than or equal to" in element-wise comparison

In the noncooperative case, the game model corresponding to the signal model under the spectral mask constraints is as follows:

Game Model (Noncooperative, Spectral Mask) for the M-User Frequency Selective Fading Channel

$$\Gamma = \left\{\Omega = \{1, 2, \ldots, M\}, \left\{\mathbf{p}_i \middle| 0 \preceq \mathbf{p}_i \preceq \mathbf{p}_i^{\max}\right\}, \{R_i\}\right\} \tag{1.14}$$

where $R_i = \sum_k \log_2(1 + (|h_{ii}(k)|^2 p_{ik})/(\sigma_i^2(k) + \sum_{j \neq i} |h_{ji}(k)|^2 p_{jk}))$ and $\sigma_i(k)^2$ is the noise power for user i on frequency bin k.

It is straightforward to see that this noncooperative game has a dominant strategy equilibrium, which is $\mathbf{p}_i = \mathbf{p}_i^{\max}$. The proof is based on the fact that if one user does not use maximum power on all the frequency bins, it can always improve its utility by changing to the strategy of using maximum power on all the frequency bins. The details of the proof are left to the readers.

In the cooperative case, the users can cooperate by adopting the joint TDM/FDM scheme [25]. In the joint TDM/FDM scheme

1. All frequency bins are shared among the users in time domain, that is, each frequency bin can be used by different users over different time intervals (TDM part).
2. All frequency bins are partitioned and allocated to the users, while only one user is allowed on any frequency bin at any given time (FDM part).

The joint TDM/FDM scheme generates a convex utility space for all users to perform NB. Since the users' utilities depend only on the lengths of time durations which the users have obtained on the frequency bins, the users' strategies can be defined in terms of their TDM/FDM coefficients as

$$\boldsymbol{\alpha}_i = [\alpha_{i1}, \ldots, \alpha_{iN}] \quad \forall i \in \Omega \tag{1.15}$$

where α_{ik} is the proportion of time that is allocated to user i on frequency bin k.

The cooperative game model can be, then, given as follows:

Game Model (Cooperative, Spectral Mask) for the M-User Frequency Selective Fading Channel

$$\Gamma = \left\{ \Omega = \{1, 2, \ldots, M\}, \left\{ \boldsymbol{\alpha}_i \middle| \boldsymbol{\alpha}_i \succ \mathbf{0}, \sum_i \boldsymbol{\alpha}_i \preceq \mathbf{1} \right\}, \{R_i(\boldsymbol{\alpha}_i)\} \right\} \tag{1.16}$$

where $R_i(\boldsymbol{\alpha}_i) = \sum_k \alpha_{ik} \log_2(1 + |h_{ii}(k)|^2 p_{ik}^{\max}/\sigma_i^2(k))$.

The corresponding NB solution can be obtained by solving the following convex optimization problem:

$$\begin{aligned} \max_{\{\boldsymbol{\alpha}_i\}} \quad & \sum_i \log\left(R_i(\boldsymbol{\alpha}_i) - R_i^{\text{NC}}\right) \\ \text{s.t.} \quad & \boldsymbol{\alpha}_i \succ \mathbf{0} \quad \forall i \\ & \sum_i \boldsymbol{\alpha}_i = \mathbf{1} \quad \forall i \\ & R_{i(\boldsymbol{\alpha}_i)} > R_i^{\text{NC}} \quad \forall i \end{aligned} \tag{1.17}$$

The objective function in (1.17) is the logarithm of the Nash function. The first two constraints are the requirements of the joint TDM/FDM, while the third constraint states that the NB solution exists if and only if all users can benefit from the joint TDM/FDM–based cooperation.

Power Allocation Game under Total Power Constraints

The games under total power constraints are different and, in fact, more complex for both noncooperative and cooperative cases. It is because the available power for all users is bounded by the total power constraints in this case.

The noncooperative game model under total power constraints for the same signal model (1.12) can be expressed as follows:

Game Model (Noncooperative, Total Power) for the M-User Frequency Selective Fading Channel

$$\Gamma = \left\{ \Omega = \{1, 2, \ldots, M\}, \left\{ \mathbf{p}_i \middle| \sum_k p_{ik} \leq P_i^{\max} \right\}, \{R_i\} \right\} \tag{1.18}$$

where

$P_i^{\max}$ is the maximum power that user i can use in total

R_i is the same as in (1.14)

Fortunately, this game is a convex game, and, therefore, it has at least one NE [26]. Thus, the existence of an NE is guaranteed. Practically, iterative water-filling (including sequential water-filling and parallel water-filling) algorithms work well for finding NE of such games. However, there is no guarantee that the water-filling algorithms can converge. There are examples in which both sequential water-filling and parallel water-filling may diverge while searching for an NE of a simple game with a unique NE [25].

The cooperative case is even more complex. Assume again that the users cooperate according to the joint TDM/FDM scheme. Then the users' strategies are composed of both the TDM/FDM coefficients and the power allocation coefficients on the frequency bins due to the total power constraints. The cooperative game model under total power constraints in this case is as follows:

Game Model (Cooperative, Total Power) for the M-User Frequency Selective Fading Channel

$$\Gamma = \left\{ \Omega = \{1, 2, \ldots, M\}, \left\{ \mathbf{p}_i, \boldsymbol{\alpha}_i \middle| \sum_k \alpha_{ik} p_{ik} \leq P_i^{\max} \right\}, \{R_i\} \right\} \tag{1.19}$$

where

$\boldsymbol{\alpha}_i$ has the same meaning as in the cooperative game under spectral mask constraints

$R_i(\boldsymbol{\alpha}_i) = \sum_k \alpha_{ik} \log_2(1 + |h_{ii}(k)|^2 p_{ik}/\sigma_i^2(k))$

The challenge in this problem lies in the fact that the maximization of the Nash function is a non-convex optimization problem. A water-filling-based algorithm can still be used to search for the NB solution in the simplified two-user version of this model [27]. The algorithm bargains in many different convex subspaces of the original utility space and obtains one NB solution in each subspace. Then the largest of the NB solutions is selected as the final NB solution of the game. However, the complexity of such an algorithm is high even for the two-user case and the algorithm cannot be extended to the M-user ($M > 2$) games.

Similar to the FDM-based cooperative game on the flat fading channel, the joint TDM/FDM–based cooperation can be inefficient when the interference among users is weak.

There are some other models for cooperative power allocation games. For example, in [28], a cooperative game similar to the game on the frequency selective fading channel with total power constraints is studied. In [28], neither TDM nor FDM is assumed. Instead, the users maximize the Nash function iteratively given the strategies of other users. The algorithm is proved to converge, however, not necessarily to the global optimal point.

Power allocation games are basic, since many complex games can be transformed to equivalent power allocation games. The power allocation games introduced in this section pave the way to the higher-level games in the following two sections. A power allocation cooperative game under both spectral mask and total power constraints is considered in [29].

1.3 Beamforming Games on MISO Channels

The games in the preceding section are all games on single-input single-output (SISO) channels. Note that games involving multiple antennas have also been studied in the literature. The focus in this section is to investigate the games played on multi-input single-output (MISO) channels where the users' strategies are defined as their beamforming vectors.

1.3.1 Beamforming on the MISO Channel: Noncooperative Games

Assume that there are M wireless users in a system, all communication channels are flat fading, and each user has N antennas at the transmitter side and a single antenna at the receiver side. All users interfere with each other if they communicate simultaneously, and this setup constitutes an MISO interference channel. The signal transmission model in this case is as follows [30].

Signal Model for the Beamforming on M-User MISO Channel

$$y_i = \mathbf{h}_{ii}^T \mathbf{w}_i s_i + \sum_{j \neq i} \mathbf{h}_{ji}^T \mathbf{w}_j s_j + n_i \quad \forall i \in \Omega \tag{1.20}$$

where

y_i is the received symbol for user i
s_i is the information symbol transmitted by user i
$\mathbf{w}_i$ is the beamforming vector of user i
$\mathbf{h}_{ji} = [h_{ji}(1), \ldots, h_{ji}(N)]$ is the $N \times 1$ channel vector between transmitter of user j and receiver of user i
n_i is the additive white Gaussian noise for user i
$(\cdot)^T$ stands for transpose

The assumptions made on $\mathbf{s}_i$ and n_i in the generalized signal model in Section 1.1.3 are also inherited here.

First, consider a two-user beamforming game and assume that the number of transmit antennas is only two (this is a practically important case). Moreover, let all users be subject to the total power constraint $\|\mathbf{w}_i\|^2 \leq P, \forall i \in \Omega$. The game model in this case can be written as follows:

Game Model (Noncooperative) for the Beamforming on Two-User MISO Channel

$$\Gamma = \left\{\Omega = \{1, 2\}, \{\mathbf{w}_i | \; \|\mathbf{w}_i\|^2 \leq P\}, \{R_i\}\right\} \tag{1.21}$$

where $R_i = \log_2(1 + |\mathbf{w}_i^T \mathbf{h}_{ii}|^2 / (|\mathbf{w}_j^T \mathbf{h}_{ji}|^2 + \sigma^2)), \{j\} = \Omega \backslash \{i\}$.

It can be shown that this game has a unique NE (which is actually a dominant strategy equilibrium), and user i's strategy in the NE is $\mathbf{w}_i^{\text{NE}} = \mathbf{h}_{ii}^* / \|\mathbf{h}_{ii}\|$ where $(\cdot)^*$ denotes complex conjugate [30]. The proof here is similar to that in the power allocation game on the frequency selective fading channel with total power constraints in the previous section.

The same result can be derived in much more general cases, for example, in the cases when

1. The number of users is arbitrary.
2. The number of antennas at each transmitter is arbitrary and can be different for different users.
3. The power constraints are different for different users.

Note that NE may be a poor solution for all users, especially when the interference among users is strong. Thus, cooperation is a better choice for both users if the communication system can afford the related overhead.

1.3.2 Cooperative Beamforming: Time Sharing, Convex Hull, and Nash Bargaining

Recall that the NB solutions are only defined for games with convex utility spaces. In the cooperative power allocation games, FDM or joint TDM/FDM schemes are used to generate convex subspaces of the original utility spaces in order to perform NB. The inefficiency is noticed when the interference among users is weak, or equivalently, when the convex sub-spaces are quite small compared to the original utility spaces. Unlike the FDM or join TDM/FDM, here a new manner of cooperation is introduced in [30], which enlarges the original utility space by generating a *convex hull* of the original space to perform bargaining.

Denote the original utility space (rate region) of the two-user game with constraints $\|\mathbf{w}_i\|^2 \le P$, $\forall i \in \Omega$ as $\mathcal{R} = \{R_1, R_2\}$. The convex hull of the game can be obtained by performing time sharing

$$\tilde{\mathcal{R}} = \left\{\beta R_1^1 + (1-\beta)R_1^2, \beta R_2^1 + (1-\beta)R_2^2\right\} \tag{1.22}$$

where
(R_1^1, R_2^1) and (R_1^2, R_2^2) are two points in $\mathcal{R}$
β is the time-sharing coefficient

In the first β portion of time, both users choose their strategies $\mathbf{w}_i^1$, $\forall i \in \Omega$ and get payoff βR_i^1, $\forall i \in \Omega$. Then in the remaining $1-\beta$ portion of time, the users choose strategies $\mathbf{w}_i^2$, $\forall i \in \Omega$ and get payoff $(1-\beta)R_i^2$, $\forall i \in \Omega$. Thus, the overall payoff for user i is $\tilde{R}_i = \beta R_i^1 + (1-\beta)R_i^2, \forall i \in \Omega$.

It can be proved that $\tilde{\mathcal{R}}$ is a convex set, which includes $\mathcal{R}$ as its subset. Therefore, the bargaining can be performed in a larger space. However, there is a price paid for generating the convex hull using time sharing. Indeed, the users have to determine their strategies twice along with the time-sharing coefficient β. Thus, the number of variables of the NB problem to be solved is increased.

The model for the two-user cooperative time-sharing game is as follows.

Game Model (Cooperative, Time Sharing) for the Beamforming on Two-User MISO Channel

$$\Gamma = \left\{\Omega = \{1,2\}, \left\{\mathbf{w}_i^l; \beta \,\middle|\, \|\mathbf{w}_i^l\|^2 \le P, l = 1,2; 0 \le \beta \le 1\right\}, \{\tilde{R}_i\}\right\} \tag{1.23}$$

Then, the NB solution can be obtained from solving the following optimization problem:

$$\begin{aligned} \max_{\{\mathbf{w}_i^l\},\beta} \quad & \sum_i \log\left(\tilde{R}_i - R_i^{\mathrm{NC}}\right) \\ \text{s.t.} \quad & \left\|\mathbf{w}_i^l\right\|^2 \le P \quad \forall i \quad \forall l \\ & 0 \le \beta \le 1 \\ & \tilde{R}_i > R_i^{\mathrm{NC}} \quad \forall i \end{aligned} \tag{1.24}$$

Cooperative solutions on MISO channels have also been studied using other bargaining theories. For example, Nokleby and Swindlehurst [31] investigate the cooperative bargaining game on the MISO channel and derive the Kalai–Smorodinsky solution.

1.3.3 Pareto-Optimality: Competition and Cooperation

One property of the NB solution is its Pareto-optimality. However, it is not the only Pareto-optimal point in the utility space. In fact, all points in the Pareto boundary of the corresponding utility space are Pareto-optimal. For multiuser games in wireless systems, the points on the Pareto boundary generally represent efficient allocations of communication resources. For the above studied games on the two-user MISO channels, an interesting result is that any Pareto-optimal point can be realized through certain balance of users' competition and cooperation [32,33]. To explain the proposition, we start by considering the NE and the zero-forcing (ZF) beamforming strategies.

Recall that the NE for the two-user noncooperative beamforming game is $\mathbf{w}_i^{\mathrm{NE}} = \mathbf{h}_{ii}^*/\|\mathbf{h}_{ii}\|$ [30]. The NE strategy can be viewed as completely competitive and selfish because the user that uses this strategy aims at maximizing its own payoff only. On the other hand, a user is considered as altruistic if it adopts the strategy that generates no interference to other users. Such a strategy is known as the ZF strategy. Note that the ZF strategy for the two-user beamforming game can be expressed as [32]

$$\mathbf{w}_i^{\mathrm{ZF}} = \frac{\mathbf{\Pi}_{\mathbf{h}_{ij}^*}^{\perp}\mathbf{h}_{ii}^*}{\left\|\mathbf{\Pi}_{\mathbf{h}_{ij}^*}^{\perp}\mathbf{h}_{ii}^*\right\|} \quad \forall i \in \Omega, \{j\} = \Omega\backslash\{i\} \tag{1.25}$$

where $\mathbf{\Pi}_{\mathbf{A}}^{\perp} = \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$ is the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{A}$.

Indeed, if both users choose the ZF strategies, they obtain nonoptimal payoffs for themselves yet avoid interfering with each other. Thus, the ZF strategies can be considered as completely cooperative and altruistic strategies. Note that in general, the NE and ZF strategies do not lead to the Pareto-optimal solutions. However, it is proved that any point on the Pareto-optimal boundary can be achieved using a certain combination of the NE and the ZF. This combination can be formally expressed as [32]

$$\mathbf{w}_i(\lambda_i) = \frac{\lambda_i\mathbf{w}_i^{\mathrm{NE}} + (1-\lambda_i)\mathbf{w}_i^{\mathrm{ZF}}}{\left\|\lambda_i\mathbf{w}_i^{\mathrm{NE}} + (1-\lambda_i)\mathbf{w}_i^{\mathrm{ZF}}\right\|} \quad \forall i \in \Omega \tag{1.26}$$

where λ_i is a parameter satisfying $0 \le \lambda_i \le 1$.

The detailed proof is omitted here. However, intuitively, one can understand it in the following way. To achieve global efficiency, each user has to combine its own utilities and all other users' utilities in its objective to be maximized. Considering NB as an example, the NB solution is Pareto-optimal in a convex utility space because every user maximizes the Nash function, which combines all players' utilities. In fact, if each user maximizes the weighted Nash function defined as

$$F = \prod_i (R_i - R_i^{\mathrm{NC}})^{\gamma_i} \tag{1.27}$$

where $\gamma_i > 0, \forall i$, then any point on the Pareto-optimal boundary can be achieved using the weighted NB by varying γ_i's. Using the ZF strategy, a user actually aims at maximizing the achievable utility of other users, while using the NE strategy, a user aims at maximizing its own utility only. Thus, the combination of the ZF and the NE strategies covers the utilities of all players in the game and can lead to Pareto-optimal solutions.

1.4 Matrix Games: Precoding Games and Others

In the previous sections, we have introduced power allocation games on SISO channels and beamforming games on MISO channels. Next, we proceed to the games on multiple-input multiple-output (MIMO) channels. It will be shown that such games can be transformed into power allocation games under the assumption of orthogonal signaling and, thus, can be solved using the aforementioned methods.

1.4.1 Precoding Games on the MIMO Channels

Consider an M-user communication system based on OFDM. The signal transmission model can be written as follows [34,35].

Signal Model for the Precoding on M-User MIMO Channel

$$\mathbf{y}_i = \mathbf{H}_{ii}\mathbf{F}_i\mathbf{s}_i + \sum_{j\neq i} \mathbf{H}_{ji}\mathbf{F}_j\mathbf{s}_j + \mathbf{n}_i \quad \forall i \in \Omega \tag{1.28}$$

where

$\mathbf{y}_i$ is the received symbol block for user i
$\mathbf{s}_i$ is the information symbol block transmitted by user i
$\mathbf{F}_i$ is the precoding matrix of user i
$\mathbf{H}_{ji}$ is the $N \times N$ channel matrix between transmitter of user j and receiver of user i
$\mathbf{n}_i$ is the additive white Gaussian noise for user i

Note that the assumptions made on $\mathbf{s}_i$ and $\mathbf{n}_i$ in the generalized signal model are inherited here.

In an OFDM system, cyclic prefixes (CP) are used to cancel the ISI and decouple the wideband frequency selective fading channel into a number of flat fading frequency bins. The channel matrix $\mathbf{H}_{ji}$ can be diagonalized as $\mathbf{H}_{ji} = \mathbf{W}\mathbf{D}_{ji}\mathbf{W}^H$ due to the CP insertion, where $\mathbf{W}$ is the $N \times N$ discrete Fourier transform (DFT) matrix and $\mathbf{D}_{ji}$ is the diagonal sampled channel matrix between transmitter of user j and receiver of user i.

The transmitters in the system are subject to both total power and spectral mask constraints, that is,

1. *Total power constraints:* $Tr\left(\mathbf{F}_i\mathbf{F}_i^H\right) \leq NP_i, \forall i \in \Omega$, where $Tr(\cdot)$ stands for the trace operator and P_i is the maximum available power for the transmission of a signal symbol.
2. *Spectral mask constraints:* $E\{|[\mathbf{W}^H\mathbf{F}_i\mathbf{s}_i]_k|^2\} = \left[\mathbf{W}^H\mathbf{F}_i\mathbf{F}_i^H\mathbf{W}\right]_{kk} \leq p_i^{\max}(k), \forall i \in \Omega, \forall k \in \{1,\ldots,N\}$, where $E\{\cdot\}$ stands for the expectation and $p_i^{\max}(k)$ is the maximum power that user i can allocate on frequency bin k.

Let $\mathfrak{F}_i$ be the strategy space of user i subject to the above constraints, then the noncooperative game model of this signal model can be written as follows:

Game Model (Noncooperative) for the Precoding on M-User MIMO Channel

$$\Gamma = \left\{\Omega = \{1,2,\ldots,M\}, \{\mathbf{F}_i|\mathbf{F}_i \in \mathfrak{F}_i\}, \{R_i\}\right\} \tag{1.29}$$

where $R_i = (1/N)\log_2\left(\left|\mathbf{I} + \mathbf{F}_i^H\mathbf{H}_{ii}^H\mathbf{R}_i^{\dagger}\mathbf{H}_{ii}\mathbf{F}_i\right|\right)$ is the information rate of user i with $\mathbf{R}_i = \sigma^2\mathbf{I} + \sum_{j\neq i}\mathbf{H}_{ji}\mathbf{F}_j\mathbf{F}_j^H\mathbf{H}_{ji}^H$ representing the noise plus interference for user i.

The above game is a matrix game. However, it is proposed that this game can be transformed into a power allocation game due to the CP insertion. Practically, it is proved that the NE of the precoding game can be achieved using the following precoding strategies [34]:

$$\mathbf{F}_i = \mathbf{W}\sqrt{diag(\mathbf{p}_i)} \quad \forall i \in \Omega \tag{1.30}$$

where $diag(\mathbf{a})$ is a square diagonal matrix with its kth diagonal elements as $[\mathbf{a}]_k$ and $\mathbf{p}_i$ can be found by solving the following power allocation game.

Game Model (Noncooperative) for the Precoding on M-User MIMO Channel, Equivalent Power Allocation Game

$$\Gamma = \left\{\Omega = \{1,2,\ldots,M\}, \left\{\mathbf{p}_i \middle| \mathbf{p}_i \preceq \mathbf{p}_i^{\max}, \sum_k \mathbf{p}_i(k) \leq NP_i\right\}, \{R_i\}\right\} \tag{1.31}$$

where $\mathbf{p}_i^{\max} = \left[p_i^{\max}(1),\ldots,p_i^{\max}(N)\right]$ is the spectral mask vector. The expressions for R_i's can also be simplified. However, we skip it here for the sake of brevity.

The equivalent game (1.31) is similar to the noncooperative power allocation game on the frequency selective fading channel (1.18). One difference is that this game considers both spectral mask and total power constraints. However, the water-filling-based algorithms are still applicable [35].

In what follows, for the cooperative case of the precoding game, only spectral mask constraints are considered for simplicity because the total power constraints will render the problem non-convex [36,37]. Adopting again the joint TDM/FDM scheme as the manner of cooperation among users, it can be observed that the diagonal structure as in (1.30) also applies to the cooperative case, and thus, the cooperative precoding game can be transformed to a cooperative power allocation game, which can be written as follows:

Game Model (Cooperative) for the Precoding on M-User MIMO Channel, Equivalent Power Allocation Game

$$\Gamma = \left\{\Omega = \{1,2,\ldots,M\}, \left\{\mathbf{p}_i, \boldsymbol{\alpha}_i \middle| \mathbf{p}_i \preceq \mathbf{p}_i^{\max}, \mathbf{0} \preceq \boldsymbol{\alpha}_i \preceq \mathbf{1}, \sum_i \boldsymbol{\alpha}_i \preceq \mathbf{1}\right\}, \{R_i\}\right\} \tag{1.32}$$

where $\boldsymbol{\alpha}_i = [\alpha_i(1), \ldots, \alpha_i(N)]$ is the TDM/FDM coefficient vector of user i as defined in (1.15).

The above game is similar to the cooperative power allocation game on the frequency selective fading channel (1.16). The difference is that this game is not a two-player game and the algorithm used for solving the two-player game cannot be extended to the multiuser case. This equivalent cooperative game can be solved in a distributed manner using a dual decomposition–based algorithm [37]. A non-convex game with both spectral mask and total power constraints is considered in [29].

1.4.2 Other Matrix Games

Besides the precoding games, some other matrix games have also been studied [38,39]. Specifically, games can be played on MIMO channels with the strategies defined as the users' signal covariance matrices subject to total power constraints [38]. For the noncooperative case, the MIMO water-filling algorithm based on singular value decomposition of the channel matrices and signal covariance matrices is adopted to obtain the NE of the game [38]. It is noticed that the algorithm, which works well practically, is not guaranteed to converge. Note here the similarity with the noncooperative power allocation game on the frequency selective fading channel under total power constraints. For the cooperative case, the gradient projection method is proposed for finding the NB solution of the game. However, the convergence can be only guaranteed to a local optimal point.

A game can also be played to allocate communication resources among the secondary users in an MIMO cognitive radio system [39]. The constraints can be set on each user's total power and the maximum interference that it can generate. It is observed that this matrix game can be transformed to a power allocation game while the interference constraints can be transformed to the rotations of the channel matrices. Moreover, the NE solution for the covariance matrices adopts the diagonal structure similar to (1.30). Again the MIMO water-filling-based algorithm can be used to derive the NE with the original channel matrices substituted by the rotated channel matrices.

1.5 Further Discussions

1.5.1 Efficiency and Fairness

The efficiency of the NE and NB solutions has been mentioned and compared many times in preceding sections. The examples lead to the conclusion that the FDM or joint TDM/FDM–based NB solutions are efficient when the potential interference among users is strong and vice versa. We have two supplementary remarks here.

First, the above conclusion is based on the assumption that the noise power is fixed for all users. Actually the efficiency of the FDM and joint TDM/FDM-based NB solutions depends on the interference to noise ratio (INR). If the interference power dominates the noise power, the FDM and joint TDM/FDM–based cooperations are efficient. Otherwise, they are inefficient because the FDM and joint TDM/FDM schemes cannot decrease the noise PSD.

Secondly, the efficiency of the FDM or joint TDM/FDM–based NB solutions is not equivalent to the efficiency of the NB solution. Indeed, the NB solution depends on the manner of cooperation assumed. For the same multiuser wireless system, different manners of cooperation can generate different NB solutions. For example, time sharing (recall the cooperative beamforming game on the MISO channel) can achieve better efficiency than joint TDM/FDM at the price of increasing the number of parameters in the corresponding optimization problem.

As for the fairness, it has been mentioned that the NB solution is related to the so-called proportional fairness. However, our previous example also shows that the NB solution favors the user with better utility in the disagreement point. On the other hand, NE are considered as completely selfish strategies. However, NE are not necessarily unfair. The fairness of NE, if any, depends on the system setup (wireless users' desired channels, interference channels, and the noise power). For example, in a symmetric system, an NE will be completely fair for all users. However, unlike the NB solution, NE may not be unique, and the discussion on fairness would be obviously meaningless in this case.

1.5.2 Uniqueness and Complexity

It is well known that in a convex utility space only one point maximizes the Nash function, while there may be multiple NE regardless of the convexity of the utility space. However, it is more accurate to conclude that the NB solution is unique if it exists. In fact, the NB solution may not exist even in a convex utility space. Again, the existence of the NB solution depends on the specific manner of cooperation and the problem setup. If all users are able to benefit from cooperation, there is a unique NB solution in the corresponding cooperative game. A detailed study can be found in [40,41]. The condition for the uniqueness of NE is much more complex, and we refer the readers to the references of noncooperative games in preceding sections.

Therefore, one difficulty in finding NE is related to the fact that it may not be unique, which adds to the complexity of the game. In many cases, the algorithms designed for finding NE are not guaranteed to converge. However, for NB, determining the manner of cooperation, which renders the corresponding utility space convex and as large as possible contributes to the main complexity.

1.5.3 Implementation: Centralized versus Distributed Structure

An NE solution has an advantage over the NB solution in developing distributed implementations. It is because in an NE each player only considers its own utility, which usually requires local information only. However, to find the NB solution, all users need to cooperate with each other and information exchange is inevitable. For example, in the joint TDM/FDM–based cooperation scheme, the TDM/FDM coefficients of all users need to be collected and broadcasted to the users. Thus, a completely distributed algorithm is not possible for most cooperative bargaining games. However, a distributed structure with a coordinator is still possible for cooperative games. Two examples are developed in [37,42]. In these examples, the original bargaining problems are decoupled into two-level problems. Then in the corresponding two-level structure, the higher level problem is solved by the coordinator using the information collected from all users, while the lower level problems are solved in parallel by individual users using local information and the broadcasted information from the coordinator. Note that the coordinator may be selected from one of the users or performed by the users in a round-robin manner.

1.6 Open Issues

1.6.1 On the User Number

Currently two-user games have been extensively studied in the literature. Generalizations to the multiuser cases, however, are still open problems. Algorithms for two-user games are not always applicable for multiuser cases. For example, the algorithm used in [24] in order to find the NB solution cannot be extended for solving the problem with the number of players $M > 2$. However, this problem exists only in cooperative games. For most noncooperative games, the water-filling-based solutions are applicable to both two-user and multiuser games.

1.6.2 On the Constraints

This applies mostly to the cooperative games due to the requirement of convex utility spaces for the NB. As mentioned before, maximization of the Nash function can be a non-convex optimization problem under certain constraints. One example is the cooperative power allocation game with total power constraints. Moreover, many games in the literature deal with other power-related constraints. Games that incorporate multiple constraints related to different practical requirements are still open for research.

1.6.3 On the Strategies

Most of the works on applications of game theory in wireless communications consider pure strategies only. Mixed strategies are seldom studied in multiuser wireless systems. Thus, the investigation of noncooperative and cooperative games based on mixed strategies remains an open problem. Moreover, players of the games introduced in this chapter have their strategies defined on a single target, for example, power. In a multiuser system, however, power allocation is usually not the only issue that needs to be solved to optimize the performance of the system. Admission control, scheduling, and others may also need to be taken into account. Thus, the users may have joint strategies defined on power, scheduling, and/or other parameters. In many applications, such games are still yet to be investigated.

1.7 Conclusion

This chapter reviews the applications of game theory for the multiuser wireless systems. Different games on different channels are considered, for both noncooperative and cooperative cases. The power allocation games are reviewed in details. These power allocation games construct the basis for higher-level beamforming and precoding games. The focuses of the discussion are on the comparison between noncooperative and cooperative games, as well as on games under different constraints. A number of similarities and differences between different games are emphasized and different performance metrics for cooperative and noncooperative games are analyzed. Throughout this chapter, we show that game theory is a powerful tool for solving many different problems in the multiuser wireless communications.

References

1. A. B. MacKenzie and S. B. Wicker. Game theory in communications: Motivation, explanation, and application to power control. In *Proceedings of IEEE Global Communications Conference*, pp. 821–826, San Antonio, TX, November 2001.
2. F. Meshkati, H. V. Poor, and S. C. Schwartz. Energy-efficient resource allocation in wireless networks. *IEEE Signal Processing Magazine*, 24(3):58–68, May 2007.

3. N. Dusit and H. Ekram. Radio resource management games in wireless networks: An approach to bandwidth allocation and admission control for polling service in IEEE 802.16. *IEEE Wireless Communications*, 14(1):27–35, February 2007.
4. A. Agustin, O. Muhoz, and J. Vidal. A game theoretic approach for cooperative MIMO schemes with cellular reuse of the relay slot. In *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 581–584, Montreal, Canada, May 2004.
5. J. O. Neel, J. H. Reed, and R. P. Gilles. Convergence of cognitive radio networks. In *Proceedings of IEEE Wireless Communications and Networking Conference*, pp. 2250–2255, Atlanta, GA, March 2004.
6. J. Virapanicharoen and W. Benjapolakul. Fair-efficient guard bandwidth coefficients selection in call admission control for mobile multimedia communications using game theoretic framework. In *Proceedings of IEEE International Conference on Communications*, pp. 80–84, Paris, France, June 2004.
7. S. Gunturi and F. Paganini. Game theoretic approach to power control in cellular CDMA. In *Proceedings of IEEE Vehicular Technology Conference*, pp. 2362–2366, Orlando, FL, October 2003.
8. S. M. Betz and H. V. Poor. Energy efficient communications in CDMA networks: A game theoretic analysis considering operating costs. *IEEE Transactions on Signal Processing*, 56(10):5181–5190, October 2008.
9. Z. Han, Z. Ji, and K. J. R. Liu. Power minimization for multi-cell OFDM networks using distributed non-cooperative game approach. In *Proceedings of IEEE Global Communications Conference*, pp. 3742–3747, Dallas, TX, November 2004.
10. H. Bogucka. Game theoretic model for the OFDM water-filling algorithm with imperfect channel state information. In *Proceedings of IEEE International Conference on Communications*, pp. 3814–3818, Beijing, China, May 2008.
11. Y. Wei and K. J. R. Liu. Game theoretic analysis of cooperation stimulation and security in autonomous mobile ad hoc networks. *IEEE Transactions on Mobile Computing*, 6(5):507–521, 2007.
12. O. Ileri, M. Siun-Chuon, and N. B. Mandayam. Pricing for enabling forwarding in self-configuring ad hoc networks. *IEEE Journal of Selected Areas in Communications*, 23(1):151–162, 2005.
13. M. Maskery, V. Krishnamurthy, and Q. Zhao. Decentralized dynamic spectrum access for cognitive radios: Cooperative design of a non-cooperative game. *IEEE Transactions on Communications*, 57(2):459–469, February 2009.
14. D. Niyato and E. Hossain. Competitive spectrum sharing in cognitive radio networks: A dynamic game approach. *IEEE Transactions on Wirless Communications*, 7(7):2651–2660, July 2008.
15. A. Attar, M. R. Nakhai, and A. H. Aghvami. Cognitive radio game for secondary spectrum access problem. *IEEE Transactions on Wirless Communications*, 8(4):2121–2131, April 2009.
16. D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, Cambridge, MA, 1991.
17. J. Nash. Two-person cooperative games. *Econometrica*, 21(1):128–140, January 1953.
18. Z. Han, Z. J. Ji, and K. J. R. Liu. Fair multiuser channel allocation for OFDMA networks using Nash bargaining solutions and coalitions. *IEEE Transactions on Communications*, 53(8):1366–1376, August 2005.
19. W. Thomson. Cooperative models of bargaining. In *Handbook of Game Theory with Economic Applications*, R. J. Aumann and S. Hart, (Eds.), vol. 2, pp. 1237–1284, Elsevier Science B.V., New York, 1994.
20. M. J. Herrero. The Nash program: Non-convex bargaining problems. *Journal of Economic Theory*, 49:266–277, 1989.
21. T. C. A. Anant and B. Mukherji. Bargaining without convexity: Generalizing the Kalai-Smorodinsky solution. *Economics Letters*, 33:115–119, 1990.
22. Z. Q. Luo and J. S. Pang. Analysis of iterative waterfilling algorithm for multiuser power control in digital subscriber lines. *EURASIP Journal on Applied Signal Processing*, 2006 (24012), 2006. Available at: http://www.hindawi.com/journals/asp/2006/024012.abs.html

23. A. Leshem and E. Zehavi. Bargaining over the interference channel. In *Proceedings of IEEE International Symposium on Information Theory*, pp. 2225–2229, Seattle, WA, July 2006.
24. A. Leshem and E. Zehavi. Cooperative game theory and the gaussian interference channel. *IEEE Journal of Selected Areas in Communications*, 26(7):1078–1088, September 2008.
25. A. Leshem and E. Zehavi. Game theory and the frequency selective interference channel. *IEEE Signal Processing Magazine*, 26(5):28–40, September 2009.
26. T. Basar and G. Olsder. *Dynamic Non-Cooperative Game Theory.* pp. 447–551, Academic Press, London, 1982.
27. E. Zehavi and A. Leshem. Bargaining over the interference channel with total power constraints. In *Proceedings of International Conference on Game Theory for Networks*, pp. 447–451, Istanbul, Turkey, May 2009.
28. J. E. Juan, L. A. DaSilva, Z. Han and A. B. MacKenzie. Cooperative game theory for distributed spectrum sharing. In *Proceedings of IEEE International Conference on Communications*, pp. 5282–5287, Glasgow, Scotland, June 2007.
29. J. Gao, S. A. Vorobyov, and H. Jiang. Cooperative resource allocation games under spectral mask and total power constraints. *IEEE Transactions on Signal Processing*, 58(8):4379–4395, August 2010.
30. E. G. Larsson and E. A. Jorswieck. Competition versus cooperation on the MISO interference channel. *IEEE Journal of Selected Areas in Communications*, 26(7):1059–1069, September 2008.
31. M. Nokleby and A. L. Swindlehurst. Bargaining and the MISO interference channel. *EURASIP Journal on Advances in Signal Processing*, 2009 (368547), 2009. Available at: http://www.hindawi.com/journals/asp/2009/368547.html
32. E. A. Jorswieck and E. G. Larsson. The MISO interference channel from a game theoretic perspective: A combination of selfishness and altruism achieves pareto optimality. In *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 5364–5367, Las Vegas, NV, March 2008.
33. E. A. Jorswieck, E. G. Larsson, and D. Danev. Complete characterization of the Pareto boundary for the MISO interference channel. *IEEE Transactions on Signal Processing*, 56(10):5292–5296, October 2008.
34. G. Scutari, D. P. Palomar, and S. Barbarossa. Optimal linear precoding strategies for wideband non-cooperative systems based on game theory—part I: Nash equilibria. *IEEE Transactions on Signal Processing*, 56(3):1230–1249, March 2008.
35. G. Scutari, D. P. Palomar, and S. Barbarossa. Optimal linear precoding strategies for wideband non-cooperative systems based on game theory—Part II: Algorithms. *IEEE Transactions on Signal Processing*, 56(3):1250–1267, March 2008.
36. J. Gao, S. A. Vorobyov, and H. Jiang. Game theoretic solutions for precoding strategies over the interference channel. In *Proceedings of IEEE Global Communications Conference*, New Orleans, LA, November 2008.
37. J. Gao, S. A. Vorobyov, and H. Jiang. Game theory for precoding in a multi-user system: Bargaining for overall benefits. In *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 2361–2364, Taipei, Taiwan, April 2009.
38. M. Nokleby, A. L. Swindlehurst, Y. Rong, and Y. Hua. Cooperative power scheduling for wireless MIMO networks. In *Proceedings of IEEE Global Communications Conference*, pp. 2982–2986, Washington, DC, November 2007.
39. G. Scutari, D. P. Palomar, and S. Barbarossa. Cognitive MIMO radio: A competitive optimality design based on subspace projections. *IEEE Signal Processing Magazine*, 25(6):46–59, November 2008.
40. Z. Chen, S. A. Vorobyov, C.-X. Wang, and J. Thompson. Nash bargaining over MIMO interference systems. In *Proceedings of IEEE International Conference on Communications*, pp. 1–5, Dresden, Germany, June 2009.

41. Z. Chen, S. A. Vorobyov, C.-X. Wang, and J. Thompson. Pareto region characterization for rate control in multi-user systems and Nash bargaining. *IEEE Transactions on Automatic Control.* Submitted. (Arxiv: 1006.1380).
42. H. Yaiche, R. R. Mazumdar, and C. Rosenberg. A game theoretic framework for bandwidth allocation and pricing in broadband networks. *IEEE/ACM Transactions on Networking*, 8(5):667–678, October 2000.