

# Modeling and Control of Hydraulic Drivetrain in Agricultural Tractor with Position Backlash in Speed Sensor Feedback

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**Abstract:** The modeling and control design of nonlinear systems with backlash is studied in many publications. However, in most cases, the backlash is considered in the input part of the plant, usually between a motor and a load, and in many cases, feedback from the motor side is available. In this paper, a hydrostatic drivetrain of an agricultural tractor powered by a diesel engine is studied. The drivetrain consists of a variable displacement pump and a direct-drive constant displacement motor. The special challenge, related to speed control, is a remarkable backlash in the wheel speed sensor, between the wheel and the sensor. The system also contains input saturation and quantization, but these nonlinearities are negligible to the backlash. The modeling of the system that contains engine, variable displacement pump with servo control, hydraulic pipes, constant displacement motor and speed sensor is presented in the paper. The dynamic model is fine-tuned with experimental data that is collected from the system by using various input signals. In the system under study, the only feedback signal is the speed sensor. The backlash is remarkable, about 55 degrees of wheel and this sets a specific challenge to design speed controller for the system. In this paper, three-mode control design is proposed together with backlash observer and switching logic. One of the modes is tuned to control speed in case the backlash gap is open, and this tuning is robust enough to avoid limit cycle. The second mode is taking care of the control in positive direction and the third one in negative direction when the backlash gap is closed. The positive and negative controllers try to keep the backlash gap closed as long as possible, in order to restrict the backlash gap opening only into the situations of driving direction changes.

**Keywords:** vehicle, agriculture, hydraulics, hydrostatic drivetrain, backlash compensation, observers, adaptive control, servo control

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## 1. INTRODUCTION

The control problem of a system with internal backlash has been studied since 1940's; Krylov & Bogoliubov (1943) and Tustin (1947). Over the years some established methods have been described in text books discussing nonlinear control systems. Studies of systems with backlash have been important for industry, as almost in all mechanical systems that contain gears some backlash appears. In the typical case, the actuator is a motor and that is connected to the load with gears, or a series of gears. Therefore in typical study the backlash type of nonlinearity is located between the two masses, a motor and a load, like in Odai et al (1998). In many studies also feedback from motor part has been used, like in Brandenburg (1986) and Marton & Lantos (2009).

The most common way to analyze the dynamic behavior of a system having internal backlash is to use a describing function; and in control design use the describing function together with linear part and controller to keep the system away from limit cycle. (Nordin and Gutman, 2002)

The other established control design is relying on a backlash observer, and a switching logic between "stiff" controller and

"soft" controller (Brandenburg and Schäfer, 1989). The stiff controller would be used when the observer finds that backlash gap is closed and the soft controller in case the backlash gap is open. This type of control can be classified into adaptive control. (Nordin and Gutman, 2002)

In this paper, a hydraulic drivetrain in an agricultural tractor is under study. The tractor is powered by a diesel engine and the transmission of power from engine to wheels is hydrostatic. The objective is to design a control system for wheel speed servo control. With open-loop control the steady-state error exists and a feedback control loop is needed. However, the only feedback measurement from the wheel contains a remarkable internal backlash in position (the objective is speed control), and this kind of nonlinearity in the feedback loop is not studied in many publications. In this paper is presented a model structure, identified model parameters, control design and achieved results.

## 2. MECHANICAL AND HYDRAULIC SYSTEM

The system under study is a prototype tractor. The tractor has four identical wheels, each of which is independently steered and driven. The tractor is equipped with 123 kW diesel engine with turbocharger. The drivetrain is hydrostatic. Each

wheel is driven with a constant displacement hydraulic motor ( $1800 \text{ cm}^3/\text{rev}$ ), direct drive. The tractor is equipped with four identical variable displacement hydraulic pumps ( $0\text{-}56 \text{ cm}^3/\text{rev}$ ), all of which are direct driven by the diesel engine. One pump is driving one wheel, all of which are identical. However, the hydraulic fluid container and the filters are shared hydraulic parts.

The diesel engine is controlled with another ECU, to keep desired speed. As mentioned, the hydraulic drive pumps are direct driven by the main shaft of the diesel engine. The other parts that are connected to the engine are an alternator, two additional hydraulic pumps for actuators and a PTO over a clutch and auxiliary equipments of the engine. The diesel engine idle speed was set to 740 RPM (revolutions per minute) and the maximum speed is 2500 RPM.

The hydraulic pump is an axial piston pump, with variable displacement. The displacement is controlled by swashplate swivel angle and the direction of flow is changed by changing the swashplate angle from negative to positive or vice versa. The pump contains an internal system for the pressure cut-off as well as for relief. The pump swashplate is controlled with electrical proportional solenoid.

The hydraulic motor is a radial piston motor, with five pistons. The total displacement is  $1800 \text{ cm}^3/\text{rev}$ . The pistons are driven with a flow distributor that is integrated into the motor. The distributor contains a rotating plate that divides flow to appropriate pistons in order to produce torque to the shaft. The distributor plate rotates about at the same speed as the motor shaft.

The speed is measured by using an optical incremental encoder that produces 2500 pulses/rev. As the rolling circumference of the wheel is 4.7 meters, one pulse corresponds to 1.8 mm in travel. As the typical driving speed is 2-3 m/s, the resolution to speed measurement is rather high. The encoder is mechanically connected to the distributor plate, not directly to the motor shaft. The only possibility to connect it to the motor shaft would require installation to the outer side of the wheel, and that is not done as it would require remarkable external chassis that would be liable to colliding into external objects. The indirect rotation measurement causes a remarkable backlash to the measurement that is seen every time that the motor drive direction is changed. The roughly identified backlash is around 55 degrees in a wheel, but it varies from wheel to wheel a bit. The backlash corresponds to 0.7 meter in travel. As the corresponding number of pulses is 380, the quantization error of measurement due to the incremental encoder is negligible.

The control system for modeling and rapid control prototyping was based on mounted ECU's that controlled both the engine and pumps and measured the encoder pulses. The proportional solenoid of the pump was controlled by using PWM signal and the resolution was 8-bit for both solenoids of a pump. The ECU was commanded by using Simulink software. Simulink and the ECU's were interconnected with a CAN bus and the sample time for the whole control system was 100 ms. The final control design

was implement on ECU, but the results shown in this paper were collected with rapid prototyping environment.

To see the actual speed of the wheel in the test setup, an external encoder with 360 ppr was installed on the outer side of the wheel and that signal was brought to rapid prototyping environment by using additional I/O board. The actual angle of the wheel was analyzed during the sensor backlash with this signal.

As a summary, the system contains a pump and motor that are quite linear and a speed sensor for feedback, which is pretty sensitive but on the other hand has a remarkable backlash that appears when the motor drive direction is changed.

### 3. MODELING

For modeling purposes the pump proportional control system was calibrated so that the input range was from -1 to +1. At both ends the swashplate of the pump is in the maximum position, and at zero value the angle is zero, and there is no flow to the motor. The calibration was done to each wheel-pump-motor system separately, even if no remarkable differences were found.

In order to model the dynamic and static behaviour, a number of tests were carried out with the system. The tests were done for each wheel separately and also in case of all-wheels driven at the same time. In open loop control, the system was excited with various input signals: a triangle from 0 to 1, a triangle from -1 to 0, a triangle from -1 to 1, steps from 0 to positive, steps from 0 to negative, and steps from negative to positive. Additionally, a response from engine speed to wheel speed was tested with constant swashplate angle and varying engine speed. It was found that the dynamic response from engine speed to actual wheel speed is negligible in comparison with the dynamics from the pump swashplate to wheel speed. Therefore in the dynamic modeling, the engine RPM is taken as static measurement that scales the flow rate from pump to motor; and the system model is SISO.

From input signals that do not cross zero, it is possible to identify the linear part of the system. After the linear part is identified, from signals crossing zero again and again, it is possible to estimate the backlash.

The identified model structure is shown in Figure 1. The inputs to the system are pump swashplate angle (from -1 to 1) and measured engine RPM, denoted below as  $\omega(t)$ . The linear behaviour was modeled as discrete transfer function (Equation 1), and a transfer delay of 0.1 s. The sensor part is shown in the feedback loop. The backlash appears in the position of the shaft, not on speed, so therefore the actual speed has to be first integrated and after the backlash it is time-derivated back to speed, the maximum backlash is denoted below as  $\alpha$ . Only one channel of the encoder was used for speed measurement, and no information about the direction was got, so therefore a sensor part contains an absolute function. As the direction is not measured from the plant, it has to be estimated. Luckily the dynamic behaviour of linear part is has a small time constant, the direction can be estimated by filtering the pump swashplate command over

the identified discrete filter, and taking the sign of this and putting it into the speed measurement, see Equation 2.

In Table 1 are represented the estimated values for each wheel.

$$H(z) = K \frac{1-a}{1-a \cdot z^{-1}} \tag{1}$$

$$\hat{y}_{sign}(z) = \text{sgn}(H(z) \cdot z^{-1} \cdot U(z)) \tag{2}$$

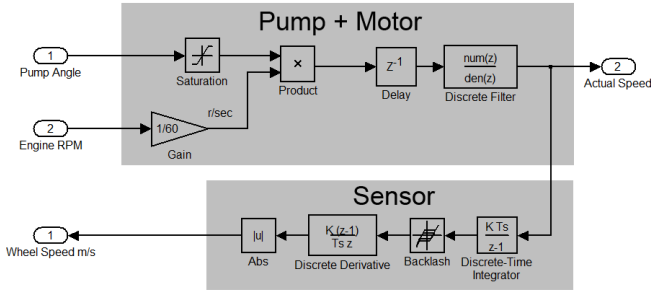


Figure 1. Model structure for the pump-motor-sensor system.

Table 1. Estimated values for model parameters

	<i>K</i>	<i>a</i>	<i>backlash</i>
Wheel 1	0.162	0.724	0.719
Wheel 2	0.162	0.723	0.730
Wheel 3	0.164	0.769	0.587
Wheel 4	0.164	0.771	0.606

In Figure 2 is presented one of the open-loop responses that was used in modeling the system; this test was carried out with engine speed 745 RPM. The input signal contains both the triangles on both sides, as well as zero crossing triangles, and steps. In Figure 3 is shown a part of Figure 2 signal. The noise amplitude is clearly dependent on the speed the backlash effect is clearly seen. The figure also shows that system gain is linear. Both in Figure 2 and Figure 3, the sign of measurement *y* is corrected by using the Equation 2.

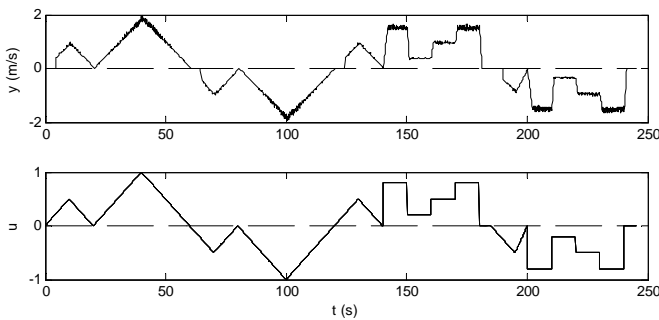


Figure 2. Open loop response and the input test signal.

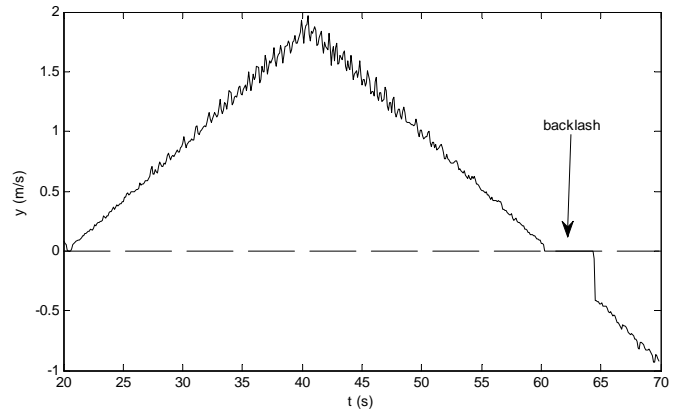


Figure 3. Backlash when input crossing zero.

Additionally, for the simulation purposes the noise of the system was modeled. By comparing the measurement data and output of the model with the same input signal, it was seen that the residual is non-autocorrelating and the noise level clearly depends on the speed of motor, or about of the swashplate angle. By dividing the modeling error with the input signal, the resulting relative modeling error is Gaussian distributed white noise. So for simulation purposes the sensor noise will follow Equation 3.

$$y_{sim}(k) = y(k) + y(k) \cdot k_{noise} \cdot w(k), \tag{3}$$

where  $k_{noise}$  is an identified parameter corresponding standard deviation and  $w$  is a Gaussian random variable. The value of  $k_{noise}$  was identified to be 0.039.

The frequency of limit cycle for this system can be analyzed in several ways. Describing function technique is a traditional method. However, this type of nonlinearity is not listed in textbooks. A trial to calculate describing function for this nonlinearity ends on failure, as one of the basic requirements for describing function is not fulfilled: in this case the nonlinearity seems to depend on input frequency. The nonlinearity is represented in Figure 4 on the left, and on the right is the amplitude response with various frequencies. It can be clearly seen that the describing function does not exist as the response depends on frequency.

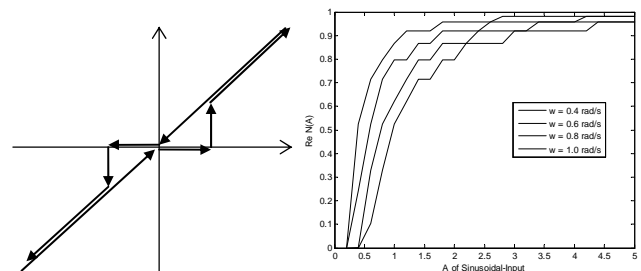


Figure 4. Nonlinear part and the frequency response of that.

The other way to analyze the frequency is to use simulation to find the closed loop margin gain that causes oscillation. By using that method, the margin gain is about 1.8 and the resulting frequency 2.0 rad/s. The third way is to compute Bode diagram numerically using simulation, and with this method, the result was 2.1 rad/s.

#### 4. CONTROL DESIGN

The main remarks from modeling were: a) the system gain is linear in case the backlash is not moving, b) the engine speed has a scaling effect on flow and actual speed, and c) the backlash is remarkable in comparison with the other nonlinearities or noises of the system. The linear behaviour of the system is so good that feedforward part of the controller is beneficial. As the backlash behaviour is known, the backlash can be observed from the input signal. Basically, if the sign of input is changed, the backlash gap will be opened, and the gap is assumed closed when the integrated input (position) over static part of the model is more than backlash.

The simple backlash observer is presented in Equation 4. The backlash is in an uncertain position, when  $\beta(k) \leq 1$ . When  $\beta(k) > 1$ , the backlash gap is considered closed.

$$\begin{cases} \beta(k) = \frac{1}{\alpha} \int_{t_0(k)}^{t(k)} K|u(t)|\omega(t)dt \\ t_0(k) = \begin{cases} t(k) & \text{if } \text{sgn } r(k) \neq \text{sgn } r(k-1) \\ t_0(k-1) & \text{else} \end{cases} \end{cases} \quad (4)$$

The proposed solution is a three-mode adaptive controller with feedforward part. The first mode is “soft”, where the control design is made taking care of not going into limit cycle. The second mode is used when driving forward, and the third when driving backwards. Three modes are used in place of typical two mode controller for the reason that the backlash is not moving unless the direction of the pump is changed. In the second mode the controller output is limited  $u(t) \in [\varepsilon, 1]$  and in the third mode the controller output is limited  $u(t) \in [-1, -\varepsilon]$ .  $\varepsilon$  is a small number; in the following experiments a value 0.005 was used. Therefore the overall control design tries not to change the direction of plant input unless required by the reference signal.

A simple three-mode switcher is presented in Equation 5. The inputs to each controller  $C_*$  are: the reference signal, the measurement feedback signal, the measured engine speed, the low level for output saturation and the high level for output saturation.

$$u(k) = \begin{cases} C_{stiff}(r(k), y(k), \omega(t), \varepsilon, 1), & \text{if } \beta(k) > 1 \text{ and } r(k) > 0 \\ C_{soft}(r(k), y(k), \omega(t), -1, 1), & \text{if } \beta(k) \leq 1 \\ C_{stiff}(r(k), y(k), \omega(t), -1, -\varepsilon), & \text{if } \beta(k) > 1 \text{ and } r(k) < 0 \end{cases} \quad (5)$$

The internal structure of each controller is presented in Figure 5. The control design incorporates feedforward part with gain scheduling from engine speed that is considered as a scaling, as it was proved in the modeling. Due to saturation also anti-windup is realised in the feedback controller. As the output saturation is in the range of real system input saturation  $[-1, 1]$  in any case, the realization is sufficient.

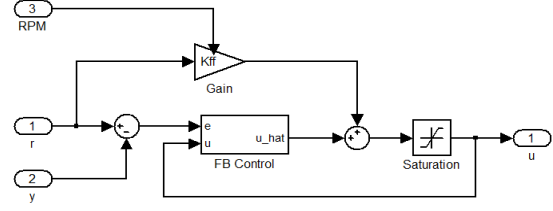


Figure 5. Internal structure of stiff and robust controllers.

As the open-loop control itself provides good enough performance, the role of feedback controller in the overall control is to compensate the steady-state error. Therefore the compensator in the feedback loop is a pure discrete time integrator, in stiff mode the gain was set to 0.16 and in robust mode to 0.04.

$$C_{stiff}^{fb}(z) = 0.16 \frac{1}{z-1}, \quad (6)$$

$$C_{robust}^{fb}(z) = 0.04 \frac{1}{z-1}, \quad (7)$$

#### 5. RESULTS

The proposed control design was tested both in simulation and with the real system.

The time response from the real system, from wheel 1, is shown in Figure 6. On top the measured output of the system is shown as bold and the reference with solid. On the left is shown a response for the ramp-type setpoint signal, and on the right for step inputs.

In Figure 7 is shown a Bode plot, a frequency response of the system with the three-mode controller. The diagram is drawn by using the sinusoidal frequency response in the simulator. The amplitude of the sinusoidal signal was 1. As computed earlier, the limit cycle of appears on frequency 2.1 rad/s, this can be seen also in the closed loop response. System behavior above 2 rad/s is steady.

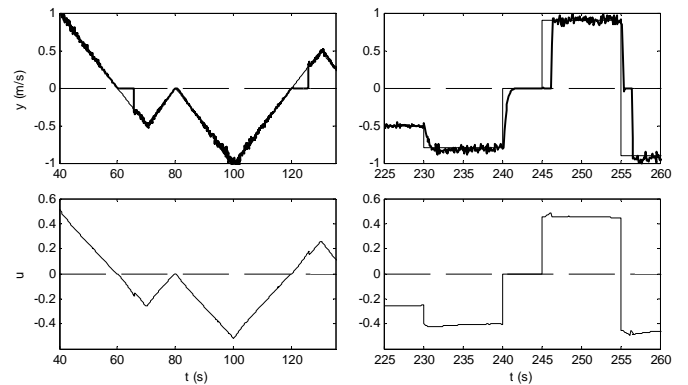


Figure 6. Closed-loop response with three-mode controller.

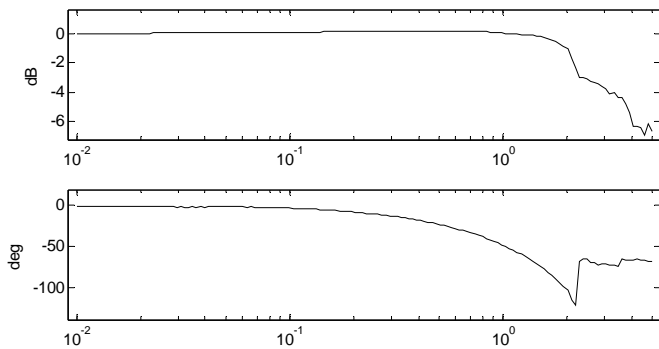


Figure 7. Bode plot of the closed system, from  $r$  to  $y$ .

## 6. CONCLUSIONS

In this paper a modeling and control design of a system with a sensor internal position backlash were presented. The specific feature of the system is the type of backlash that appears on the feedback loop in the sensor, and the backlash is in position of the angle sensor, not in angular velocity directly.

The modeling was done by using data driven identification and the system model was validated also with an external sensor. The dynamic model explains the hydraulic pump and hydraulic motor as a combined system. The dynamical differences between four wheels were not remarkable.

The control design is based on a three-mode adaptive controller, which was proposed as a solution in this paper. The mode is switched based on a backlash observer design and on the other hand based on the sign of the reference signal. Besides feedback control design, a feedforward part was used to achieve performance and the engine speed was taken into the control loop by using gain scheduling.

The presented results show that with the proposed control design both the performance and robustness are good.

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