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PARTICLE ACCELERATION AND TURBULENCE TRANSMISSION IN RELATIVISTIC PARALLEL SHOCKS

by

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Turku, February 2006

Joni Tammi

Contents

Abstract

This dissertation concentrates on two closely related and somewhat overlapping topics: particle acceleration, and turbulence transmission related to shock waves. Emphasis is on relativistic shocks propagating parallel to a magnetic field, and the effect of the thickness of the shock front is also considered.

Discussion is started with short reviews of the basic properties of astrophysical shocks and turbulence, and continued with considering their relations to particle acceleration. This introductory part is meant to give a non-specialist reader a picture of current knowledge concerning particle acceleration theory within the aforementioned limitations, as well as sufficient background information needed to understand the context of the research papers discussed in Chapter 5 and presented in the end of the thesis.

The research papers included in this thesis present results obtained using both analytical and numerical methods; for the latter, a numerical simulation code, QSHOCK (for a description, see Paper IV and `http://www.iki.fi/joni.tammi/qshock`), developed mainly by the author, has been used. First-order Fermi acceleration, in relativistic modified shocks, is studied numerically in Papers I and II, with our results confirming those found earlier by other authors, namely that parallel relativistic shocks with thickness determined by ion dynamics need a sufficiently strong energisation mechanism for the electrons in order to be able to accelerate them efficiently enough to account for observations. Paper III presents a way of calculating the downstream turbulence conditions from those of the upstream, for a relativistic step shock, and demonstrates that for shocks with low-to-moderate Alfvénic Mach number this leads to increased first-order acceleration efficiency. In Paper IV the results of Paper III are applied to a numerical simulation, and this is observed to lead to second-order acceleration in the downstream region. This mechanism is also found to be a promising candidate for the energisation process required by Papers I and II. Paper V presents results for first-order acceleration in modified shocks, taking into account turbulence transmission. Results show that even relatively thick shocks can produce very hard particle spectra, if the scattering-centre compression is sufficiently enhanced. This enhancement was found to follow from turbulence transmission analysis with certain upstream conditions in step-shocks of all speeds, and for non-to-mildly relativistic thick shocks (Paper VI).

List of publications

I *Simulations on the effect of internal structure of relativistic shock fronts on particle accereration*
Joni Virtanen and Rami Vainio
in High Energy Blazar Astronomy, ASP Conference Proceedings, Vol. 299, p. 157. Edited by L. O. Takalo and E. Valtaoja. ISBN: 1-58381-146-X (2003)

II *Monte Carlo simulations of electron acceleration in parallel relativistic shocks*
Joni Virtanen and Rami Vainio
in The 28th International Cosmic Ray Conference proceedings, p. 2023, Edited by T. Kajita, Y. Asaoka, A. Kawachi, Y. Matsubara and M. Sasaki (2003)

III *Alfvén-wave transmission and test-particle acceleration in parallel relativistic shocks*
Rami Vainio, **Joni Virtanen** and Reinhard Schlickeiser.
Astronomy & Astrophysics, 409, 821 (2003)
Erratum: Astronomy & Astrophysics, 431, 7 (2005)

IV *Stochastic acceleration in relativistic parallel shocks*
Joni Virtanen and Rami Vainio
The Astrophysical Journal, 621, 313 (2005)

V *Particle acceleration in thick parallel shocks with large compression ratio*
Joni Virtanen and Rami Vainio
Astronomy & Astrophysics, 439, 461 (2005)

VI *Turbulence transmission in parallel relativistic shocks using ray tracing*
Joni Tammi and Rami Vainio
submitted to Astronomy & Astrophysics

CHAPTER 1
Introduction

It is commonly accepted that shock waves have a central role in the production of energetic charged particles, observed either directly as cosmic rays or by the radiation they produce. The exact physical processes, and the way these particles are accelerated almost to the speed of light in shocks is, however, less certain. Although the theoretical understanding of how some of the basic mechanisms work has been improving for long, their details and limitations in more complex cases are still debated.

This thesis aims at contributing to the answer of the general question: ”How are charged particles accelerated to ultrarelativistic energies in shocks?” The particular question we try to answer is: ”How is particle acceleration in a parallel relativistic shock affected if the shock is not assumed to be a discontinuous step, and if the effect of the shock to the plasma turbulence is taken into account?” The latter question contains the ideas of non-steplike shocks and turbulence transmission in particle acceleration studies. Neither of these ideas are new, but both have been largely neglected in relativistic applications.

Problems in studying particles, shock structure and turbulence together lie in nonlinearity. Consider the following simplified picture (see Chapters 2–4 for detailed discussion and references). First of all, particles are accelerated in shocks; the basic mechanisms for this are quite well known (although the general opinion of the applicability of different models in different physical environments is still far from consensus). However, accelerated particles can shape the structure of the shock front, thus affecting their own acceleration efficiency. Effects of this kind of nonlinearity have been studied for long, and its consequences are known to some extent.

Secondly, the transport of particles is controlled by turbulence in the plasma – the effects and limitations are, again, well known. But the turbulence also is bound to undergo changes when hit by, and transmitted through, a relativistic shock wave, and in certain cases the changes in the turbulence also affect the acceleration of particles. For non-relativistic shocks this possibility has been studied to some level, but for relativistic case only a few special cases have been considered. The form of the shock front – whether steplike or modified – has further effects on the turbulence transmission, and these effects are, in turn, reflected in the acceleration process, in addition to the effects of the modification itself.

Thirdly, in addition to the effect of turbulence on particles, the particles themselves can affect the turbulence, or they can even be the source of the turbulence in the first place. Furthermore, the turbulence can affect the way particles get injected into the

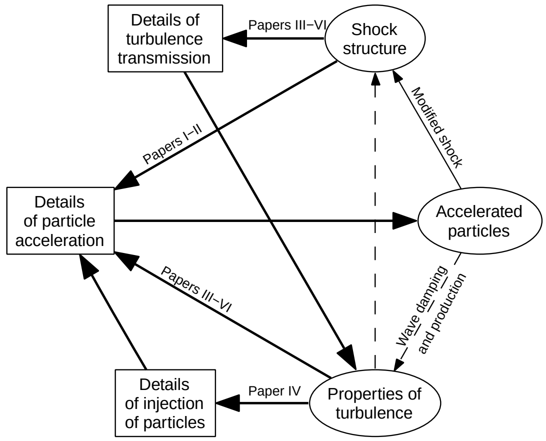

Figure 1.1: A sketch of the relations of some basic factors affecting the acceleration of charged particles in parallel shocks when turbulence is taken into account. Boxes on the left stand for general physical processes, ellipses on the right mark physical components, and the different lines point direction of effects. Thick lines show effects included and studied in the papers of this thesis, the thin solid line shows the effect of the assumed shock-front modification, and the dashed lines show relations not included in this work.

acceleration mechanism. Waves can even contribute to the modification of the shock front, thus affecting, again, the transmission of waves, bringing in additional sources of nonlinearity.

At this point, not even taking into account energy losses or radiation, the situation is extremely nonlinear (for a simplified sketch, see Figure 1.1), and far beyond the current capabilities of any analytical theory or model. The same applies widely also to numerical approaches, excluding maybe the most simple approximations. For this reason it is crucial to try to build the picture of a general particle acceleration theory piece by piece, starting from the foundation of the most basic and underlying mechanism, upon which different, finer pieces, are then laid and interlocked to each other.

In the papers of this thesis the interfaces between the pieces of shock structure, turbulence, and particle acceleration are tried to fit each other and the general picture of current theories, described in the following introductory part. The key results of the papers are presented in the scientific context, in Chapters 2–4, and discussed in more detail in Chapter 5. Chapter 6 makes some final remarks and general conclusions about these studies, and after that, the papers are presented in their original form.

The discussion and the research papers presented in this dissertation are kept on the theoretical level; we concentrate on the physical processes as such, and reserve the interpretation of observations and measurements, as well as the modelling of real observable sources, to future work. For general reviews concerning objects where particle acceleration and shocks are considered to be present, see, e.g., Blandford and Eichler (1987), Axford (1994), Jones and Ellison (1991), Protheroe and Clay (2004) or Schlickeiser (2002, Chapters 1–3 and 5–7).

CHAPTER 2
Astrophysical shocks

Astrophysical shock waves can be formed, e.g., by an object (consider, for example, a plasma ”cloud” denser than the surrounding medium) moving through a medium at a speed larger than the signal speed of the medium (typically sound or Alfvén speed). The matter ahead of the shock has no possibility of ”detecting” the forthcoming object, yet it has to have means to adapt to it. Nature has solved this problem with a shock wave: matter piles and compresses in front of the obstacle, thus forming a denser region where the heated and slowed matter flows, now sub-sonically, in the reference frame of the object. At the boundary of uncompressed and compressed material a shock front is formed. Alternatively, e.g., sudden release of large amount of energy into a small volume, e.g., by an explosion (such as a supernova, for instance), can cause a different kind of a shock, a blast wave. In this case, the blast of the explosion expands outwards, hitting the material, just as the shock wave ahead of a moving obstacle. Again, and also generally speaking, the shock wave is the surface between the unshocked material ahead of the shock and the compressed matter behind it.

The former type of shocks can be formed, for example, in the *hot spots* of relativistic jets of micro- and macroquasars where the jet drills into interstellar or -galactic matter. For the latter, an obvious example is the blast wave of a supernova.

2.1 Shock geometry and coordinate systems

When studying physics that take place right around the shock, it is very convenient to change to a coordinate system in which the shock itself is at rest. In this *shock frame* an observer, now ”flying” along with the shock front, sees the unshocked plasma flowing towards the shock with an *upstream flow speed* equal but opposite to the speed of the shock as seen in the upstream, undergoing compression at the front, and flowing out at a slower *downstream flow speed*. The pre-shock region is called the *upstream* while the post-shock half-space is called the *downstream*; the corresponding coordinate frames, moving in to and out of the shock, are called the *upstream* and *downstream rest frames*, because in them the upstream/downstream matter is at rest. Generally, the *local plasma (rest) frame* is the frame moving at the same speed as the plasma flow. Following the common convention of denoting different physical upstream quantities with subscript 1 and the downstream ones with subscript 2, we use V_1 and V_2 for the up- and downstream flow speeds, respectively (see Figure 2.1).

The word *shock* is here used to mean the region where the flow speed drops from the

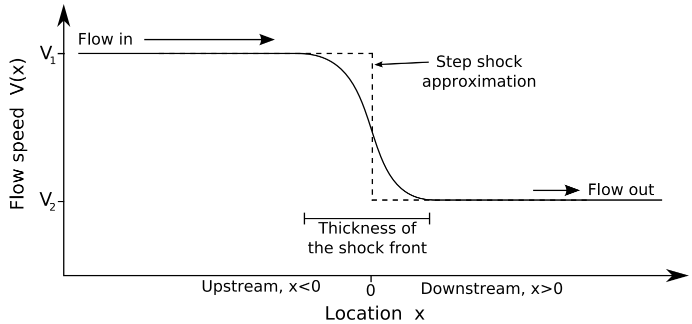

Figure 2.1: The flow profile and direction of the flow as measured in the shock frame.

upstream speed to that of the downstream. The crudest approximation of the shock front can be thought of as a discontinuity in the flow parameters. This kind of *step shock* is, obviously, quite unphysical in the smallest scales, but since, typically, the transition takes place on scales much shorter than what is seen by, e.g., the accelerating particles, the approximation usually simplifies analyses much more than restricts them. Generalisation of the step shock picture, the so-called *modified shock* structure, is introduced in the following sections.

This thesis concentrates on parallel shocks, i.e., cases in which the normal of the shock plane is parallel to the large-scale magnetic field direction. Due to the conservation laws for the magnetic field, the magnetic field strength and alignment remain constants throughout the shock transition, and, thus, the large-scale electric field vanishes. In general, of course, the angle θ between the shock normal $\mathbf{n}$ and the magnetic field direction $\mathbf{B}$ is not exactly zero, i.e., the shock is either *oblique* ($0 < \theta < \pi/2$) or *perpendicular* ($\theta = \pi/2$). However, in certain oblique cases it is possible to make a boost along the magnetic field to the frame where the shock is stationary. In this so-called *de Hoffmann–Teller frame* (after de Hoffmann and Teller 1950) (*dHT*) the electric field vahishes and the plasma flows along the magnetic field lines on both sides of the shock. Such a boost to the *dHT* frame is, of course, only possible if the speed required is less than the speed of light, i.e., the speed of the intersection of the magnetic field line and the shock front, $V_{\rm IS} = V_{\rm sh}/\cos\theta$, is below the speed of light. For relativistic shocks this means that the shock normal and magnetic field have to aligned within $\theta \lesssim 1/\Gamma_1$, where

$$\Gamma_1 = 1/\sqrt{1 - V_1^2/c^2}$$

is the Lorentz factor of the shock with speed V_1.

2.2 Compression ratio

While the physics of the plasma itself is mainly beyond the scope of this Thesis (for a review of basic plasma physics in shocks in the context of particle acceleration, see, e.g.,

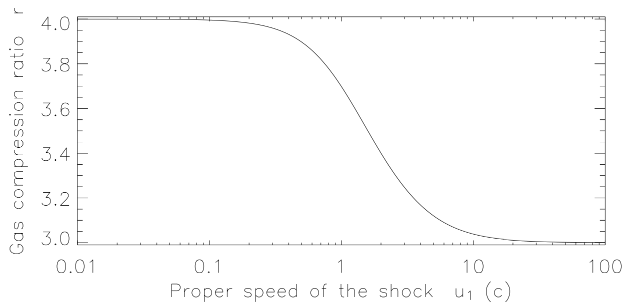

Figure 2.2: Compression ratio of a parallel shock, r, as a function of its proper speed u_1. From Paper IV.

the review of Jones and Ellison 1991, and for especially relativistic magnetohydrodynamic shocks, Appl and Camenzind 1988 and references therein), the following must be emphasised for it is crucial in all shock-related physics – especially for turbulence transmission and particle acceleration: from the fundamental conservation laws (for a review see, e.g., Kirk and Duffy 1999) of magnetic flux, energy, momentum and particle number, one can calculate the jump plasma parameters for a shock, i.e., solve the conditions at a given point from a given upstream state. In general, all the physical downstream quantities can be expressed in terms of the velocity difference between the upstream and downstream flows (Appl and Camenzind 1988).

The flow speed difference, most conventionally written in the form of the compression ratio $r = V_1/V_2$, depends on the adiabatic index of the gas, $\hat{\gamma}$, which has a value of $5/3$ for non-relativistic monatomic gas, and tends to $4/3$ as the plasma gets relativistic. The consequent compression ratio for a strong parallel shock varies between 4 (non-relativistic) and 3 (ultrarelativistic; its value is plotted in Fig. 2.2 as a function of shock *proper speed* $u_1 = \Gamma_1 V_1$. Throughout the studies of this thesis the Synge equation of state, appropriate for dissipation-free ideal gas, is used (see Appendix A of Kirk and Duffy 1999); for other possibilities for the compression, see Kirk and Duffy (1999).

2.3 From steplike to modified shocks

While keeping a distance to plasma physics, we must consider certain additional factors known to affect the acceleration-related physics at the shock. One important aspect concerning the shock is the possibility of modification of the shock front structure. The thickness and the form of the flow speed profile in this kind of a *modified shock* heavily affects the particle acceleration efficiency and, thus, the possible changes in the structure have to be taken into account when modelling acceleration. Although the particle acceleration itself will be discussed in Chapter 4, the basic interactions including accelerated particles are presented here.

Although *step shocks* and discontinuous transitions from upstream to downstream are frequently used in modelling, this is, typically, done for simplicity and approximation; the transition naturally has a finite thickness and is continuous, but since in many cases, the spatial extent of the shock front has been suspected to be of the order of the proton Larmor radius (see, e.g., Parker 1961, Appl and Camenzind 1988 or Achterberg et al. 2001; the idea is also supported by *in situ* observations of non-relativistic cases in the interplanetary space, in Earth's bow shock, as well as in numerical simulations – see for instance, Pinter 1980, Smith 1983, Vainio 2003), they appear thin or even steplike for those rigid particles. However, as discussed in Chapter 4, for light-weighted electrons (with mass only about one two-thousandths of that of a proton) the mean free path can be up to orders of magnitude shorter than for heavy protons. Consequently, the spatial scales experienced by protons as discontinuous steps, can be seen as wide and smooth structures by electrons. This leads to the *problem of injection* as described in Chapter 4 and problems with electron acceleration, but, for now, the important thing to acknowledge is that the steplike velocity profile is not necessarily appropriate, especially for electrons in a plasma where the kinematics are controlled by heavier particles.

There are also other, perhaps more pressing reasons for leaving the step-shock approximation, namely the back-reactions of accelerated particles to the shock front (for example Eichler 1979, Blandford 1980, Drury and Völk 1981, Axford et al. 1982, Baring and Kirk 1991, Kang and Jones 2005, Jones and Kang 2005). As the particles accelerate in the shock, the ratio of their pressure to that of thermal gas starts to become important; at some point the increased cosmic-ray pressure starts to slow down the upstream plasma before it hits the shock, thus making the transition wider. On the other hand, if particles can obtain sufficiently high energies and escape the system, they can carry away energy and reduce the pressure of the cosmic ray gas, thus enabling higher compression ratios than what is expected from basic plasma physics (see, e.g., Eichler 1984). Accurate description of a cosmic-ray-modified shock is very complicated due to nonlinear and counter-affecting processes like these, as well as others caused by, e.g., magnetohydrodynamic waves (see the next Chapter).

As was mentioned in the Introduction, the shock-affecting nature of these phenomena are taken into account in studies of this thesis, but the exact mechanisms and nonlinear behaviour of the shock front structure are beyond the scope of this particle-acceleration oriented study. In the papers of this thesis we mostly use a hyperbolic tangent profile (Schneider and Kirk 1987; for an exception, see Paper II) for the flow speed profile, and include the effects of shock modification in the variation of the thickness of the profile.

CHAPTER 3
Waves and turbulence

Magnetohydrodynamic (MHD) turbulence is ubiquitous in space plasmas. It is found everywhere in space – either 'frozen in' the plasma following its movements, or flowing as waves through the space – where ionised gas is present. Turbulence is also an essential part of the theory of particle acceleration, for it is the agent responsible for scattering. I.e., the nature of waves and turbulence is one of the main factors that influence the transport and acceleration of charged particles. Especially in shocks, the properties of the turbulence will often dictate the outcome of particle acceleration (see Chapter 4). For this reason, a thorough consideration of the turbulence is needed when building a comprehensive model to explain the micro-physics behind the observations.

In this Chapter the main features of the scattering turbulence will be described, with special emphasis put on the way the waves are affected by shock waves travelling through the plasma. The consequent effects the turbulence has on particle transport are dealt with in the next Chapter.

3.1 Modelling the turbulence

Magnetised low-density plasmas, such as the plasmas relevant to most of the high-energy astrophysics, contain various MHD wave modes; of these the most relevant one is typically considered to be the weakly damped incompressional Alfvén wave, although the use of the fast magnetosonic wave modes could in some cases be justified as well (see, e.g., Yan and Lazarian 2004). The damping and dissipation of the Alfvénic mode is, however, in general much weaker than the damping and dissipation of the other low-frequency modes (see, e.g., Lazar et al. 2003), so concentrating on it probably provides a good and justified starting point without too much loss of generality.

Current knowledge of MHD turbulence is not even near the level of complete understanding. Nevertheless, some success has been made and some level of universality observed, both in observations and theory, and both for heliospheric and interstellar plasmas (e.g., Armstrong et al. 1995, Maron and Goldreich 2001, Lithwick and Goldreich 2001, Horbury et al. 2005); it has been observed that the turbulence can, in many cases, be modelled with an energy spectrum $E(k) \propto k^{-q}$ with the well-known Kolmogorov spectral index $q = 5/3$ in the inertial range of wavenumbers k. For a detailed review, see, e.g., Krommes (2002) or Mac Low (2005).

In the papers in this thesis we apply a simple *quasilinear* approach to the turbulence modelling, with the total field **B** being decomposed into a slowly varying large-scale field $\mathbf{B_0}$ and a fluctuating small-scale field $\delta\mathbf{B}$ superposed on the large-scale one. This approach is particularly handy when dealing with parallel shocks, where the background field $\mathbf{B_0}$ is constant throughout the shock and the large scale electric field vanishes. The fluctuations are modelled as Alfvén waves propagating at Alfvén speed V_A (measured in the local plasma frame) along the steady background field line, in both parallel and anti-parallel directions.

Within the quasilinear approximation, we make another common restrictive assumption concerning the amplitude of the turbulence: the turbulence is assumed to be 'weak', i.e., the fluctuating part of the magnetic field is assumed to be always weaker than the constant background field. There are a few main reasons for this simplification, all trying to keep the modelling relatively simple, yet as general as possible. Firstly, too powerful turbulence leads quickly to nonlinear effects between the waves and the shock, complicating the wave transmission analysis considerably – in its current state, the turbulence theory is not yet ready to account for these effects. Secondly, the quasilinear treatment of particle scattering is applicable only if the fluctuating field remains essentially weaker than the large scale background field. Lastly, from the observational point of view, if the magnetic field is very tangled, the emission produced by the particles moving in it can no longer be modelled with the simple syncrotron model, but it becomes so-called *jitter radiation* (Medvedev 2000) – also this effect has to be taken into account when constructing a general theory for turbulence and radiation from accelerated particles, but for now such additional complications are excluded from our studies. In the analysis, the smallness of the fluctuating part of the magnetic field comes in by requiring the ratio $\epsilon = (\delta B/B_0)^2$ to be small with respect to unity.

In addition to being already existing in the plasma, the turbulent waves can also be self-generated by a shock wave and particles, as first described by Bell (1978a) for Alfvén waves created by cyclotron resonance due to particles streaming ahead of the shock. These waves are then caught by the shock and transmitted to the downstream, thus providing a scattering turbulence field throughout the shock. In the simplest case, then, the properties of the upstream turbulence, whether pre-existing or self-generated, and from that initial turbulence, the properties of the whole shock and downstream wave fields are solved. This kind of *turbulence transmission* analysis provides the tools needed for treating the effects of a shock on the turbulence.

3.1.1 Parameters for turbulence modelling

To clarify the analysis, we define here the main characteristic speeds appearing frequently in the transmission calculations. First of all, the plasma itself streams at a local flow speed toward the shock (in the upstream) or away from it (in the downstream); the flow speed is measured in the shock frame, and its sign is always positive. The waves flow through the plasma at local Alfvén speed V_A parallel or anti-parallel to the flow speed, measured in the local plasma frame. So, in the shock frame the waves are moving with speed $(V_{\mathrm{flow}} \pm V_{\mathrm{wave}})/(1 \pm V_{\mathrm{flow}} V_{\mathrm{wave}}/c^2)$, where plus is for forward and minus

	Upstream plasma frame	**Shock rest frame**	**Downstream plasma frame**
US Plasma flow	0 ○	V_1 →	$V_1 - V_2$ →
DS Plasma flow	$-(V_1 - V_2)$ ←	V_2 →	0 ○
US Forward waves	$V_{A,1}$ →	$V_1 + V_{A,1}$ →	
US Backward waves	$-V_{A,1}$ ←	$V_1 - V_{A,1}$ →	
DS Forward waves		$V_2 + V_{A,2}$ →	$V_{A,2}$ →
DS Backward waves		$V_2 - V_{A,2}$ →	$-V_{A,2}$ ←

Table 3.1: Velocities for the plasma flows and for Alfvén waves as measured in the upstream, downstream and the shock rest frames. A nonrelativistic case is chosen for clarity – relativistic correction has to be taken into account for the general velocity addition formula. $V_{1[2]}$ is the far-upstream [downstream] speed, and $V_{A,1[2]}$ is the Alfvén speed in the upstream [downstream] plasma frame. The Alfvénic Mach number is chosen to be $M = 3$, and $r = 4$ and $V_{A,2} = V_{A,1}/\sqrt{r}$ following the assumption of nonrelativistic speeds. Arrows show relative speed in a given frame – flow direction in the shock frame is chosen to be the positive direction (cf. Fig. 2.1).

for backward waves. The flow and wave speeds in the case of a low *quasi-Newtonian Alfvénic Mach number* $M \equiv u_1/u_{A,1}$ (where $u_{A,1}$ is the the proper speed of the Alfvén waves in the upstream) are showed for the non-relativistic case in Table 3.1 for comparison. Note that for this low Mach number the downstream backward waves are almost standing in the shock frame.

In super-Alfvénic shock waves the shock speed is always larger than the wave speed, as measured in the upstream frame. This is equivalent to saying that in the upstream region, in the shock frame, both the forward and backward waves are always propagating towards the shock (see Fig. 3.1), although, depending on the wave speed, the backward waves can approach the shock much more slowly than the forward waves. The same applies also in the downstream: restricted by the evolutionary argument (see, e.g., Kirk and Duffy 1999) the Alfvén speed has to stay below the local flow speed also in the downstream. This defines the *critical Mach number* $M_c \equiv \sqrt{r}$ below which the shock-frame speed of the backward waves would exceed the downstream flow speed, making it able for the backward waves to propagate back to the upstream region.

We also introduce here the *normalised cross-helicity* (hereafter referred to simply as the cross-helicity) $H_c(x, k)$ of the turbulence. It is a practical measure of the turbulence, indicating the relation of the forward and backward wave intensity spectra $I^{\pm}(x,k)$ as

$$H_{\mathrm{c}}(x,k) = \frac{I^+ - I^-}{I^+ + I^-},$$

having values between −1 (for only backward waves) and +1 (only forward waves). In the case of vanishing cross-helicity ($H_{\mathrm{c}} = 0$) both the forward and backward waves are in equipartition.

3.2 Transmission of turbulence through a shock front

A common approach in particle acceleration studies has been the assumption of the turbulence being frozen in to the plasma. This allows one to use the flow speeds for the speeds of the scattering centres, and enables the disregarding of the complicating turbulence transmission analysis (i.e., not taking into account the effects the shock has on the turbulence). However, in some cases, the initial assumption of frozen-in turbulence is no longer valid, and one has to take the waves and their transmission into account. This happens when the speed of the waves is non-negligible compared to the speed of the plasma flow.

Although the transmission coefficients for finite-amplitude Alfvén waves through a step shock have been known for decades (McKenzie and Westphal 1969), this particular field of study has not been subject to large interest, even if already some early applications of it showed the possible importance and its effects to particle acceleration scheme (see, e.g., Bell 1978a, Achterberg and Blandford 1986, Jones 1993).

In the papers of this thesis we solved the transmission of the scattering Alfvén wave field for both the step-like and the modified relativistic parallel shocks. The step-shock case had already been studied quite extensively at the non-relativistic limit by Campeanu and Schlickeiser (1992) and Vainio and Schlickeiser (1998, 1999, 2001); the analysis was then extended to also cover the relativistic case in Paper III. Furthermore, the transmission in a thick shock was solved for the non-relativistic case in Paper V, and extended to the relativistic regime in Paper VI.

The analysis methods are different for thick and steplike shocks (or, for waves shorter and longer than the thickness of the shock front, respectively). Furthermore, while for nonrelativistic cases the outcome is similar in both cases, for relativistic shocks the results differ remarkably. In the next Sections the main results are reviewed for step- and thick shocks separately, and finally a combination of the two different transmission mechanisms are considered.

3.2.1 Transmission in step-shocks

The transmission coefficients for finite-speed Alfvén waves, or shocks whose Alfvénic Mach number is not extremely large, were solved for parallel non-relativistic step shocks by Vainio and Schlickeiser (1998), following and correcting the previous analysis of

Campeanu and Schlickeiser (1992). They found that when transmitted through a shock with low-to-moderate Alfvénic Mach number ($M \lesssim 10$), backward waves were amplified much more than the forward waves. In addition, part of the waves were found to be reflected by the shock, i.e., a certain fraction of the upstream forward waves were transformed to downstream backward waves, and vice versa. Consequently, downstream forward waves (I_2^+) consist of waves that were forward already in the upstream and were simply transmitted through the shock, and waves that were backward in the upstream, but whose propagation direction was reflected at the shock; and vice versa for the downstream backward waves (I_2^-):

$$I_2^{\pm} = T_{\pm}^2 I_1^{\pm} + R_{\mp}^2 I_1^{\mp} \tag{3.1}$$

The ratio of these *transmitted* and *reflected* waves for both of the upstream wave modes can be calculated from the transmission and reflection coefficients, $T_{\pm}$ and $R_{\pm}$ respectively, for forward (+) and backward (−) waves. In the non-relativistic case, the coefficients depend only on the Alfvénic Mach number and the shock compression ratio as (Vainio and Schlickeiser 1998, Paper III)

$$T_{\pm} = \frac{\sqrt{r}(\sqrt{r}+1)}{2}\frac{M \pm 1}{M \pm \sqrt{r}} \qquad \text{and} \qquad R_{\pm} = \frac{\sqrt{r}(\sqrt{r}-1)}{2}\frac{M \pm 1}{M \mp \sqrt{r}}.$$

From Equation (3.1) it is easily seen that when $M \to M_{\rm c} \equiv \sqrt{r}$, the coefficients that constitute to the downstream backward waves (T_- and R_+) go to infinity. This means that right behind the shock front the waves are propagating predominantly backwards. It can be also seen that for super-Alfvénic strong shocks there are always both wave modes present in the downstream.

When extended to take into account relativistic jump conditions (Appl and Camenzind 1988) the behaviour found by Vainio and Schlickeiser (1998) for non-relativistic shocks was found to be present also in relativistic shocks (Paper III). For relativistic step shocks, it was observed, the cross-helicity of the downstream wave, H_{c2}, approached -1 as the Mach number approaches the critical value $M_c = \sqrt{r}$. In other words, as the *wave speed* in the downstream increases and approaches the *plasma flow speed* in the downstream, larger and larger part of the waves were flowing anti-parallel to the flow direction as seen in the downstream plasma frame, i.e., backward, towards the shock.

The description of transmission can be further extended, for instance, by taking into account the pressure of the waves. This was done for the non-relativistic case by Vainio and Schlickeiser (1999); they included the waves in the jump condition calculations and were able to remove the mathematical singularity present in the earlier analysis (Vainio and Schlickeiser 1998). In earlier studies it was observed that when $M \to M_c$, the amplified intensity of the backward waves went to infinity. Inclusion of the effect of the transverse wave fields was shown to affect the compression ratio of the gas so that in the calculation of wave transmission, the singularity vanishes, and the transmission coefficients remains finite and the shock solution evolutionary for all $M > 1$.

A significant effect due to (i) the change in the wavenumber and intensity, and (ii) the different reflection properties for different waves, is the following: for example, in a case where the upstream cross-helicity is zero, most of the waves in the downstream are propagating backward, following the shock. Exactly the same was later observed in Paper III for relativistic step shocks. From the particle point of view, this leads to a situation where the effective scatterer speed,

$$V_{kj} = \frac{V_j + H_{cj} V_{Aj}}{c(1 + H_{cj} V_{Aj} V_j / c^2)},$$

(where V_{Aj} is the Alfvén speed, and $j = 1[2]$ denotes quantities measured in the upstream[downstream]), in the downstream decreases, leading to an increased scattering-centre compression ratio

$$r_k \equiv \frac{V_{k1}}{V_{k2}}.$$

In the case of vanishing upstream cross-helicity, $H_{c1} = 0$, the average scatterer speed in the upstream is simply the flow speed V_1, and the scattering-centre compression ratio becomes

$$r_k = \frac{V_1(1 + H_{c2} V_{A2} V_2 / c^2)}{V_2 + H_{c2} V_{A2}}.$$

The same phenomenon also occurs in the case of degenerate upstream cross helicity ($H_{c1} = \pm 1$); the only exception is the case where all the waves in the upstream are streaming parallel to the flow into a shock with relatively low proper-speed. In the latter case the transmission is not able to increase the backward wave intensity enough for high-M cases, and the scattering-centre compression ratio is somewhat lower than the compression ratio of the gas. See the top panels of Fig. 3.1.

Here it is stressed that the results of wave transmission calculations apply for turbulence right behind the shock; further away in the downstream the wave field can be dissipated by energy transport from waves to particles (e.g., Ko 1992) or wave–wave interactions (e.g., Skilling 1975b, Vainio and Spanier 2005).

3.2.2 Transmission in thick shocks

For those high-wavenumber waves whose wavelength is essentially shorter than the thickness of the shock front, the transmission model described in the previous section applies no more. For these waves the shock is not a steplike structure, but instead a smooth gradual transition in the background flow parameters.

While for the step-shock approximation the transmission was calculated over a discontinuous change in the wave and plasma parameters, for a thick shock the transmission coefficients are easily solved by means of ray tracing in a slowly changing medium: waves are treated as quasiparticles, and the properties of the flux of these wave quanta passing through the changing medium are calculated from the conservations of wave action and shock-frame frequencies of each ’particle’ (for a detailed description of the method and its applicability to waves in a slowly changing medium, see, e.g., Dewar

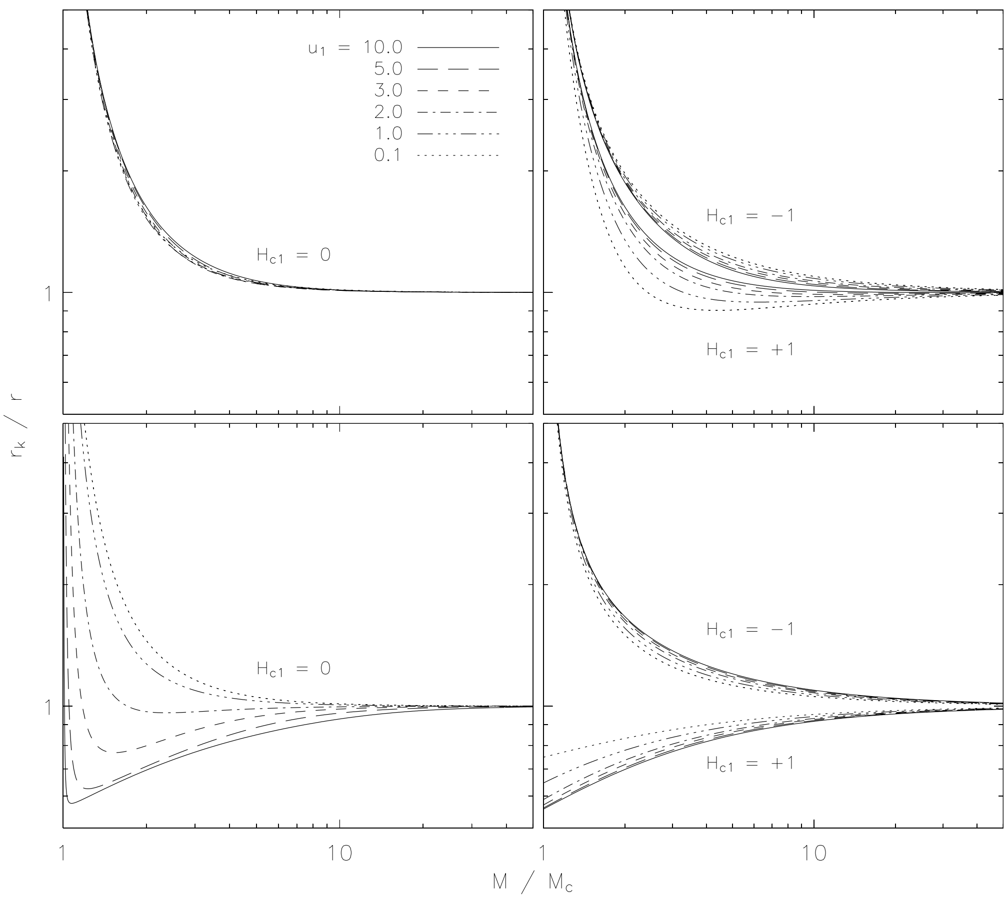

Figure 3.1: Change of the scattering-centre compression ratio r_k due to turbulence transmission as a function of the Alfvénic Mach number M (scaled with $M_c = \sqrt{r}$) for both steplike (top) and thick (bottom) shocks. The leftmost panels show results for vanishing upstream cross-helicity, while the panels on the right contain cases of degenerate upstream cross-helicities; ($|H_{c,1}| = 1$). Different lines mark different shock proper speeds $u_1 = V_1\Gamma_1$. Lower panels are adapted from Paper VI.

1970, 1972). This was done for Alfvén waves in parallel shocks in Papers V–VI first at the non-relativistic limit and later extending to fully relativistic cases.

In the non-relativistic case, the resulting downstream cross-helicity behaves similarly to the step-shock cases: as $M \to M_c$ the waves tend to propagate predominantly backwards. In the relativistic case, however, due to lack of wave reflection in thick shocks, the fraction of backward waves is reduced while the forward mode starts to dominate until, at the ultrarelativistic limit for low-to-medium Mach numbers, practically all waves are forward. As a consequence, the scattering-centre compression ratio decreases. This is

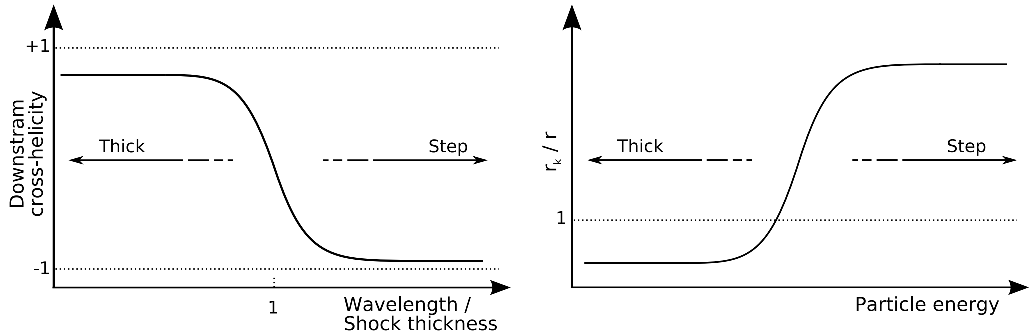

Figure 3.2: Schematic depiction of result of wave transmission in a relativistic low-Mach-number modified shock. The downstream cross helicity for waves of different length is shown on the left, while on the right the consequent effects to the scattering-centre compression ratio is shown as a function of particle energy.

shown in the bottom panels of Figure 3.1 for cases with upstream cross-helicity $H_{c1} = 0$ and ± 1 separately. In this figure, $r_k \to \infty$ as $M \to M_c$, but this is due to the mathematical singularity described earlier and the effect most probably vanishes, if the compression ratio of the gas is determined taking the wave pressure into account.

3.2.3 General model for transmission

So far, the wave transmission coefficients have been determined for waves with wavelength either much larger (Paper III) or smaller (Paper VI) than the length scales of the shock front. We still lack, however, a method for treating waves with wavelength comparable to the shock thickness, in order to have a general model for transmission. Although the transmission coefficients for the intermediate wavelength waves are likely to be significantly harder to obtain than for the two opposite cases found so far, the existing approximations should still provide a starting point for constructing a general model. In addition to the work presented here, also possibilities for wave reflection in the thick-shock case (e.g., Laitinen 2005, and references therein) should be considered.

Although such a general model is yet to be created, already a crude application to the acceleration of particles in a shock with scattering-centre compression ratio depending on the energy of the particle, could provide interesting results. Particles with low energy would be initially scattered off short-wavelength waves (see the next Chapter for explanation of this relation) that see the shock as a thick structure, and as the particle energy increases above an energy corresponding to a resonant wavelength equal to the shock thickness, the particle would continue to scatter off waves for which the shock is a thin step. Effectively, this would mean that for low-energy particles the scattering-centre compression ratio would be different than for the high-energy particles (see the discussion in Paper VI). See Figure 3.2 for a sketch of the cross-helicity of the transmitted waves with respect to the wavelength (scaled with the shock thickness).

CHAPTER 4
Particle acceleration

This Chapter concentrates on mechanisms considered to be responsible for the acceleration of charged particles in relativistic parallel shocks. Emphasis is given to the Fermi mechanisms (Sections 4.2 and 4.3) due to reasons explained later; other relevant mechanisms are discussed in Section 4.4. For a review of the development of shock acceleration theories, as well as for a general view of acceleration mechanisms in parallel and non-parallel shocks, see, for example, reviews of Drury (1983), Blandford and Eichler (1987), Jones and Ellison (1991) and Kirk and Duffy (1999).

4.1 Particle transport

A brief review of the basic properties of the transport of charged energetic particles in a magnetic field and turbulence follows. The description is not meant to be comprehensive, but just to explain the most essential aspects needed to understand the underlying physics for mechanisms discussed in this Chapter. Throughout this work turbulence is taken to consist of Alfvén waves. For a detailed description of shock-related particle transport and acceleration theory in turbulent plasma of this kind, see, e.g., Skilling (1975a) and Schlickeiser (1989 and 2002, Chap. 12 and 16).

4.1.1 Turbulence and the quasilinear theory of scattering

In extremely thin astrophysical plasmas particle collisions are very rare, and instead of particle–particle interactions, charged particles interact with Alfvén waves. These interactions occur in gyroresonance, and the particle ”sees” waves with wavelengths equal to its gyroradius (Jokipii 1966). A particle's gyrofrequency is inversely proportional to its energy and, thus, the distance it travels in a constant magnetic field during one gyromotion increases with energy. This, again, means that as the particle's energy increases, it will see waves with longer and longer wavelengths, leading to a clear dependence of particle transport on the spectrum of turbulence. This effect will have significant consequences, as will be shown later.

Now assuming these resonant interactions with waves to cause a particle to change it's direction by some amount, and following the weak-turbulence approximation introduced in the previous Chapter, one can model the scattering event with a small change in the particle's *pitch angle*, i.e., the angle between the particle's velocity vector and the magnetic field. Alternatively, one can consider a sphere centred on the particle and with radius vector corresponding to particle's momentum: this way the elastic scatterings cause the tip of the particle's momentum vector to perform random motion on the surface of the sphere. If the subsequent scatterings are independent of each other, this will lead to diffusion in pitch angle.

A quasilinear approach to the scattering (see, e.g., Jokipii 1966, Schlickeiser 1989, Schlickeiser and Vainio 1999, for a review) takes the total magnetic field to consist of a steady large-scale field and superposed small-amplitude disturbances (see Section 3.1). Particle transport in this kind of field is described by a Fokker–Planck-type equation, and the problem of particle transport reduces to solving the Fokker–Planck coefficients for diffusion in pitch-angle and in momentum (see Schlickeiser 1989). The first one of these is closely related to the so-called *first-order Fermi acceleration*, and the last one to the *second-order* process, both named after Fermi for his pioneering work and being the first to suggest such processes to be capable of accelerating cosmic-rays (Fermi 1949, 1954).

4.2 First-order Fermi acceleration

In the first-order Fermi, a mechanism particle gains energy from subsequent shock crossings. In the simplest case, the particle is scattered elastically off small irregularities travelling with the one-dimensional plasma flow. When a particle crosses the shock front. e.g., from upstream to downstream, it successively scatters on both sides of the shock and in the scattering-centre rest frames moving with substantially different speeds. Because of this, when the particle comes in for the second scattering, its energy in the new scattering-centre frame is somewhat higher than it was in the previous frame. As a result, the particle's scattering-centre-frame energy has been boosted by an amount depending on the velocity difference between the scattering centres on different sides of the shock.

Now, if the scattering in the downstream is sufficiently efficient in order to turn the particle around before it gets advected too far away to the downstream, the particle may return to the shock and re-cross it back to the upstream, and try to escape from the shock (see Begelman and Kirk 1990, for discussion). However, because the speed of a relativistic shock is close to that of light, the particle would have to propagate exactly anti-parallel to the incoming flow in the turbulent medium in order to escape the following shock, so basically all particles in the upstream are caught again by the shock and made to cross back to the downstream. This way these repeated crossings and re-crossings can lead to huge increases of energy for those particles that undergo many such cycles (see Fig. 4.1; although it shows, for simplicity, the mechanism in the non-relativistic case, where the particle speed, v, can be much higher than the flow speed, a similar graph could be drawn also for the relativistic case). The increase of energy on shock crossing is of first order in the quantity $\Delta V/v$ – hence the name ”first-order” process.

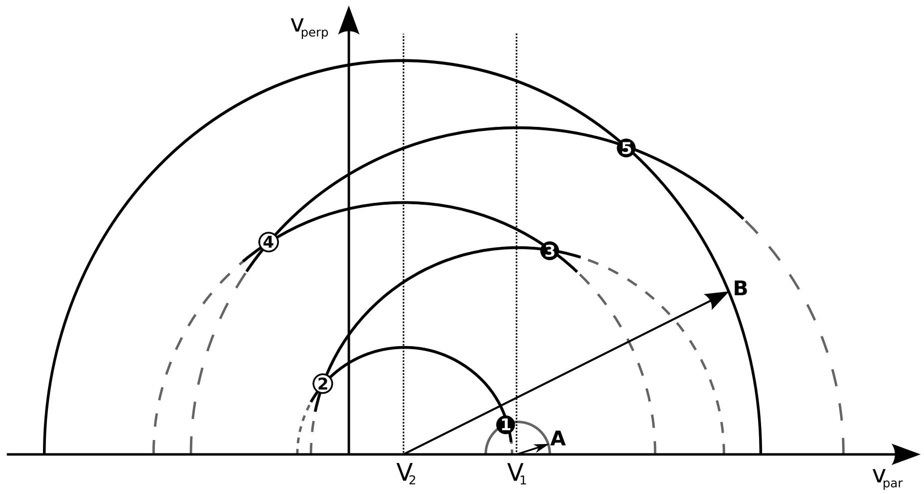

Figure 4.1: First-order acceleration process in non-relativistic step shock with scattering centres frozen-in to the flow. The x- and y-axes show particle's velocity components parallel and perpendicular to the flow. Arcs centred on the upstream or downstream flow speeds show the total speed of the particle scattering elastically in the respective flow rest frame. Arrow A shows speed of a particle coming into the shock with a small initial velocity in the upstream frame. Points with odd numbers mark occasions where the particle crosses the shock front parallel to the flow and starts to feel the downstream-scattering-centre speed, while the even numbers mark returning to the upstream. In this example the particle returns to the upstream twice and after that, gets absorbed in the downstream. In the end its flow-frame speed (arrow B) has increased by large amount.

4.2.1 Diffusive shock acceleration

The first-order mechanism was revised in the late 1970's by various authors, who suggested it to work via scatterings off Alfvén waves frozen-in to the plasma (Axford et al. 1977, Krymsky 1977, Bell 1978a,b, Blandford and Ostriker 1978). It was later extended to cover, e.g. energy losses due to radiation (Schlickeiser 1984, Webb et al. 1984, Kirk et al. 1988), as well as nonlinear modifications (e.g. Blandford 1980, Drury et al. 1982, Drury 1983, Berezhko and Ellison 1999). For non-relativistic shocks, the analysis relies on the *diffusion approximation*, which requires the angular distribution of particles to be close to isotropy in the local plasma frame. When this assumption is made, the particle transport equation with only first-order acceleration present can be reduced to a diffusion equation in space. The mechanism is also referred to as *diffusive shock acceleration* and it leads to a simple power-law energy distribution $\mathrm{d}N/\mathrm{d}E \propto E^{-\sigma}$ of the accelerated particles (e.g. Jones and Ellison 1991, Kirk and Duffy 1999) with spectral index σ determined solely by the compression ratio of the scattering centres, r_k:

$$\sigma = \frac{r_k + 2}{r_k - 1}. \tag{4.1}$$

Using the assumption of scattering centres frozen-in to the plasma, r_k reduces to the compression ratio of the flow $r = V_1/V_2$. For non-relativistic strong parallel shocks (with $r_k = 4$) this yields $\sigma = 2.0$.

4.2.2 Extension to relativistic shocks

The assumption of isotropic particle distribution needed for the diffusive approximation limits the approach to cases where the particles have velocities much larger than the shock speed. It is not applicable to e.g. relativistic shocks, where extreme anisotropies due to relativistic relative speeds violate the requirement of isotropy. In order to model acceleration in relativistic shocks, the pitch-angle distribution and energy distribution of the particles have to be solved either semi-analytically (Schneider and Kirk 1989, Kirk and Schneider 1989, Kirk et al. 2000), by numerical simulations (e.g. Peacock 1981, Kirk and Schneider 1987b, Ellison et al. 1990, Bednarz and Ostrowski 1998, Lemoine and Pelletier 2003, Meli and Quenby 2003b, and Papers I,II, IV and V of this thesis), or by developing fully analytical methods to also take the anisotropies into account (Blasi and Vietri 2005, Keshet and Waxman 2005, Spanier et al. 2006).

Even the basic analysis in the relativistic case is much more complex than for non-relativistic shocks due to the particle anisotropies and relativistic corrections, and only recently analytical solutions for particle spectra have been found. However, already in the earlier relativistic studies it was found that there are certain characteristic differences between the relativistic and non-relativistic case: relativistic shocks were found to produce spectra flatter than that from non-relativistic acceleration for fixed r (Kirk and Schneider 1987a,b), and their acceleration timescales are shorter (Quenby and Lieu 1989, Ellison et al. 1990, Bednarz and Ostrowski 1996).

For the spectral index of the accelerated particles, value $\sigma \approx 2.2$ has been observed to appear from different simulations and semi-analytical studies (e.g. Bednarz and Ostrowski 1998, Gallant et al. 2000, Kirk et al. 2000, Achterberg et al. 2001) dealing with ultrarelativistic step shocks. This value was recently found also from analytical calculations (Keshet and Waxman 2005) at the ultrarelativistic limit, in the case of isotropic scattering and scattering-centre compression ratio being that of the gas. In this case, a relation similar to the non-relativistic equation (4.1) can be written as

$$\sigma = \frac{r_k + 2}{r_k - 1}\left[1 - \frac{V_1^2}{c^2}\frac{2r_k - 1}{r_k^2(r_k + 2)}\right],$$

which gives $\sigma = 2.22\ldots$ for $V_1 \to c$ and $r_k \to 3$. This is in good accordance with expectations from earlier simulations and models. Next steps of generality – analytical solutions including also anisotropic scattering – are being developed (e.g. Spanier et al. 2006).

4.2.3 Average energy gain and the effects of a scattering model

As explained above, a particle's energy is boosted at the shock crossing. In a relativistic shock, the average change of particle energy for subsequent downstream-upstream-downstream cycles depends on the pitch-angle distribution of the incoming particles: those particles who come in for the first time and have more or less isotropic distribution in the upstream plasma frame, get their energy boosted by factor of $\sim \Gamma_1^2$, whereas for latter cycles the energy is only doubled (e.g. Gallant and Achterberg 1999, Baring 1999, Bednarz and Ostrowski 1999). This decrease in energy gain for the latter cycles is due to the aforementioned fact that the particles that have crossed the shock from downstream to upstream, are flying ahead of the shock and their pitch-angle distribution in the upstream plasma frame is heavily peaked and pointing away from the shock. If the particles can isotropise in the upstream before the shock catches them again, also the latter cycles could be boosted by higher efficiency. This is easily managed in a non-relativistic shock, but for relativistic cases very rapid and effective scattering would be required (see,e.g. Quenby and Lieu 1989).

Generally speaking, the scattering events can be divided into small- and large-angle scatterings. Scattering off small disturbances leads to a small change in particle's pitch angle, but also larger scatterings can occur, for instance, in the presence of highly turbulent plasma (see, e.g. Kirk and Schneider 1988). Typically, relativistic first-order Fermi acceleration models assume only pitch-angle scattering, but if taken into account, the large-angle scattering will change the resulting particle spectra significantly (e.g. Kirk and Schneider 1988, Meli and Quenby 2003b). This is because the large-angle scatterings isotropise the particle populations more effectively, and dominant large-angle scattering in the upstream can lead to an energy boost of the order of Γ_1^2 again, instead of factor of ~ 2. In addition to flat spectrum, this leads to distinctive "steps" superposed on the power-law (see, e.g. Meli and Quenby 2003b). The spectral flattening is the strongest when large-angle scattering is present on both sides of the shock, but the effects are also visible if the downstream scattering is due to only small-angle scattering (Kirk and Schneider 1988). Although the large-angle scattering has significant effects on the acceleration process, it is not likely to be present in relativistic shocks (e.g. Bednarz and Ostrowski 1996, Meli and Quenby 2003b), and typically only small-angle scattering (or pitch-angle diffusion) is considered in simulations and modelling.

Another common assumption made in particle transport calculations is to assume the scattering event to be independent of pitch angle. Although this is not very accurate in many cases, it is still a good starting point, and simplifies the numerical treatment considerably. For this reason this assumption has been made also in all of the papers presented in this thesis. Inclusion of the anisotropic scattering is expected to lead to mild softening of the accelerated particle spectrum, as reported by Kirk et al. (2000) and Lemoine and Revenu (2006).

4.2.4 First-order mechanism in modified shocks

As described in Section 2.3, the step-shock scenario is merely an approximation of the real shock structure and it is not very physical to the smallest scales. In order to study particle acceleration at all relevant spatial scales, the possibility that a shock wave is not necessarily seen as a plain discontinuity by the particles, has to be taken into account.

For non-relativistic shocks the back-reactions of accelerated particles on the shock front and acceleration in these modified shocks have been studied quite extensively since the first diffusive shock acceleration theories by various authors (e.g. Eichler 1984, Ellison and Eichler 1984, Webb et al. 1985, Webb 1989, Drury and Völk 1981, Drury et al. 1982, Axford et al. 1982, Drury 1983, Duffy 1992, 1994, Duffy et al. 1994, Jones 1993), and the study has been later extended to also cover relativistic cases (e.g. Schneider and Kirk 1989, Kirk and Schneider 1989, Baring and Kirk 1991, Ellison et al. 1990, Ellison and Double 2002).

If the mean free path of accelerating particles is comparable or smaller than the thickness of the shock transition and the flow (or scattering-centre) speed changes only a little between two successive scatterings, the particle receives a lower energy boost per the two scatterings than what it would have received had it crossed the whole shock. In addition, it is more difficult for an upward-propagating particle to reach the ”far-upstream speed” V_1, because the particle, now, would have to retain its pitch-angle (as measured in the local plasma frame) very close to -180° over many scatterings in order to get feel of the whole velocity difference $\Delta V = (V_1 - V_2)/(1 - V_1 V_2/c^2)$ across the shock. Odds are that that the particle's direction is deviated enough before it has managed to get to the 'far upstream' (where $V(x) \simeq V_1$), and thus the particle sees only a part of ΔV on that shock 'crossing' cycle. This leads to a weaker energy gain and a strong dependence of the accelerated particle spectrum and the shock thickness (e.g. Drury 1983, Schneider and Kirk 1989, for non-relativistic and relativistic shocks, respectively). In some cases, it is possible to solve the relation of the produced particle spectrum and the structure of the shock front. Drury et al. (1982) and Drury (1983) applied the diffusion approximation and were able to write an analytical relation between the thickness and the scattering-centre compression ratio, and the spectral index of the high-energy tail of the particle spectrum. This relation was later found to also apply well for relativistic shocks, provided that the scattering-centre compression ratio is high (Paper V). Later, some semi-analytical methods for finding the spectral index in relativistic shocks have been developed, e.g., by Schneider and Kirk (1987). In addition to these, various Monte Carlo based simulations have been applied (see, e.g., Ellison et al. 1990, Paper I).

For relativistic modified shocks the particle acceleration efficiency decreases rapidly as the shock thickness increases, and shocks wider than just a fraction of particle's mean free path do not seem to be able to accelerate low-energy particles efficiently enough in order to produce spectra hard enough to account for observations (Schneider and Kirk 1987, Ellison 1992, Paper I). This is problematic especially for electrons, because, as discussed in Section 2.3, if the shock thickness is determined by ion dynamics, and if particle back-reactions widen the shock structure even more, the transition easily gets much wider than the mean free path of thermal upstream electrons. If there is, however, some mechanisms working in the downstream that heat and energise the particles (Paper II), or if the compression ratio of the scattering centres is essentially higher than that of the gas (Paper V), the accelerated particle spectra can be very flat.

The problem of getting low-energy particles energised enough in order to get them to rise from the thermal bulk and to be injected into the acceleration process is also problematic for step shocks (see, e.g. Kirk and Schneider 1989, Kirk and Dendy 2001). In the diffusive acceleration scheme, the problem is to get the particle speeds sufficiently high compared to the flow speeds, and for relativistic cases the main problem is to get the particles to resist the relativistic downstream flow in order to re-cross the shock and get to the first-order process. The role of injection and its relation to the shock structure have been studied extensively by many authors (see, e.g. Eichler 1979, 1984, Axford et al. 1982, Ellison and Eichler 1984, Blandford and Eichler 1987, Kirk and Schneider 1989), but the subject is still somewhat open. Some progress has nevertheless been made (see Malkov and Völk 1995, 1998, for the diffusion approximation case), and recently some possibilities of sufficient energisation from the initial relativistic crossing (Achterberg et al. 2001, Ellison and Double 2002) or post-shock acceleration in the shock-induced turbulence (Paper IV) have been suggested.

In addition to being steeper, the spectra from modified shocks can also show another feature different from basic step-shock-accelerated spectra. Namely if the scattering mean free path increases with energy, low-energy particles see the shock transition thicker than the high-energy ones, thus accelerating less efficiently. As the mean free path of the particle increases, the effective shock thickness from the particle's point of view decreases. Above some energy, when the particle's mean free path is comparable to the shock thickness, the particle sees the shock as a step. As a result, the accelerated particle spectrum hardens around this energy, and leads to an asymptotic spectral index corresponding to the step-shock case (Blandford 1980, Kirk and Schneider 1989, Ellison 1992, Paper II).

4.2.5 Effect of turbulence transmission

As was explained in Section 3.2, the compression felt by the particles is not necessarily equal to that of the plasma. As was shown by Vainio and Schlickeiser (1998) for non-relativistic and in Paper III for relativistic step shocks, and in Papers V & VI for modified shocks, the scattering-centre compression ratio (and thus the acceleration efficiency) can be heavily altered if the motions of the waves with respect to the plasma are taken into account.

In shocks with a high Alfvénic Mach number, the wave speeds are, by definition, small compared to the flow speed, and the transmission does not have significant effects. For lower Mach numbers, say $M \lesssim 10M_\mathrm{c}$ ($M_\mathrm{c} \equiv \sqrt{r}$), however, the transmission leads to intensified first-order acceleration efficiency (Papers III & V) due to the increased scattering-centre compression ratio (see Section 3.2.1). An exception for this is the case of relativistic thick shocks, where the scattering-centre compression ratio was found to decrease unless the upstream field has a degenerate cross-helicity, $H_{c1} = -1$ (see Fig. 3.1). A more general picture of the effects of transmission in modified shocks was discussed in Section 3.2.3, where the behaviour of the scattering-centre compression ratio was estimated. The expected change due to the transmission was a convex shape of the spectrum due to the increased effective compression ratio above some energy. This effect is similar to the aforementioned hardening of the spectrum due to increased mean free path. Careful study of the effects due to both the waves and particles, and their relation to the shock thickness, would be a natural future extension for the work presented in this thesis.

4.3 Stochastic acceleration

In the first-order Fermi process, the velocity difference of the scattering centres comes from the velocity gradient at the shock. In presence of e.g. counter-streaming Alfvén waves, a particle can either gain or lose energy depending on the direction of the wave with respect to the propagation direction of the particle. The probability for the energy-gaining scattering is, however, slightly larger than that for the energy-losing one, so the net effect of many scatterings is an increase of the mean energy proportional to factor $(V/v)^2$, thus making it a 'second-order' process (see, e.g. Ostrowski and Siemieniec-Oziębło 1997). Due to the stochastic nature, the name *stochastic acceleration* is also commonly used.

Stochastic acceleration follows directly from the particle transport equation in small-amplitude turbulence (see, e.g. Skilling 1975a, Schlickeiser 1989), and it is always present in the turbulent downstream of super-Alfvénic shocks (Dung and Schlickeiser 1990, Vainio and Schlickeiser 1998). However, partly because in many cases the faster first-order mechanism tends to dominate over the slower second-order one, and partly due to the rapid development of the first-order acceleration theory in late 70's and it's good agreement with many observations, the second-order process received much less attention for some time. Nevertheless, despite of the success of the first-order process in explaining some central observations, it fails in being able to account for all cases (for examples, see e.g. Schlickeiser et al. 1993, Ostrowski 1994, and references therein) and, indeed, taking the stochastic process into account has been shown to improve the models in certain cases (Krülls 1992, Ostrowski 1994, Schlickeiser and Dermer 2000, Dermer and Humi 2001).

As was mentioned, stochastic acceleration is unavoidable whenever the turbulence is non-degenerate, i.e. when there are both wave modes, forward and backward, present and the normalised cross-helicity $|H_c| < 1$ (Chapter 3). Efficiency of the acceleration depends much on the turbulence, of course: the total energy density of the turbulence (e.g. Petrosian and Liu 2004), as well as the flattening of the turbulence spectrum (e.g. Krülls 1992, Paper IV) lead to a more efficient stochastic acceleration. In general, the stochastic process is able to produce very flat spectra on its own (Ostrowski and Schlickeiser 1993, Ostrowski 1994), and to also re-shape spectra created by the first-order mechanism (Paper IV). Especially the former characteristic makes the stochastic process appealing in explaining observations requiring spectra much harder than what is possible for the first-order mechanism, which is not able to produce spectra flatter than $\sigma = 1$. See Section 5.3 for discussion concerning these sources.

4.3.1 Stochastic acceleration in relativistic shocks

Although stochastic acceleration is able to work even in a simple shockless plasma flow (e.g. in AGN jet, see Wang 2002), it becomes especially interesting when applied to shocks. Firstly, as explained in Chapter 3, a strong shock with low-to-moderate Alfvénic Mach number can amplify the waves as they cross the shock transition and produce downstream turbulence with cross-helicity suitable for the stochastic acceleration (Krülls 1992, Ostrowski and Schlickeiser 1993, Paper IV). Secondly, in addition to producing flat spectra beyond the capabilities of the first-order process, stochastic acceleration could also provide relief to the injection problem discussed in the last section (Petrosian and Liu 2004, Paper IV). Namely, as the process heats and accelerates also low-energy particles, it can raise them to energies required for the injection into the first-order process (energy at least a couple times than that of the thermal particles, sufficiently long mean free path, etc.). An example of this kind of phenomenon was observed in the test-particle simulations of Paper IV (see Figures 6 and 7 of that paper), where particles injected, with low energy, into downstream of the shock were accelerated stochastically and finally propagated into the shock where they were accelerated further by the first-order process.

As pointed out by Schlickeiser et al. (1993), the total effectiveness of the stochastic process in the downstream of a shock depends mainly on three factors: the scattering-centre compression ratio, the extent of the turbulent Alfvén wave field in the downstream, and the strength of the momentum diffusion. Although full analysis for the transport of waves and particles in shocks can be extremely complicated and nonlinear, approximative solutions can be found when taking into account that two of the aforementioned factors, namely the scattering-centre compression ratio and strength of momentum diffusion, can be solved for, once the magnetic field density, upstream turbulence and the equation of state for the plasma, are known. In Paper IV we studied stochastic acceleration in relativistic step shocks numerically, and included turbulence transmission (from Paper III). We found the process to be capable of significantly transforming the particle spectra produced by the first-order process at the shock, and also to produce flat (and even inverse!) spectra from particles injected in the downstream further away from the shock. The study showed the second-order process to be capable of very strong acceleration within time-scales certainly short enough in order to affect the observable spectrum. Those simulations, however, did not take into account the third factor of Schlickeiser et al. (1993), namely the extent of the turbulent region. Including wave damping and a realistic model for the turbulent downstream is likely to change the results.

4.4 Other acceleration mechanisms

In addition to the collisionless Fermi processes, there are also other mechanisms related to shocks, and capable of accelerating particles to high energies, and even power-law distributions. In this Section we discuss some of these. We limit ourselves to mechanisms taking place around parallel relativistic shocks. This choice restricts the discussion to objects like the relativistic jets of micro- and macroquasars and blazars, and jets and blast waves of supernovae and gamma-ray bursts, although even in these cases parallel shocks represent only a minor sub-class.

The most interesting of different mechanisms are those capable of producing power-law energy spectra. One such process, very similar to the first-order Fermi process, on one hand, but still working in a totally different manner, is the collisional *converter mechanism* (Stern 2003, Derishev et al. 2003). It lets the accelerating charged particles, e.g., protons or electrons/positrons, in the downstream to re-cross the shock to the upstream through temporary change to a neutral form (e.g., neutron or inverse-Compton-scattered high-energy photon), and get re-injected into the shock after another change of state back to charged protons or electrons/positrons. The neutral state particle is free from magnetic scattering and can easily cross the shock back to the upstream before changing back into the original form. In the upstream, the particle can again scatter, and depending how far ahead of the shock it is, it can meet the shock with an incoming angle much larger than what is possible for a charged particle trying to escape the shock to the upstream. This leads to the possibility of multiple Γ_1^2 reflections instead of the only one for the first shock encounter. This, then, can lead to very flat accelerated particle spectra, making the converter mechanism especially promising for shocks with very high Lorentz factors.

Particle-in-cell simulations have recently revealed possibilities of effective acceleration due to plasma instabilities caused by colliding and interpenetrating plasmas (e.g. Nishikawa et al. 2003, Haugbølle 2005). In some cases when these instabilities develop to current channels in which particles can accelerate, this has been observed to lead to particle acceleration and even to power-law energy distributions (Hededal et al. 2004). In addition to current channels, electrostatic fields generated by distortion of the Alfvén waves can also lead to electron acceleration (Tsiklauri et al. 2005).

For a recent review of other different mechanisms applicable to acceleration in more general sources, see e.g. Kirk (2005).

CHAPTER 5
Discussion of the papers

In this Chapter the original papers of this thesis are discussed. The papers are here grouped according to their subject and relation to the other papers.

Papers I and II study acceleration in shocks of different thicknesses (steplike and modified) when the turbulence is assumed to be frozen-in to the plasma. Paper III then develops a way of calculating the turbulence field behind a relativistic step shock in the case where the waves are *not* static in the plasma frame. These results are applied to the first-order Fermi acceleration in step shocks in Paper III, and to the second-order process in Paper IV. Paper V then studies the first-order process in thick shocks (cf. Papers I and II), but now including the effects from turbulence transmission. Paper VI finally develops the wave transmission analysis for thick shocks of all speeds.

5.1 First-order acceleration in modified shocks

Paper I Simulations on the effect of internal structure of relativistic shock fronts on particle accereration by J. Virtanen & R. Vainio, *in High Energy Blazar Astronomy, ASP Conference Proceedings, Vol. 299, p. 157. Edited by L. O. Takalo and E. Valtaoja. ISBN: 1-58381-146-X (2003)*

Paper II Monte Carlo Simulations of Electron Acceleration in Parallel Relativistic Shocks by J. Virtanen & R. Vainio, *in The 28th International Cosmic Ray Conference proceedings, p. 2023, Edited by: T. Kajita, Y. Asaoka, A. Kawachi, Y. Matsubara and M. Sasaki (2003)*

These papers present numerical results from kinetic test-particle Monte Carlo simulations used to study how the first-order Fermi acceleration in relativistic parallel shocks is affected by the thickness of the shock front. In addition, the effect of the energy dependence of the particle mean free path was studied.

In Paper I we introduce a simple model for the scattering frequency using the quasi-linear approach. Approximating the shock thickness to be roughly equal to the mean free path of an upstream proton with energy $\Gamma_1 m_{\rm p} c^2$, we wrote the equation for the shock thickness W as a function of the spectral index of the turbulence power-law, q, as $W = (m_{\rm p}/m_{\rm e})^{2-q} \simeq 1836^{2-q}$, where $m_{\rm p}$ and $m_{\rm e}$ are the rest masses of the proton the electron, accordingly; the unit of length was chosen to correspond to the mean free path of an electron with Lorentz factor equal to Γ_1. Simulations were run for this "W depends on q" case, and additionally for a case were q was fixed to the Kolmogorov index 5/3 and the thickness was a free parameter.

We then simulated the outcoming power-law of the accelerated particles in a shock with thickness W varying over three orders of magnitude in the aforementioned units, from $\approx \frac{1}{100}$ to 10, using the smooth flow profile of Schneider and Kirk (1989). The simulations were run for shocks with speed ranging from $0.9\,c$ to $0.999\,c$ (Lorentz factors Γ_1 varying roughly from 2.2 to 22.4). Test-particles were injected in the upstream with a small injection energy, and then let to isotropise via small-angle scatterings while drifting into the shock. As for the energy loss mechanisms, the particles were taken to lose energy only via synchrotron emission. The results show particle spectral indices increasing very fast as the shock thickness grows larger than just a fraction of the mean free path. This suggests that parallel shocks would have to be very thin in order to accelerate particles to spectra with $\sigma \lesssim 3$, and that, in general, a relativistic parallel shock would not seem suitable for accelerating low-energy electrons to energies high enough to meet the requirements of observations. We later learned that Ellison (1992) had applied simulations similar to ours, and that he was led to the same conclusions.

In Paper II we studied the effects of injection using a new injection mechanism. We injected particles right behind the shock in the downstream with their initial velocity towards the upstream. We used two different injection energies: in the first case the electrons were given energy equal to the energy of cold upstream electrons as seen from the downstream, while in the second case the electrons received an energy corresponding to 20% of the downstream proton thermal energy. This latter method simulates the case where there exists some downstream thermalisation mechanism that heats the electrons and injects them back into the shock with an energy essentially higher than the energy of the electrons that have just crossed the shock once and return to the shock without any further energisation. The shock thickness was kept equal to unity in the abovementioned units for all simulations. In addition to the smooth hyperbolic tangent profile presented in Paper I, we applied a modified profile that was obtained from self-consistent Monte Carlo simulations of Vainio (2003). The turbulence spectral index q was given two values: $5/3$ for comparison of results from Paper I, and 2 for which the particle mean free path is energy-independent.

It was observed that, in contrast to low-energy injection in Paper I, in presence of higher-energy injection the particles can easily accelerate to very high energies and flat power-law spectra. While for the energy-independent mean free path (turbulence corresponding to $q = 2$), a spectral index of 3.2 was obtained, for the case where the mean free path increased with energy ($q = 5/3$) the energy spectrum had a convex non-power-law form with the high-energy part corresponding to a power-law with $\sigma \approx 2.2$ (see Section 4.2.4). This was the case for both flow profiles used. Although there were small differences in the resulting spectrum between the two flow profiles (cf. Schneider and Kirk 1987), the exact form of the transition did not seem to have significant effect when compared to the effects of the energy dependence of the mean free path or the injection method.

For forthcoming work, a very simple extension to the studies done so far would be to simulate the particle acceleration in multiple shocks. Preliminary simulations have shown that subsequent acceleration in two or more shocks can easily lead to very hard particle spectra. Astronomically this kind of modelling would be of interest when studying internal shocks in gamma-ray bursts or in parsec-scale jets in active galaxies, where the particle population in the upstream of a shock can already be energised and heated by a previous shock wave. In this case, even the upstream particle could have very high injection energies, so an efficient injection into the first-order process even in a thick shock would not necessarily be a problem.

The greatest restriction of the present model is, however, limiting the study to parallel shocks. Extending the model to also work for oblique shock geometries would open a totally new field of application, and it is also a very natural extension for the current simulation code. In addition to the well-known basic properties of acceleration in oblique shock waves (see e.g. Meli and Quenby 2003a, for a review), the combination of the oblique geometry and modified shock structure could turn up to be interesting (see e.g. Ellison and Double 2004). Namely, while for a step shock the alignment of the upstream and the downstream magnetic field direction and density change instantaneously at the shock front, in a modified shock the changes have to be calculated continuously across the whole transition. This kind of gradual changes could have interesting effects for the particle transport and resulting spectrum, when compared to a simple step shock case.

5.2 Turbulence transmission in parallel shocks

Paper III Alfvén-wave transmission and test-particle acceleration in parallel relativistic shocks by R. Vainio, J. Virtanen & R. Schlickeiser, *Astronomy & Astrophysics, 409, 821 (2003); 431, 7 (2005)*

Paper V Particle acceleration in thick parallel shocks with large compression ratio by J. Virtanen & R. Vainio, *Astronomy & Astrophysics, 439, 461 (2005)*

Paper VI Turbulence transmission in parallel relativistic shocks using ray tracing by J. Tammi & R. Vainio, *Astronomy & Astrophysics, submitted*

Paper III extended the study of Alfvén wave transmission in parallel step shocks (Vainio and Schlickeiser 1998) to relativistic speeds. Paper VI did the same for transmission in thick shocks, previously analysed at the non-relativistic limit in Paper V[1].

As explained in Section 3.2.1 and shown in Figure 3.1, the transmission coefficients for relativistic parallel shocks behave very similarly to those at the non-relativistic limit. The main result of Paper III was, that regardless of the shock speed, for low-to-moderate Alfvénic Mach numbers, the wave field is dominated by the backward mode. This was shown to lead to increased first-order acceleration, as was confirmed analytically as well

[1] Due to a typo there is r^2 instead of r^3 in the denominator of the last term of Eq. (1) in Paper V.

as by using numerical simulations. Although in Paper III this was shown only for vanishing upstream cross-helicity ($H_{c1} = 0$), the qualitative effect in the relativistic case remains the same also for degenerate values of $H_{c1} = \pm 1$ (see Figure 3.1).

For thick shocks (or for waves much shorter than the shock thickness) the non-relativistic case in Paper V showed exactly the same behaviour: a resulting backward wave field, and significantly enhanced acceleration. This coherence of transmission for non-to-ultrarelativistic steplike shocks and non-relativistic thicker ones breaks, however, when relativistic thick shocks are brought in: as the shock speed increases, the downstream mean cross helicity for low-M shocks tends to +1, i.e., the waves flow predominantly forward when $H_{c1} = 0$. In the case of $H_{c1} = +1$ *all* waves in the downstream are flowing parallel to the flow due to a lack of wave reflection during the crossing (cf. the case of step shocks). In Paper VI the transmission through a thick shock was solved for relativistic speeds and the main result was the aforementioned qualitative difference between relativistic and the non-relativistic cases.

An obvious shortcoming of this kind of a ”infinitely thin shock vs. infinitely thick shocks” separation, as was discussed in Section 3.2.3. However, now that these first building blocks at the opposite sides of relativistic wave transmission analysis have been laid, the natural next step towards a general model can be taken. On this basis additional ’blocks’ can also be constructed. For example, reflection of waves from smooth gradients (see e.g. Laitinen 2005, and references therein) is expected to change the results of Paper VI, and including the pressure from the waves to calculating the shock structure will probably alter the transmission for the lowest Mach number shocks (see Vainio and Schlickeiser 1999). Furthermore, once a general transmission model for Alfvén waves in parallel shocks has been achieved, performing similar analysis for fast magnetosonic waves would lead to even more complete description of turbulence transmission in shocks.

It is stressed that the present transmission analyses are based on the assumption of strictly parallel geometry. Effort should be put on extending the transmission analysis to non-parallel shocks (both steplike and modified), especially when the current particle acceleration model used in this thesis is modified to also work in the oblique cases.

5.3 Particle acceleration and turbulence transmission

Paper III Alfvén-wave transmission and test-particle acceleration in parallel relativistic shocks by R. Vainio, J. Virtanen & R. Schlickeiser, *Astronomy & Astrophysics, 409, 821 (2003); 431, 7 (2005)*

Paper IV Stochastic Acceleration in Relativistic Parallel Shocks by J. Virtanen & R. Vainio, *The Astrophysical Journal, 621, 313 (2005)*

Paper V Particle acceleration in thick parallel shocks with large compression ratio by J. Virtanen & R. Vainio, *Astronomy & Astrophysics, 439, 461 (2005)*

In Papers III—V we applied the effect of turbulence transmission to first- and second-order Fermi acceleration in shocks of different kind.

In addition to the wave transmission calculations, Paper III also contains results from numerical simulations of the first-order acceleration in step shocks, with scattering-centre compression ratio calculated using the wave transmission analysis. The results showed the expected hardening of the particle spectral index when the effective compression ratio was used, and good agreement with the diffusive approximation was obtained for the non-relativistic case. Similar simulations were later performed for an extensive set of thick shocks (Paper V). There we compared our simulation results with analytical predictions of Drury et al. (1982) for diffusive acceleration in modified shocks with thicknesses ranging over four orders of magnitude, as well as Keshet and Waxman (2005) for ultrarelativistic step shocks, and found excellent agreement with both in the applicable parts of the parameter space.

The analytical model of Drury et al. (1982) was found to provide quite good estimate for the high-energy power-law spectral index. In the non-relativistic cases the simulations showed a perfect match with the theory, even for relativistic speeds, where the use of the diffusion approximation is not justified, there was a good agreement in those cases where the scattering-centre compression ratio was high. A clear result for the thick-shock cases was that in the presence of an increased scattering-centre compression ratio even very thick shocks are able accelerate particles to power laws with $\sigma \sim 2$ (or flatter!), if the scattering-centre compression is sufficiently large.

However, the simple transmission model for relativistic shocks does not suggest these high compressions for very thick shocks. Preliminary studies for the general transmission model, however, do not rule out efficient compression even in those cases, so the case of first-order acceleration in low-Mach-number modified shocks remains yet to be solved until a more general way of dealing with the transmission is available.

Probably the most interesting application was studied in Paper IV, where we let the transmitted wave field affect the particles as separate sources of scattering. This led, expectedly, to stochastic acceleration. We simulated different cases of relativistic parallel shocks for different turbulence properties and magnetic fields, and demonstrated that in small-M cases, the second-order Fermi mechanism has remarkable effects on the first-order-accelerated particle spectrum. Furthermore, the stochastic acceleration of thermal particles in the downstream can even provide the high energies for particles injected into the first-order mechanism, thus potentially offering relief to the problem of injection by providing the energisation process required in Paper II for high-energy injection from the downstream side.

For future work, the Alfvén wave transmission could be worth applying to some astronomical objects where the observations seem to require accelerated electron populations with spectra harder than the 'universal' $\sigma \approx 2.2$.

As was pointed in Paper IV, plain stochastic acceleration of low-energy particles in the case of continuous acceleration can lead to very flat (or even inverse) particle spectral indices corresponding to synchrotron spectra with photon spectral index $-0.5 < \alpha < 0$ in the GHz–THz regime. This would suggest some flat-spectrum sources, for instance, would be interesting objects to model with stochastic acceleration. This requires, of course, the inclusion of more detailed loss mechanisms (at least the inverse Compton mechanism in addition to the synchrotron mechanism used in Paper IV) and careful consideration of the source geometry and radiation production, in addition to light-travel-time effects. A more realistic model would also require restricting the acceleration efficiency in the downstream by taking into account the damping and turbulent dissipation of the waves (see e.g. Ko 1992 and Vainio and Spanier 2005, respectively).

One thing that has not been considered, so far, is the combined effect of the two sources for convexity in the particle spectrum from modified shocks. Namely, as was discussed separately for wave transmission and particle acceleration, both the particle mean free path and the scattering-centre compression ratio can cause an upward bend in the accelerated particle spectrum. How strong an effect this combination can have on the spectrum, and what kind of radiation could be expected from such a source (low-M requirement suggests strong magnetic field and low density), are questions to be answered.

CHAPTER 6
Conclusions

This dissertation presents studies of particle acceleration of first- and second-order Fermi type in parallel relativistic shocks. Compared to the 'traditional' approach, two significant extensions have been made here: we have *not* assumed the shock front to be a discontinuous step in the plasma parameters, *nor* have we neglected the possibility for Alfvén waves to have non-negligible speeds compared to the speed of plasma. These extensions, alone, as well as together, have been shown to be able to have significant effects on the particle acceleration efficiency, when compared to results excluding them. Our results are in accordance with the previous ones found in the literature in those parts where the comparison is possible; in cases where they extend to previously unknown parameter space, they show significant results that are both applicable and extendable in many ways.

While we have confirmed the inability of moderately thick modified shock to a strong first-order acceleration of low-energy electrons (Paper I), we have showed them to be capable of efficient acceleration in the case of a sufficiently strong injection mechanism (Paper II) and scattering-centre compression ratio higher than that of the plasma flow (Paper V).

We have also shown that if the speed of the Alfvén waves differs notably from the speed of the underlying flow, the compression felt by the accelerating particles at the shock is not necessarily that of the gas, but can be many times higher, or, in some cases, even lower (Paper III, Paper VI). This underlines the significance of the turbulence transmission analysis in objects and applications where the wave speeds cannot be safely neglected.

Turbulence transmission at shock waves was also shown to be able to provide suitable conditions for stochastic acceleration in the downstream of a relativistic shock with low-to-moderate Alfvénic Mach number (Paper IV). The second-order process was found to be able to significantly change the energy spectrum of the particles accelerated in the shock by the first-order mechanism. In addition, it was shown to also accelerate the low-energy particles in the downstream. In the case of a continuous injection from the thermal bulk, this was shown to lead to very hard (or even inverse) particle spectra, and even to injection of high-energy particles into the first-order acceleration at the shock.

6.1 Limitations of the current model and some directions for future work

Although he have omitted the assumptions of static turbulence and steplike shocks, the current work is still limited by the assumption of parallel geometry. Both the turbulence transmission analysis as well as the simulations of particle acceleration rely on this assumption, and for highly relativistic shocks this requirement can become very restricting. This is because of the Lorentz-booster transverse magnetic fields, which make it harder to justify the treatment of including only parallel magnetic field components for large values of Γ_1.

Especially for turbulence transmission analysis, the assumption of parallel shock geometry sets certain limitations. For example, increasing obliqueness would decrease the relative speed of the Alfvén waves with respect to the shock normal, but it can also lower the gas compression ratio of a low-Mach-number shock. The resulting complexity depending on the shock obliqueness has been avoided in this study by the assumption of parallel shock geometry.

Furthermore, it must be stressed that the particle acceleration simulations and the wave transmission calculation are based on the test-particle and test-wave approaches. The current model includes the effects the shock has on particles and acceleration, as well as the effects the shock thickness and speed have on waves. At its present stage, however, the model does not include the micro-physical and nonlinear effects that particles have on waves, and that particles and waves have on the shock front. The shock-modifying effects are only taken into account by an assumption of the thick shock in an *ad hoc* manned. Due to the complicated nature of this *ménage à trois* between the shock structure, particles and waves, complete non-linear treatment is far beyond the scope of this thesis. Instead, we have limited ourselves by treating the shock structure more or less as a free parameter. This approach has, however, turned out to be a practical starting point for studies concerning the acceleration and turbulence in different kinds of shocks.

Due to the simplified test-wave approach in the turbulence transmission analysis, the results obtained describe the conditions immediately behind the shock; further away interactions with particles and other waves are likely to affect the turbulence spectrum and, thus, also the properties of particle acceleration. Examining these effects in detail is crucial for the forthcoming work.

Throughout this thesis the turbulence is also assumed to consist of small-amplitude Alfvén waves. The requirement of small amplitude is, clearly, a restriction. However, because the nature of the turbulence, in cases applicable to relativistic shocks, is still a great mystery, it is hard to say how strict a limitation this is. The assumption of the Alfvénic nature of the turbulence is not a restriction *per se*, but extending the analyses to also account for fast magnetosonic waves will offer a more general foundation for the future studies of shock–turbulence–particles interactions.

Bibliography

Achterberg, A. and Blandford, R. D.: 1986, *MNRAS* **218**, 551

Achterberg, A., Gallant, Y., Kirk, J. G., and Guthmann, A. W.: 2001, *MNRAS* **328**, 393

Appl, S. and Camenzind, M.: 1988, *A&A* **206**, 258

Armstrong, J. W., Rickett, B. J., and Spangler, S. R.: 1995, *ApJ* **443**, 209

Axford, W. I.: 1994, *ApJS* **90**, 937

Axford, W. I., Leer, E., and McKenzie, J. F.: 1982, *A&A* **111**, 317

Axford, W. I., Leer, E., and Skadron, G.: 1977, *Proc. 15th ICRC* **11**, 132

Baring, M.: 1999, in *Proc. 26th ICRC*, p. 5

Baring, M. G. and Kirk, J. G.: 1991, *A&A* **241**, 329

Bednarz, J. and Ostrowski, M.: 1996, *MNRAS* **283**, 447

Bednarz, J. and Ostrowski, M.: 1998, *Phys. Rev. Lett.* **80**, 3911

Bednarz, J. and Ostrowski, M.: 1999, *MNRAS* **310**, L11

Begelman, M. C. and Kirk, J. G.: 1990, *ApJ* **353**, 66

Bell, A. R.: 1978a, *MNRAS* **182**, 147

Bell, A. R.: 1978b, *MNRAS* **182**, 443

Berezhko, E. G. and Ellison, D. C.: 1999, *ApJ* **526**, 385

Blandford, R. and Eichler, D.: 1987, *Physics Reports* **154**, 1

Blandford, R. D.: 1980, *ApJ* **238**, 410

Blandford, R. D. and Ostriker, J. P.: 1978, *ApJL* **221**, L29

Blasi, P. and Vietri, M.: 2005, *ApJ* **626**, 877

Campeanu, A. and Schlickeiser, R.: 1992, *A&A* **263**, 413

de Hoffmann, F. and Teller, E.: 1950, *Phys. Rev.* **80**, 692

Derishev, E. V., Aharonian, F. A., Kocharovsky, V. V., and Kocharovsky, V. V.: 2003, *Phys. Rev. D* **68(4)**, 043003

Dermer, C. D. and Humi, M.: 2001, *ApJ* **556**, 479

Dewar, R. L.: 1970, *Physics of Fluids* **13**, 2710

Dewar, R. L.: 1972, *ApJ* **174**, 301

Drury, L. O.: 1983, *Reports on Progress in Physics* **46**, 973

Drury, L. O., Axford, W. I., and Summers, D.: 1982, *MNRAS* **198**, 833

Drury, L. O. and Völk, J. H.: 1981, *ApJ* **248**, 344

Duffy, P.: 1992, *A&A* **262**, 281

Duffy, P.: 1994, *ApJS* **90**, 981

Duffy, P., Drury, L. O., and Völk, H.: 1994, *A&A* **291**, 613

Dung, R. and Schlickeiser, R.: 1990, *A&A* **240**, 537

Eichler, D.: 1979, *ApJ* **229**, 419

Eichler, D.: 1984, *ApJ* **277**, 429

Ellison, D. C.: 1992, *AIP Conf. Proc.* **264**, 331

Ellison, D. C. and Double, G. P.: 2002, *Astrop. Phys.* **18**, 213

Ellison, D. C. and Double, G. P.: 2004, *Astrop. Phys.* **22**, 323

Ellison, D. C. and Eichler, D.: 1984, *ApJ* **286**, 691

Ellison, D. C., Reynolds, S. P., and Jones, F. C.: 1990, *ApJ* **360**, 702

Fermi, E.: 1949, *Phys. Rev.* **75**, 1169

Fermi, E.: 1954, *ApJ* **119**, 1

Gallant, Y. and Achterberg, A.: 1999, *MNRAS* **305**, L6

Gallant, Y., Achterberg, A., Kirk, J. G., and Guthmann, A.: 2000, *AIP Conf. Proc.* **526**, 524

Haugbølle, T.: 2005, *Ph.D. thesis*, Niels Bohr Institute

Hededal, C., Haugbølle, T., Frederiksen, J., and Nordlund, Å.: 2004, *ApJL* **617**, L107

Horbury, T. S., Forman, M. A., and Oughton, S.: 2005, *Plasma Physics and Controlled Fusion* **47**, B703

Jokipii, J. R.: 1966, *ApJ* **146**, 480

Jones, F. C. and Ellison, D. C.: 1991, *Space Sci. Rev.* **58**, 259

Jones, T. W.: 1993, *ApJ* **413**, 619

Jones, T. W. and Kang, H.: 2005, *Astrop. Phys.* **24**, 75

Kang, H. and Jones, T. W.: 2005, *ApJ* **620**, 44

Keshet, U. and Waxman, E.: 2005, *Phys. Rev. Lett.* **94(11)**, 111102

Kirk, J. G.: 2005, *Proc. XXII Texas Symposium, eConf: C041213, 0045*

Kirk, J. G. and Dendy, R. O.: 2001, *J. Phys. G.* **27**, 1589

Kirk, J. G. and Duffy, P.: 1999, *J. Phys. G.* **25**, 163

Kirk, J. G., Guthmann, A. W., Gallant, Y. A., and Achterberg, A.: 2000, *ApJ* **542**, 235

Kirk, J. G. and Schneider, P.: 1987a, *ApJ* **315**, 425

Kirk, J. G. and Schneider, P.: 1987b, *ApJ* **322**, 256

Kirk, J. G. and Schneider, P.: 1988, *A&A* **201**, 177

Kirk, J. G. and Schneider, P.: 1989, *A&A* **225**, 559

Kirk, J. G., Schneider, P., and Schlickeiser, R.: 1988, *ApJ* **328**, 269

Ko, C.-M.: 1992, *A&A* **259**, 377

Krommes, J. A.: 2002, *Phys. Rep.* **360**, 1

Krülls, W. M.: 1992, *A&A* **260**, 49

Krymsky, G. F.: 1977, *Doklady Akademii Nauk SSSR* **243**, 1306

Laitinen, T.: 2005, *Astrophysics and Space Sciences Transactions* **1**, 35

Lazar, M., Spanier, F., and Schlickeiser, R.: 2003, *A&A* **410**, 415

Lemoine, M. and Pelletier, G.: 2003, *ApJl* **589**, L73

Lemoine, M. and Revenu, B.: 2006, *MNRAS* p. 77

Lithwick, Y. and Goldreich, P.: 2001, *ApJ* **562**, 279

Mac Low, M.: 2005, in *The Magnetized Plasma in Galaxy Evolution*, pp 41–47

Malkov, M. A. and Völk, H. J.: 1995, *A&A* **300**, 605

Malkov, M. A. and Völk, H. J.: 1998, *Advances in Space Research* **21**, 551

Maron, J. and Goldreich, P.: 2001, *ApJ* **554**, 1175

McKenzie, J. F. and Westphal, K. O.: 1969, *Planetary and Space Science* **17**, 1029

Medvedev, M. V.: 2000, *ApJ* **540**, 704

Meli, A. and Quenby, J. J.: 2003a, *Astrop. Phys.* **19**, 649

Meli, A. and Quenby, J. J.: 2003b, *Astrop. Phys.* **19**, 637

Nishikawa, K.-I., Hardee, P., Richardson, G., Preece, R., Sol, H., and Fishman, G. J.: 2003, *ApJ* **595**, 555

Ostrowski, M.: 1994, *A&A* **283**, 344

Ostrowski, M. and Schlickeiser, R.: 1993, *A&A* **268**, 812

Ostrowski, M. and Siemieniec-Oziębło, G.: 1997, *Astrop. Phys.* **6**, 271

Parker, E. N.: 1961, *J. Nucl. E.* **2**, 146

Peacock, J. A.: 1981, *MNRAS* **196**, 135

Petrosian, V. and Liu, S.: 2004, *ApJ* **610**, 550

Pinter, S.: 1980, *Bulletin of the Astronomical Institutes of Czechoslovakia* **31**, 368

Protheroe, R. J. and Clay, R. W.: 2004, *PASP* **21**, 1

Quenby, J. J. and Lieu, R.: 1989, *Nature* **342**, 654

Schlickeiser, R.: 1984, *A&A* **136**, 227

Schlickeiser, R.: 1989, *ApJ* **336**, 243

Schlickeiser, R.: 2002, *Cosmic ray astrophysics*, Springer. ISBN 3-540-66465-3, 2002

Schlickeiser, R., Campeanu, A., and Lerche, L.: 1993, *A&A* **276**, 614

Schlickeiser, R. and Dermer, C. D.: 2000, *A&A* **360**, 789

Schlickeiser, R. and Vainio, R.: 1999, *Astrophysics and Space Science* **264**, 457

Schneider, P. and Kirk, J. G.: 1987, *ApJl* **323**, L87

Schneider, P. and Kirk, J. G.: 1989, *A&A* **217**, 344

Skilling, J.: 1975a, *MNRAS* **172**, 557

Skilling, J.: 1975b, *MNRAS* **173**, 245

Smith, E. J.: 1983, *Space Sci. Rev.* **34**, 101

Spanier, F., Schlickeiser, R.and Keshet, U., and E., W.: 2006, *submitted to ApJ*

Stern, B. E.: 2003, *MNRAS* **345**, 590

Tammi, J. and Vainio, R.: 2006, *submitted to A&A*

Tsiklauri, D., Sakai, J.-I., and Saito, S.: 2005, *A&A* **435**, 1105

Vainio, R.: 2003, *ASP Conf. Proc.* **299**, 143

Vainio, R. and Schlickeiser, R.: 1998, *A&A* **331**, 793

Vainio, R. and Schlickeiser, R.: 1999, *A&A* **343**, 303

Vainio, R. and Schlickeiser, R.: 2001, *A&A* **378**, 309

Vainio, R. and Spanier, F.: 2005, *A&A* **437**, 1

Vainio, R., Virtanen, J. J. P., and Schlickeiser, R.: 2003, *A&A* **409**, 821

Virtanen, J. and Vainio, R.: 2003a, *Proc. 28th ICRC* **OG 1.4**, 2023

Virtanen, J. and Vainio, R.: 2003b, *ASP Conf. Proc. Vol.* **299**, 157

Virtanen, J. J. P. and Vainio, R.: 2005a, *A&A* **439**, 461

Virtanen, J. J. P. and Vainio, R.: 2005b, *ApJ* **621**, 313

Wang, J.-C.: 2002, *Chinese Journal of Astronomy and Astrophysics* **2**, 1

Webb, G. M.: 1989, *ApJ* **340**, 1112

Webb, G. M., Bogdan, T. J., Lee, M. A., and Lerche, I.: 1985, *MNRAS* **215**, 341

Webb, G. M., Drury, L. O., and Biermann, P.: 1984, *A&A* **137**, 185

Yan, H. and Lazarian, A.: 2004, *ApJ* **614**, 757

Abbreviations of journal names

A&A	Astronomy and Astrophysics
AIP Conf. Proc.	AIP Conference Proceedings
ApJ	Astrophysical Journal (the)
ApJL	Astrophysical Journal Letters (the)
ApJS	Astrophysical Journal Supplement Series (the)
ASP Conf. Proc.	ASP Conference Proceedings
Astrop. Phys.	Astroparticle Physics
ICRC	International Cosmic Ray Conference, proceedings
J. Nucl. E.	Journal of Nuclear Energy
J. Phys. G	Journal of Physics G, Nuclear Physics
MNRAS	Monthly Notices of the Royal Astronomical Society
PASP	Publications of the Astronomical Society of the Pacific
Phys. Rev.	Physical Review
Phys. Rev. Lett.	Physical Review Letters
Space Sci. Rev.	Space Science Reviews

Original papers