From Him all the body, by being harmoniously joined together and being made to cooperate through every joint that gives what is needed,...makes for the growth of the body...

– EPHESIANS 4:16.

University of Alberta

COOPERATIVE LINEAR PRECODING FOR SPECTRUM SHARING IN MULTI-USER WIRELESS SYSTEMS: GAME THEORETIC APPROACH

by

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Abstract

Future wireless communications expect to experience a spectrum shortage problem. One practical solution is spectrum sharing. This thesis studies precoding strategies to allocate communication resources for spectrum sharing in multi-user wireless systems from a game-theoretic perspective. The approaches for the precoding games are developed under different constraints. It is shown that the precoding game with spectral mask constraints can be formulated as a convex optimization problem and a dual decomposition based algorithm can be exploited to solve it. However, the problem is non-convex if users also have total power constraints. This study shows that an efficient sub-optimal solution can be derived by allocating the bottleneck resource in the system. The sub-optimal solution is proved to be efficient and can even achieve an identical performance to that of the optimal solution in certain cases, but with significantly reduced complexity.

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List of Symbols

$\mathbf{x}$	column vector
$\mathbf{X}$	matrix
$\mathcal{B}$	set
$\lvert x \rvert$	magnitude of number x
$\lvert \mathcal{B} \rvert$	cardinality of set $\mathcal{B}$
$\bigcap$	intersection (set operation)
$\lVert \mathbf{x} \rVert$	Euclidian norm of $\mathbf{x}$
$\mathbf{X}^H$	Hermitian transpose of $\mathbf{X}$
$\mathbf{X}^T$	transpose of $\mathbf{X}$
$[\mathbf{X}]_{i,j}$	element at the intersection of row i and column j of $\mathbf{X}$
$det(X)$	determinant of $\mathbf{X}$
$diag(\mathbf{x})$	operator which forms a diagonal matrix such that $[diag(\mathbf{x})]_{i,i} = [\mathbf{x}]_i$
$Tr\{\mathbf{X}\}$	trace of $\mathbf{X}$
$E\{\cdot\}$	expectation operation
$\mathbf{I}$	identity matrix
$\mathbf{0}$	all-zero matrix
$(x)_+$	defined as $(x)_+ = \max\{x, 0\}$
$\nabla^2 f$	Hessian of f

List of Abbreviations

CP	cyclic prefix
DSE	dominant strategy equilibrium
FFT	fast Fourier transform
IFFT	inverse fast Fourier transform
ISI	inter-symbol interference
ISM	industrial scientific and medical
MIMO	multiple-input multiple-output
NE	Nash equilibrium
NB	Nash bargaining
NF	Nash function
OFDM	orthogonal frequency-division multiplexing
QoS	quality of service
SNR	signal to noise ratio
SVD	singular value decomposition
TDM/FDM	time-division multiplexing/frequency-division multiplexing
WLAN	wireless local area network
ZMCSCG	zero mean circularly symmetric complex Gaussian

Chapter 1

Introduction

Wireless communications have experienced a significant development during the past two decades thanks to the improvements of radio and electronics technologies. The boom of wireless cellular networks and wireless local area networks (WLANs) has indicated a bright future of wireless communications.

In wireless cellular systems, the number of cellular users has experienced a rapid growth over the last ten years. Currently, the third generation (3G) standards, which support more diverse applications and higher data rates, have been launched in many countries and areas in the world. Indeed, the progress of wireless cellular systems not only provides cellular users with better wireless services but also creates a huge commercial success in the wireless market. In the future, the Next-generation (NextG) wireless systems are expected to provide wireless users with higher-quality higher-speed services and to construct a basis for ubiquitous wireless communications. A wireless user will be able to enjoy comprehensive and secure services including voice, data and multimedia transmission anytime and anywhere [1].

Another huge progress appears in WLANs. The first generation of WLANs based on the standard IEEE 802.11-1997 achieves little success in the market due to the problem of inter-operability and high cost. On the contrary, the second generation of WLANs based on 802.11b is much more successful. Many products have been developed based on 802.11b and subsequent standards 802.11a, 802.11g and 802.11n [2]. These products provide higher data rate and better coverage, while at the same time, they are competitive in price. WLANs are popular now in homes, campuses and companies as an extension of the wired LANs. The trademark *Wi-Fi* for certified products based on the IEEE 802.11 standards can be found all over the world, and a WLAN card is deployed in almost every laptop.

There is no doubt that wireless communications, including all the technologies and applications, have a promising future. However, many challenges still exist. New applications, such as video-conferencing and online gaming are expected to be accessible in future wireless networks, while existing services are expected to be able to provide better quality of service (QoS). High data rate is a precondition to support high-quality multimedia transmission. For example, one objective of the NextG is to achieve data rate of 100 Mbit/s for mobile users at high velocity and 1 Gbit/s for stationary users, compared to the data rate of around 10 Mbit/s for stationary users in 3G. These expectations and objectives lead to the requirement of larger bandwidth. One can predict that the demand for higher QoS will always exist. In other words, there will be an ever-growing demand for bandwidth in future wireless networks. However, the radio spectrum resource (from 3KHz to 300GHz) is limited. Thus, the growing demand of bandwidth will result in a spectrum shortage problem which will become more and more severe with the explosive growth of wireless services and applications.

In fact, the spectrum shortage already appears at the current stage. Radio spectrum is controlled by the regulatory organizations in different regions. The competition for spectral licenses is intense and the scarcity of the spectrum can be illustrated by the following fact. In April 2000, the spectrum auction in the United Kingdom for five 3G mobile wireless licenses raised around 34 billion dollars for 120 MHz spectrum band [3]. The license-free frequency bands, for example, the 2.4-2.5 GHz industrial scientific and medical (ISM) radio band, are also very crowded. The WLANs based on 802.11b operate in the 2.4 GHz ISM band, and they may receive interference from other wireless systems, such as Bluetooth devices.

Another challenge resulted from the demand of larger bandwidth is the frequency selective fading of the communication channels [4]. A signal transmitted into a wireless channel may arrive at the receiver through different paths due to scattering and reflection. The received signal is a sum of several delayed and attenuated versions of the transmitted signal from different paths. The signal from the direct path is therefore distorted. In the time domain, the neighboring symbols interfere with each other due to the delayed spread, which results in the inter-symbol interference (ISI). In the frequency domain, different frequency components of the received signal experience different fading. For a signal with bandwidth much less than the *channel coherent bandwidth*, the fading over different frequency components can be considered as equal and the ISI can be neglected. The channel, in this case, is called a flat fading channel. However, for a wideband signal whose bandwidth is larger than or comparable to the channel coherent bandwidth, the channel response varies largely across the frequency domain and the resulting ISI can significantly degrade the system performance [5]. The channel, in this case, is called a frequency selective fading channel.

Both the spectrum shortage problem and the wideband frequency selective fading problem need to be solved to meet the demands for future wireless communications. For the spectrum shortage problem, there is no other solution but improving spectrum efficiency because the total amount of spectrum resource is fixed. Recently cognitive radio has emerged as a promising solution for the spectrum shortage problem based on the observation that most parts of the spectrum are not utilized efficiently and left idle for a large percentage of time [6], [7], [8]. Indeed, the fixed spectrum allocation under the regulatory polices leads to the result that some of the spectrum bands are overloaded while others may be under-utilized. In cognitive radio, it is proposed that unlicensed users can sense the spectrum and detect the idle spectrum bands, referred to as *spectrum opportunities*. Moreover, they can make use of the idle spectrum bands in a manner that causes no or little interference to the licensed users. By allowing unlicensed users to *share* the licensed bands, the spectrum efficiency can be improved. However, another problem emerges. Since there are probably more than one unlicensed user trying to use the spectrum, they may interfere with each other. An efficient and fair spectrum allocation is required for multiple unlicensed users to share the idle spectrum bands, as otherwise all of their transmissions will be degraded significantly. The unlicensed users may belong to different systems and thus there is probably no central controller to coordinate their transmissions. Thus, the users are driven by their own selfishness in sharing the spectrum and they aim at maximizing their own benefits. It is obvious that every user will try to use more spectrum resource. However, if the users are too greedy such that everyone tries to use all spectrum opportunities, the interference can be high and damage the benefits of all users.

Cognitive radio is only one example in which spectrum sharing is exploited to improve the spectrum efficiency. There are many other examples in which the spectrum resources in wireless systems are shared by multiple users and the users may potentially interfere with each other [9], [10]. Thus, it is important and beneficial to study the problem of spectrum sharing among wireless users. The question is: if the wireless users are selfish but rational, will there be stable results of the competition for spectrum resources? And if yes, what it is and how to achieve the best result for all users. Game theory is a viable candidate to deal with these issues. Initially developed in economics, game theory is recognized as a useful tool in many science and engineering fields. Game theory studies individuals' decision-making strategies when their decisions affect each other. For the spectrum sharing problem, the interference among wireless users bounds them together into the situation that everyone's benefit depends on all users' actions. This relationship among users makes game theory applicable and useful.

Modeling the spectrum sharing problem as a game and the wireless users as players of the game, the problem can be studied for different scenarios according to whether there is a cooperation among users or not. If there is no voluntary cooperation among users, spectrum sharing among unlicensed users can be studied using the *equilibrium* concept in game theory. An equilibrium can be considered as a stable result of a non-cooperative game where no player can improve its benefit unilaterally. However, an equilibrium can be highly inefficient for all the players due to the lack of cooperation. One famous example is the 'Prisoner's Dilemma' [11]. Thus, we are more interested in the cases in which the users are willing to cooperate with each other. Bargaining solutions in cooperative game theory can be used to study such cases. Nash axiomatic bargaining, originally proposed in 1950s, is the most popular bargaining solution for cooperative games. The game solution derived by Nash axiomatic bargaining, if exists, represents a fair and efficient distribution of benefits among game players. Thus, spectrum sharing among different users can be investigated from a cooperative game theoretic perspective by introducing Nash axiomatic bargaining.

The ISI problem on frequency selective fading channels also needs to be considered. Compared to spectrum shortage problem, ISI has been studied in wireless communications for a long time. A traditional solution to mitigate ISI is using *equalizer* at the receiver side. However, the equalization amplifies the channel noise while equalizing the amplitude distortion. Moreover, it suffers from error propagation. Therefore, *orthogonal frequency-division multiplexing* (OFDM) technique has been suggested and popularized as an effective method to combat the ISI with a simple transceiver structure [12]. For example, IEEE 802.11a and 802.11n standards adopt the OFDM technique. The essence of the OFDM technique is to divide the wideband frequency selective fading channel into many frequency bins with small bandwidth and to transmit data in those frequency bins so that the ISI in the transmission on each frequency bin can be neglected.

In OFDM systems, wireless users can allocate their communication resources such as power and data across frequency bins using *precoding*. The information about how a certain user exploits its spectrum is contained in its precoding scheme. Thus, the status of a spectrum sharing game and the interference in the game are determined by all users' precoding schemes. In the spectrum sharing game, a user adjusts its precoding scheme according to precoding schemes of other users (in the non-cooperative case) or according to a certain agreed principle of cooperation (in the cooperative case). In the context of game theory, precoding schemes here are recognized as the players'/wireless users' *strategies*. Modeling the precoding schemes of users as strategies in the spectrum sharing game, one should find how users should share the public resource (the spectrum band) and allocate the individual resources (for example, transmission power) to boost the benefits of all users.

The purpose of this thesis is to study wireless users' precoding schemes for sharing a wideband spectrum from a game theoretic perspective. The particular objective is to derive precoding strategies using the cooperative Nash axiomatic bargaining, which corresponds to a fair and efficient allocation of available communication resources for the wireless users. The main contribution of this work is threefold, as follows.

First, a general structure for precoding matrices in OFDM systems on frequency selective fading channels is considered. It is shown that the precoding matrices adopt a strictly diagonal form under the total power and/or spectral mask constraints, compared to the precoding structure of a diagonal matrix multiplied by a non-diagonal constant matrix in multiple-input multiple-output (MIMO) systems. Based on this result, a cooperative two-user precoding game and the extension to M-user precoding game are studied under spectrum mask constraints, where the users cooperate with each other based on time-division multiplexing/frequency-division multiplexing (TDM/FDM). It is shown that the problem of finding precoding strategies can be transformed to the problem of finding portions of time that each user obtains on each frequency bin.

Second, it is shown that the cooperative precoding games can be formulated as convex optimization problems when only spectrum mask constraints are applied. For cooperative games, all users have to exchange information in cooperation. So a purely distributed algorithm is not applicable. However, it is shown that the problem can be decoupled using dual decomposition, and the bargaining among users can be physically realized using a distributed structure with a coordinator.

Last, the two-user cooperative game is considered in the case when both the total power and spectral mask constraints are present. The problem of finding the TDM/FDM based bargaining solution is shown to be non-convex in this case. It is proposed that wireless systems can be categorized into two types according to their bottleneck resources (power or bandwidth). For each type of system, an algorithm is found to obtain a sub-optimal solution by allocating the bottleneck resource. It is proved that the sub-optimal solutions are efficient under mild conditions and can even achieve, in some cases, an identical performance to that given by the optimal solutions, while the complexity of finding such sub-optimal solutions can be significantly less compared to the complexity of finding the optimal solutions.

Chapter 2

Background and Related Works

2.1 Preliminaries on game theory

Mathematically, an M-player game can be modeled as

$$\Gamma = \Big\{\Omega, \{s_i|i \in \Omega\}, \{u_i|i \in \Omega\}\Big\} \tag{2.1}$$

where $\Omega = \{1, 2, ..., M\}$ is the set of all players, s_i is the strategy of player i, and u_i is the utility (payoff) for player i as a function of $\{s_1, s_2, ..., s_M\}$. Therefore, *Players*, *strategies* of the players, and corresponding *payoffs* of the players are three key elements of a game. The rationality and selfishness assumption suggests that all players aim at maximizing their own utilities. The payoff for any single player is subject to the collective strategies of all players in the game.

Depending on whether players collaborate or not, a game can be cooperative or non-cooperative. For non-cooperative games, *equilibrium* is the basic concept. An equilibrium $\{s_i\}_{i=1}^{M}$ is a strategy set composed of the best stable strategies for all players of the game [13]. An Nash equilibrium (NE) is a typically considered equilibrium, which satisfies the condition

$$u_i({s_i}^{\mathrm{NE}}, s_{-i}^{\mathrm{NE}}) \geq u_i({s_i}', s_{-i}^{\mathrm{NE}}), \quad \forall {s_i}', \forall i \tag{2.2}$$

where ${s_i}^{\mathrm{NE}}$ is the strategy of player i in the NE, ${s_{-i}}^{\mathrm{NE}}$ is the combination of strategies of all players except player i in the NE, and ${s_i}'$ stands for any possible strategy of player i. An NE can be viewed as a set of stable strategies under which no player can increase its utility by unilaterally deviating from his current strategy. If there exists a unique NE of a game, it can be used as a non-cooperative solution of the game. One significant advantage of the NE is that it is widely applicable. However, there are also two problems with the NE. First, more than one NE may exist for a given game, which renders difficulty in predicting the outcome of the game. Second, the NE almost always leads to an inefficient outcome for all players.

A special case of the NE is the so-called *dominant strategy equilibrium* (DSE), in which each player has a fixed best strategy regardless of the strategies of other players. Mathematically, the DSE should satisfy the following condition

$$u_i({s_i}^{\mathrm{D}}, s_{-i}) \geq u_i({s_i}', s_{-i}), \quad \forall s_{-i}, \forall {s_i}', \forall i \tag{2.3}$$

where ${s_i}^{\mathrm{D}}$ is the strategy of player i in the DSE, s_{-i} is any combination of strategies of all players except player i, and ${s_i}'$ stands for any possible strategy for player i.

Unlike the NE, the DSE is unique if it exists, and each player has a fixed best strategy for whatever choices other players make. Thus, it can be inferred that it is less likely to exist than NE. Similar to the NE, the DSE may also be quite inefficient for all players. The inefficiency of the NE and DSE is due to the fundamental reason that there is no coordination among players.

If the players are willing to cooperate, a better outcome of the game can be expected. Cooperative game theory deals with games in which all users cooperate with each other. One of the most popular cooperation approaches is based on the so-called *Nash bargaining axioms* [14]. The axioms propose that the solution of a cooperative game should satisfy:

- Linearity. A linear transformation of all players' utility functions leads to the same transformation of the game solution.

- Symmetry. The game solution does not depend on the numbering of the players.

- Independence on irrelevant alternatives. Restricting the utility space of the game $\{u_i | i \in \Omega\} \rightarrow \boldsymbol{R}^M$ to a subspace which still contains the solution point of the original game does not change the solution, i.e., this point is also the solution of the new game.

- Pareto-optimality. The solution is not weakly dominated by any point in the utility space.

Characterized by these four axioms, there is a unique point in a convex utility space that maximizes the Nash function (NF) defined as

$$\mathfrak{F} = \prod_{i \in \Omega} (u_i - u_i^{'}) \tag{2.4}$$

where $u_i^{'}$ is the utility obtained by user i in the non-cooperative case. The point $(u_1^{'}, ...u_M^{'})$ is known as the disagreement point which all users will resort to if the cooperation breaks up.

Nash axiomatic bargaining focuses on describing properties of the final solution of a cooperative game. However, how to cooperate so as to reach this solution is not specified. Thus, the Nash bargaining (NB) solution in a specific game depends on the manner of cooperation. For example, wireless users may perform TDM or FDM in a cooperative game. The only limitation is that the utility space should be convex.

It has been observed that the NB solution, if exists, may provide supplementary benefits for all users as compared to the non-cooperative solution. Moreover, the benefits among users are distributed based on the so-called proportional fairness [15]. There are several other results on cooperative game theory, such as Kalai-Smorodinsky and Egalitarian solutions which also deal with convex games [16]. The extension of the Nash axiomatic bargaining, Kalai-Smorodinsky, and Egalitarian solutions to certain non-convex problems are studied in [17], [18].

2.2 Linear precoding

Precoding is a signal processing technique used to enhance the quality of wireless transmission when the channel state information is known at the transmitter side.

Precoding can be performed in a non-linear or linear manner. Non-linear precoding can be viewed as a special equalization performed at the transmitter side, which involves a non-linear structure to feed back the channel information. One example is Tomlinson-Harashima precoding [19]. Linear precoding simply performs a linear transformation of the data to be transmitted. As compared to non-linear precoding, linear precoding has lower complexity and can be applied to systems with an arbitrary number of antennas. Thus, it is widely considered in the literature and it better fits current practical applications.

Linear precoding has been extensively studied in single-user MIMO wireless systems. Although the objective in this work is to investigate precoding strategies in multi-user systems on a wideband frequency selective fading channel where wireless users are assumed to be equipped with a single antenna at both transmitter and receiver sides, there exists an equivalence between MIMO channels and frequency selective fading channels. Thus, some results on precoding derived for MIMO systems are first reviewed as a background for linear precoding. The equivalence between MIMO channels and frequency selective fading channels will be also explained later while deriving particular precoding techniques.

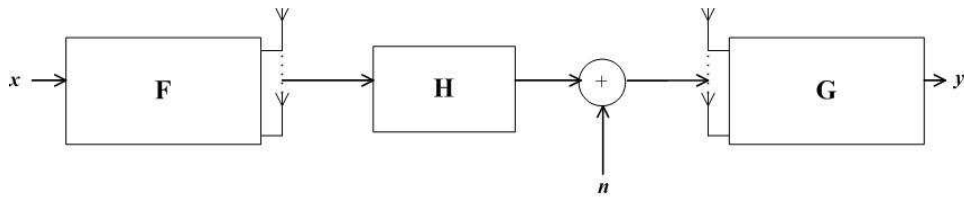

Figure 2.1: Diagram of precoding in a MIMO system.

Consider precoding for a single-user MIMO wireless system on a flat fading channel, where the transmitter and receiver are equipped with M_t and M_r antennas, respectively. The diagram of precoding in a MIMO system is shown in Figure 2.1. The signal model of the communication system with precoding can be written as [20]

$$\mathbf{y} = \mathbf{GHFx} + \mathbf{Gn} \tag{2.5}$$

where $\mathbf{H}$ is the $M_r \times M_t$ channel matrix with its (i, j)th element being the channel gain from the jth transmit antenna to the ith receiver antenna, $\mathbf{y}$ is the $D \times 1$ vector of the received symbols with $D = rank(\mathbf{H})$ being the number of data streams to be transmitted on the MIMO channel, $\mathbf{F}$ is the $M_t \times D$ precoding matrix, $\mathbf{G}$ is the $D \times M_r$ decoding matrix, $\mathbf{x}$ is the $D \times 1$ vector of symbols to be transmitted, and $\mathbf{n}$ is the zero-mean noise vector of dimension $M_r \times 1$. It is assumed that the transmitted symbols on different data streams are uncorrelated and the signal and noise are uncorrelated, i.e., $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}$ and $E\{\mathbf{x}\mathbf{n}^H\} = \mathbf{0}$.

The optimal $\mathbf{F}$ and $\mathbf{G}$ have been designed in many works [20], [21], [22], [23]. There are two main results on the optimal structure of $\mathbf{F}$ and $\mathbf{G}$.

First, it has been shown that the optimal $\mathbf{F}$ adopts a diagonal structure in many scenarios. Specifically, assuming that the noise covariance is $\mathbf{R}_n$, the optimal precoding matrix can be written as

$$\mathbf{F} = \mathbf{V}\mathbf{B} \tag{2.6}$$

where $\mathbf{V}$ and $\mathbf{B}$ are obtained from the eigenvalue decomposition of $\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}$. Here $\mathbf{B}$ is the $D \times D$ diagonal matrix constructed from non-zero eigenvalues of $\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}$ and the M_t columns of $\mathbf{V}$ form a basis of the column space of $\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}$. The above structure is proved to be optimal when the transmitter has a power constraint and the design objective is to maximize the mutual information on the MIMO channel or to minimize the weighted mean square symbol estimation error. In fact, the same result also holds for many other precoding designs with objectives expressed as functions of the mean square symbol estimation error or signal-to-noise ratio (SNR). The diagonal structure of the precoding matrix decouples the MIMO channel into eigen-channels and allocates transmission power on these eigen-channels. It is proved that such decoupling of MIMO channel combined with proper power allocation, i.e. water-filling, on the eigen-channels is capacity achieving [24].

Second, it is proved that the Wiener filter, defined as

$$\mathbf{G} = \mathbf{F}^H\mathbf{H}^H(\mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H + \mathbf{R}_n)^{-1} \tag{2.7}$$

is an optimal linear receiver which is capacity-lossless and minimizes the mean square symbol estimation error [21]. It can be seen from (2.7) that the optimal decoding matrix **G** depends on the precoding matrix **F**. Particularly, given the precoding matrix, channel matrix, and noise covariance, the optimal **G** can be found. Thus, only **F**, or equivalently **B**, needs to be optimized in the precoding design problem.

Frequency selective fading channels are similar to MIMO channels in some aspects. Specifically, on a flat fading MIMO channel, each receive antenna picks up a combination of signals simultaneously transmitted from different transmit antennas. Similarly, on a frequency selective fading channel, a single receive antenna receives a combination of signals transmitted at different time instances. Thus, a frequency selective fading channel is mathematically equivalent to a MIMO channel in time domain.

For more details, consider a frequency selective fading channel with channel length L (where channel length is a parameter of frequency selective fading channel which depends on the channel delay spread and the signal symbol duration). Signal blocks of length N are transmitted through this channel. Then, the discrete sampled channel can be written as [25]

$$\mathbf{H} = \begin{pmatrix} h_0 & 0 & 0 & 0 & \dots & 0 \\ h_1 & h_0 & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & h_{L-1} & \dots & h_0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & h_{L-1} & \dots & h_0 \end{pmatrix}_{N\times N} \tag{2.8}$$

where h_l $(l = 0, 1, ...L-1)$ are the sampled channel impulse responses. From the channel matrix (2.8), it can be seen that this frequency selective fading channel is mathematically equivalent to an $N \times N$ MIMO channel. The difference is that on the frequency selective channel the N symbols in a block are transmitted in serial while on the equivalent MIMO channel the N symbols are transmitted from N antennas simultaneously.

There is one more obvious difference between the two cases. Unlike the channel matrix of the MIMO channel, there are only L different non-zero elements in the channel matrix of the frequency selective fading channel. Owing to this feature, the frequency selective fading channel can be divided into flat fading frequency bins (like in OFDM systems) by adding a cyclic prefix (CP) into the signal block.

OFDM is a multi-carrier modulation scheme used in wideband wireless systems to eliminate the ISI. In OFDM systems, the transmitter performs an inverse fast Fourier transform (IFFT) after the signal symbol block passes the precoder and yields

$$\mathbf{s} = \mathbf{D}^H \mathbf{F} \mathbf{x} \tag{2.9}$$

where $\mathbf{x} = [x(1), ..., x(N)]^T$ is the $N \times 1$ symbol block to be transmitted, $\mathbf{F}$ is the $N \times N$ precoding matrix, and $\mathbf{D}^H$ is the $N \times N$ IFFT matrix with its (i, j)th element given as

$$\mathbf{D}(i, j) = \frac{1}{\sqrt{N}} e^{\frac{j2\pi(i-1)(j-1)}{N}}. \tag{2.10}$$

Let us assume that the channel length is L. A CP consisting of the last $L - 1$ symbols of $\mathbf{s}$ is then inserted into $\mathbf{s}$. The resulting vector of dimension $(N+L-1)\times 1$ is $\mathbf{s}' = [s(N - L + 2), ..., s(N), s(1), ..., s(N)]^T$. The symbols in $\mathbf{s}'$ are transmitted through the frequency selective fading channel, and $N+2L-2$ symbols are received at the receiver due to the delay spread of the channel. The receiver picks up the symbols starting from the Lth received symbol to the $(N + L - 1)$th symbol, and forms a received vector $\mathbf{y}'$. Then, $\mathbf{y}'$ can be written as [5]

$$\mathbf{y}' = \mathbf{H}'\mathbf{s}' + \mathbf{n} \tag{2.11}$$

where $\mathbf{n}$ is the additive Gaussian noise and $\mathbf{H}'$ is the channel matrix given by

$$\mathbf{H}' = \begin{pmatrix} h_{L-1} & \dots & h_0 & 0 & 0 & \dots & 0 \\ 0 & h_{L-1} & \dots & h_0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & h_{L-1} & \dots & h_0 & 0 \\ 0 & 0 & \dots & 0 & h_{L-1} & \dots & h_0 \end{pmatrix}_{N\times N+L-1}. \tag{2.12}$$

Because of the CP insertion, the first $L-1$ symbols in $\mathbf{s}'$ are the same as the last $L-1$ symbols. Thus, signal model (2.11) can be rewritten as

$$\mathbf{y}' = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{2.13}$$

where the equivalent channel matrix $\mathbf{H}$ is a circulant matrix. The eigen decomposition of the channel matrix $\mathbf{H}$ can be found as

$$\mathbf{H} = \mathbf{D}^H \boldsymbol{\Omega} \mathbf{D} \tag{2.14}$$

where $\boldsymbol{\Omega}$ is a diagonal matrix with its elements being the sampled frequency responses of the channel.

The fast Fourier transform (FFT) is performed on $\mathbf{y}'$ at the receiver side, followed by the multiplication by the decoding matrix $\mathbf{G}$. As a result, a new vector $\mathbf{y} = \mathbf{G}\mathbf{D}\mathbf{y}'$ is obtained. Thus, the signal model for the entire transmission process is

$$\mathbf{y} = \mathbf{G}\mathbf{D}\mathbf{H}\mathbf{D}^H \mathbf{F}\mathbf{x} + \mathbf{G}\mathbf{D}\mathbf{n}. \tag{2.15}$$

Note that matrix $\mathbf{D}$ has two properties. First, it is a unitary matrix so that $\mathbf{D}\mathbf{D}^H = \mathbf{I}$. Second, the elements of $\mathbf{D}\mathbf{n}$ are uncorrelated zero mean circularly symmetric complex Gaussian (ZMCSCG) if $\mathbf{n}$ is a vector of uncorrelated ZMCSCG noises, i.e., multiplication by $\mathbf{D}$ does not change the property of the noise. Then, equation (2.15) can be rewritten as

$$\mathbf{y} = \mathbf{G}\boldsymbol{\Omega}\mathbf{F}\mathbf{x} + \mathbf{n}' \tag{2.16}$$

where $\mathbf{n}' = \mathbf{G}\mathbf{D}\mathbf{n}$ is the noise vector after the decoder.

Thus, the OFDM scheme decouples the frequency selective fading channel into N flat fading frequency bins. The equivalence between MIMO channels and frequency selective fading channels appears again in the channel diagonalization. The results on precoding design for MIMO channels are, therefore, still valid for OFDM systems on frequency selective fading channels. These results will be used as a basis in this work. However, it will be shown in Chapter 3 that there is a slight difference between the structures of the precoding matrices in these two cases. Moreover, in this work, the objective is to study precoding strategies in multi-user systems, instead of single-user systems. Thus, the channel resource will no longer belong to a single user and will be allocated as a result of user competition, which makes the problem more complex.

2.3 Game theory in multi-user wireless systems

In multi-user wireless systems, all users compete for resources. Operating on the same frequency band, users interfere with each other if they communicate simultaneously. The conflicting objectives of the users make it almost impossible for any user to gain more profit without harming other users. There are many information-theoretic studies on multi-user wireless systems and interference channels (see [26], [27], [28] and the references therein). However, these information-theoretic studies generally focus on finding the rate regions of multi-user systems and do not advise how to actually achieve the best rates for all users simultaneously. It is, however, clear that the performance of multi-user systems must depend on the balance among users during the competition for resources. Moreover, the points in the achievable rate region are not all equivalent, stable, or even feasible if the selfish nature of users is taken into account. In such cases, it is reasonable to assume that all users will compete for the maximum achievable benefits at all times, which may render difficulties in the implementation of any prescribed regulations against the selfishness of users. For example, although an outcome corresponding to the case when one user is forced to sacrifice its performance for the benefits of other users can be theoretically obtained, it is hard to make sure in practice that the sacrificed user will not actually deviate from the strategy which is unfair for him.

For multi-user systems, the resource sharing problem can be investigated from a game-theoretic perspective. Without coordination among users, the existence of stable outcomes, corresponding to the so-called *equilibrium*, can be analyzed. On the other hand, if there is a voluntary cooperation among users, extra benefits for all users can be gained. These benefits can be distributed among all users. In both the non-cooperative and cooperative cases, the efficiency of resource utilization can be boosted and the system stability can be guaranteed.

Focusing on the interactions among players with conflicting objectives, game theory has found applications in various areas of signal processing and communications including OFDM, ad-hoc, and cognitive radio networks [29], [30], [31]. However, the applications of cooperative game theory for multi-user systems and interference channels are quite recent [15], [32], [33]. A popular application of game theory is, for example, the multi-user power allocation problem [34], [35], [36]. A two-user power allocation game on a flat fading channel is investigated in [37]. It is argued that certain points in the utility space of the game (i.e., the information-theoretic rate region of the interference channel) are not achievable from a game-theoretic perspective. It is also shown that the NB solution obtained based on the TDM scheme improves significantly the benefits of all players as compared to the non-cooperative NE solution. The study is extended to the N-player case on a frequency selective channel in [38], where spectral mask constraints are used to limit the transmission power of each user on each frequency bin. The NB solution is derived based on the TDM/FDM scheme. Different from the flat fading channel case, the allocation of frequency bins becomes a major problem on the frequency selective channels. It is shown that at most one frequency bin needs to be shared by any two users. A similar problem is studied in [39].

A more complex game is the power allocation game on the frequency selective channels with total power constraints which limit the total transmission power of each user. It can be proved that the latter problem is non-convex. A water-filling based algorithm is proposed to search for the NB solution in the case of only two users [40]. The proposed algorithm bargains in many different convex subspaces of the original utility space and obtains one NB solution in each subspace. Then the largest of the NB solutions is selected as the final NB solution of the game. However, the complexity of such an algorithm is high even for the two-user case and the algorithm can not be extended to M-user ($M > 2$) games.

A matrix-valued precoding game is analyzed in [41] and [42], focusing on the non-cooperative case. A multi-user frequency selective interference channel is considered and the optimal precoding matrix for each user in the non-cooperative case is derived based on the NE. It is shown that the matrix-valued precoding game can be solved from an equivalent vector-valued non-cooperative power allocation game, and the resulted precoding matrices adopt a diagonal structure similar to (2.6). The existence and uniqueness of the NE is guaranteed when all communication links are uncorrelated, i.e. they are sufficiently far away from each other. The NE is shown to be more efficient when the interference is relatively low as compared to noise.

Similar to the aforementioned work [42], most of the research efforts in the literature deal with the non-cooperative case and aim at finding an NE as an optimal solution. Corresponding non-cooperative games do not coordinate users, and typically, allow for low-complexity and distributed solutions. However, non-cooperative games often lead to quite inefficient results for all users due to the lack of coordination. Different from the aforementioned work on the non-cooperative precoding, the study in this thesis focuses on developing cooperative precoding strategies for multi-user wireless systems using cooperative game theory. In the following sections, the cooperative precoding strategies in multi-user systems will be analyzed based on the NB theory. Part of this thesis has been reported in [43] and [44], and also summarized in [45].

Chapter 3

System Model

Consider an M-user wireless system in which all users transmit on the same wide-band frequency selective fading channel with channel length L. Assuming block transmission with block length N for all users, the general signal model (OFDM modulation not assumed) for the M-user frequency selective fading channel can be then written as

$$\mathbf{y}_i = \mathbf{G}_{ii}\mathbf{H}_{ii}\mathbf{F}_i\mathbf{s}_i + \mathbf{G}_{ii}\sum_{j=1,j\neq i}^{M}\mathbf{H}_{ji}\mathbf{F}_j\mathbf{s}_j + \mathbf{G}_{ii}\mathbf{n}_i \tag{3.1}$$

where $\mathbf{s}_i$ is the $N \times 1$ information symbol block of user i, $\mathbf{F}_i$ is the $N \times N$ precoding matrix of user i, $\mathbf{H}_{ji}$ is the $N \times N$ channel matrix between transmitter j and receiver i, $\mathbf{n}_i$ is the $N \times 1$ additive Gaussian noise vector with covariance $E\{\mathbf{n}_i\mathbf{n}_i^H\} = \sigma_i^2\mathbf{I}$. The information symbols are assumed to be uncorrelated and all have unit-energy, i.e., $E\{\mathbf{s}_i\mathbf{s}_i^H\} = \mathbf{I}$. The information symbols and the noise are also assumed to be uncorrelated, i.e. , $E\{\mathbf{s}_i\mathbf{n}_i^H\} = \mathbf{0}$.

If OFDM modulation is adopted for each user and the block length N is larger than L, the signal model can be rewritten according to equation (2.16) as following

$$\mathbf{y}_i = \mathbf{G}_{ii}\mathbf{\Omega}_{ii}\mathbf{F}_i\mathbf{s}_i + \mathbf{G}_{ii}\tilde{\mathbf{n}}_i \tag{3.2}$$

where $\tilde{\mathbf{n}}_i = \sum_{j=1,j\neq i}^{M}\mathbf{\Omega}_{ji}\mathbf{F}_j\mathbf{s}_j + \mathbf{D}\mathbf{n}_i$ is the $N \times 1$ interference plus noise vector for user i before the decoder, $\mathbf{\Omega}_{ji}$ is the $N \times N$ diagonalized channel matrix from the transmitter of user j to the receiver of user i with its kth element being the sampled frequency response of the kth frequency bin.

It can be seen that both the desired communication channel $\mathbf{H}_{ii}$ and the interference channels $\mathbf{H}_{ji}$ $(j = 1, ...i-1, i+1, ...M)$ are diagonalized for user i due to the CP insertion and the multiplication by matrices $\mathbf{D}^H$ and $\mathbf{D}$ at the transmitter and receiver sides, respectively.

Let us consider the general case in which all users treat the interference as additive noise. Then the noise covariance for user i before the decoder is

$$\mathbf{R}_{-i} = E\{\tilde{\mathbf{n}}_i \tilde{\mathbf{n}}_i^H\} = \sigma_i^2 \mathbf{I} + \sum_{j=1, j\neq i}^{M} \mathbf{\Omega}_{ji} \mathbf{F}_j \mathbf{F}_j^H (\mathbf{\Omega}_{ji})^H. \tag{3.3}$$

As mentioned in the previous chapter, the Wiener filter is an optimal capacity-lossless linear receiver. Thus, the decoding matrix can be found using the Wiener filter

$$\mathbf{G}_i = \mathbf{F}_i^H \mathbf{H}_{ii}^H (\mathbf{H}_{ii} \mathbf{F}_i \mathbf{F}_i^H \mathbf{H}_{ii}^H + \mathbf{R}_{-i})^{-1}. \tag{3.4}$$

Since the Wiener filter $\mathbf{G}_i$ does not affect the capacity of the channel, the maximum information rate that user i can achieve can be calculated before the decoder and be written as [24]

$$R_i = \log\left(det\left(\mathbf{I} + \mathbf{F}_i^H (\mathbf{\Omega}_{ii})^H \mathbf{R}_{-i}^{-1} \mathbf{\Omega}_{ii} \mathbf{F}_i\right)\right). \tag{3.5}$$

Considering the wireless users as players of the precoding game, the choices of precoding matrices as players' strategies, and the corresponding information rates of users $R_i's$ as players' payoffs, the game model of the precoding problem can be written as

$$\Gamma = \Big\{\Omega = \{1, 2, ...M\}, \{\mathbf{F}_i | i \in \Omega\}, \{R_i | i \in \Omega\}\Big\}. \tag{3.6}$$

In practice, all users attempt to maximize their information rates under certain power constrains. For the case of frequency selective fading channels, spectral mask constraints are usually considered to limit the power that user i can allocate on each frequency bin. These powers are denoted as $p_i^{\max}(k)$ $(\forall i \in \Omega, \forall k \in \{1, 2, ..., N\})$. Without loss of generality, $p_i^{\max}(k)$ $(\forall i \in \Omega)$ are assumed to be identical for all users and equal to $p_{\max}(k)$. Although spectral mask constraints actually bound the total available power for the users, i.e. $\sum_k p_{\max}(k)$, this bound might be loose. Thus, sometimes a total power constraint is also needed on the top of the spectral mask constraints.

The aforementioned two power constraints for the users can be mathematically written as follows [42].

Constraint 1 on spectral mask:

$$E\{|[\mathbf{F}_i\mathbf{s}_i]_k|^2\} = [\mathbf{F}_i\mathbf{F}_i^H]_{kk} \leq p_{\max}(k). \tag{3.7}$$

Constraint 2 on total transmission power $P_i^{\max}$:

$$E\{\|\mathbf{F}_i\mathbf{s}_i\|^2\} = Tr\{\mathbf{F}_i\mathbf{F}_i^H\} \leq P_i^{\max}. \tag{3.8}$$

The sets of precoding matrices which satisfy (3.7) and (3.8) are denoted as $\mathcal{F}^1$ and $\mathcal{F}^2$, respectively.

For further developments, three general assumptions need to be made:

1 . All wireless users treat the interference from other users as noise.

2 . The channel information of the desired channel $\mathbf{H}_{ii}$ is known at both the transmitter and receiver sides of user i.

3 . The total power constraints are tight when they are taken into account, i.e., $P_i^{\max} < \sum_k p_{\max}(k)$.

In the following chapters, the users are assumed to cooperate with each other in the precoding games based on different manners of cooperation, i.e., TDM/FDM or FDM/time sharing. These two manners of cooperation have one common ground that they are both orthogonal signaling schemes. The main reason for considering orthogonal signaling is that the capacity region of a general communication channel on which users interfere with each other is not derived yet in the literature. Moreover, the users may need to exchange the interference information to achieve a desirable performance if they are allowed to interfere with each other. This will add to the overhead in the communication system and will significantly complicate the transmitter and receiver designs. Therefore, orthogonal signaling, which is simple and practical, is a reasonable choice for users to perform cooperation.

The following proposition regarding the structure of the precoding matrices provides the basis for the discussion in the next two chapters.

Proposition 3.1: The precoding matrix for each user adopts a strictly diagonal structure $\mathbf{F}_i = \mathbf{\Lambda}_i$ in the precoding games under constraints 1 and/or 2 if the cooperation among users is based on orthogonal signaling.

Proof: The noise covariance in (3.3) is equivalent to $\mathbf{R}_{-i} = \sigma_i^2\mathbf{I}$ when the cooperation among users is based on orthogonal signaling. Thus, R_i is simplified to

$$R_i = \log\left(det\left(\mathbf{I} + \frac{1}{\sigma_i^2}\mathbf{F}_i^H(\mathbf{\Omega}_{ii})^H\mathbf{\Omega}_{ii}\mathbf{F}_i\right)\right). \tag{3.9}$$

The Hadamard's inequality (which is used in [42] to prove the optimality of a similar structure for precoding matrices in the non-cooperative games) suggests that the determinant in (3.9) is maximized when $\mathbf{F}_i$ is diagonal.

Moreover, the power constraints given in (3.7) and (3.8) are irrelevant to the non-diagonal elements of $\mathbf{F}_i$. Therefore, the optimal structure for precoding matrices is the diagonal structure. ■

Note that the structure suggested by Proposition 3.1 is different from the multiplication of a constant non-diagonal matrix and a diagonal matrix for precoding in MIMO channels. Recall that precoding on MIMO channels adopts a diagonal structure $\mathbf{F} = \mathbf{VB}$, as given in (2.6). Then, Proposition 3.1 states that precoding in OFDM systems on frequency selective fading channels does not have the non-diagonal multiplier $\mathbf{V}$ under constraints 1 and 2. The fundamental reason is that precoding contributes to the channel diagonalization on MIMO channels. Indeed, for the case of MIMO channel, the non-diagonal multiplier $\mathbf{V}$ in the precoding matrix is derived from the singular value decomposition (SVD) of the channel and is used to diagonalize the channel, while the diagonal matrix $\mathbf{B}$ is used to allocate transmission resources. On a frequency selective fading channel, the channel diagonalization is realized by the CP insertion, IFFT and FFT operations. Thus, there is no need to have a non-diagonal multiplier in precoding matrices. Therefore, precoding in OFDM systems on the frequency selective fading channel based on orthogonal signaling is equivalent to the allocation of transmission resources, such as power allocation and channel allocation.

In Chapter 4, precoding strategies under spectral mask constraints will be investigated and the NB solution will be derived. In Chapter 5, both total power constraints and spectral mask constraints will be considered and sub-optimal solutions will be studied instead of the optimal solutions, which have unacceptably high complexity.

Chapter 4

Precoding Games with Spectral Mask Constraints

4.1 Cooperative precoding strategies: two-user game

We study the precoding games with spectral mask constraints in this chapter, beginning from a simplified two-user game [1]. First consider the disagreement point of the two-user cooperative game. Particularly, the NE of the non-cooperative game is a typical choice of the disagreement point. It is straightforward to see that the NE solution of the two-user game with spectral mask constraints is

$$\mathbf{F}_i^{\mathrm{NE}} = \sqrt{diag(\mathbf{p}_{\max})}, \quad i = 1, 2 \tag{4.1}$$

where $\mathbf{p}_{\max} = [p_{\max}(1), ..., p_{\max}(N)]$ and $p_{\max}(k)$ $(\forall k)$ are the spectral mask constraints. It shows that each user exploits maximum allowable power on all frequency bins to maximize its rate. In fact $\{\mathbf{F}_1^{\mathrm{NE}}, \mathbf{F}_2^{\mathrm{NE}}\}$ constitutes a DSE here.

After the disagreement point is fixed, the manner for cooperation between users needs to be specified. A joint TDM/FDM approach is one of the desirable choices.

[1] A version of this chapter has been published in proceedings of the IEEE Global Telecommunications Conference 2008 (GLOBECOM'08) [43]. Related results and further extensions have been published in proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing 2009 (ICASSP'09) [44].

Although the TDM/FDM rate region is not the capacity region of the interference channel, it is convex and can be implemented with low complexity. Joint TDM/FDM prescribes that any frequency bin can be used by only one user at any time instant but it may be shared by different users throughout the operation time.

The following proposition on the structure of the NB solution for precoding matrices in the TDM/FDM cooperative game is in order.

Proposition 4.1: The NB solution for precoding strategies for the two-player TDM/FDM cooperative precoding game on the frequency selective fading channel can be obtained through time sharing of at most two sets of diagonal precoding matrices denoted as $\{\mathbf{F}_1^1, \mathbf{F}_2^1\}$ and $\{\mathbf{F}_1^2, \mathbf{F}_2^2\}$. The following conditions should be satisfied

$$\begin{aligned} &\mathbf{F}_1^l + \mathbf{F}_2^l = \sqrt{diag(\mathbf{p}_{\max})} \\ &\mathbf{F}_1^l \mathbf{F}_2^l = \mathbf{0} \\ &|Tr\{\mathbf{F}_1^1 - \mathbf{F}_1^2\}|^2 = |Tr\{\mathbf{F}_2^1 - \mathbf{F}_2^2\}|^2 \in \mathfrak{P} \end{aligned} \tag{4.2}$$

where $l \in \{1, 2\}$ and $\mathfrak{P} = \{p_{\max}(1), p_{\max}(2), ..., p_{\max}(N)\}$ is the set of spectral mask constraints on the frequency bins.

Proof: The diagonal structure of $\mathbf{F}_1^l$ and $\mathbf{F}_2^l$ $(l \in \{1, 2\})$ follows from Proposition 3.1. The three conditions in (4.2) are based on the TDM/FDM assumption. First consider the FDM part. Given any division of the frequency bins, both users will use maximum allowable power on all frequency bins allocated to them. Thus, the first condition is derived. The second condition is based on the fact that only one user is allowed on any given frequency bin at any time. Thus, there must be one user which allocates zero power on any given frequency bin at any time. The third condition is based on the fact that the NB solution can be achieved by sharing at most a single frequency bin between the two users, which has been also observed in [38].

The proof can be given by contradiction using the optimality of the NB solution. Assume that the NB solution can be achieved only by sharing two or more frequency bins between the two users and consider the case when two frequency bins m and n are shared. In the sharing scheme, user 2 uses a fraction α_1 of the time in frequency bin m and a fraction α_2 of the time in frequency bin n. Let $R_i(m)$ and $R_i(n)$ be the rates that user i can obtain by exclusively using frequency bins m and n, respectively. Without loss of generality, we assume that $R_2(m)/R_1(m) \geq R_2(n)/R_1(n)$. Then either of the following cases must happen:

i) If $\alpha_1 R_2(m)+\alpha_2 R_2(n) \geq R_2(m)$, there exists $\gamma \in [0, 1)$ such that $\alpha_1 R_2(m)+\alpha_2 R_2(n) = R_2(m) + \gamma R_2(n)$;

ii) If $\alpha_1 R_2(m)+\alpha_2 R_2(n) < R_2(m)$, there exists $\gamma \in [0, 1)$ such that $\alpha_1 R_2(m)+\alpha_2 R_2(n) = \gamma R_2(m)$.

Case (i) corresponds to a new sharing scheme according to which only frequency bin n is shared between both users, and user 2 exploits a fraction γ of time on frequency bin n. In the new scheme, user 2 obtains the same rate on frequency bins m and n as that in the original scheme. Then the rate that user 1 can obtain on frequency bins m and n in the new scheme is

$$\begin{aligned}(1-\gamma)R_1(n) &= \left(1 - \frac{(\alpha_1 - 1)R_2(m) + \alpha_2 R_2(n)}{R_2(n)}\right) R_1(n) \\ &= (1-\alpha_1)\frac{R_1(n)R_2(m)}{R_2(n)} + (1-\alpha_2)R_1(n) \\ &\geq (1-\alpha_1)R_1(m) + (1-\alpha_2)R_1(n). \qquad (4.3)\end{aligned}$$

The last inequality follows from the assumption that $R_2(m)/R_1(m) \geq R_2(n)/R_1(n)$. It can be seen from (4.3) that the rate that user 1 can obtain in the new scheme is equal to or larger than that in the original scheme. This contradicts the assumption that the NB solution can be achieved only by sharing two frequency bins between both users.

A similar result can be derived in case (ii). Moreover, for the case that more than two frequency bins are shared, the above proof can be used iteratively to obtain the same result.

These results show that the optimal solution can be obtained by sharing at most a single frequency bin between the users. Thus, $\mathbf{F}_i^1, i \in \{1, 2\}$ can be obtained by adding/deleting a single diagonal element of $\mathbf{F}_i^2, i \in \{1, 2\}$ and the third condition in (4.2) follows. ■

Proposition 4.1 states that the TDM/FDM based cooperation on N frequency bins can be realized by the time sharing of two diagonal precoding matrices under spectral mask constraints. From the proof of Proposition 4.1, it can be seen that only one frequency bin needs to be shared. Denote this frequency bin as $k^\star$ and assume that $\mathbf{F}_1^1(k^\star, k^\star) = \sqrt{p_{\max}(k^\star)}$ (therefore $\mathbf{F}_1^2(k^\star, k^\star) = \mathbf{F}_2^1(k^\star, k^\star) = 0$ and $\mathbf{F}_2^2(k^\star, k^\star) = \sqrt{p_{\max}(k^\star)}$). Assuming that user 1 shares frequency bin $k^\star$ for α portion of time ($0 \leq \alpha \leq 1$), the information rate of user i ($i = 1, 2$) can be written as

$$\begin{aligned} R_i &= \alpha \log\left(det\left(\mathbf{I} + \frac{1}{\sigma_i^2}{\mathbf{F}_i^1}^H (\mathbf{\Omega}_{ii})^H \mathbf{\Omega}_{ii} \mathbf{F}_i^1\right)\right) \\ &+ (1-\alpha)\log\left(det\left(\mathbf{I} + \frac{1}{\sigma_i^2}{\mathbf{F}_i^2}^H (\mathbf{\Omega}_{ii})^H \mathbf{\Omega}_{ii} \mathbf{F}_i^2\right)\right). \end{aligned} \tag{4.4}$$

The problem is now converted to finding $k^\star$ and α which maximize the Nash function (2.4). The problem can be simplified as follows. The information rate for user i given in (4.4) is the summation of user i's information rates on all frequency bins. Denoting the set of frequency bins allocated to user i exclusively as $\mathcal{B}_i$, the sum rate can be rewritten as

$$R_i = \left((i-1) + (-1)^{i+1}\alpha\right) R_i(k^\star) + \sum_{m \in \mathcal{B}_i} R_i(m), \quad \forall i \tag{4.5}$$

where

$$R_i(m) = \log(1 + |\mathbf{\Omega}_{ii}(m)\mathbf{F}_i(m)|^2/\sigma_i^2), \quad \forall i \tag{4.6}$$

is the information rate that user i can obtain on frequency bin m by using precoding strategy $\mathbf{F}_i$. Note that (4.5) and (4.6) can be further rewritten as

$$R_i = \sum_{k=1}^{N} \alpha_i^k R_i(k), \quad \forall i \tag{4.7}$$

where

$$R_i(k) = \log(1 + |\mathbf{\Omega}_{ii}(k)|^2 p_{\max}(k)/\sigma_i^2), \quad \forall i \tag{4.8}$$

is the information rate that user i can obtain on frequency bin k by using it exclusively for all the times, and α_i^k is the time portion during which the frequency bin k

is allocated to user i. Then the NB solution can be found from solving the following convex optimization problem

$$\begin{aligned}
\max_{\{\alpha_i^k\},\forall i,\forall k} \quad & \log(R_1 - R_1^{\mathrm{NE}}) + \log(R_2 - R_2^{\mathrm{NE}}) \\
\textbf{subject to}: \quad & 0 \le \alpha_i^k \le 1,\ \forall i \in \{1,2\}, \forall k \in \{1,2,...,N\} \\
& \alpha_1^k + \alpha_2^k \le 1,\ \forall k \\
& R_i > R_i^{\mathrm{NE}},\ \forall i
\end{aligned} \tag{4.9}$$

where R_i^{NE} is the utility that user i can obtain in the non-cooperative game.

Note that the objective function of problem (4.9) is derived by taking logarithm of the NF function. The property of the log function that it does not change the maximum of a function to which it applies is used. The third constraint in (4.9) guarantees that both users can achieve higher rates through the TDM/FDM based cooperation. Otherwise the users resort to the disagreement point and the cooperation breaks up. Note that in the solution of the above problem $0 < \alpha_i^k < 1$ only for $k = k^\star$.

4.2 Cooperative precoding strategies: an extension to M-user game

The TDM/FDM scheme is still assumed here as the cooperation scheme for the M-user game. Unlike the two-user case, where the NB solution of the precoding game can be simply formulated as a time sharing between two sets of precoding matrices, it is much more complex to coordinate the users' precoding matrices in the M-user game. The structure used for the two-user game can hardly be applied here, especially when the number of users is large. To solve this problem, we first partition the time into time slots each of length T to make it easier for the users to perform time sharing. Moreover, considering the case when the number of users or the channel states change over time, the partitioning of time slots enables a timely update of the bargaining solution if time slots are small enough.

The cooperative solution in this case can be obtained through the following steps:

- *Step 1* -Initialization: users are in non-cooperative state and the NE solution for precoding matrices is obtained.
- *Step 2* -Computation: the cooperative NB solution for the precoding matrices is calculated.
- *Step 3* -Implementation: Implement the NB solution for one time slot. If any changes of the number of users or the channel states are detected during this time slot, go back to step 1 in the next slot; otherwise, repeat step 3.

As an extension of Proposition 4.1, the following proposition is in order.

Proposition 4.2: Precoding matrices corresponding to the NB solution of the M-player TDM/FDM cooperative game on the frequency selective fading channel have the form

$$\mathbf{F}_i = \mathbf{\Gamma}_i(t)\sqrt{diag(\mathbf{p}_{\max})}, \quad \forall i \in \{1, 2, ..., M\} \tag{4.10}$$

where $\mathbf{\Gamma}_i(t)$ is a diagonal matrix with its kth diagonal element

$$\mathbf{\Gamma}_i^k(t) = \begin{cases} 1, & \text{if } t \in [b_i^k, e_i^k] \\ 0, & \text{if } t \notin [b_i^k, e_i^k] \end{cases}, \quad \forall i \tag{4.11}$$

with b_i^k and e_i^k representing, respectively, the starting and ending time moments between which frequency bin k is allocated to user i in a time slot $[0, T]$. The following conditions are then satisfied

$$\begin{gathered} \sum_i \mathbf{\Gamma}_i(t) = \mathbf{I}, \quad \forall t \in [0, T] \\ \mathbf{\Gamma}_i(t)\mathbf{\Gamma}_j(t) = \mathbf{0}, \quad \forall i, \forall j \neq i, \ \forall t \in [0, T] \end{gathered} \tag{4.12}$$

where $t \in [0, T]$ is the time instant in a current slot.

The proof is similar to the proof of Proposition 4.1 and is omitted here. As compared to the three conditions for precoding in the two-user game given by (4.2), we find that there are only two conditions required in this problem. The reason is that the sharing of frequency bins among multiple users is more complex. Unlike the two-user game in which at most one frequency bin needs to be shared, different groups of users may share different frequency bins in the M-user game. Thus, the third condition in (4.2) no longer holds for M-user game. However, the other two conditions for the two-user game are inherited here. The first condition in (4.12) states that no frequency bin should be vacant at any time, while the second one requests that no frequency bin be used by more than one user at any time.

It is the length of $[b_i^k, e_i^k]$, denoted as $\alpha_i^k = e_i^k - b_i^k$, rather than the specific values of b_i^k and e_i^k, that affects the payoffs of the users. Once the time portions α_i^k are fixed, the order of using frequency bins is not important to the users. Thus, the key problem is to calculate the fractions of time α_i^k $(\forall i, \forall k)$ that user i obtains on a frequency bin k. Mathematically, this problem can be formulated as the following optimization problem

$$\begin{aligned} \max_{\{\alpha_i^k\}} \quad & \prod_i (R_i - R_i^{\mathrm{NE}}) \\ \text{s.t.} \quad & 0 \le \alpha_i^k \le 1, \ \forall i, \ \forall k, \\ & \sum_i \alpha_i^k \le 1, \ \forall k, \\ & R_i > R_i^{\mathrm{NE}}, \ \forall i \end{aligned} \tag{4.13}$$

where

$$R_i = \sum_k \log(1 + |\mathbf{\Omega}_{ii}(k)\mathbf{F}_{ii}(k)|^2/\sigma^2) = \sum_k \alpha_i^k \log(1 + |\mathbf{\Omega}_{ii}(k)|^2 \mathbf{p}_{\max}(k)/\sigma^2) \tag{4.14}$$

is the sum of information rates that user i can obtain on all frequency bins.

To avoid a centralized channel estimation and information exchange overhead among users on the cooperation stage, we next develop a distributed algorithm for solving (4.13). In Chapter 6 we will show by simulations that the algorithm converges to the optimal solution.

4.3 Distributed algorithm for finding the NB solution

From practical point of view, it is preferable to decompose the original problem (4.13) and solve it in a distributed manner. To achieve that, first note that the problem (4.13) can be rewritten as

$$\begin{aligned}
\max_{\{\alpha_i^k\}} \quad & \sum_i \log(R_i - R_i^{\text{NE}}) \\
\textbf{s.t.} \quad & 0 \le \alpha_i^k \le 1, \ \forall i, \ \forall k, \\
& \sum_i \alpha_i^k \le 1, \ \forall k, \\
& R_i > R_i^{\text{NE}}, \ \forall i
\end{aligned} \tag{4.15}$$

which is a convex optimization problem with a coupling constraint. Therefore, it can be solved through dual decomposition.

The Lagrange dual problem to the problem (4.15) is given as

$$\begin{aligned}
\max_{\{\alpha_i^k\}} \quad & \sum_i \log(R_i - R_i^{\text{NE}}) - \sum_k \lambda^k (\sum_i \alpha_i^k - 1) \\
\textbf{s.t.} \quad & 0 \le \alpha_i^k \le 1, \ \forall i, \ \forall k, \\
& R_i > R_i^{\text{NE}}, \ \forall i, \\
& \lambda^k \ge 0, \ \forall k.
\end{aligned} \tag{4.16}$$

The problem (4.16) can be further converted to a two-level optimization problem with the lower level subproblems given as

$$\begin{aligned}
\max_{\{\alpha_i^k\}} \quad & \log(R_i - R_i^{\text{NE}}) - \sum_k \lambda^k \alpha_i^k \\
\textbf{s.t} \quad & 0 \le \alpha_i^k \le 1, \ \forall k, \\
& R_i > R_i^{\text{NE}}
\end{aligned} \tag{4.17}$$

for each user $i \in \{1, 2, ..., M\}$, and the higher level master problem given as

$$\begin{aligned}
\min_{\{\lambda^k\}} \quad & \sum_i U_i(\boldsymbol{\lambda}) + \sum_k \lambda^k \\
\textbf{s.t.} \quad & \lambda^k \ge 0, \ \forall k
\end{aligned} \tag{4.18}$$

where $U_i(\boldsymbol{\lambda})$ is the maximum value of the objective function in (4.17) given $\boldsymbol{\lambda} = [\lambda^1, \lambda^2, ..., \lambda^N]$.

The dual problem (4.17) and (4.18) can be solved based on a distributed structure with a coordinator. Since the original problem is convex, the strong duality holds and the solutions of the dual problem (4.16) and the original problem (4.15) are the same if Slater's condition is satisfied [46]. For our specific problem, we have the following proposition.

Proposition 4.3: The Slater's condition is guaranteed to be satisfied as long as the NB solution exists.

Proof: Since the constraints of the problem (4.15) are all linear, the Slater's condition reduces to two parts with the first part requiring that the feasible domain of $f = \sum_i \log(R_i - R_i^{\text{NE}})$ be open and the second part requiring that the feasible domain of the whole problem be non-empty.

It is straightforward to verify that the first part is satisfied. The second part is equivalent to the requirement of the existence of the NB solution. Thus, the Slater's condition is guaranteed to be satisfied if the NB solution exists. ■

Note that Proposition 4.2 can be used to further simplify the dual problem (4.17). Substituting (4.10) into the objective function of the sub-problem (4.17) and assuming, for simplicity but without loss of generality, that T=1, the lower level subproblems can be rewritten as

$$\begin{aligned} \max_{\alpha_i^k} \quad & \log(\sum_{k=1}^{N} \alpha_i^k R_i^k - R_i^{\text{NE}}) - \sum_k \lambda^k \alpha_i^k \\ \text{s.t.} \quad & 0 \leq \alpha_i^k \leq 1, \ \forall k, \\ & \sum_{k=1}^{N} \alpha_i^k R_i^k > R_i^{\text{NE}} \end{aligned} \tag{4.19}$$

where $R_i^k = \log(1 + |\boldsymbol{\Omega}_{ii}(k)|^2 p_i^{\max}(k)/\sigma^2)$ is the rate on frequency bin k for user i.

The lower level subproblems are solved distributively by the corresponding users. The Hessian of the objective function $f_i = \log\left(\sum\limits_{k=1}^{N} \alpha_i^k R_i^k - R_i^{\text{NE}}\right) - \sum\limits_k \lambda^k \alpha_i^k$

Table 4.1: Dual decomposition algorithm for NB.

1. The coordinator initializes $\boldsymbol{\lambda}$=$\boldsymbol{\lambda}^0$=$[\lambda_1^0, \lambda_2^0, ..., \lambda_N^0]$ and broadcasts it to all users.
2. Each user solves (4.19) according to the present value of $\boldsymbol{\lambda}$ and transmits its solutions for $\alpha_i^k, k \in \{1, .., N\}$ to the coordinator.
3. The coordinator updates $\boldsymbol{\lambda}$ according to the gradient of the master problem (4.18) as $\hat{\lambda}^k = [\lambda^k - \delta(1 - \sum_i \alpha_i^k)]_+ \ (\forall k)$.
4. If $\lvert\hat{\lambda}^k - \lambda^k\rvert \leq \xi \ (\forall k)$, stop; otherwise broadcast $\hat{\boldsymbol{\lambda}}$ and go to step 2.

can be written as

$$\nabla^2 f_i = -\frac{1}{\left(\sum_{k=1}^{N} \alpha_i^k R_i^k - R_i^{\mathrm{NE}}\right)^2} \mathbf{r}\mathbf{r}^T \tag{4.20}$$

where $\mathbf{r} = [R_1^1, ..., R_1^N, ..., R_i^1, ..., R_i^N, ..., R_M^1, ..., R_M^N]^T$. It is straightforward to see that $\nabla^2 f_i$ is negative definite since $R_i^k > 0(\forall i, \forall k)$. Thus, each Lagrange problem (4.19) is guaranteed to be strictly convex and a unique solution exists. More importantly, the information required for solving the ith subproblem, i.e., R_i^k and R_i^{NE}, is local to user i.

A coordinator is needed to solve the higher level master problem. Since the overhead of the information exchange and computation is not significant, one user may be, for example, selected as the coordinator, or all users may perform the functions of the coordinator in a round-robin manner.

The complete process of solving the dual problem is summarized in the implementation algorithm shown in Table 4.1, where δ and ξ are the step length and the stopping threshold, respectively. The complexity of finding the bargaining solution using the proposed algorithm is $O(N^3)$, which is determined by solving the lower level subproblems (4.17).

Note that the coefficients λ_k $(k = 1, 2, ..., N)$ have specific physical meanings. Indeed, the coefficient λ_k represents the risk that cooperation among users breaks up due to a conflict on sharing frequency bin k. Thus, in the lower level subproblems, the objective for each user consists of two parts. Taking user i as an example, it can be described as follows. On one hand, a larger α_i^k is preferred to increase the total information rate of user i. On the other hand, if α_i^k becomes too large, the cooperation may break up and the payoff of user i will return to the inferior competitive solution.

Chapter 5

Precoding Games with Total Power and Spectral Mask Constraints

5.1 Bandwidth-dominant and power-dominant systems

The games investigated in the preceding chapter consider spectral mask constraints only. Thus, the diagonal elements of the precoding matrices $\mathbf{F}_i(\forall i)$ satisfy $\mathbf{F}_i(k,k) = \sqrt{p_{\max}(k)}$ when frequency bin k is allocated to user i and $\mathbf{F}_i(k,k) = 0$ otherwise. The corresponding optimization problem for finding the NB solution for the precoding matrices/strategies is convex due to the inherent structure of $\mathbf{F}_i$ and can be transformed to the problem of finding coefficients $\alpha_i^k(\forall i, \forall k)$. The objective of this chapter is to investigate the precoding strategies with both spectral mask and total power constraints in the cooperative games. First, it can be shown that the problem of finding the NB solution for the precoding matrices/strategies based on TDM/FDM cooperation is non-convex in this case. The optimization problem can be formulated then by modifying (4.15) as

$$\begin{aligned}
\max_{\{\alpha_i^k, \mathbf{p}_i\}} \quad & \sum_i \log(R_i - R_i^{\mathrm{NE}}) \\
\mathbf{s.t.} \quad & 0 \le \alpha_i^k \le 1, \ \forall i, \ \forall k, \\
& \sum_i \alpha_i^k \le 1, \ \forall k, \\
& \sum_k \alpha_i^k \mathbf{p}_i(k) \le P_i^{\max}, \forall i \\
& R_i > R_i^{\mathrm{NE}}, \ \forall i
\end{aligned} \tag{5.1}$$

where

$$R_i = \sum_k \log(1 + |\mathbf{\Omega}_{ii}(k)\mathbf{F}_{ii}(k)|^2/\sigma_i^2) = \sum_k \alpha_i^k \log(1 + |\mathbf{\Omega}_{ii}(k)|^2 \mathbf{p}_i(k)/\sigma_i^2) \tag{5.2}$$

is the sum information rate that user i can obtain. It is easy to see that the above optimization problem is non-convex. Specifically, the Hessian matrix $\mathbf{H}_{f_i}$ of $f_i(\boldsymbol{\alpha}_i, \mathbf{p}_i) = \sum_k \alpha_i^k \mathbf{p}_i(k)$ can be written as

$$\mathbf{H}_{f_i} = \nabla^2 f_i(\boldsymbol{\alpha}_i, \mathbf{p}_i) = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix} \tag{5.3}$$

where $\boldsymbol{\alpha}_i = [\alpha_i^1, ..., \alpha_i^N]$. Thus, the Hessian matrices $\mathbf{H}_{f_i}$ of $f_i(\boldsymbol{\alpha}_i, \mathbf{p}_i)$ $(\forall i)$ are orthogonal matrices, i.e., they satisfy $\mathbf{H}_{f_i}\mathbf{H}_{f_i}^T = \mathbf{I}\,(\forall i)$. The eigenvalues of orthogonal matrices can only be 1 or -1. Moreover, it is known that the summation of all eigenvalues of $\mathbf{H}_{f_i}$ equals $Tr\{\mathbf{H}_{f_i}\}$, which is zero for any i. Therefore, $\mathbf{H}_{f_i}$ $(\forall i)$ must have both eigenvalues 1 and -1, and the number of eigenvalues 1 is equal to the number of eigenvalues -1. Thus, $\mathbf{H}_{f_i}$ $(\forall i)$ are indefinite and the constraints $\sum_k \alpha_i^k \mathbf{p}_i(k) \le P_i^{\max}$ $(\forall i)$ are non-convex. The non-convexity of the optimization problem 5.1 follows.

The third constraint in (5.1) is the total power constraint. Unlike the case in (4.13), power allocation vectors $\mathbf{p}_i$ $(\forall i)$ are also optimization variables to be found here. However, $\mathbf{F}_i$ is still diagonal according to Proposition 3.1 and can be written as

$$\mathbf{F}_i = diag(\sqrt{\mathbf{p}_i}). \tag{5.4}$$

Thus, the problem of finding $\mathbf{p}_i\ (\forall i)$, which is equivalent to finding $\mathbf{F}_i\ (\forall i)$, is considered below.

For simplicity, the two-user case is considered throughout this chapter, and the disagreement point is chosen at the origin of the utility space instead of the NE point because finding the NE solution, in this case, is itself a complicated problem currently under research.

To find the optimal solution of the corresponding precoding game, each user should consider both the power allocation and the frequency bin allocation in (5.1). The complexity of solving the problem(5.1) is high even for the simple two-user case, especially when the number of frequency bins N is large. Therefore, it is necessary to resort to sub-optimal solutions. Moreover, the total power constraints for all users will render the TDM/FDM based cooperation inefficient for some system configurations while in other system configurations efficient results can still be obtained using TDM/FDM. Thus, this thesis suggests to categorize wireless systems into two classes according to their bottleneck resources, which can be the available bandwidth or power. Such a classification allows us to develop different sub-optimal precoding strategies for each class of systems. Such sub-optimal strategies will guarantee that the sub-optimal solutions are efficient or even the same as the optimal solutions, while at the same time the corresponding algorithm for finding such solutions has low complexity.

In order to do the classification, two definitions need to be given first.

Definition 5.1. Pareto-optimality. A point $\mathbf{x}$ (indexed by its elements) is Pareto-optimal, or equivalently, Pareto-efficient in a space $\mathcal{S}$ if and only if $\mathbf{y} = \mathbf{x}$ for all $\mathbf{y}$ satisfying $\mathbf{y} \succeq \mathbf{x}$ in $\mathcal{S}$.

The NB solution is one of the Pareto-optimal points in the utility space. For the two-user cooperative precoding game, there is a well known algorithm for obtaining the TDM/FDM based NB solution if only spectral mask constraints are present [37], [39]. According to this algorithm, one first arranges the frequency bins such that $R_1(k)/R_2(k) \geq R_1(j)/R_2(j)\ (\forall j, \forall k < j)$, where $R_i(k)$ is the information rate that user i can achieve in frequency bin k by allocating $p_{\text{max}}(k)$ on it (and allowing no interference from other users). Given any integer $\hat{k} \in [1, N]$, let

$$\begin{aligned} \alpha_1^k = 1, \alpha_2^k = 0, p_1(k) = p_{\max}(k), p_2(k) = 0, \forall k < \hat{k} \\ \alpha_1^k = 0, \alpha_2^k = 1, p_2(k) = p_{\max}(k), p_1(k) = 0, \forall k > \hat{k} \\ \alpha_1^{\hat{k}} = \beta, \alpha_2^{\hat{k}} = 1 - \beta, p_1(\hat{k}) = p_{\max}(\hat{k}), p_2(\hat{k}) = p_{\max}(\hat{k}). \end{aligned} \tag{5.5}$$

Then the corresponding result, i.e., the point $\mathbf{R} = [R_1, R_2]$, is guaranteed to be Pareto-optimal in the game's utility space for any $0 \le \beta \le 1$. Varying $\hat{k}$ and β, all Pareto-optimal points can be obtained including the NB solution of the game.

Definition 5.2. Pareto-boundary. The set of all Pareto-optimal points in a convex space $\mathcal{S}$ forms the Pareto-boundary of $\mathcal{S}$.

Thus, the NB solution for the two-user cooperative precoding game with only spectral mask constraints can be found by searching on the Pareto-boundary (instead of the entire utility space) of the game. The above algorithm is based on the principle that frequency bins which are "better" for a certain user should be allocated to this user prior to the other frequency bins which are "inferior".

However, the aforementioned principle may fail and lead to highly inefficient solutions if the total power constraints are also imposed. Consider the following simple example. Assume that there are four frequency bins and $[R_1(k), R_2(k)]$ are $[0.5, 0.1]$ for $k = 1$, $[2, 1]$ for $k = 2$, $[1, 3]$ for $k = 3$, and $[0.3, 1]$ for $k = 4$. Assume also that $\mathbf{p}_{\max} = [1, 1, 1, 1]$ and $P_i = 1.5$ for both users. Then according to the aforementioned principle, we obtain the following resource allocation $\alpha_1^1 = \alpha_2^4 = 1, \alpha_1^2 = \alpha_2^3 = 0.5, \alpha_1^3 = \alpha_1^4 = \alpha_2^1 = \alpha_2^2 = 0$ and $\mathbf{p}_1(1) = \mathbf{p}_1(2) = \mathbf{p}_2(3) = \mathbf{p}_2(4) = 1, \mathbf{p}_1(3) = \mathbf{p}_1(4) = \mathbf{p}_2(1) = \mathbf{p}_2(2) = 0$. Note that the total power constraints $\sum\limits_k \alpha_i^k \mathbf{p}_i(k) \le P_i^{\max}$ $(i = 1, 2)$ are used to derive the TDM/FDM coefficients ($\alpha_1^2 = \frac{P_1 - p_{\max}(1)\alpha_1^1}{p_{\max}(2)} = 0.5$ and $\alpha_2^3 = \frac{P_2 - p_{\max}(4)\alpha_2^4}{p_{\max}(3)} = 0.5$). The resulting utilities are $R_1 = 1.5$ and $R_2 = 2.5$, and $(1.5, 2.5)$ is obviously not Pareto-optimal in the utility space of the game. For example, the strategies according to which frequency bin 2 is allocated to user 1 and frequency bin 3 is allocated to user 2 for the whole time provide higher rates than the allocation performed according to the aforementioned principle. It is because this principle assumes unlimited total power and considers only comparative advantage on each frequency bin, but not the absolute advantage.

Thus, the total power constraints render a different bargaining problem since there is a need to coordinate between the power allocation and the frequency bin allocation. However, the problem can be viewed from another perspective. Particularly, we can first consider the solutions for the bargaining game with only spectral mask constraints (denoted as game $\mathcal{G}1$), and then add the total power constraints to the game (denoted as game $\mathcal{G}2$).

Observation 5.1. The total power constraints do not enlarge the utility space of the game. The Pareto-optimal solutions for $\mathcal{G}1$ are still Pareto-optimal for $\mathcal{G}2$ if they are achievable.

Denote the Pareto-boundary of the TDM/FDM utility space of $\mathcal{G}1$ as $\mathcal{P}$. Then, the following proposition is in order.

Proposition 5.1: Assume that the frequency bins are ordered such that $\frac{R_1(k)}{R_2(k)} \geq \frac{R_1(j)}{R_2(j)}$ if $k < j$. A non-empty subset of $\mathcal{P}$ can be achieved in $\mathcal{G}2$ under constraints on P_i and $\mathbf{p}_i^{\max}$ (note that the spectral mask constraints $\mathbf{p}_i^{\max}$ can be different for the two users here) if and only if there exist $1 \leq \tilde{k} \leq N$ and $0 \leq \tilde{\alpha} \leq 1$ such that

$$\frac{P_1 - \sum_{k=1}^{\tilde{k}-1} p_1^{\max}(k)}{p_1^{\max}(\tilde{k})} \geq \tilde{\alpha} \geq \frac{\sum_{m=\tilde{k}}^{N} p_2^{\max}(k) - P_2}{p_2^{\max}(\tilde{k})}. \tag{5.6}$$

Proof: First note that in $\mathcal{G}1$ any resource allocation scheme satisfying (5.5) will result in a Pareto-optimal point of the utility space, and vice versa. Thus, it is equivalent to prove that (5.6) is the sufficient and necessary condition to satisfy (5.5) under total power constraints. Assuming that (5.6) is satisfied and letting $\hat{k} = \tilde{k}$ and $\beta = \tilde{\alpha}$ in (5.5), the resulting total powers actually used by the two users are $P_1^{'} = \sum_{k=1}^{\tilde{k}-1} p_1^{\max}(k) + \tilde{\alpha} p_1^{\max}(\tilde{k})$ for user 1 and $P_2^{'} = \sum_{k=\tilde{k}+1}^{N} p_2^{\max}(k) + (1-\tilde{\alpha}) p_2^{\max}(\tilde{k})$ for user 2. Using (5.6), it is easy to verify that $P_1 \geq P_1^{'}$ and $P_2 \geq P_2^{'}$. Therefore, the sufficiency is proved. The necessity can be proved similarly using contradiction. ■

Observation 5.2. Given a two-user wireless communication system, L frequency bins, and power constraints for both users, we add C frequency bins into the system. Then, the larger C is, the smaller the chance that a non-empty subset of $\mathcal{P}$ can be achieved in $\mathcal{G}2$.

The above observation follows straightforward from the expression (5.6). Therefore, if we categorize the wireless systems into bandwidth-dominant systems and power-dominant systems according to whether condition (5.6) is satisfied, then Observation 5.2 states that a wireless system gradually changes from bandwidth-dominant (in which the available bandwidth is the bottleneck for improving the performance of the system) to power-dominant (in which the limited power of the users is the bottleneck for improving the performance of the system) as the number of available frequency bins increases.

5.2 Bandwidth-dominant systems: TDM/FDM based bargaining

In wireless communications practice, most systems are bandwidth-dominant. Denote the Pareto-boundary of the TDM/FDM utility space of $\mathcal{G}2$ as $\mathcal{P}_2$. Then for the bandwidth-dominant systems, the bargaining can be restricted in $\mathcal{P}' = \mathcal{P}_2 \cap \mathcal{P}$. The resulted NB solution can be sub-optimal for the cooperative TDM/FDM NB game because the power allocation (which is not the dominant factor for bandwidth-dominant systems) is not optimized jointly. Denote the NB solution of $\mathcal{G}2$ derived by bargaining only in $\mathcal{P}'$ as S'_{NB}, and the optimal NB solution of $\mathcal{G}2$ as S^{opt}_{NB}. Also denote the TDM/FDM utility spaces of $\mathcal{G}1$ and $\mathcal{G}2$ as $\mathcal{U}_1$ and $\mathcal{U}_2$, respectively. Then, the following proposition regarding the optimality of S'_{NB} is in order.

Proposition 5.2: $S'_{NB} = S^{opt}_{NB}$ if $S^{opt}_{NB} \in \mathcal{P}'$. If $S'_{NB} \neq S^{opt}_{NB}$, then $S^{opt}_{NB} \notin \mathcal{P}$ but $S'_{NB} \in \mathcal{P}$, which means that S^{opt}_{NB} is not Pareto-optimal in $\mathcal{U}_1$ but S'_{NB} is Pareto-optimal in $\mathcal{U}_1$.

Proof: The first part of the proposition follows from the *independence on irrelevant alternatives* property of the NB as discussed in Chapter 2. This property states that bargaining in a convex subset which contains the NB solution of the original set results in the same NB solution. Thus, it is clear that if $S'_{NB} \neq S^{opt}_{NB}$, then $S^{opt}_{NB} \notin \mathcal{P}'$. Since $\mathcal{P}'$ is the achievable subset of $\mathcal{P}$ in $\mathcal{G}2$, it is, thus, impossible that $S^{opt}_{NB} \in \mathcal{P}$, but $S^{opt}_{NB} \notin \mathcal{P}'$. Combining these two points, the second part of the proposition is proved. ■

Proposition 5.2 leads to the following two conclusions about the optimality of S'_{NB} in the bandwidth-dominant systems:

1. S'_{NB} can be identical to the optimal TDM/FDM based NB solution.
2. S'_{NB} is guaranteed to be Pareto-optimal in $\mathcal{U}_1$ (which is larger than $\mathcal{U}_2$) even if the optimal NB solution is not Pareto-optimal.

5.3 Power-dominant systems: FDM/sampled time sharing based bargaining

Let us now consider the case of the power-dominant wireless communication systems. The numeric example at the beginning of this chapter is, in fact, an example of a power-dominant system.

In the power-dominant systems, the bandwidth is over-supplied and the frequency bin allocation is no longer the dominant factor in the problem. Instead, the importance of power allocation increases significantly. Thus, power allocation according to (5.5) using maximum power on all allocated frequency bins is not reasonable in such systems.

Observation 5.3. The use of maximum allowable power on all allocated frequency bins generally results in non-optimal solution for $\mathcal{G}2$.

It is straightforward to verify Observation 5.3. Specifically, denote the set of all frequency bins as $\mathcal{B}$, the set of frequency bins which user 1 occupies using maximum allowable power as $\mathcal{B}_1^m$, the set of frequency bins which user 2 occupies using maximum allowable power as $\mathcal{B}_2^m$. Then, the first user may improve its utility by water-filling on $B - \mathcal{B}_2^m$ (instead of using maximum allowable power on $\mathcal{B}_1^m$), while the second user's utility is kept the same. Note that the general term 'water-filling' is used here to represent the meaning of finding the solution of the following convex problem

$$\begin{aligned}
\max_{p_j} \quad & \sum_j \log(1 + \varepsilon_j p_j) \\
\textbf{s.t.} \quad & \sum_j p_j = P, \\
& p_j \leq p_{\max}(j), \ \forall j
\end{aligned} \tag{5.7}$$

which is a single-user multi-channel power allocation problem with constant $\varepsilon_j = \frac{|\mathbf{\Omega}(j)|^2}{\sigma^2}$ being a measure of channel j which depends on the channel gain and channel noise.

Thus, power-dominant games will be played in a different way. First, the manner of cooperation is assumed to be the FDM/time sharing scheme instead of the TDM/FDM scheme. Given any two sets of strategies $\mathbf{s}_x$, $\mathbf{s}_y$ and the corresponding points x and y in the utility space, time sharing can achieve any point in the line section between x and y by playing $\mathbf{s}_x$ for a time portion of ξ and playing $\mathbf{s}_y$ for a time portion of $1 - \xi$ (where $0 \leq \xi \leq 1$). Thus, time-sharing is always used to obtain the *convex hull* of a non-convex utility space.

FDM/time sharing scheme considers time-sharing between points corresponding to different frequency bin allocation schemes, and one frequency bin can be allocated to only one user in any of the two allocation schemes. However, users' power allocation schemes on the frequency bins are changed. The power allocation, which is the dominant problem in the power-dominant systems, is then based on the water-filling procedure. To obtain the optimal solution based on the FDM/time-sharing scheme, water-filling should first be performed for all allocation schemes of the frequency bins between the users, and the resulted 2^N points in the utility space are recorded. Then the time-sharing is used to obtain a minimum convex space containing all these points (where the complexity is $O(4^N)$) and the NB solution can be derived. The associated high complexity is not reasonable for the two-user game. Instead, we consider the FDM/sampled time sharing and find a sub-optimal

Table 5.1: Algorithm for the power-dominant system.

1. User i performs water-filling on $\mathcal{B}$ and obtains a set of frequency bins $\tilde{\mathcal{B}}_i$. Then $\mathcal{B}_c = \tilde{\mathcal{B}}_1 \bigcap \tilde{\mathcal{B}}_2$ is the set of frequency bins under competition.
2. In the first round, user 1 is allocated $\tilde{\mathcal{B}}_1$, and user 2 performs water-filling on $\mathcal{B} - \tilde{\mathcal{B}}_1$. In round j (j goes from 2 to $L = \lvert\mathcal{B}_c\rvert$), user 1 selects $j-1$ frequency bins with smallest channel gains in $\mathcal{B}_c$ (denoted as $\mathcal{B}_s^j$) and performs water-filling on $\mathcal{B} - \mathcal{B}_s^j$. Then user 2 performs water-filling on the other frequency bins. After the Lth round, there will be L points in the utility space.
3. Perform L rounds of the aforementioned step 2 for user 2. Starting from the state that user 2 is allocated $\tilde{\mathcal{B}}_2$, and user 1 performs water-filling on $\mathcal{B} - \tilde{\mathcal{B}}_2$. Obtain another L points in the utility space.
4. Denote the $2L$ points as a set $\mathcal{T}$. Find the Pareto-boundary $\mathcal{P}_{\mathcal{T}}$ of $\mathcal{S}_{\mathcal{T}}$, where $\mathcal{S}_{\mathcal{T}}$ is the minimum convex space containing $\mathcal{T}$.
5. Bargain on $\mathcal{P}_{\mathcal{T}}$ and obtain the solution S'_{NB} which maximizes (2.4) on $\mathcal{P}_{\mathcal{T}}$.

solution which can be derived using the algorithm described in Table 5.1.

Let $\mathcal{WF}^i(\mathcal{X})$ be the water-filling operator for user i on the set of frequency bins $\mathcal{X}$. It returns the maximum rate that user i can obtain by optimizing its power allocation scheme on $\mathcal{X}$. Then, the following proposition is in order.

Proposition 5.3: The difference $|d|$ between the logarithm of the NF for the optimal solution S_{NB}^{opt} based on the FDM/time sharing scheme and the logarithm of the NF for the solution S'_{NB} based on FDM/sampled time sharing obtained using the above algorithm in Table 5.1 is bounded by

$$|d| \leq \min\left(\log\left(\frac{\mathcal{WF}^2(\mathcal{B}_2^{opt2})}{\mathcal{WF}^2(\mathcal{B} - \tilde{\mathcal{B}}_2)}\right), \log\left(\frac{\mathcal{WF}^1(\mathcal{B}_1^{opt1})}{\mathcal{WF}^1(\mathcal{B} - \tilde{\mathcal{B}}_1)}\right)\right) \tag{5.8}$$

where $\mathcal{B}_1^{opt1}$ and $\mathcal{B}_2^{opt2}$ correspond to the frequency bin allocations in the optimal solution as defined in details in the following proof.

Proof: Assume that the optimal solution $\mathbf{R}^{opt} = (\lambda R_1^{opt1}+(1-\lambda)R_1^{opt2}, \lambda R_2^{opt1}+(1-\lambda)R_2^{opt2})$ is obtained by time sharing of two points, which are (R_1^{opt1}, R_2^{opt1}) and (R_1^{opt2}, R_2^{opt2}) in the utility space, and the time sharing coefficients are λ and $1-\lambda$, respectively. Also denote the corresponding frequency bins allocated to the users as $(\mathcal{B}_1^{opt1}, \mathcal{B}_2^{opt1})$ and $(\mathcal{B}_1^{opt2}, \mathcal{B}_2^{opt2})$. Assume that $R_1^{opt1} > R_1^{opt2}$ (then $R_2^{opt1} < R_2^{opt2}$ due to the Pareto-optimality). Two points can be found, which are $\mathbf{R}^1 = (R_1^1, R_2^1)$ and $\mathbf{R}^2 = (R_1^2, R_2^2)$ generated by the first and the last L rounds, respectively, where $R_1^1 > R_1^{opt1}$ and $R_2^2 > R_2^{opt2}$. Denote the corresponding frequency bin allocation sets as $(\mathcal{B}_1^1, \mathcal{B}_2^1)$ and $(\mathcal{B}_1^2, \mathcal{B}_2^2)$.

The difference between the logarithm of the NF in (2.4) for $\mathbf{R}^{opt}$ and $\mathbf{R}^1$ is then

$$\begin{aligned}
|d_1| &= \log(\lambda R_1^{opt1} + (1-\lambda)R_1^{opt2}) \\
&+ \log\left(\lambda R_2^{opt1} + (1-\lambda)R_2^{opt2}\right) - \log(R_1^1) - \log(R_2^1) \\
&= \log\left(\lambda\frac{R_1^{opt1}}{R_1^1} + (1-\lambda)\frac{R_1^{opt2}}{R_1^1}\right) \\
&+ \log\left(\lambda\frac{R_2^{opt1}}{R_2^1} + (1-\lambda)\frac{R_2^{opt2}}{R_2^1}\right) \\
&< \log\left(\lambda\frac{R_1^{opt1}}{R_1^1} + (1-\lambda)\frac{R_1^{opt1}}{R_1^1}\right) \\
&+ \log\left(\lambda\frac{R_2^{opt2}}{R_2^1} + (1-\lambda)\frac{R_2^{opt2}}{R_2^1}\right) \\
&< \log\left(\lambda\frac{R_2^{opt2}}{R_2^1} + (1-\lambda)\frac{R_2^{opt2}}{R_2^1}\right) = \log\left(\frac{R_2^{opt2}}{R_2^1}\right)
\end{aligned} \tag{5.9}$$

where the two inequalities follow from the assumptions $R_1^{opt1} > R_1^{opt2}, R_2^{opt1} < R_2^{opt2}$ and $R_1^1 > R_1^{opt1}$, respectively.

A further relaxation can be made using the fact that

$$R_2^{opt2} = \mathcal{WF}^2(\mathcal{B}_2^{opt2}) \tag{5.10}$$

and

$$R_2^1 = \mathcal{WF}^2(\mathcal{B} - \mathcal{B}_1^j) \geq \mathcal{WF}^2(\mathcal{B} - \tilde{\mathcal{B}}_2) \tag{5.11}$$

where $1 \leq j \leq L$ denotes the index of the round in which R_2^1 is obtained by user 2.

Thus, we have

$$|d_1| < \log\left(\frac{\mathcal{WF}^2(\mathcal{B}_2^{opt2})}{\mathcal{WF}^2(\mathcal{B}-\tilde{\mathcal{B}}_2)}\right). \tag{5.12}$$

Similarly, the difference between the logarithm of the NF for $\mathbf{R}^{opt}$ and $\mathbf{R}^2$ satisfies

$$|d_2| < \log\left(\frac{\mathcal{WF}^1(\mathcal{B}_1^{opt1})}{\mathcal{WF}^1(\mathcal{B}-\tilde{\mathcal{B}}_1)}\right). \tag{5.13}$$

Note that neither $\mathbf{R}^1$ nor $\mathbf{R}^2$ is assumed to be the sub-optimal solution returned by the algorithm. Instead, $\mathbf{R}^1$ and $\mathbf{R}^2$ are only two of the $2L$ points generated in the first and the last L rounds, respectively. Thus, the sub-optimal solution returned by the algorithm is expected to be superior than both of the two points. Therefore, $|d| \leq \min(|d_1|, |d_2|)$. ■

Although the solution can be sub-optimal, it may coincide with the optimal solution. One example is when $\mathcal{B}_c$ is empty. From the expression of $|d|$, it can be seen that if the channel gains in $\mathcal{B}-\tilde{\mathcal{B}}_i$ do not drop seriously compared to those in $\tilde{\mathcal{B}}_i$ for at least one user, the sub-optimal solution is efficient. In fact, the sub-optimal solutions derived are the same with the optimal solutions for most of the cases, as will be shown in the simulations.

5.4 The two-user algorithm

The overall algorithm combining both cases of the bandwidth-dominant and power-dominant systems for the two-user cooperative game is given in Table. 5.2.

In the bandwidth-dominant case, the complexity of searching on $\mathcal{P}'$ is $O(N)$. In the power-dominant case, the complexity of the algorithm is determined by the time sharing part, which is $O(L^2)$. In the latter case, the advantage in the complexity, i.e., $O(L^2)$ compared to $O(4^N)$ for the optimal solution (where the time consumed on water-filling is neglected in both cases) becomes obvious, especially when N becomes larger. At the same time, the sub-optimal solutions are efficient and are identical to the optimal solutions for most of the cases, as will be shown in the simulations in the next chapter.

Table 5.2: Two-user algorithm for finding the sub-optimal solution of the NB game.

1. Check condition (5.6): *Condition satisfied, go to step 2.* *Otherwise, go to step 3.*
2. System is bandwidth-dominant: *Search on the Pareto-boundary* $\mathcal{P}'$, *return* S'_{NB}.
3. System is power-dominant: *Derive* $\tilde{\mathcal{B}}_1$, $\tilde{\mathcal{B}}_2$, $\mathcal{B}_c$. *Play the* $2L$ *rounds and obtain* $\mathcal{T}$ *and* $\mathcal{P}_{\mathcal{T}}$. *Search on* $\mathcal{P}_{\mathcal{T}}$, *return* S'_{NB}.

Chapter 6

Simulations

6.1 Precoding with spectral mask constraints

In the first example, assume that two users have four available frequency bins to share. The noise power σ^2 is 0.01 for both users on all frequency bins. The channel gains of the desired channels $\mathbf{\Omega}_{11}$ and $\mathbf{\Omega}_{22}$ are generated as Rayleigh random variables with mean $\mathbf{1}$. The channel gains of the interference channels $\mathbf{\Omega}_{12}$ and $\mathbf{\Omega}_{21}$ are generated as Rayleigh random variables both with means $\mathbf{0.7}$ and $\mathbf{0.2}$, respectively. The spectral mask constraint $\mathbf{p}_{\text{max}}$ is also generated as a random vector with mean $\mathbf{1}$. In Fig. 6.1, the NB solution computed according to Propositions 4.2 is shown, while the NE solution is also shown for comparison and the boundary of the TDM/FDM rate region is indicated in the figure. Fig. 6.2 displays the values of the NF under different FDM/TDM frequency bin allocation schemes, where k is the frequency bin being shared and α is the fraction of time that user 1 uses the frequency bin k. The largest value of the NF in Fig. 6.2 corresponds to the optimal scheme that provides the NB solution.

In the second example, the distributed algorithm for the M-user game developed in Section 4.3 is tested. Four users are assumed in the system to share six frequency bins. For a given step length $\delta = 0.2$ and stopping threshold $\xi = 10^{-5}$, the iterations of bargaining process are shown in Fig. 6.3. The four curves on the

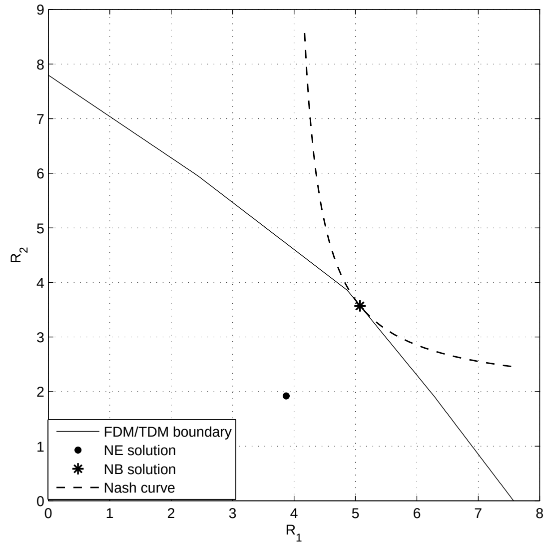

Figure 6.1: The FDM/TDM rate region, and the NE and NB solutions on the frequency selective channel.

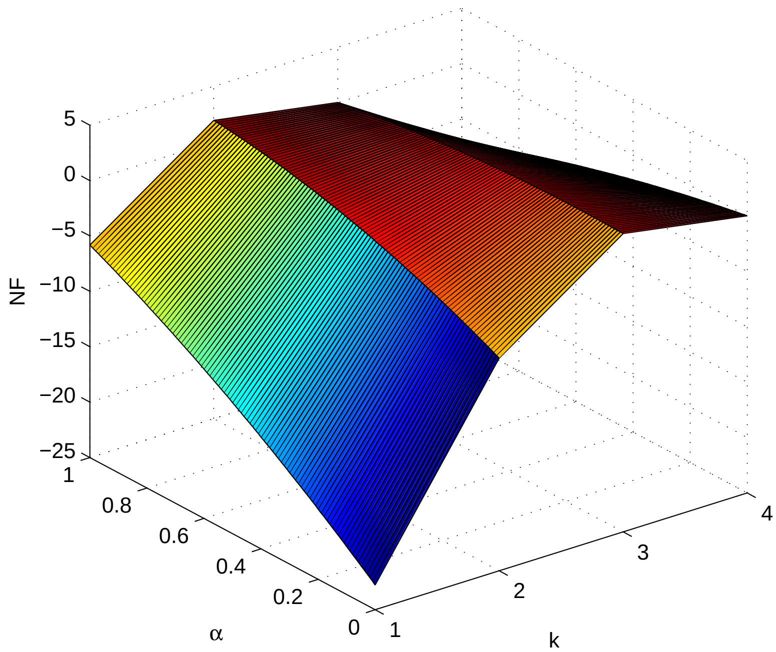

Figure 6.2: The NF under different FDM/TDM frequency bin allocation schemes.

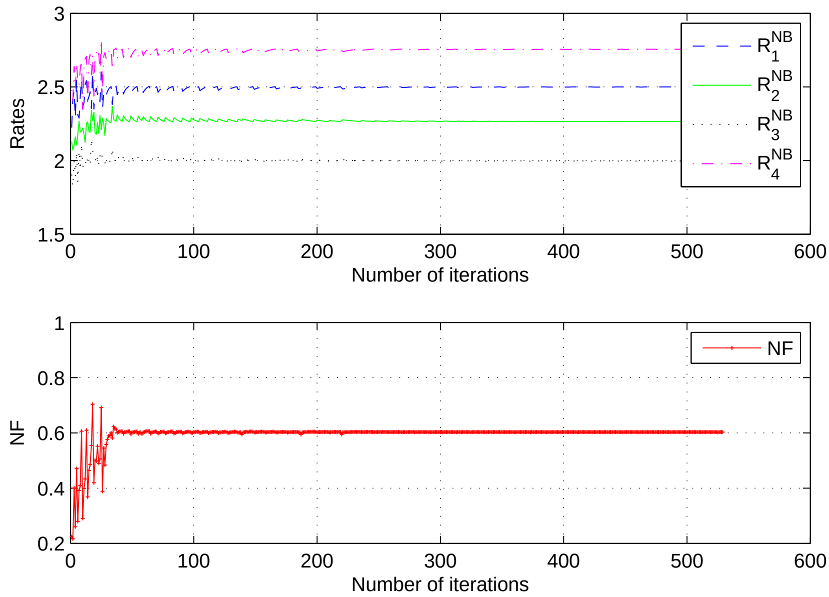

Figure 6.3: Instantaneous information rates of users and the corresponding values of NF versus number of iterations.

Table 6.1: Comparisons between NE and NB

User	NE Solution	NB solution	Increased by
1	1.1296	2.2707	101.02%
2	1.4014	2.4906	77.72%
3	1.2952	2.3992	85.24%
4	1.6957	2.4175	42.56%

upper side of the figure show the instantaneous information rates that the corresponding users can achieve, and the curve at the bottom is the corresponding value of the NF, i.e., the objective function of the optimization problem (4.13). The NB and NE solutions and the comparisons between them in one simulation are shown in Table 6.1. It can be seen that all users obtain supplementary benefit from cooperation. The corresponding final allocation of time portions on each frequency bin for each user is shown in Fig. 6.4. It can be seen that frequency bins 1, 2, 3, and 4 are occupied exclusively by users 3, 4, 1, and 2, respectively. Frequency bins 5 and 6 are shared by users 1 and 4, and users 2 and 3, respectively.

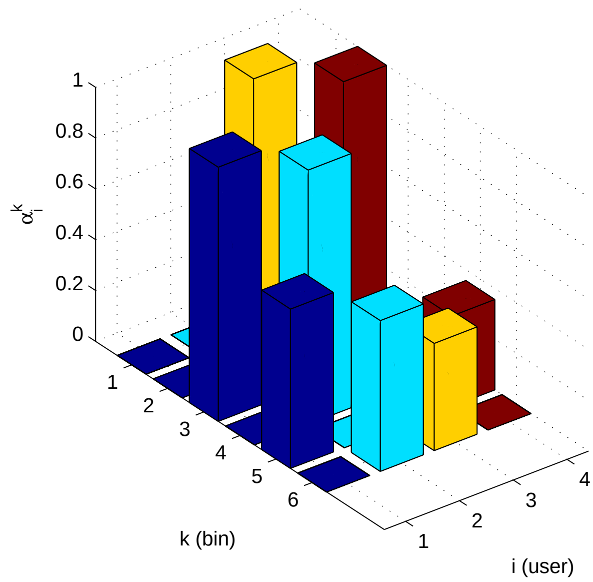

Figure 6.4: Allocation of time portions on frequency bins $\{\alpha_i^k\}$.

Fig. 6.5 depicts the effect of the step length on the convergence speed of the algorithm. With the values of $\delta \in \{0.1, 0.2, 0.3\}$, the values of the NF are shown in the corresponding sub-figures. It can be seen that the algorithm is time-efficient with a good choice of step length.

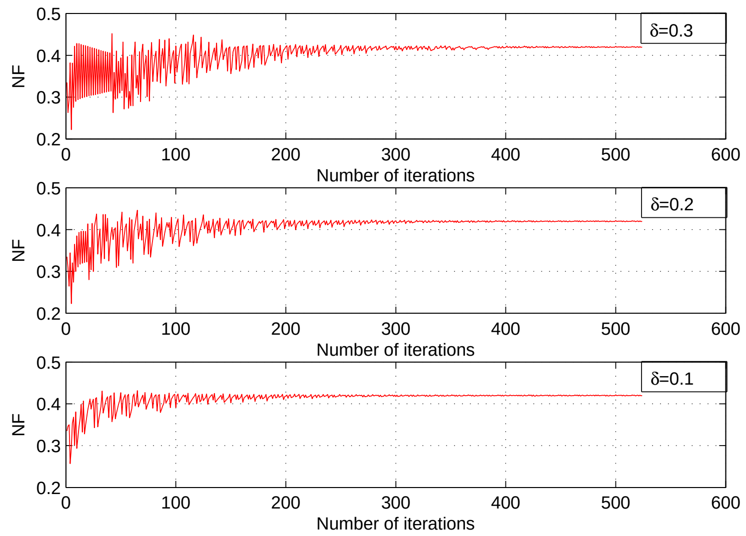

Figure 6.5: The NF versus number of iterations under different step lengths, $\delta \in \{0.3, 0.2, 0.1\}$.

6.2 Precoding with total power and spectral mask constraints

The multi-user wireless systems in which users have both the total power and spectral mask constraints are categorized to power-dominant and bandwidth-dominant systems in Chapter 5. For the bandwidth-dominant systems, the algorithm for finding the sub-optimal NB solution inherits the algorithm for finding the optimal TDM/FDM based NB solution in the precoding games without total power constraints. This case is, thus, simpler as compared to the case in the power-dominant systems, where a different algorithm is proposed to find the sub-optimal NB solution. Therefore, the simulation for the precoding with total power and spectral mask constraints mainly focuses on the power-dominant case.

Fig. 6.6 shows the system classification versus users' total power constraints and the number of frequency bins N in the system. The total power values $P_i^{\max}$ for both users are set to be the same and vary from 1 to 51. The number of frequency bins increases from 1 to 256. A simplified case is simulated where the spectral

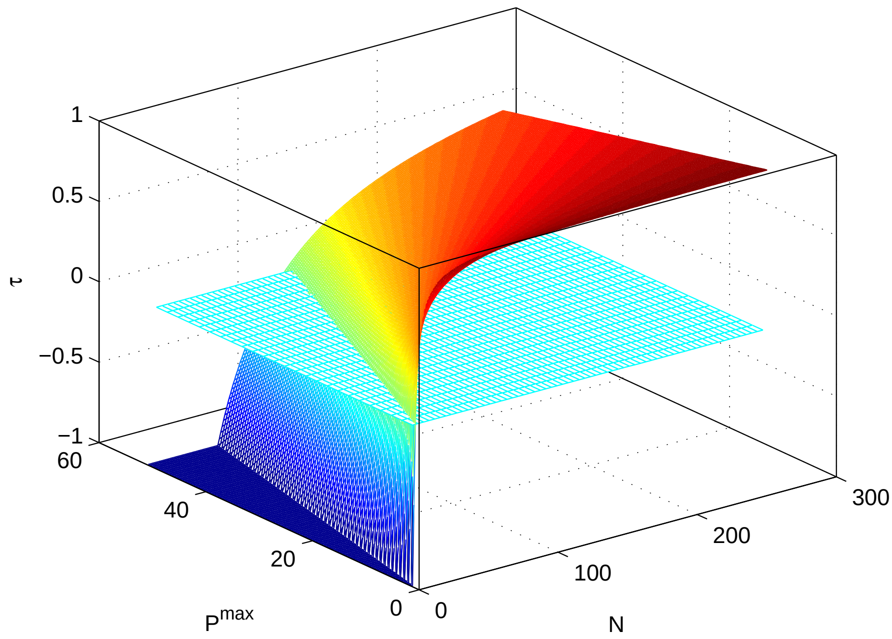

Figure 6.6: System classification versus values of total power constraints and number of frequency bins.

mask constraints are set to be $p_s = 1$ on all frequency bins for both users. The total bandwidth b_i that user i can cover is calculated as $b_i = \min(k_i + \alpha_i, N)$, where k_i ($k_i \in \{0, 1, ...N-1\}$) and α_i ($0 \leq \alpha_i < 1$) satisfy $P_i^{\max} = (k_i + \alpha_i)p_s$. Then, the variable $\tau = 1 - (\alpha_1 + k_1 + \alpha_2 + k_2)/N$ stands for the system property according to Proposition. 5.6. The system is bandwidth-dominant if $-1 \leq \tau \leq 0$ and is power-dominant if $0 < \tau < 1$. From the figure it can be seen that the system changes from bandwidth-dominant to power-dominant when new frequency bins are added into the system if the total power constraints of the users are fixed. On the other hand, given the number of frequency bins, the system changes from power-dominant towards bandwidth-diminant when the total power constraints of the users are relaxed.

Two samples of the optimal and sub-optimal NB solutions in power-dominant systems are demonstrated in Figs. 6.7 and 6.8. Two users are assumed to share eight frequency bins and the total power constraints are set to be $P^{\max} = 4$ for both

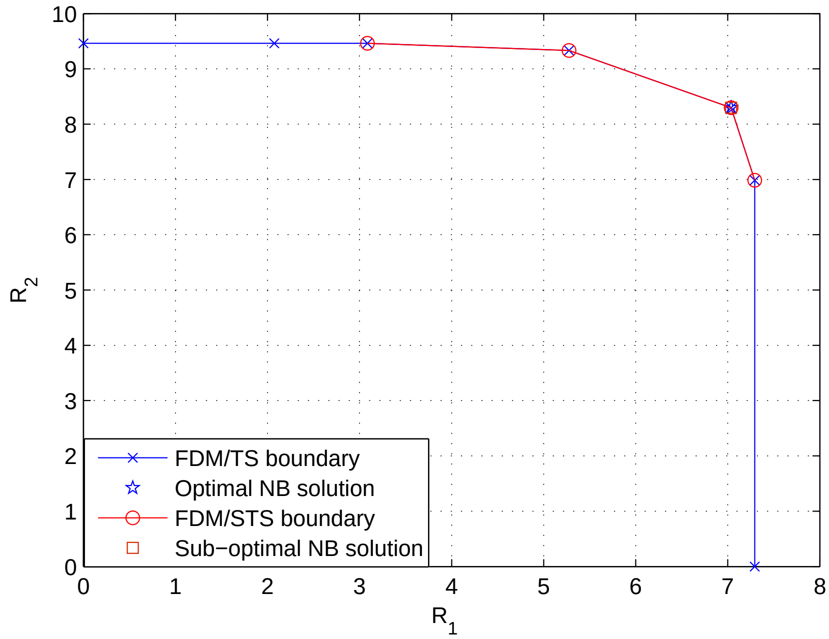

Figure 6.7: The optimal versus sub-optimal solutions and rate regions:sample 1.

users. The channel gains on all frequency bins are randomly generated. Fig. 6.7 demonstrates the case when the optimal and sub-optimal NB solutions are the same while Fig. 6.8 depicts the case when the sub-optimal solution is different from the optimal one.

Fig. 6.9 shows the accuracy of the sub-optimal solutions in 300 simulation, where the number of frequency bins varies from 4 to 9 (50 simulations for each case). The total power constraints of the users are set to $P^{\max} = 2$ for each user, and the spectral mask constraints are set to $1 + x(k)$ (where $x(k)$ is a random variable in the internal $[0.2, 0.25]$ for frequency bin k) to guarantee that the system is power-dominant. The channel gains on the frequency bins are randomly generated for both users. It can be seen from the figure that the sub-optimal solutions are identical to the optimal solutions for most of the cases. Note that although, for some cases, the difference between the optimal and sub-optimal solutions may appear large in the utility space as shown in Fig. 6.9, the difference between the values of the NF for the optimal and sub-optimal solutions is very small.

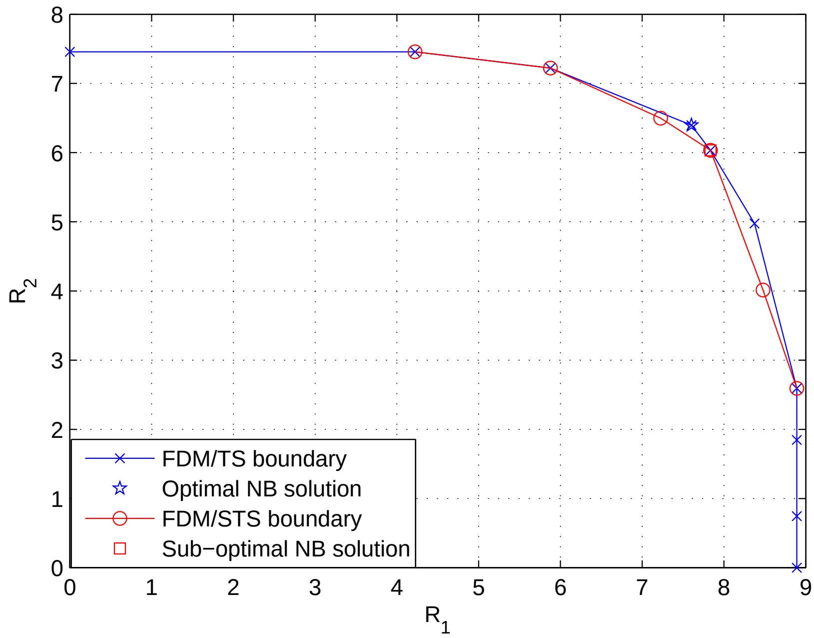

Figure 6.8: The optimal versus sub-optimal solutions and rate regions:sample 2.

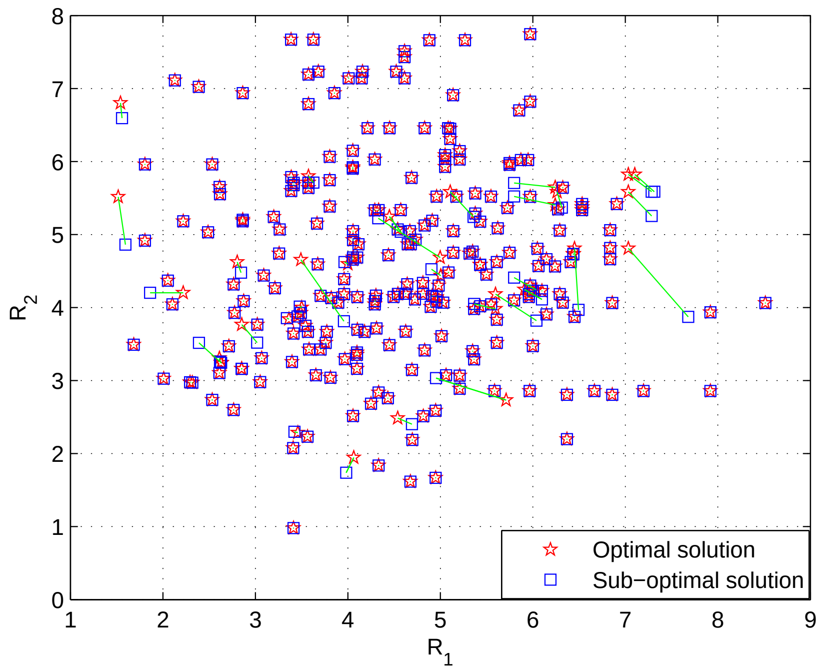

Figure 6.9: The accuracy of the sub-optimal solutions.

Chapter 7

Conclusions and Future Works

Spectrum shortage and wideband frequency selective fading are two major problems in wireless communications. Spectrum sharing is a practical and effective solution to improve the spectrum efficiency and, thus, alleviate the spectrum shortage problem. Allowing different users to share the same spectrum resource, the users may potentially interfere with each other and all of their transmissions can be degraded. Since the users are driven by their selfishness, game theory is a powerful tool to study the spectrum sharing problem. On the other hand, the problem of wideband frequency selective fading, can be dealt with by using OFDM. In OFDM systems, precoding is used to allocate the available transmission resources for the users. Modeling the precoding schemes as strategies in a game, the spectrum sharing problem is solved by finding efficient precoding game strategies.

In this thesis, the cooperative precoding strategies on frequency selective fading channels are derived. It is found that the precoding matrices adopt a strictly diagonal structure if OFDM modulation is exploited in the wireless system. Based on this structure, the optimal precoding matrices for the two-user cooperative game with spectral mask constraints are obtained by using the NB approach. The results are extended to the M-user game as well. It is shown that the problem of finding precoding strategies can be transformed to the problem of finding the time portions that each user obtains on each frequency bin. Formulated as a convex optimization problem, this problem can be solved in a distributed manner using the dual decomposition method, which physically realizes the process of bargaining among users. The simulation results demonstrate the superiority of the NB solution over the NE solution for precoding strategies. The impact of the spectrum allocation schemes on the system performance is also studied. The convergence of the distributed algorithm for the M-user game is demonstrated.

The precoding strategies are also studied for the two-user game with both spectral mask and total power constraints. It is shown that the problem is non-convex in this case. Finding the optimal solution requires joint optimization of the allocation of frequency bins (which is the public resource in the wireless system) and each user's transmission power (which is the individual resource). The complexity of finding the optimal solution is unacceptably high. Therefore, it is proposed that such systems be categorized into two classes according to their bottleneck resources, i.e., bandwidth or power. Based on such classification, the sub-optimal algorithms are derived for each class of the systems by concentrating on the allocation of the key resource. The classification guarantees that the solutions obtained by the sub-optimal algorithms are efficient or even identical to the optimal solutions, while the complexity of the sub-optimal algorithms is significantly reduced.

This work can be extended in three directions. First, cooperation without orthogonal signaling can be considered. Orthogonal signaling, such as TDM and/or FDM, is a simple and practical manner for users to cooperate. However, it is inefficient when the interference among the users is weak as compared to the noise power. For example, the TDM/FDM utility region is a subset of the general utility region when the interference among users is low. If we do not limit the cooperation to be based on orthogonal signaling, larger utility region can be achieved. However, it again comes with a tradeoff between complexity and optimality. The problem of finding the NB solution of the precoding matrices without orthogonal signaling is non-convex even when only spectral mask constraints are applied. Moreover, time sharing must be used to obtain the convex hull of the general utility region which is not convex in most cases. The complexity of finding the optimal solution is then unacceptably high, especially when the number of users is large. For example, each user needs to calculate three precoding matrices to perform time sharing and obtain the optimal solution if there are three users in the game. Nevertheless, efficient sub-optimal solutions with reduced complexity can be considered.

Second, the two-user game with total power and spectral mask constraints can be extended to multi-user games. For bandwidth-dominant systems, the extension is straightforward. The dual decomposition based algorithm in Section 4.3 can be used to find the NB solution for precoding strategies in multi-user bandwidth-dominant systems. Power-dominant systems are more complex since the complexity of frequency bin allocation (and also time sharing) grows fast with the number of users.

Last but not least, new constraints can be added into the problem. The games investigated in this thesis only consider power constraints. In fact, other constraints, such as interference constraints can be incorporated into the problem. For example, considering the case when there exist limitations of the maximum interference that any user can generate to other users, the structure of precoding matrices will no longer be diagonal and the problem becomes more complex. Specifically, allowing limited interference among users means that there is no orthogonal signaling in the system. Therefore, the problem of cooperation without orthogonal signaling must be solved first to proceed with this problem.

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